

Algorithm:

It is a step by step procedure for solving a computation problem.

- | Algorithm | Program |
|---|---------------------------------------|
| → Written at design phase | → Development / Implementation phase. |
| → Written by someone having domain knowledge. | → Written by a programmer. |
| → Written in an English. | → Written in a Programming language.. |
| → Hardware and OS independent | → Depends on Hardware and OS. |

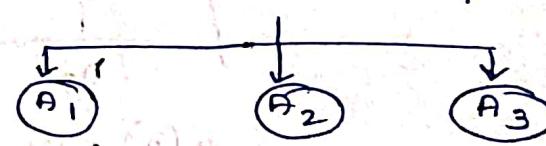
How to write an algorithm

Summation of two numbers

Algorithm - Sum - of - two - numbers (a, b) → no datatype {
 $\text{Sum} \leftarrow a + b.$

}

Problem P_1



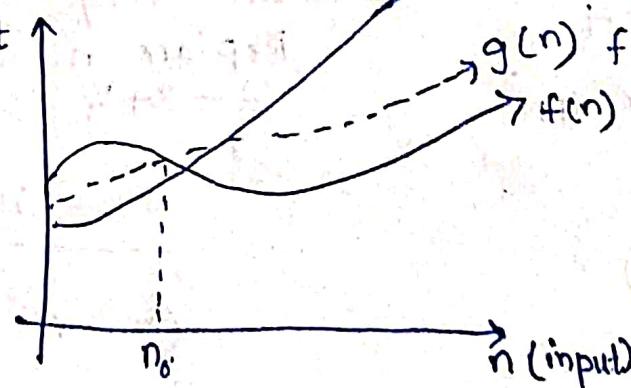
A, Time and Space Complexity

$O(n^2)$

$f(n)$

$$g(n) \quad f(n) \leq c \cdot g(n) \quad \begin{cases} n > n_0 \\ c \geq 1 \end{cases}$$

Big O:



$$f(n) = O(g(n))$$

$$f(n) = 3n + 2, g(n) = n$$

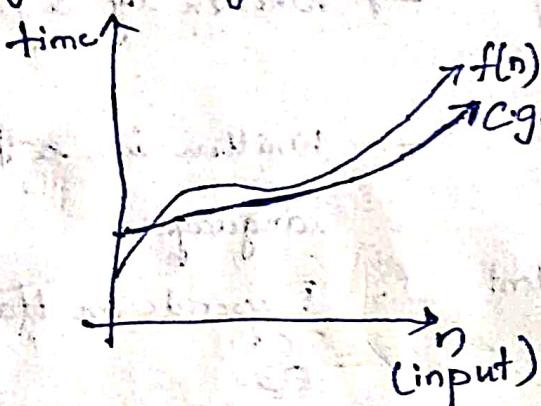
$$3n+2 \leq c \cdot n \quad \text{for ex } c=10.$$

$$3n+2 \leq 10n \rightarrow \forall n > 1 \\ \downarrow \\ 3n+2(n) = 5n$$

$$3n+2 \leq 5n$$

Big(O) denotes highest upper bound:

Big Omega:- Highest lower bound.



$$f(n) = 3n + 2$$

$$g(n) = n$$

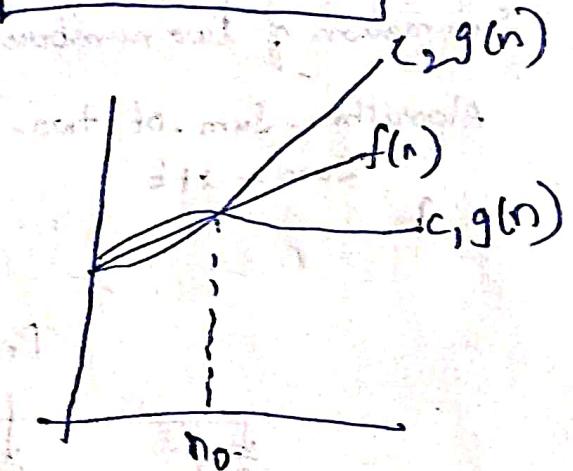
$$\text{find if } f(n) = \Omega(g(n))$$

$$f(n) \geq cg(n), \forall n > 1$$

$$f(n) = \Omega(g(n))$$

$$f(n) = \Theta(n)$$

↓ Average.



$$f(n) = 2n^2 + 3n + 4$$

O, Ω , Θ

$$O(n) = 2n^2 + 3n + 4$$

$$= 2n^2 + 3n^2 + 4n^2$$

$$\Rightarrow 9n^2$$

$$f(n) = O(n^2)$$

$$g(n) \leq f(n) \leq c_2 g(n)$$

$$\Omega = 2n^2 + 3n + 4 \geq n^2$$

$$\text{Replace } n \text{ with } 1 \\ 2 + 3 + 4 \geq n$$

$$\Omega = n^2$$

$$\Theta = n^2$$

$$f(n) = n!$$

$$\Omega(f(n)) = 1$$

$$O(f(n)) = n^n$$

we cannot determine the Average Case

O

$$f(n) = \log(n!) \quad \text{and } x \text{ is } n!$$

$$= \log(1 \times 2 \times 3 \dots n) \xrightarrow{\text{replace everything with } n} \log n^n$$

$$\log n^n$$

Cannot determine O

$$\log^1 \leq \log n! \leq \log n^n$$

$$\log^1 \leq \log n! \leq \underline{n \log n}$$

$$1. \quad 2^n, n^2$$

apply log

$$\log_2 n$$

$$n \log 2$$

$$\log n^2$$

$$2 \log n$$

$$5. \quad n^2, n \log n$$

$$n \times n$$

$$n \log n$$

$$1 \times n$$

$$\log n$$

$$128 < 8 \leftarrow \log$$

$$\log$$

$$128 > 7 \leftarrow \log_2$$

$$n^2 > n \log n$$

$$\Rightarrow 128, \log 128$$

$$2^n > n^2 \Rightarrow (n, 2^n), \text{ so } \log 2^7$$

$$\text{So } 2^n > n^2$$

$$2. \quad 3^n, 2^n \rightarrow \log$$

$$\log 3^n \quad \log 2^n$$

$$n \log 3 \quad n \log 2$$

$$\log 3 > \log 2$$

$$3^n > 2^n$$

$$3. \quad n^2, n \log n$$

$$n^2, 2 \log n$$

$$\log n \quad 100 \log(\log n)$$

$$\log_2 128 \quad 100 \log(\log_2 128)$$

$$128 < 700 \log(128)$$

$$n = 1024$$

$$\begin{array}{ll} \log_2 1024 & 100 \log(\log_2 1024) \\ 1024 & 100 \log_2(2^{10}) \\ 1024, 1000 \\ \text{So } \boxed{n > \log_n 100} \end{array}$$

$$7. \sqrt{\log n} \quad \log(\log n)$$

$$n = 2^{64}$$

$$\sqrt{\log_2 64} \quad \log(\log_2 64)$$

$$\sqrt{64} \quad \log_2(64)$$

$$8 \quad \log_2 2^6$$

$$8 > 6$$

$$\boxed{\sqrt{\log n} > \log(\log n)}$$

Summary for function:

$$C < \log(\log n) < \log n < n^{1/3} < n^{1/2} < n < n \log n < n^2 < n^3 < n^4 < 2^n < n^n$$

Algorithm

```
Algorithm
    ↓
    Iterative
    for (int i = 0 to n) {
        & O. println("abc");
    }
```

```
Algorithm
    ↓
    Recursive
    f(x) {
        f(x-1);
    }
```

1. `for(int i=0; i<n; i++){
 s.o.println("abc");
}`

$$T.C \rightarrow O(n)$$

2. `for(int i=0; i<n; i++){
 for(int j=0; j<n; j++){
 s.o.println("abc")
 }
}`

\rightarrow we are concerned the last loop will execute.

$$\begin{array}{ccccccc} & 1 & 2 & 3 & \dots & n \\ j & n & n & n & & n \\ \Rightarrow & n^2 & & & & & \\ \Rightarrow & T.C = O(n^2) & & & & & \end{array}$$

3. `for(int i=1; i<=n; i++){
 for(int j=1; j<=i; j++){
 for(int k=1; k<=100; k++){
 s.o.println("abc");
 }
 }
}`

$$\begin{aligned} & i \quad 1 \quad 2 \quad 3 \quad \dots \quad n \\ & K \quad 1 \times 100 \quad 2 \times 100 \quad \dots \quad n \times 100 \\ & = 100[1 + 2 + 3 + \dots + n] \\ & = 100 \left[\frac{n(n+1)}{2} \right] \end{aligned}$$

$$\rightarrow T.C = O(n^2)$$

4. `for(int i=1; i<n; i++){
 for(int j=1; j<i; j++){
 for(int k=1; k<=n/2; k++){
 s.o.println("abc");
 }
 }
}`

$$\begin{aligned} & i \quad 1 \quad 2 \quad 3 \quad \dots \quad n \\ & j \quad 1 \quad 2 \quad 3 \quad \dots \quad n^2 \\ & K \quad 1 \times \frac{n}{2} \quad 2 \times \frac{n}{2} \quad 3 \times \frac{n}{2} \quad \dots \quad \frac{n^2 \times n}{2} \\ & = \frac{n}{2} [1 + 2^2 + 3^2 + \dots + n^2] \\ & = \frac{n}{2} \left[\frac{n(n+1)(2n+1)}{6} \right] \end{aligned}$$

$$\boxed{T.C = O(n^4)}$$

5. `for (int i=1; i<n; i=i*2){ i=1 i=2 i=4, i=8
 s.o.println("abc"); n n/2 n/4 n/8}`

\downarrow

The stopping condition when

b) `for (int i=1; i<n; i=i*3){`

`s.o.println("abc");
 }`

$$T.c = O(\log_3 n)$$

$$\begin{aligned} i &\geq n \\ 2^k &\geq n \end{aligned}$$

$$\log_2 2^k = \log n$$

$$k = O(\log n)$$

6. `for (int i=0; i<n; i++) {`

`s.o.println("abc");
 }`

\Rightarrow

$$i < n$$

$$i < \sqrt{n}$$

$$\Rightarrow T.c = O(\sqrt{n})$$

7. `for (int i= $\frac{n}{2}$; i<=n; i++) {`

`for (int j=1; j<=n/2; j++) {`

`for (int k=1; k<=n; k=k*2){`

`s.o.println("abc");
 }`

`}`

$$i \rightarrow n/2$$

$$j \rightarrow n/2$$

$$k \rightarrow \log n$$

$$T.c = n^2 \log n$$

8. `for (int i=n/2; i<=n; i++) {`

`for (int j=1; j<=n; j=j+2) {`

`for (int k=1; k<=n; k=k*2)`

`s.o.println("abc");
 }`

$$j \rightarrow \log n$$

$$k \rightarrow \log n$$

$$T.c = n \log n^2$$

9. `for(int i=0; i<=n; i++) {
 for (int j=1; j<=n; j=j+i)
 { for (int k=1; k<n; k++)
 s.o.println("abc");
 }
 }
}`

$$= n \left[1 + \frac{1}{2} + \frac{1}{3} + \dots + 1 \right]$$

$$= n (\log n)$$

i	0	1	2	\dots	n
j	0	$n/2$	$n/3$	\dots	$\frac{n}{2}$

i	0	1	2	4	6	8	\dots	$\frac{n}{2}$
-----	---	---	---	---	---	---	---------	---------------

`for (int i=0; i<n; i=i+2)
{
 s.o.println("abc");
}

 $\underline{O(n/2)} \Rightarrow O(n)$`

Summary:

1. `for (int i=0; i<n; i++)` $\rightarrow O(n)$
2. `for (i=0; i<n; i=i+2)` $\rightarrow O(n/2) \rightarrow O(n)$
3. `for (i=1; i<n; i=i*2)` $\rightarrow O(\log_2 n)$
4. `for (i=1; i<n; i=i*3)` $\rightarrow O(\log_3 n)$
5. `for (i=n; i>1; i=i/2)` $\rightarrow O(\log_2 n)$

1) $i = 0$

while ($i < n$) {
 stmt;
 $i++$;
}

$0, 1, 2, \dots, k$
 $k > n$
 $\Rightarrow O(n)$

3) $a = 1$

while ($a < b$)
{
 stmt;
 $a = a * 2$;
}

$1, 2, 4, \dots, 2^k$

$$2^k > b$$

2) $i = 1, k = 1$

while ($k < n$) {

 stmt;

$k = k + i$;

$i++$;

}

$$T.C = (\log_2 b)$$

$i = 1, 2, 3, \dots, m$

$k = 1+2, 1+2+3, \dots, 1+2+3+\dots+m$

\downarrow

$$1 + \frac{m(m+1)}{2} (\Rightarrow n) + (1+n)r = (n)r$$

$$m^2 + m > n$$

$$m > \sqrt{n}$$

$O(\sqrt{n})$

Time Analysis of Recursive Programs:-

```
→ func(int x) {
    func(x-1)
```

{

* whenever we have a function like the function calling itself is called Recursive function.

1) $A(n)$

{

if ($n > 1$)return $A(n/2) + A(n/2)$ Step 1:- Derive Recurrence Relation; Substitute A with T

$$T(n) = T(n/2) + T(n/2)$$

$$T(n) = 2T(n/2) + c$$

2) $A(n)$ { if ($n > 1$) $\Rightarrow c = 1$ return $A(n-1);$

}

$$T(n) = T(n-1) + c \Rightarrow T(n-1) + 1$$

Step 2: Calculate Time complexity by using any of one below methods.

1. Back Substitution method.

2. Recursion Tree.

3. Masters Theorem.

Back Substitution:

$$T(n) = T(n-1) + 1 \rightarrow ①$$

Substitute (or) Calculate $n-1$

$$T(n-1) = T(n-2) + 1 \rightarrow ②$$

Calculate $n-2$ from eq. ①

$$T(n-2) = T(n-3) + 1 \rightarrow ③$$

Now, Substitute eq_② in eq_①

$$T(n) = T(n-2) + 1 + 1 \rightarrow ④$$

Sub eq_③ in eq_④:

$$T(n) = T(n-3) + 1 + 1 + 1$$

$$T(n) = T(n-3) + 3.$$

$$T(k) = T(n-k) + k \quad \begin{matrix} n-k=1 \\ k=n-1 \end{matrix}$$

$$T(n-1) = n-1 + T(1)$$

$$= n-1 + 1$$

$$T(n-1) = O(n)$$

3) $T(n) = n + T(n-1); n > 1 \rightarrow ①$

$$= 1; n = 1$$

$$T(n-1) = n-1 + T(n-2) \rightarrow ②$$

$$T(n-2) = n-2 + T(n-3) \rightarrow ③$$

Sub ② in ①

$$T(n) = n + n-1 + T(n-2)$$

$$T(n) = n + n-1 + n-2 + T(n-3)$$

$$T(n) = 3n-3 + T(n-3) \quad \text{or} \quad T(n) = n + n-1 + n-2 + \dots$$

$$= nk - k + T(n-k) \quad + T(n-(k+1))$$

$$n-k = 1 \Rightarrow k = n-1$$

$$n-(k+1) = 1 \Rightarrow n-k-1 = 1$$

$$T(n-1) = k(n-1) + T(n-k)$$

$$\Rightarrow n-k = 2$$

$$= (n-1)(n-1) + T(n-(n-1))$$

$$\Rightarrow k = n-2$$

$$= (n-1)^2 + T(1)$$

$$T(n) = n + (n-1) + (n-2) + \dots$$

$$T.C = O(n^2)$$

$$+ T(n-(n-2+1))$$

$$= n + (n-1) + n-2 + \dots + 1$$

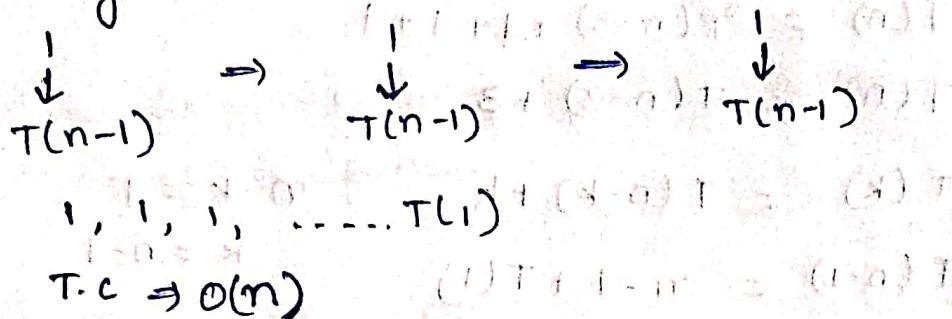
$$= \frac{n(n+1)}{2}$$

$$\therefore T(n) = O(n^2)$$

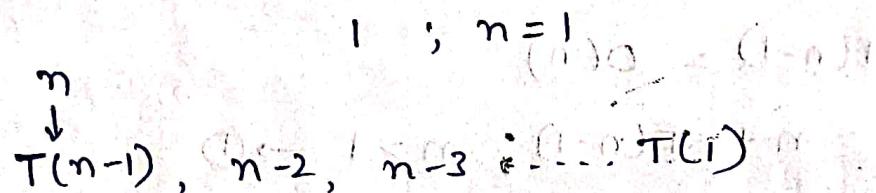
Recursion Tree:

$$1) T(n) = 1 + T(n-1)$$

We are doing a constant amount of work and then calling $T(n-1)$



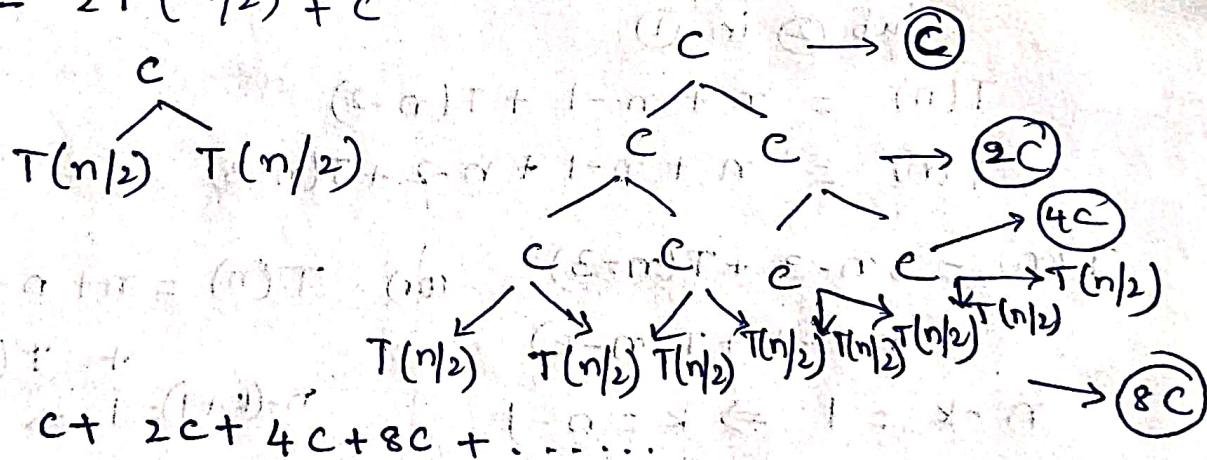
$$2) T(n) = n + T(n-1), n \geq 1$$



$$\Rightarrow n + (n-1) + (n-2) + \dots + 1$$

$$\Rightarrow \frac{n(n+1)}{2} \Rightarrow O(n^2)$$

$$3) T(n) = 2T(n/2) + C$$



$$\text{Sum of first G.P numbers} = \frac{a(r^n - 1)}{r - 1}, \text{ say } n = k+1$$

$$a = 2^0 = 1, r = 2, n = k+1 = c \cdot \frac{2^{k+1} - 1}{2 - 1}$$

$$\text{Sum (or)} \quad n = 2^{k+1} - 1$$

$$n+1 = 2^{k+1}$$

$$\log_2(n+1) = \log_2(2^{k+1}) \Rightarrow k+1 = \log_2(n+1)$$

$$K = \log_2(n+1) - 1$$

Substituting the value of K

$$T.C = C * 2^{\log_2(n+1) - 1}$$

$$= C * (n+1 - 1)$$

$$T.C = O(n)$$

Masters Theorem:

$$T(n) = aT\left(\frac{n}{b}\right) + \Theta(n^k \log^p n)$$

where n = size of the problem

a = number of subproblems in the recursion and $a >= 1$

n/b = size of each subproblem

$b > 1$, $k >= 0$ and p is a real number.

Then

1. if $a > b^k$, Then $T(n) = \Theta(n \log^k a)$

2. if $a = b^k$, Then

a) if $p > -1$ then $T(n) = \Theta(n \log^k a \log^{p+1} n)$

b) if $p = -1$, then $T(n) = \Theta(n \log^k a \log \log n)$

c) if $p < -1$ then $T(n) = \Theta(n \log^k a)$

3. if $a < b^k$, then

a) if $p \geq 0$, then $T(n) = \Theta(n^k \log^p n)$

b) if $p < 0$, then $T(n) = \Theta(n^k)$

$$1) T(n) = 3T\left(\frac{n}{2}\right) + n^2$$

$$a=3, b=2, k=2, p=0$$

$$b^k = 2^2 = 4 \quad a < b^k \text{ & } p=0$$

$$\text{so, } T(n) = \Theta(n^k)$$

$$= \Theta(n^2)$$

$$2) T(n) = 2T\left(\frac{n}{2}\right) + C$$

$$a=2, b=2, k=0, P=0$$

$$b^k = 2^0 = 1$$

$$\Rightarrow \Theta(n^{\log_b a}) \Rightarrow \Theta(n^{\log_2 2}) \\ \Rightarrow \Theta(n)$$

$$3) T(n) = 4T\left(\frac{n}{2}\right) + n^2$$

$$a=4, b=2, P=0, k=2$$

$$a = b^k$$

$$\Theta(n^{\log_b a} (\log^{P+1} n)) \Rightarrow \Theta(n^{\log_2 4} \log n) \\ \Rightarrow \Theta(n^2 \log n)$$

$$4) T(n) = T\left(\frac{n}{2}\right) + n^2$$

$$a=1, b=2, P=0, k=2$$

$$1 < 2^2 \Rightarrow 1 < 4 \Rightarrow a < b^k$$

$$\Theta(n^k \log^P n) \Rightarrow \Theta(n^2 \log^0 n)$$

$$\Rightarrow \Theta(n^2)$$

$$5) T(n) = 16T\left(\frac{n}{4}\right) + n$$

$$a=16, b=4, P=0, k=1$$

$$16 > 4^1$$

$$\Rightarrow \Theta(n \log_b a)$$

$$\Rightarrow \Theta(n \log_4 16)$$

$$\Rightarrow \Theta(n \log_4 4^2)$$

$$\Rightarrow \Theta(n^2)$$

$$6) T(n) = 2^n T\left(\frac{n}{2}\right) + n^n$$

$$T(n-1) = T(n/2) = 2^{n/2} T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^{\frac{n}{2}} \quad \text{---}$$

$$T(n) = 2^n \left[2^{n/2} T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^{\frac{n}{2}} \right] + n^n$$

$$T(n) = 2^{\frac{(n+2)}{2}} T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^{\frac{n}{2}} \times 2^n + n^n$$

$$= 2^{\frac{3n}{2}} T\left(\frac{n}{4}\right) + n^{\frac{n}{4}} \times 2^{\frac{3n}{2}} + n^n$$

$$T(n) = 2^n \left[2^{n/2} + [2^{n/4} T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^{n/4}] + \left(\frac{n}{2}\right)^{n/2} + n^n \right]$$

$\text{So } O(n) = 2^n$

Arrays:

Collection of similar data elements stored at contiguous memory location.

`int int int int int`

`int [] arr = new int[5];`

Accessing array Elements:

0	1	2	3	4	5
0	1	2	3	4	5

`int [] arr = new int[5]`

i.e. create an array of size 5.

Elements in an array can be Accessed through indexing

`arr[1] → 1 | | | |`

Different Operations on Array:

a) Insertion

1	2	3	4	5
---	---	---	---	---

Insert + 99

1	2	99	2	4	5
---	---	----	---	---	---

If we need insert an element at particular index, we need to right shift the elements. So T.C is $O(n)$

b) Accessing:

We need to iterate over the array.

Best Case Scenario T.C $\rightarrow O(1)$

Worst Case Scenario T.C $\rightarrow O(n)$

c) Deletion:

10	11	3	4
----	----	---	---

d) Accessing:

Elements can be accessed through indexing.

(for T.C = O(1))

What happens in memory?

\rightarrow Memory address are hexadecimal. For simplicity we can consider it as decimal.

int [] arr = new int [4];

100	00	00	00	112
-----	----	----	----	-----

Reserve 4 integer spaces.

\rightarrow Integer takes 4 bytes, the next memory Address can be calculated using

= Initial Mem.Address + (size of datatype) \times Element Number

$$= 100 + 4 \times 1$$

$$= 104$$

100	104	108	112
1	2	3	4

\rightarrow If the heap size is 100, but I have declared Max size as the size of the array, then we will encounter

Out of Memory error.

i.e. If the size of the array is more than the available heap size, then expect Out of Memory Error.

→ If we are trying to access an element index which is more than the array size then expect Array Index Out of Bound Exception.

Problems on Arrays:-

→ Print an Array :-

0	1	2	3
---	---	---	---

→ Iterate through the Array

→ Access the element through the index and print it.

```
for (int i=0; i<A.length; i++) {  
    System.out.println("Element at " + i + " is " + A[i]);  
}
```

→ Sum of Array:

→ Max of Array:

$$A[] = [1, 4, 2, 3]$$

```
int max = Integer.MIN_VALUE;
```

```
for (int i=0; i<A.length; i++) {
```

```
    if (A[i] > max) {
```

$$\text{max} = A[i]$$

```
}
```

```
}
```

```
return max;
```

Second Max of Array:

1	4	2	5	5
---	---	---	---	---

```

max = Integer.MIN_VALUE;
secondMax = Integer.MIN_VALUE;
for(int i=0; i < A.length; i++){
    if (arr[i] > max){
        secondMax = max;
        max = arr[i];
    }
    else if (arr[i] > secondMax && arr[i] < max)
        secondMax = arr[i];
}

```

Dry Run:

1) $s_{\text{max}} = -\infty$ 4) $5 > 4$
 $\text{max} = 1$ $s_{\text{max}} = 4$
 $\text{max} = 5$
 2) $4 > 1$
 $s_{\text{max}} = 1$ 5) $5 = 5$
 $\text{max} = 4$ so in else if
 3) $2 < 4$
 so in else if
 $2 > 1 \text{ & } 2 < 4$ $5 > 4 \text{ & } 5 < 5$
 $s_{\text{max}} = 2, \text{ max} = 4$
 So $s_{\text{max}} = 4$

Third Max Element:

```

max = Integer.MIN_VALUE;
secondMax = Integer.MIN_VALUE;
thirdMax = Integer.MIN_VALUE;
for(int i=0; i < A.length; i++){
    if (A[i] > max){
        thirdMax = secondMax;
        secondMax = max;
        max = A[i];
    }
    else if (A[i] > secondMax && A[i] < max)
        {
            thirdMax = secondMax;
            secondMax = A[i];
        }
    else if (A[i] > thirdMax && A[i] < secondMax)
        {
            thirdMax = A[i];
        }
}

```

→ Is Ascending

[1, 2, 3, 4, 5] → O/P: true

for (int i=0; i < A.length; i++) {

if (A[i] < A[i+1]) {

P++;

q++;

}

else {

return false;

}

return true;

→ nth element of Fibonacci Series

Fibonacci Series - 0, 1, 1, 2, 3, 5, 8 ...

if n = 4

If we create a new array

int[] ans = new int[n+1];

i.e. with extra space and ans[0] = 0, ans[1] = 1

ans:

0	1	2	3	4
---	---	---	---	---

ans[2] = ans[1] + ans[0];

ans[x] = ans[x-1] + ans[x-2]

for (int i=2; i < ans.length; i++)

{

ans[i] = ans[i-1] + ans[i-2];

}

return ans[n];

T.C → O(n)

S.C → O(n)

Without Extra Space:

```

firstElement = 0           if (n == 0) return 0;   else if (n == 1)
secondElement = 1          return 1;

for (int i = 2; i < n; i++) {
    ThirdElement = firstElement + secondElement;
    firstElement = secondElement;
    secondElement = ThirdElement;
}

return ThirdElement;

```

Merge Two Sorted Arrays:

arr1 [1 | 3 | 5 | 7] arr2 [2 | 4 | 6 | 8]

Op: [1 | 2 | 3 | 4 | 5 | 6 | 7 | 8] To Sort

$$\begin{aligned}
 T.C \rightarrow O(n) + O(n) &= O(2n) = O(n) + O(n \log n) \\
 S.C \rightarrow O(n) &\approx O(n \log n)
 \end{aligned}$$

→ Use 2 Pointer Approach:

1. Create 2 pointers pointing to the first element of both arrays.
2. Compare the elements being held by the pointers.
 - if ($\text{arr1}[i] < \text{arr2}[j]$) {
 - add the element to resultant array
 - $i++$
} else {
 - add the element to array
 - $j++$
}
3. Handle the case if any of the pointer reaches the end of array by appending all the values of the other array.

```

int[] result = new int[m+n];
int i=0; int j=0; int k=0;
while (i < m && j < n) {
    if (arr1[i] < arr2[j])
        {
            result[k] = arr1[i];
            k++; i++;
        }
    else
        result[k] = arr2[j];
        k++; j++;
}
while (i < m)
{
    result[k] = arr1[i];
    i++; k++;
}
while (j < n)
{
    result[k] = arr2[j];
    j++; k++;
}

```

without Extra Space:-

1	3	5	7
---	---	---	---

↑*i*

int i = array1.length;
 j = 0;
 while (j < array1.length && i > 0) {
 if (array1[i] > array2[j]) {
 int temp = array1[i];
 array1[i] = array2[j];
 array2[j] = temp
 i--; j++;
 } else { break; }
 }

2	4	6	8
↑			

Arrays.sort(array1);
 Arrays.sort(array2);
 $O(n) + O(n \log n)$
 $T.C = O(n \log n), S.C = O(1)$

Maximum Subarray Sum (or) Largest Continuous Sum:

arr →

-1	3	4	-2
----	---	---	----

 maxSum, Sum
 ↑↓ Initial val of maxSum = $-\infty$
 $\text{Sum} = \text{arr}[0]$;

Algo:-

1. Find out Subarrays
2. Calculate sum of each subarray
3. And update the maxSum.

Pseudo Code:

```

maxSum = 0
for (i = 0 to len(arr))
{
    sum = 0
    for (j = i to len(arr))
    {
        sum += arr[j];
        maxSum = Math.max(sum, maxSum);
    }
    return maxSum;
}
    
```

Time Complexity: $O(n^2)$, Space Complexity: $O(1)$

Kadane's Algorithm:-

↳ To find the largest sum in a Subarray with T.C $O(n)$

5	6	-3	7	-13	12
---	---	----	---	-----	----

 Initial maxSum, sum = arr[0]

$$\text{maxSum} = 5$$

$$\textcircled{1} = 11 > 5, \text{ So } \text{max} = 11$$

$$\textcircled{2} = 11 < 8 \text{ So } \text{max} = 11$$

$$\textcircled{3} = 11 < 15 \text{ So } \text{max} = 15$$

$$\textcircled{4} = 15 > -2 \text{ So } \text{max} = 15$$

$$\textcircled{1} 5 + 6 = 11$$

$$\textcircled{2} 5 + 6 - 3 = 8$$

$$\textcircled{3} 5 + 6 - 3 + 7 = 15$$

$$\textcircled{4} 5 + 6 - 3 + 7 - 13 = -2$$

$$\textcircled{5} \text{ As Sum is -ve, So } \text{sum} = \textcircled{2}$$

CodeEx Algo:

- ① Calculate cumulative sum, $\text{sum} = \text{arr}[0]$, $\text{maxSum} = \text{arr}[0]$
- ② update MaxSum by Comparing sum & MaxSum
- ③ If sum is -ve value, $\text{sum} = \text{arr}[i]$:

```
int sum = arr[0];
```

```
int maxSum = arr[0];
```

```
for (int i = 1; i < arr.length; i++) {
```

```
    if (sum > 0) {
```

```
        sum += arr[i];
```

```
}
```

```
else {
```

```
    sum = arr[i];
```

```
}
```

```
    maxSum = Math.max(maxSum, sum);
```

```
return maxSum;
```

PairSum: * No Duplicates.

0	1	2	3	4	5
1	2	3	4	5	<u>$k=6$</u>

↑ off: [0, 4], [1, 3]

Return the indexes of the pairs.

```
int i = 0; j = arr.length - 1;
```

```
while (i < arr.length && j > i) {
```

```
    int value = arr[i] + arr[j];
```

```
    if (value == k) {
```

```
        S.O. print (i + " " + j); i++; j--;
```

```
}
```

```
    else if (value < k) {
```

```
        i++;
```

```
    } else {
```

```
        j--;
```

Count Frequency of Each Element:

→ There is a concept called Frequency distribution.

i/p: 1, 2, 2; 1, 1, 2, 5, 2

O/P: $1 \rightarrow 3$

$$2 \rightarrow 4$$

$$5 \rightarrow 1$$

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② Algorithm:

- Create a frequency array with max value of array.
 - Traverse the array and update the value in freq. array with index from main array.
 - Traverse the array if count > 0:

```

int max = Integer.MIN_VALUE;
for(int i=0; i<arr.length; i++) {
    max = Math.max(arr[i], max);
}

int[] freqArray = new int[max+1];
for(int i=0; i<arr.length; i++) {
    int index = arr[i];
    freq[index] += 1;
}

for(int i=0; i<freqArray.length; i++) {
    if(freqArray[i] > 0)
        System.out.print(" " + i + ":" + freqArray[i]);
}

```

Best Time to Buy and Sell Stock

I/P: 7 1 2 3 4 5
 M T W Th F Sat
 C.P. = 1, S.P. = 6, $P = 6 - 1 = 5$

minArray[] = [7 1 1 1 1 1]
 ans Array[] = [0 0 4 2 5 3]

→ Before each day what was the minimum price of each stock.

maxProfit

→ Buy it on that day price & Sell it with minimum day price of stock of the previous day.

i.e. Buy it on lowest Price and sell it on the highest Price.

[7 4 5 3 1]

7 1 5 3 6 4

[7 4 4 3 1]

[7 1 0 0 0]

minArray

[0 0 1 0 0]

[0 0 4 2 5 3]

ansArray

for (int i = 0; i < arr.length; i++)

{

minSoFar = Math.min(minSoFar, arr[i]);

int profit = arr[i] - minSoFar;

max Profit = Math.max(maxProfit, profit);

}

O(n)

Segregate the array:

I/P: -1, 1, -2, 3, 4, -5

O/P: -1, -2, -3, 1, 3, 4

Similar approach or Move Zeros to End.

Missing First Positive Number:

Tip: -1, -1, 2, 3, 4

use frequency distribution approach:

0	1	2	3	4
0	1	1	1	0

TC $\rightarrow O(n)$

ans: ① SC $\rightarrow O(n)$

10	11	12
----	----	----

missing number \rightarrow ①

1	2	3
---	---	---

missing number \rightarrow length + 1

$$\rightarrow 3 + 1 = 4$$

First missing +ve number will always lie b/w 1 to n+1.

Coloring:

0	1	2
4	2	3

int index = arr[i]

$$= 2$$

0	1	2
2	3	4

$$abs = 2$$

$$index = 2 - 1 = 1$$

int n = nums.length;

for (i = 0; i < n; i++)

if (arr[i] <= 0 || arr[i] > n)

{

arr[i] = n + 1;

}

}

for (i = 0; i < n; i++)

int id = Math.abs(arr[i]);

if (id > n) continue; id--;

if (arr[id] > 0) arr[id] = -arr[id];

0	1	2
F	E	F

$$3 - 1 = 2$$

$$4 - 1 = 3$$

0	1	2
4	2	3

$$index + 1$$

$$0 + 1$$

$$= 1$$

for (int i=0; i<n; i++)

{

 if (arr[i] > 0)

 return i+1;

}

 return n+1;

(1) 0

(2) 1234

Q. दो विवरणों के बीच से

कौन कौन से विवरण अधिक विश्वासनीय है ?

उत्तर : दोनों विवरणों में एक विश्वासनीयता है।

लिखित उत्तर : दोनों विवरणों में एक विश्वासनीयता है।

प्रश्न : दो विवरणों के बीच से कौन कौन से विवरण अधिक विश्वासनीय है ?

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उत्तर : दोनों विवरणों में एक विश्वासनीयता है।

प्रश्न : दो विवरणों के बीच से कौन कौन से विवरण अधिक विश्वासनीय है ?

उत्तर : दोनों विवरणों में एक विश्वासनीयता है।

प्रश्न : दो विवरणों के बीच से कौन कौन से विवरण अधिक विश्वासनीय है ?

Majority Element

i.e. element present more than $n/2$ times

if $\rightarrow \{5, 1, 4, 1, 1\}$ $n = 5 \Rightarrow n/2 = 5/2 = 2$

Count of 1 is 3

so majority element is 1.

a) Using Frequency Distribution:

freqArr	0	1	2	3	4	5
	0	1	0	0	0	0

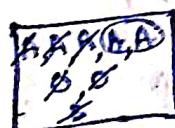
$\times 3$ 1 1

\rightarrow for (int i=0; i < freqArr.length; i++)

```

    {
        if (arr[i] > n/2) {
            return i;
        }
    }
  
```

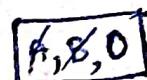
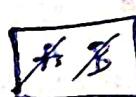
b) Moore's Voting Algorithm:-



8 elements are there in the bag.

$$8/2 = 4$$

\rightarrow If one element occurs $n/2$ times, all the other elements will occur less than $n/2$ times.



\Rightarrow no majority element.

arrIndex = 0,

Count = 1

for (int i=0; i < n).

2, 2, 1, 3, 1, 2, 2

{ if (arr[i] == arr[arrIndex]) {

= 2 votes = 2 ?

votes = 3

votes = 2

votes = 1

votes = 0

majorityElement = 1,

votes = 1

votes = 0

majorityElement = 1,

votes = 1

votes = 0

Sort an array of 0, 1, 2 (Dutch National Flag)

{1, 1, 1, 0, 2, 0, 1; 0}

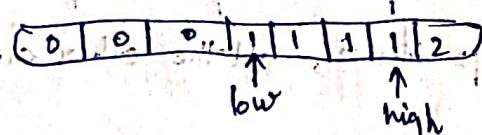
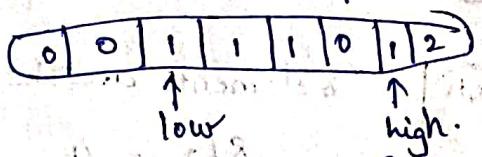
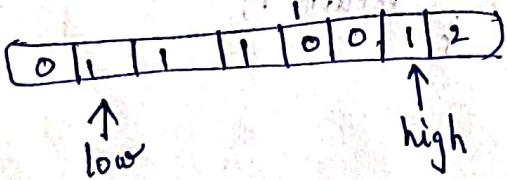
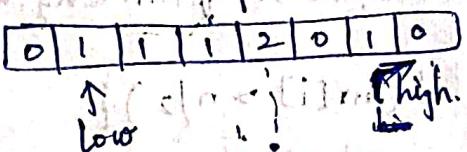
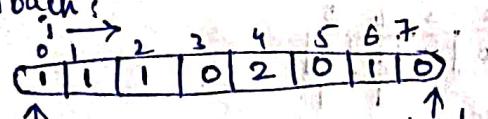
a) Approach 1:

Count 0 = 3, Count 1 = 4, Count 2 = 1

and then update the new array.

[0 0 0 1 1 1 1 2]

b) Two Pointer Approach:

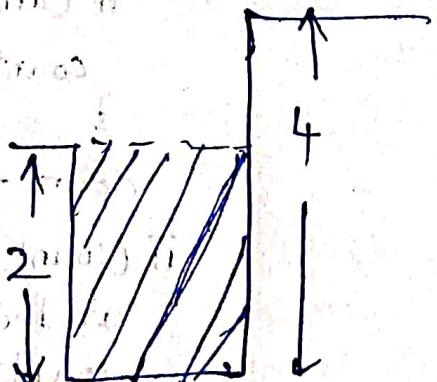
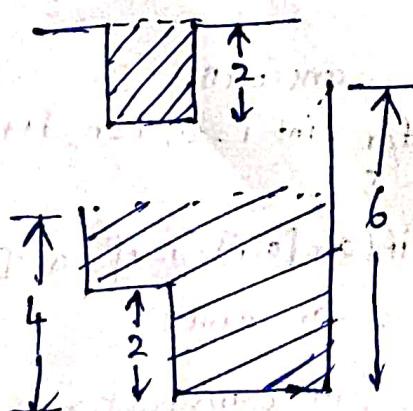
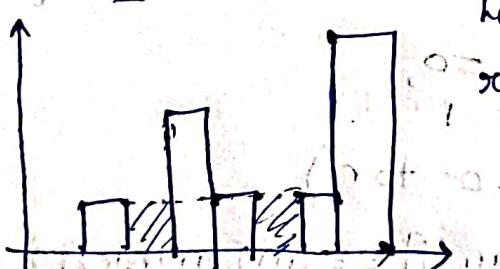


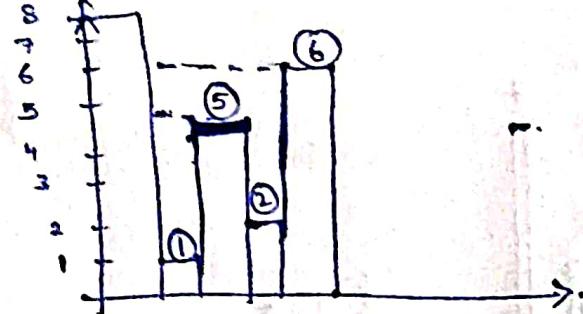
```

while(i < high) {
    if(arr[i] == 1)
        i++;
    else if(arr[i] == 0)
        swap(arr[i], arr[low])
        low++;
    else
        swap(arr[i], arr[high])
        high--;
}
  
```

Trapping Rain Water:

how much water can be trapped after raining?

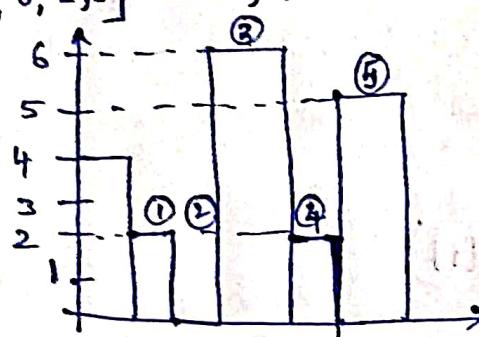




$$\min(l, r) - 1 \Rightarrow 5$$

$$\min(l, r) - 5 \Rightarrow 1 + = 9$$

$$[4, 2, 0, 6, 2, 5] \quad \min(5, 6) - 2 \Rightarrow 3$$



$$\therefore \text{ans} = \min(l_{\max}, r_{\max}) - \text{arr}(i)$$

$$1) \min(4, 6) - 2 = 4 - 2 = 2$$

$$2) \min(4, 6) - 0 = 4 - 0 = 4$$

$$3) \min(6, 5) - 6 = 5 - 6 = -1 \times$$

$$4) \min(6, 5) - 2 = 5 - 2 = 3$$

$$5) \min(6, 5) - 5 = 5 - 5 = 0.$$

$$\text{arr}[] = \boxed{4 \ 2 \ 0 \ 6 \ 2 \ 5}$$

$$2+4+3 = \underline{\underline{9}}$$

$$l_{\max} = \boxed{4 \ 4 \ 6 \ 6 \ 6}$$

$$r_{\max} = \boxed{6 \ 6 \ 6 \ 6 \ 5 \ 5}$$

$$\text{ans} = \text{Math}.\min(l_{\max}(i) - r_{\max}(i)) - \text{arr}[i];$$

b) Two Pointer Approach: $T: C \rightarrow O(n^2)$, $S.C \rightarrow O(n)$

$$\text{leftMax} = 0, \text{rightMax} = \text{arr}[0], \text{length}-1; \quad \boxed{4 \ 2 \ 0 \ 6 \ 2 \ 5}$$

while ($\text{left} < \text{right}$)

{
 $\text{leftMax} = \text{Math}.\max(\text{leftMax}, \text{height}[\text{left}]);$

$\text{rightMax} = \text{Math}.\max(\text{rightMax}, \text{height}[\text{right}]);$

 if ($\text{leftMax} < \text{rightMax}$)

$\text{ans} += \text{leftMax} - \text{height}[\text{left}];$

$\text{left}++;$

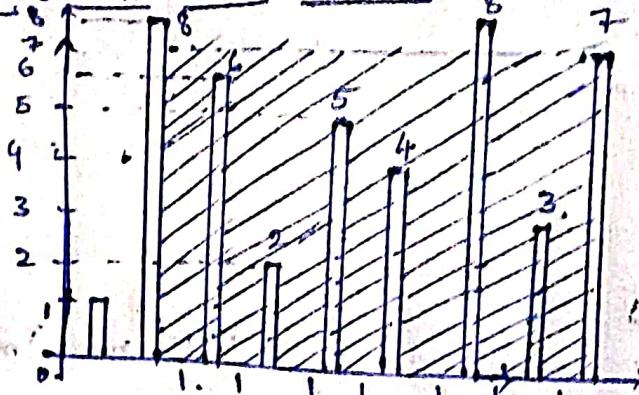
 } else {

$\text{ans} += \text{rightMax} - \text{height}[\text{right}];$

$\text{right}--;$

 }

Container with Most Water:



$$l = 7$$

$$b = 7$$

$$A = l \times b / 2$$

$$\text{height}[] = [1, 8, 6, 2, 5, 4, 3, 7] \quad 27 \times 7 = 49$$

The max area of water

```
for(int i=0; i<arr.length; i++) {
```

```
    for(int j=i+1; j<arr.length; j++) {
```

$O(n^2)$ int length = Math.min(arr[i], arr[j])

int breadth = j - i;

currentArea = l * b;

MaxArea = Math.max(currentArea, MaxArea);

Approach(2)

while(left < right) {

int

if (height[left] < height[right]) {

left++;

}

else {

right++;

}

return maxArea;

$O(n)$

30/9/2023

Sliding Window

```
arr = [2 3 5 2 9 7] 1
```

→ Find sum of all sub-arrays of size 3.

Sub-array:- any contiguous elements form a sub-array.

$$2,3,5 \approx 10$$

$$3,5,2 = 10$$

$$5, 2, 9 = 16$$

$$297 = 16$$

$$971 = 17$$

Brute Force:

```
for( int i=0; i<arr.length; i++ ) {
```

for (j = i; j < k; j++) {

sum + = arr[j]

15

S.O. phi (sum);

$$2 + \boxed{3 + 5} = 10$$

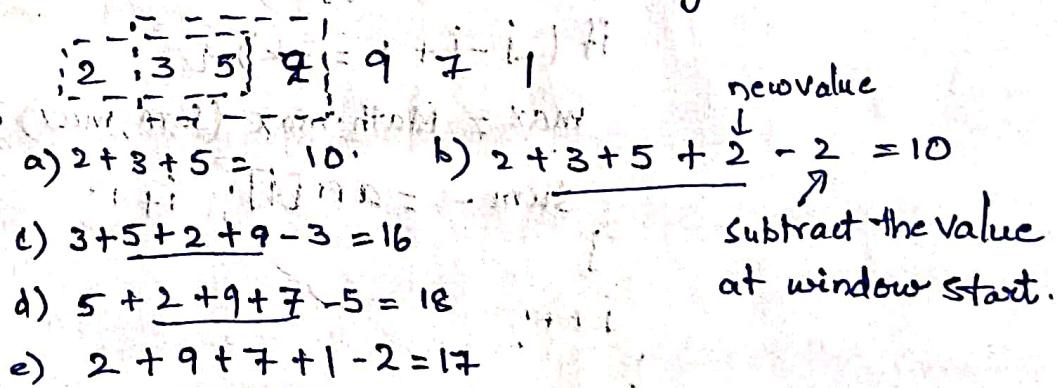
$$3 + \boxed{5 + 2} = 10$$

$$5 + \boxed{2 + 9} = 16$$

$$2 + \boxed{9 + 7} = 18$$

Step 1:- Make your window reach till the window size.

Step 2: Once you have reached the window size, do your calculation.



→ Consider Sliding Window Technique, you see Sub-array, Maximize (or) Minimize.

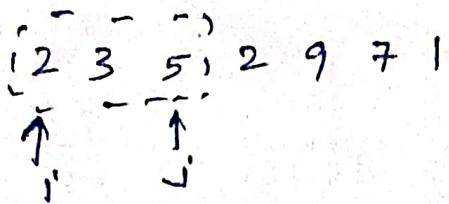


Fixes

(Window size K=3)

 Variable

(window size, needs to be calculated)



→ Initialize two variables

$i \rightarrow$ window start, $j \rightarrow$ window end
(Traversing variable)

while ($i < arr.length$) {

 sum += array[i];

 if ($j - i + 1 = k$) {

 sum -= array[i];

$i++$;

}

$j++$;

}

 Print the Sum here.

→ Maximum Sum of K Consecutive Elements:

 int i = 0, j = 0, sum = 0, Max = Integer.MIN_VALUE;

 while ($j < arr.length$) {

 sum += arr[j];

 if ($j - i + 1 = k$) {

 max = Math.max (sum, max);

 sum -= arr[i];

$i++$;

 }

$j++$;

 }

 return max;

→ Given a string, and non-empty string P. find all p's anagrams in s.
String contains only lowercase letters.

If P: s = "cbaebabcd", P = "abc", D/P: ?

client → listen

→ You are able to create a meaningful word then the word is said to be an anagram.

→

s = cabaebabed, P = abc

$$\text{Count} = \phi \times 2$$

ex ①:

a a b c → a=2, b=1, c=1

b a c a → a=2, b=1, c=1

These are anagrams.

ex ②:

a b c d → a=1, b=1, c=1, d=1

a b b c → a=1, b=2, c=1

Not anagrams.

P. Arr

0	1	2
1	1	1

→ update the frequency of character

S. Arr

0	1	2
1	1	1

→ We need to iterate over the P array & check the count of each character.

Approach ①: for (int i=0; i<Parr.length; i++) {
 if (Parr[i] != Sarr[i]) {
 return false;
 }
 return true;

Approach 2:

```

int k = P.length(); int j=0, int i=0;
int[] sArr = new int[26];
int[] pArr = new int[26];
int count = 0;
for(int l=0; l<P.length(); l++) {
    pArr[P.charAt(l) - 'a'] += 1; p = abc
}
while(j < s.length()) {
    sArr[s.charAt(i) - 'a'] += 1; boolean isAnagram(sArr, pArr)
    if (j-i+1 == k) { i = true; for(int i=0; i<pArr.length; i++)
        if (isAnagram(sArr, pArr)) { if (pArr[i] != sArr[i]) {
            count++; return false;
        }
    }
    if (i >= k) { return true;
    }
    sArr[s.charAt(i) - 'a'] -= 1; i++;
}

```

Longest SubArray of Size k: (Variable Window Size)

Given an array, find the longest subarray of size k.

I/p: {4, 1, 1, 1, 2, 3, 5} k=8, O/p: 5

That longest array is 1, 1, 1, 2, 3, so the length is 5.

4	1	1	1	2	3	5
↑	↑	↑	↑			
i	j	m	n			

i = start index, j = end index, m = max sum, n = current sum

```

int i=0, int j=0; int currentSum = 0;
int longestWindow = Integer.MIN_VALUE;
while (j < arr.length) {
    currentSum += arr[j];
    if (currentSum == k) {
        longestWindow = Math.max(longestWindow, j-i+1);
    } else {
        while (currentSum > k) {
            currentSum -= arr[i];
            i++;
            if (currentSum == k) {
                longestWindow = Math.max(longestWindow, j-i+1);
            }
        }
    }
    j++;
}
return longestWindow;
}

```

→ Smallest SubArray:

Find the length of Smallest SubArray whose length is $> k$.

$$Arr = \{4, 2, 2, 7, 8, 1, 10\};$$

$$K = 8$$

$$O/P: 1$$

```

int i=0; int j=0;
int minLength = Integer.MAX_VALUE;
int currentWindowSum = 0;
while (j < arr.length) {
    currentWindowSum += arr[i];
    while (currentWindowSum > k)
        {
            minLength = Math.min(minLength, j-i+1);
            currentWindowSum -= arr[i];
            i++;
        }
    j++;
}
return minLength;

```

→ Max Consecutive One's if at most 1 flip is allowed.

I/P: {1, 1, 0, 0, 1, 1, 1, 1} k=1, O/P: 6

```

int zeroCount = 0; int i=0; int j=0;
int maxConsecutiveOne = 0;
while (j < arr.length) {
    if (arr[j] == 0) {
        zeroCount++;
    }
    while (zeroCount > k) {
        if (arr[i] == 0) {
            zeroCount--;
        }
        i++;
    }
    maxConsecutiveOne = Math.Max(maxConsecutiveOne, j-i+1);
    j++;
}
return maxConsecutiveOne;

```