Digital Assignment - 1

1)

Under what conditions on a, b, and c, the following system has (i) no solution (ii) infinite solutions.

$$-2x + y + z = a,\ x - 2y + z = b,\ x + y - 2z = c$$

2)

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$, $X = [x_1 \;\; x_2 \;\; x_3]^T$ and $b = [1 \;\; 6 \;\; 4]^T$. Using Gauss-Jordan method, find the inverse of the matrix A. Hence solve $AX = b$.

3)

Find the LU-decomposition/factorization of $A = \begin{bmatrix} 2 & -6 & 10 \\ 1 & 5 & 1 \\ -1 & 15 & -5 \end{bmatrix}$. Hence find the solution of $AX = b$, where $X = [x_1 \;\; x_2 \;\; x_3]^T$, and $b = [4 \;\; 4 \;\; 6]^T$.

8. Let V be the vector space containing all real valued continuous functions over $\mathbb{R}$. Verify that the set W of solutions of differential equation: $2\frac{d^2y}{dx^2} - 9\frac{dy}{dx} + 2y = 0$ is a subspace of V.

9. Express $(1, -2, 5)$ as a linear combination of the vectors $\{(1, -2, 5), (1, 2, 3), (2, -1, 1)\}$.

10. In $\mathbb{R}^3$, let $W = \{(x, y, z) | x - y - z = 0\}$. Prove that W is a subspace of $\mathbb{R}^3$ and hence find a basis for this subspace.

11. Verify the following sets of functions are linearly independent in the vector space $\mathcal{C}[-\pi, \pi]$

    (a) $\{x, e^x, e^{-x}\}$

    (b) $\{x|x|, x^2\}$.