Recursion:

* when a function calls itself is known as Recursive function.

```
fun (int x)
{
    ----
    ----
    ----
    fun (x-1);  → calling the same func.
}
```

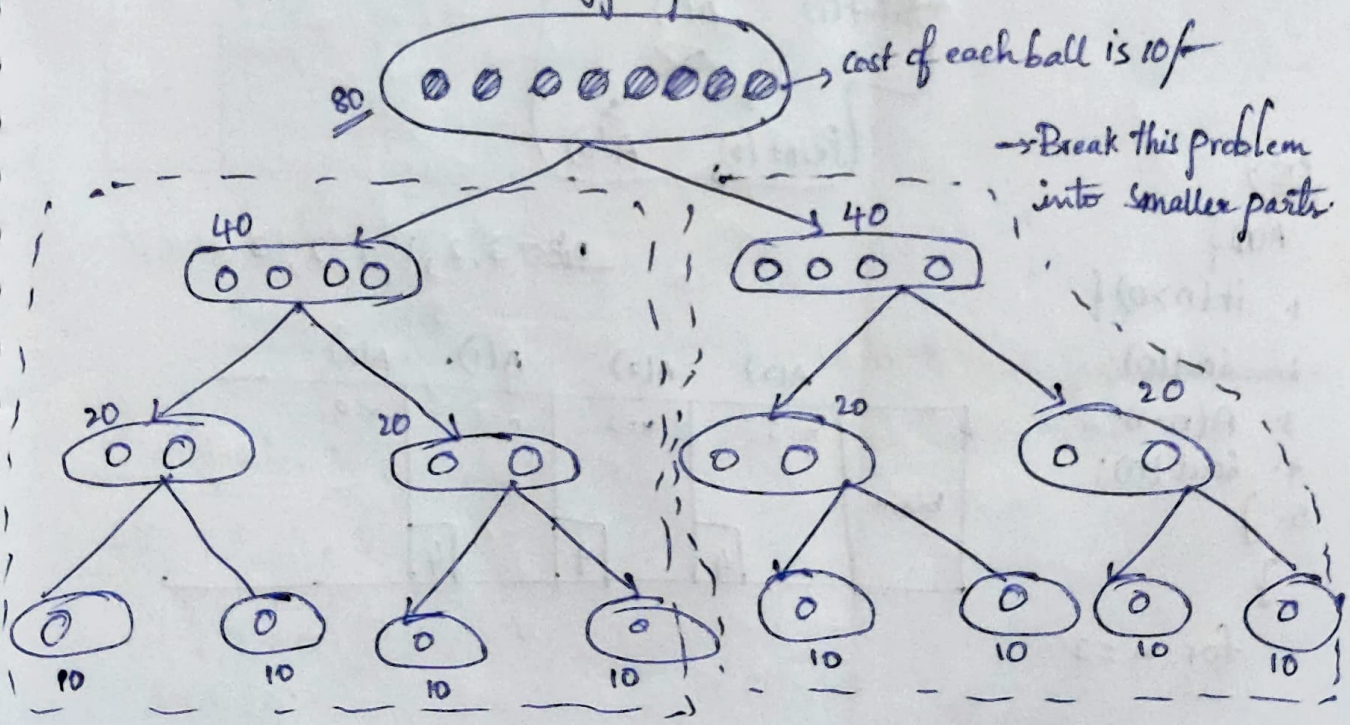
→ If the same function is called within a function it is called Direct Function. as shown in above example.

```
→ A (int x){
      B(x-1);   → This is indirect function.
  }
```

→ If $f(x) = x^2$, can you find $f(f(x))$ → This is Recursion

$\Rightarrow f(x^2) = (x^2)^2 = x^4$

→ for example, we have a bigger problem.



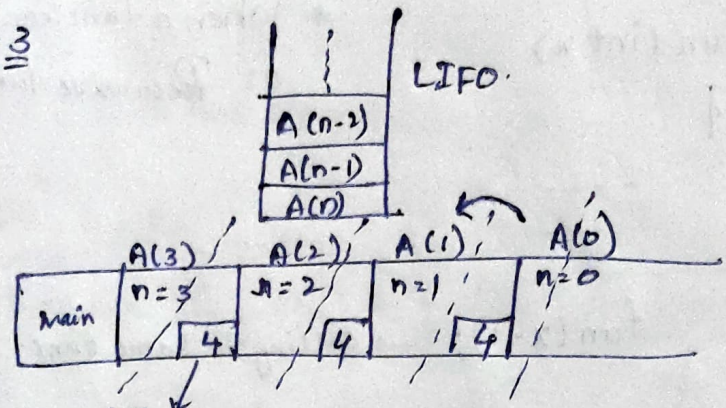
→ Tracing the Recursion:

```
A(n){
1. if(n>0)
2. Sout(n-1);
3. A(n-1);
4. }
5.}
```

n = 3

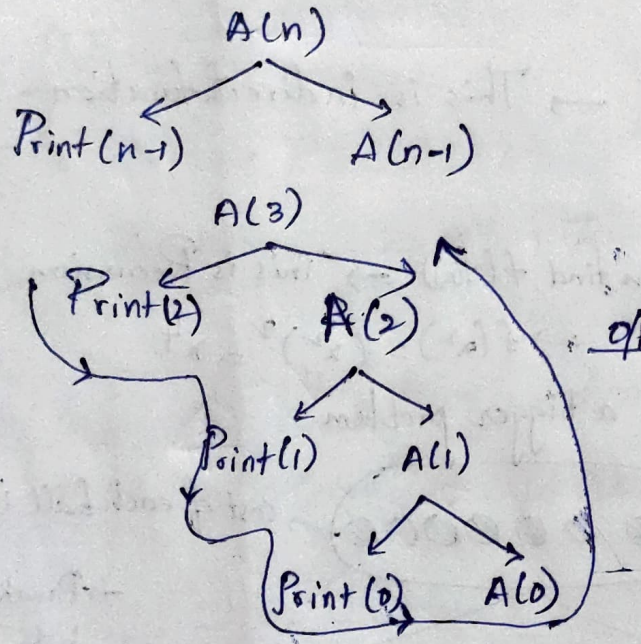
o/p: 2, 1, 0

① Recursive Stack.



Instruction pointer
[The Next line to get executed once you are done with recursive call]

② Tree Method:



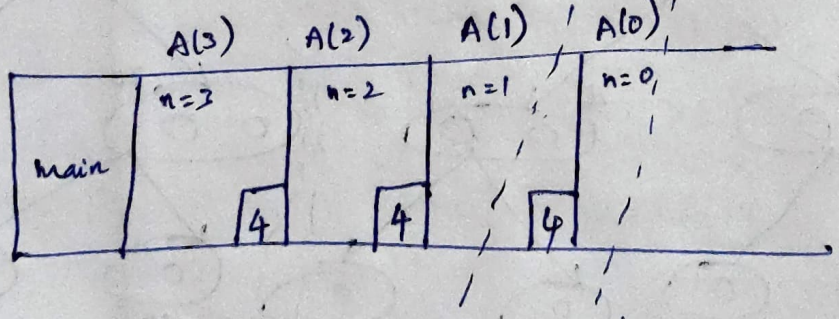
o/p:- 2, 1, 0

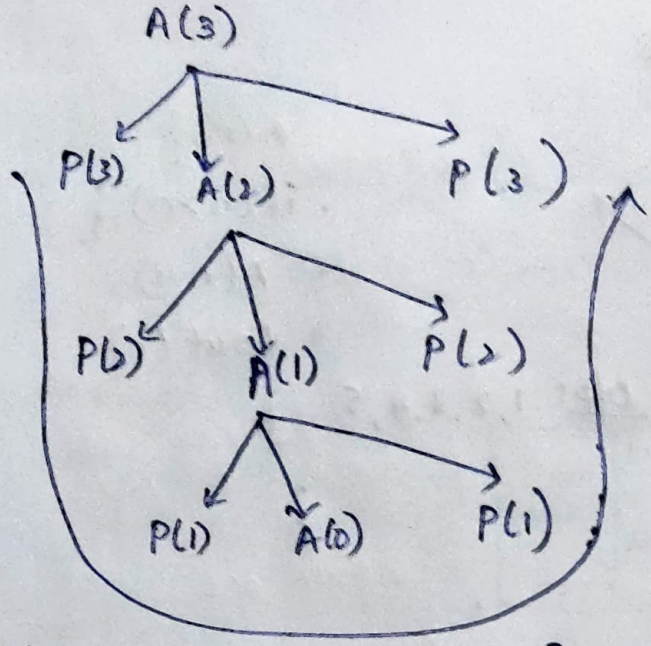
ex: 2

```
A(n){
1. if(n>0){
2. Sout(n);
3. A(n-1);
4. Sout(n);
5. }
}
```

for n = 3

o/p: 3, 2, 1, 1, 2, 3

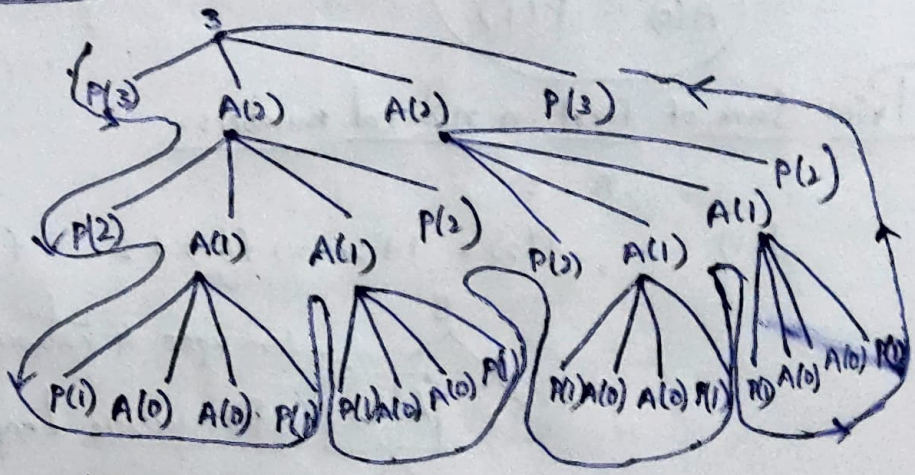




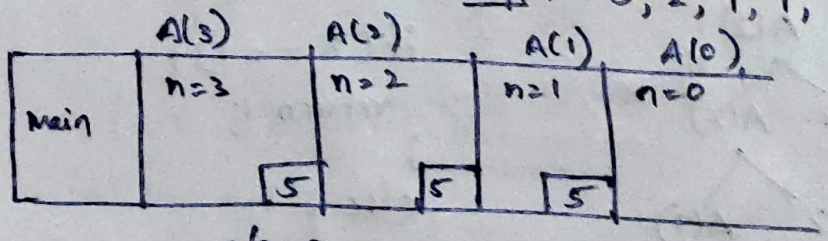
o/p :- 3, 2, 1, 1, 2, 3.

ex:3

```
A(n){
1 if(n>0){
2  Sout(n);
3  A(n-1);
4  A(n-1);
5  Sout(n);
  }
}
```



o/p: 3, 2, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 2, 3



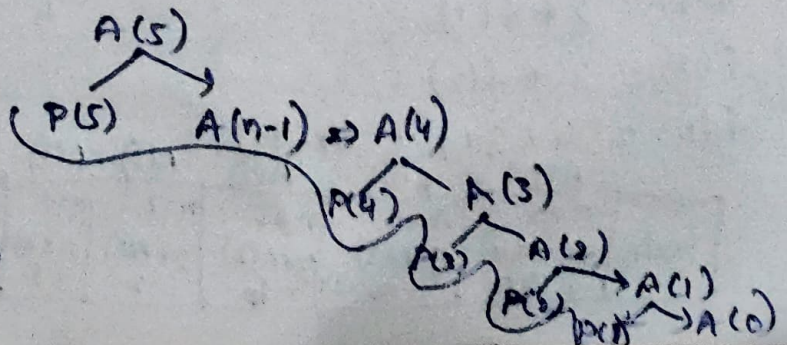
o/p:- 3, 2, 1, 1, 1, 2, 2, 1, 1, 1, 1, 2, 3

Base Condition: → Smallest possible work that you are aware of.

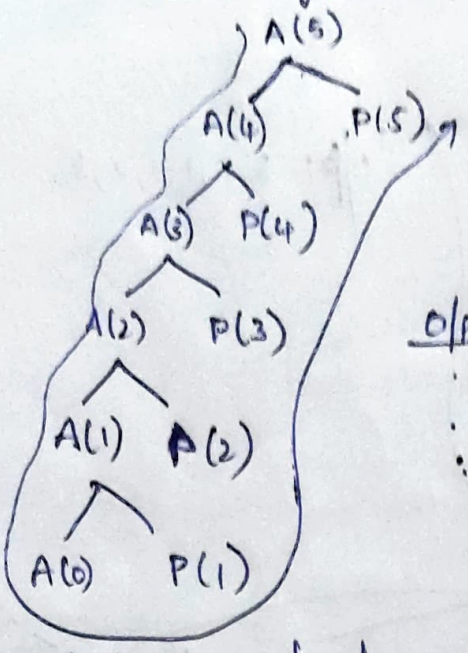
Print Number in Descending Order:

o/p: 5, 4, 3, 2, 1

```
if(n>0){
Sout(n);
PrintDescending(n-1);
}
```



Print Numbers in Ascending Order:



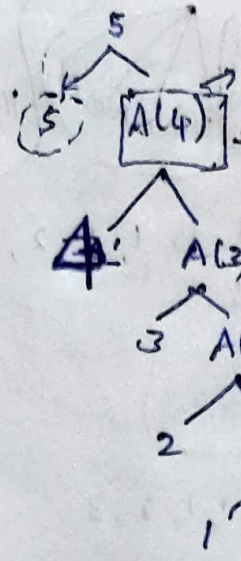

```
A(n){
if(n>0),{
  A(n-1)
  sout(n)
}
}
```

O/P: 1,2,3,4,5

Print Sum of first n natural numbers:

n = 5

f(1) = 1 , f(2) = 1+2 ⇒ f(1)+2 f(3) = f(2)+3



Sum upto 4 natural numbers.

→ Recursive leap of faith.

```
sum(n) {
if(n==1){
  return 1;
}
else{
  return n+sum(n-1);
}
}
```

Factorial:

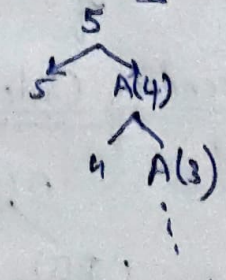
n = 5 , O/P : 120

f(0) = 1 , f(1) = 1

f(2) = 2 * f(1)

f(3) = 3 * f(2)

f(4) = 4 × 3 × 2 = 4 × f(3)

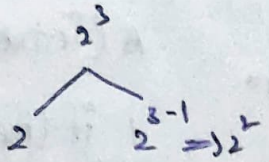



```
A(n)
{
if(n==0){
  return 1;
}
else
{
  return n*A(n-1);
}
}
```

	A(5)	A(4)	A(3)	A(2)	A(1)	A(0)
main	n=5 24×A(4) =120	n=4 4×A(3) 4×6=24	n=3 3×A(2) =6	n=2 2×A(1) =2	n=1 1×A(0) =1	n=0 1

→ Power of 2 number

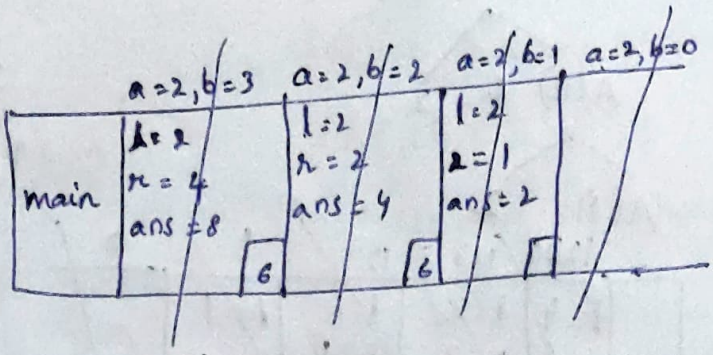
i/p :- $2^{\wedge}3$ o/p : 8



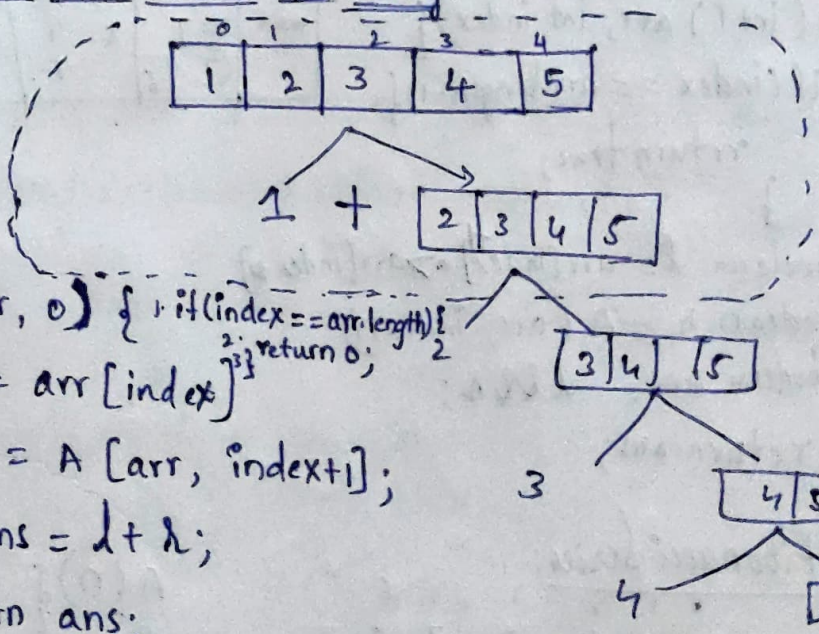
$2^0 = 1$, $2^1 = 2$

```
A(int a, int b){
1.   if(b==0){
2.   return 1;
3    }
4    int l = a;
5    int r = A(a, b-1)
6    int ans = l * r;
7.   return ans;
     }
```

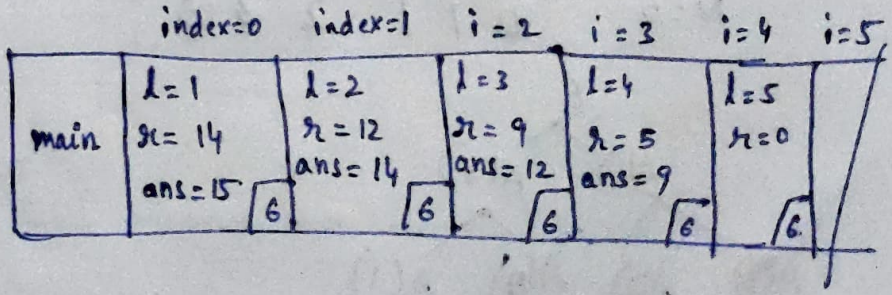
```
if (exponent == 1)
    return base;
```



→ Sum of all values inside an Array:

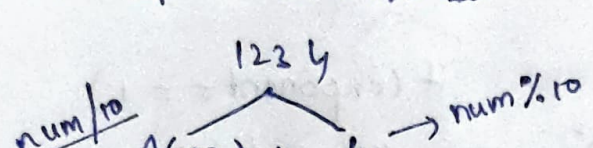


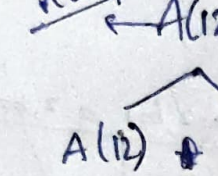

```
A [arr, 0) { 1. if(index == arr.length){
                2.   return 0;
             3  }
4. int l = arr [index];
5. int r = A [arr, index+1];
6. int ans = l + r;
7. return ans;
}
```



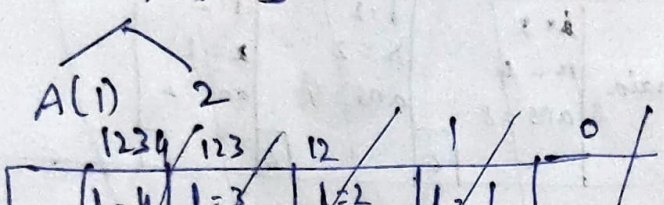
Sum of digits:

I/p: 1234 : o/p: 1+2+3+4 = 10






```
A(int num){
1  if(num==0){
2      return 0;
3  }
4  int l = num%10;
5  int r = A(n/10);
6  int ans = l+r
7  return ans;
}
```

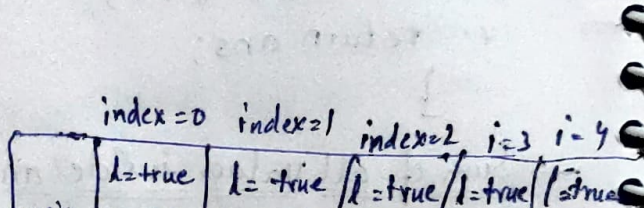


Check whether array is Sorted:

1	2	3	4

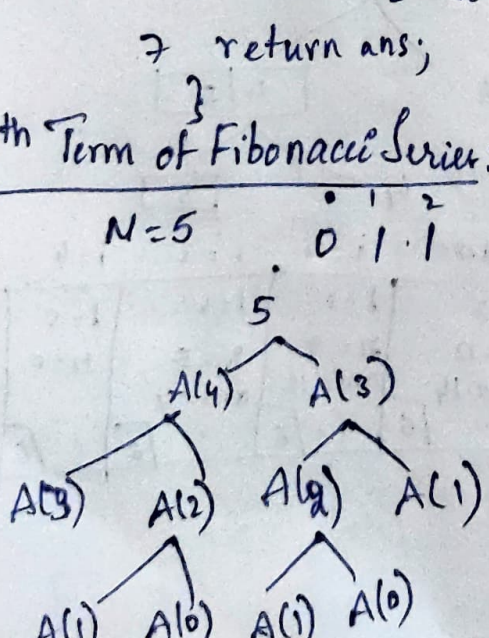
o/p: True/false.

```
A(int[] arr, int index){
1  if(index == arr.length-1){
2      return true;
3  }
4  boolean l = arr[index] <= arr[index+1]
5  boolean r = A(arr, index+1)
6  boolean ans = l && r;
7  return ans;
}
```



Nth Term of Fibonacci Series:

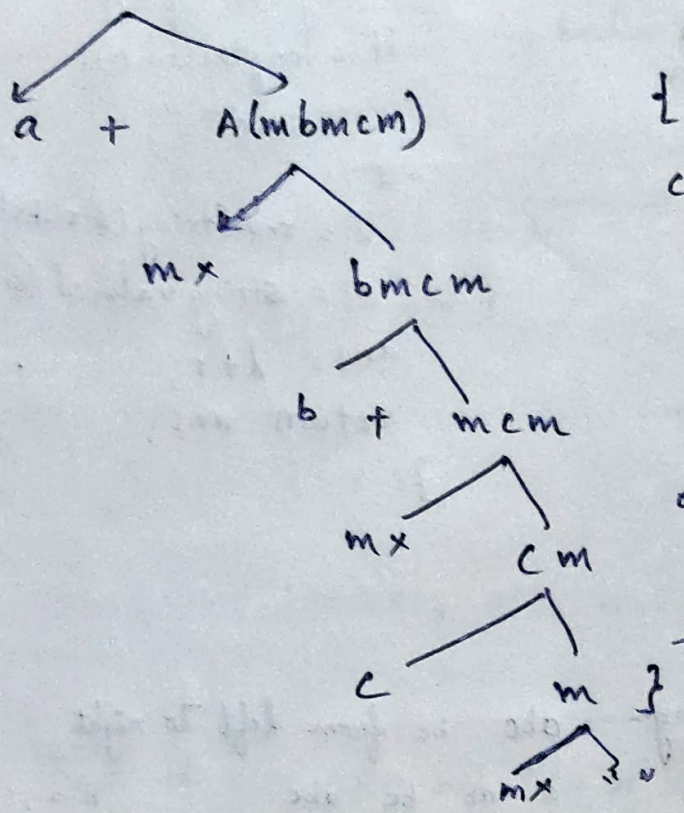
N=5 — 0 1 1 2 3 5




```
A(n){
if(n<=1){
return n;
}
else
{
return A(n-1) + A(n-2);
}
```

→ Remove 'm' from String:

ambmcm → o/p: abc



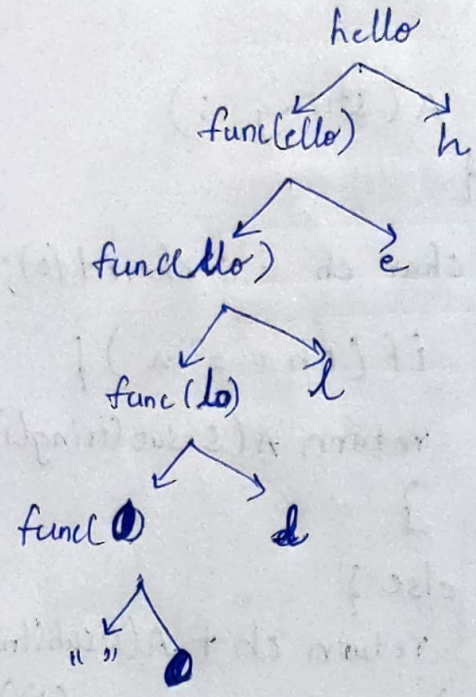

```
A(string s)
{
char ch = s.charAt(0);
if(ch == 'm') {
return A(s.substring(1));
}
else {
return ch + A(s.substring(1));
}
}
```

→ First and Last Occurrence of a character in a String:

String s = "abaacdaefaah", element = 'a' o/p: first: 0
last: 10

```
func (String s, int first, int last, char elem, int index)
{
if(index == s.length())
{
so.pln(first);
last = last != -1 ? last : first;
so.pln(last);
}
char ch = s.charAt(index);
if (ch == element)
{
if (first == -1) {
first = index;
}
func( s, first, last, eleme, index++);
}
```

String Reversal: "hello" → "olleh"




```
revString (String s){
    if (s.length() == 0){
        return "";
    }
    l = revString (s.substring(1));
    r = String.valueOf (s.charAt(0));
    ans = l + r;
    return ans;
}
```

* Subsequence of a String:-

abc i.e from left to right

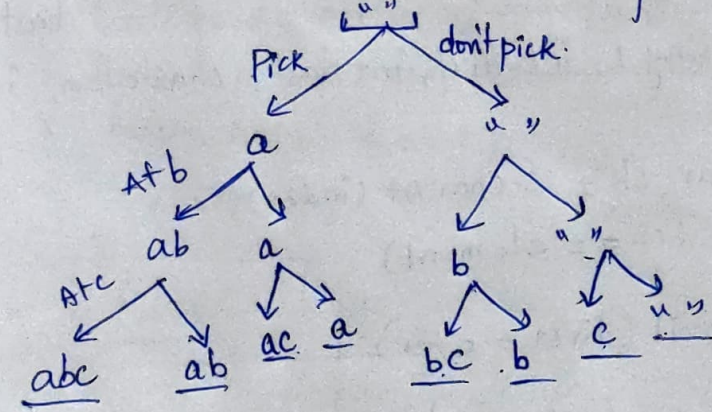
a ab bc abc

b ac

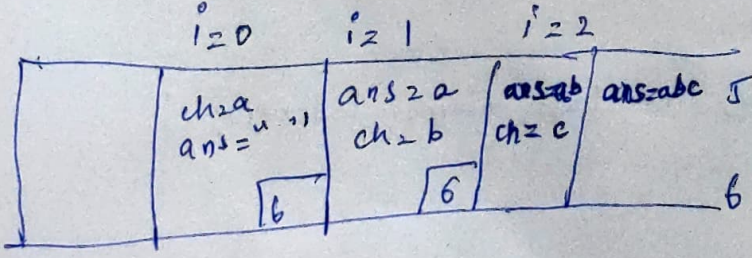
c

ba, ca, cba ✗

→ first start with empty string, at each instance we will have 2 options pick the current character (or) dont pick the character.




```
subseq (str, index, newStr)
{
1   if (index == str.length())
    { S.O.println (newStr);
2   return;
3   }
4   char current = str.charAt(index);
    // to be included
5   subseq (str, index+1, newStr + current);
    // not to be included
6   subseq (str, index+1, newStr);
```



→ Remove Duplicates in a String.

S = "abccccddab" → o/p:- abcd

	a	b	c	d	e	
S_{Arr}[]	F	F	F	F	F	..
	0	1	2	3		

→ boolean array

```
func(String, 0, "", SArr[]) {
    char ch = s.charAt(index);
    if(SArr[ch-'a'] == 'F')
    {
      ans += ch;
      SArr[ch-'a'] = 'T';
    }
    func(String, index+1, ans, SArr)
}
```

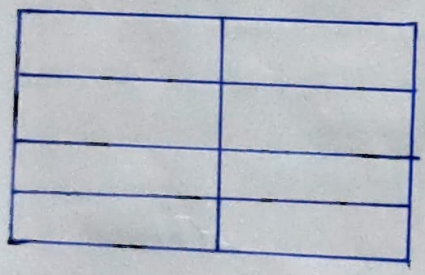
base condition

```
if(index == s.length()){
  s.o.plu(ans);
  return;
}
```

→ Given a floor of size n x m and tiles of size 1 x m. The problem is to count the number of ways to tile the given floor using 1 x m tiles. A tile can be placed either horizontally (or) Vertically. Both n and m are positive integers and 2 <= m.

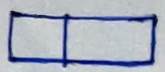
n x m.

n = 4, m = 2



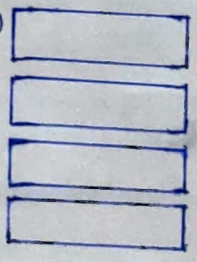
Tile → 1 x m

→ 1 x 2

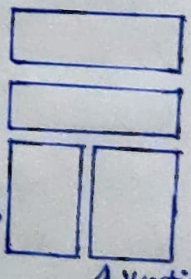


Possible-Arrangement

a)



b)

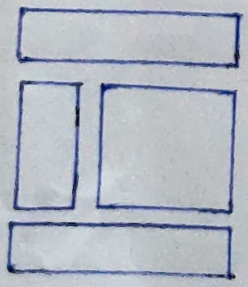


vertical

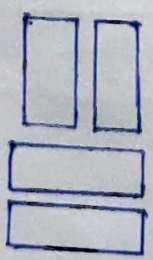
↑ vertical

~~horizontal~~

c)



d)



e)