Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

1 Signal Flow Graph

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 Bode Plot

2.1 Introduction

2.2 Example

3 Second order System

3.1 Damping

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4 Routh Hurwitz Criterion

4.1 Routh Array

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5 State-Space Model

5.1 Controllability and Observability

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5.4 Example

6 Nyquist Plot

6.1. Find the range of k such that given characteristic equation

$$s(s^3 + 2s^2 + s + 1) + k(s^2 + s + 1) = 0 \quad (6.1.1)$$

is stable.

Solution: The General form of characteristic equation :

$$1 + G(s)H(s) = 0 \tag{6.1.2}$$

6.2. We draw nyquist plot for open loop transfer function, which is G(s)H(s). When system is marginally stable nyquist plot ((G(s)H(s))) passes through (-1,0),then 1+G(s)H(s) nyquist plot passes through (0,0). We will make real and imaginary part of characteristic equation to 0.So,that we get k for system to be marginally stable. For the system to be stable , the range of k becomes any value greater than k mimimum.

$$Realpart = \omega^4 - \omega^2(k+1) + k = 0 \tag{6.2.1}$$

$$Imaginarypart = -\omega^3 + \omega(w+1) = 0 \tag{6.2.2}$$

By equating real and imaginary to 0 .We get,

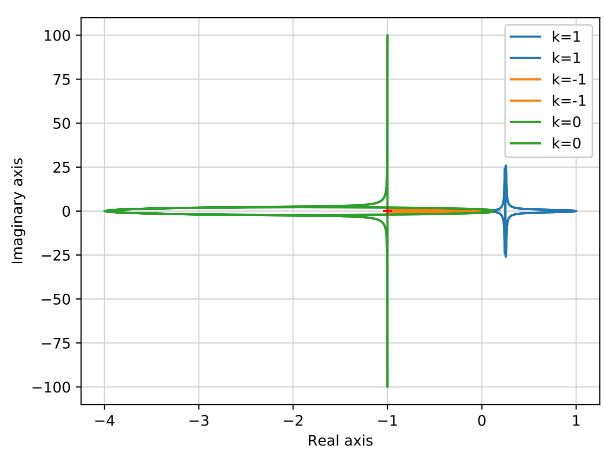
$$k = 0 \tag{6.2.3}$$

So,we got minimum value of k is 0 for system to be stable. Then the range of k is

$$0 < k < \infty \tag{6.2.4}$$

6.3. For a nyquist plot,no.of clock wise encirclement's around the point(-1,0) for a open loop transfer function gives the total no. right hand side zeros plus total no.of right hand side poles, which gives us a idea about stability of system.

6.4. Nyquist plot for different values of k.



In plot, we can see at k=0 ,the plot passes through (-1,0) and at k = 1 no,of encirclements about (-1,0) is 0 which implies system is stable as no.of right hand side zeros(positive values) are 0 and also verifies our above result of k range.

Code for Nyquist plot

```
codes/ee18btech11042_1.py
```

6.5. Verify using Routh hurwitz criterion

Solution: Routh -hurwitz criterion says system is marginally when no.of sign changes is 0 in matrix and any row of matrix is completely 0. From this , we get minimum value of k for system to be stable

$$s^4 + 2s^3 + s^2(k+1) + s(k+1) + k = 0 \tag{6.5.1}$$

$$\begin{vmatrix} s^4 \\ s^3 \\ s^2 \\ s^1 \\ s^0 \end{vmatrix} \begin{vmatrix} 1 & k+1 & k \\ 2 & k+1 & 0 \\ \frac{k+1}{2} & k & 0 \\ \frac{(k-1)^2}{2} & 0 & 0 \\ k & 0 & 0 \end{vmatrix}$$

6.6. For the system to be stable,all values of that matrix should be greater than or equal to 0.So,minimum value of k is,

$$k = 0 \tag{6.6.1}$$

The range of k system to be stable

$$0 < k < \infty \tag{6.6.2}$$

6.7. Verify it using following routh -hurwitz code.

```
codes/ee18btech11042_2.py
```

7 Compensators

7.1 Phase Lead

7.2 Example

8 Gain Margin

8.1 Introduction

8.2 Example

9 Phase Margin

10 Oscillator