

Control Systems

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10 Oscillator

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

1 SIGNAL FLOW GRAPH

1.1 Mason's Gain Formula

1.2 Matrix Formula

2 BODE PLOT

2.1 Introduction

2.2 Example

3 SECOND ORDER SYSTEM

3.1 Damping

3.2 Example

4 ROUTH HURWITZ CRITERION

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

4.4 Example

5 STATE-SPACE MODEL

5.1 Controllability and Observability

5.2 Second Order System

5.3 Example

5.4 Example

6 NYQUIST PLOT

6.1. Find the range of k such that given characteristic equation

$$s(s^3 + 2s^2 + s + 1) + k(s^2 + s + 1) = 0 \quad (6.1.1)$$

is stable.

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Solution: The General form of characteristic equation :

$$1 + G(s)H(s) = 0 \quad (6.1.2)$$

6.2. For a system to be marginally stable , nyquist plot passes through (-1,0). Generally we will draw nyquist plot for open loop gain $G(s)H(s)$. So, $1+G(s)H(s)$ passes through (0,0)

$$\text{Realpart} = \omega^4 - \omega^2(k+1) + k = 0 \quad (6.2.1)$$

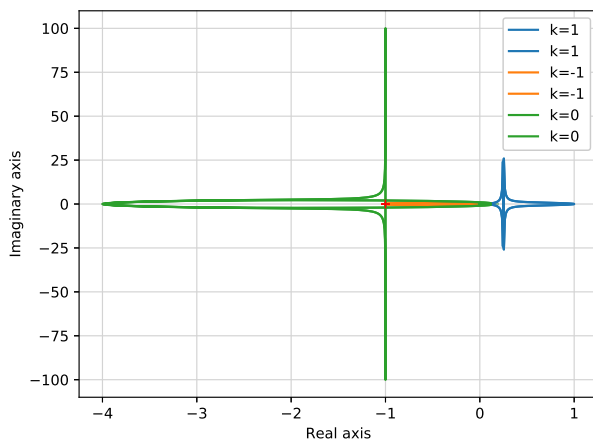
$$\text{Imaginarypart} = -\omega^3 + \omega(w+1) = 0 \quad (6.2.2)$$

By equating real and imaginary to 0 .We get,

$$k = 0 \quad (6.2.3)$$

6.3. For a nyquist plot, no. of clock wise encirclement's around the point(-1,0) for a open loop transfer function gives the total no. right hand side zeros ,

6.4. We can verify it through nyquist plot



The following python code generates the nyquist plot :

```
codes/ee18btech11042_1.py
```

Verify it through routh hurwitz criterion.

$$s^4 + 2s^3 + s^2(k+1) + s(k+1) + k = 0 \quad (6.4.1)$$

$$\begin{vmatrix} s^4 & 1 & k+1 & k \\ s^3 & 2 & k+1 & 0 \\ s^2 & \frac{k+1}{2} & k & 0 \\ s^1 & \frac{(k-1)^2}{2} & 0 & 0 \\ s^0 & k & 0 & 0 \end{vmatrix}$$

6.5. For the system to be stable, all values of that matrix should be greater than or equal to 0. So, minimum value of k is,

$$k = 0 \quad (6.5.1)$$

The range of k system to be stable

$$0 < k < \infty \quad (6.5.2)$$

verify it using following routh -hurwitz code.

```
codes/ee18btech11042_2.py
```

7 COMPENSATORS

7.1 Phase Lead

7.2 Example

8 GAIN MARGIN

8.1 Introduction

8.2 Example

9 PHASE MARGIN

10 OSCILLATOR