

Control Systems

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Frequency Response

For a system having transfer function

$$G(s) = \frac{-s + 1}{s + 1}, \quad (1)$$

a unit step is applied at time $t = 0$. The value of the response of the system at $t = 1.5\text{sec}$ is:

Solution

We know that,

$$x(t) = u(t) \quad (2)$$

Where $u(t)$ is a unit step input. The Laplace transform $x(t)$ is:

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt \quad (3)$$

From this,

$$X(s) = \frac{1}{s} \quad (4)$$

Solution

We know that,

$$Y(s) = X(s)H(s) \quad (5)$$

in Laplace domain. So,

$$Y(s) = \frac{-s + 1}{s(s + 1)} \quad (6)$$

By doing partial fractions,

$$\frac{-s + 1}{s(s + 1)} = \frac{A}{s} + \frac{B}{s + 1} \quad (7)$$

By this,

$$A = 1, B = -2 \quad (8)$$

From this,

$$Y(s) = \frac{1}{s} + \frac{-2}{s+1} \quad (9)$$

The inverse Laplace transform of $Y(s)$ is:

$$y(t) = u(t) - 2e^{-t}u(t) \quad (10)$$

$u(t) = 1$ for all $t > 0$. So, at $t = 1.5$ sec,

$$y(1.5) = 1 - 2e^{-1} \quad (11)$$

$$y(1.5) = 0.5537 \quad (12)$$

The value of the response of the system at $t = 1.5$ sec is 0.5537

