* Graph:

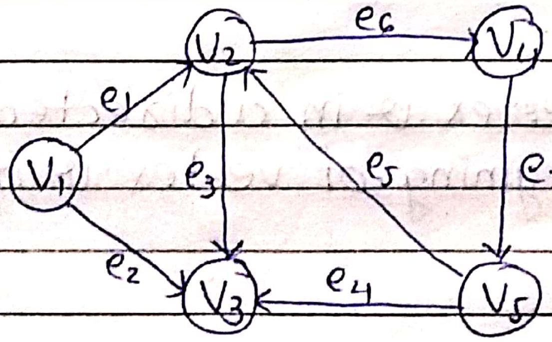
→ A Graph is writtenas $G = G(V, E)$ is a collection of two components
1. Finite set of vertices V which is also known as points or nodes.
2. Finite set of edges E also call line or arc connected the pair of vertices

→ In other words in graph there is a mapping from set of edges E to the set of vertices V such that $e \in E$ is associated with ordered or unordered pair of element of V.

* Directed graph or digraph:

→ A graph said to be directed graph or digraph if it's required to associated direction with each edge of the graph.

ex.



$V = \{ V_1, V_2, V_3, V_4, V_5 \} \cancel{V_6}$

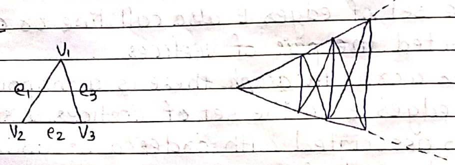
$E = \{ e_1, e_2, e_3, e_4, e_5, e_6, e_7 \}$

$= \{ (V_1, V_2), (V_1, V_3), (V_2, V_3), (V_5, V_2), (V_5, V_3), (V_2, V_4), (V_4, V_5) \}$

* Finite graph :

→ if the graph as finite no of edge & vertex then it's known as finite graph otherwise it's known infinite graph.

ex.



* Indegree :

→ The degree of vertex v in a directed graph is the no. of edges ending at vertex v it's denoted by indeg(v).

* Outdegree :

→ The outdegree of vertex v in a directed graph is the no. of edges begining of vertex v it's denoted by outdeg(v)

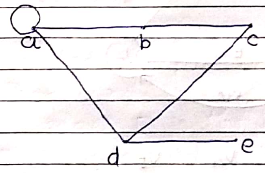
ex.

vertex	V_1	V_2	V_3	V_4	V_5
indeg	0	2	3	1	1
outdeg	2	2	0	1	2

indeg(v_1) = 0
outdeg (v_5) = 2

* degree of vertex :

→ The degree of vertex is denoted by $d(v)$ or $deg(v)$ is the no. of edges connected with vertex v.



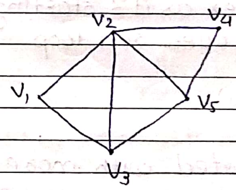
d (a) = 5
d (b) = 2
d (c) = 3
d (d) = 3
d (e) = 1

→ A vertex is said to be even vertex when it's degree is even.

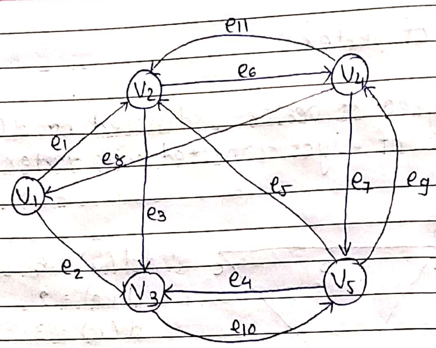
→ A vertex is said to be odd vertex when it's degree is odd.

ex. d (a) is odd
d (b) is even

ex.



$d(V_1) = 2$ even
$d(V_2) = 4$ even
$d(V_3) = 3$ odd
$d(V_4) = 2$ even
$d(V_5) = 3$ odd



	V_1	V_2	V_3	V_4	V_5
in	1	3	3	2	2
out	2	2	1	3	3

= { (v_1,v_2), (v_1,v_3), (v_4,v_1), (v_2,v_4), (v_2,v_3), (v_5,v_2), (v_5,v_4), (v_4,v_2), (v_3,v_5), (v_4,v_5), (v_5,v_3) }

* Self loop:

→ An edge haveing same vertex as it's starting & ending point (vertex) it's called self loop.

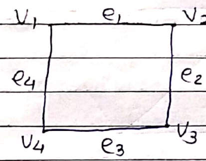
* Parallel edge: (Multiple edge)

→ more than one edge associated with more than one pair of vertices such edges are called parallel edges

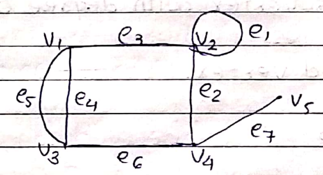
* Simple graph

→ A graph that has neither self loop nor parallel edge it's called simple graph.

ex.



ex.

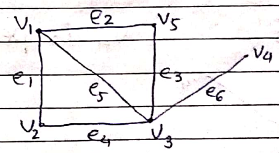


self loop = e_1
parallel edge = e_4 & e_5

* Incidence :

→ when a vertex v_i is and end vertex of some edge e_j, v_i & e_j are said to be incidence with (on or to) each other.

ex



e_3, e_5 & e_4 & e_6 incidence vertex with v_3

V_4 : e_6 $\quad$ V_1 : e_1, e_2

V_5 : e_2, e_3 $\quad$ V_2 : e_1, e_4

* Adjacence:

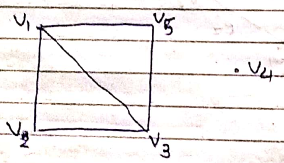
→ Two non-parallel edge are said to be adjacence if they are incidence on a common vertex

ex. $\{e_1, e_4\}$ v_2 — e_1, e_2, e_5 are adjacence
$\{e_1, e_2, e_5\}$ v_1

* Isolated vertex:

→ A vertex having no incidence edge is called isolated vertex or
isolated vertex is vertex with degree 0.

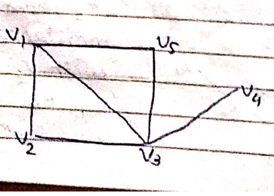
ex. v_4 is isolated vertex.



* Pendant vertex:

→ A vertex is degree 1 is called pendant

ex.

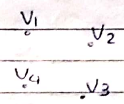


V_4 is pendant

* Null graph:

→ A graph without any edge is known as null graph

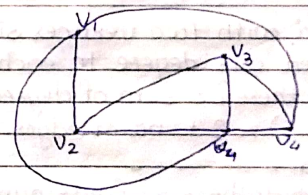
ex.



* Regular graph:

→ A graph in which all the vertex are of equal degree

ex.



$d(v_1) = 3$ $d(v_3) = 3$
$d(v_2) = 3$ $d(v_4) = 3$

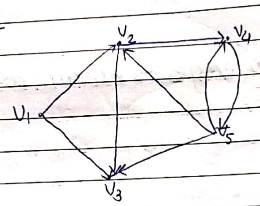
all are equal

Theorem-1:

If $G(V, E)$ is a directed graph then the sum of outdegree of the vertices of digraph is equal to sum of indegree of the vertices which equals to no. of edges in a digraph.

$e = 8$

ex.



	v_1	v_2	v_3	v_4	v_5
in	0	2	3	2	1
out	2	2	0	1	3

Formula:

Σ indeg(v) = Σ outdeg(v) = no. of edges for all $v \in V$

$8 = 8 = 8$

Theorem-2:

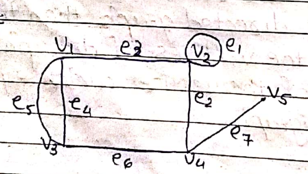
Prove that sum of degree given graph is the twice the no of edges in a graph. (handshaking)

Proof:

$\Rightarrow$ each edge associated with two vertices since each edge distributed one degree to each vertex an adjacence of it. therefor sum of degree of vertex in graph $G(V,E) = 2 *$ no. of edges.

$$\boxed{\sum_{i=1}^{n} d(v_i) = 2 * e}$$ where n = finite number

ex.



from graph

$d(v_1) = 3$

$d(v_2) = 4$

$d(v_3) = 3$

$d(v_4) = 3$

$d(v_5) = 1$

$n = 5 \quad e = 7$

$$\sum_{i=1}^{5} d(v_i) = d(v_1) + d(v_2) + d(v_3) + d(v_4) + d(v_5)$$
$$= 3 + 4 + 3 + 3 + 1$$
$$= 14$$
$$= 2 * 7$$

$$\sum_{i=1}^{n} d(v_i) = 2 * e$$

Theorem-3:

The no. of vertices of odd degree in a graph is always even.

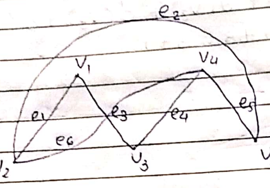
$\rightarrow$ By using handshaking theorem we know that for a G $\sum_{i=1}^{n} d(v_i) = 2 * e$

$$\underbrace{\sum_{i=1}^{n} d(v_i)}_{\text{odd degree}} + \underbrace{\sum_{i=1}^{n} d(v_i)}_{\text{even degree}} = 2 * e$$

$$\sum_{\substack{i=1 \\ \text{odd}}}^{n} d(v_i) = 2*e - \text{even}$$

$$\sum_{\substack{i=1 \\ \text{odd}}}^{n} d(v_i) = \text{even} \rightarrow (2)$$

$\rightarrow$ By using eq^n (2) is prove that the no of vertices of odd degree is always even ex. Verify handshaking theorem as well as theorem 3.



e = no. of edges = 6 n = no. of vertex = 5

d(V1) = 2 d(V3) = 2 d(V5) = 2

d(V2) = 3 d(V4) = 3

→ $\sum_{i=1}^{n} d(V_i) = 2 * e$

$\sum_{i=1}^{n} d(V_i) = d(V_1) + d(V_2) + d(V_3) + d(V_4) + d(V_5)$

$= 2 + 3 + 2 + 3 + 2$

$= 12$

$= 2 * 6$

$= 2 * e$

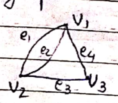
→ In this graph V_2 & V_4 are of odd degree

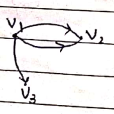
→ Total no. of vertex having odd degree = 2

→ Hence, the theorem that is the total no. of vertices is even.

* Multi Graph:

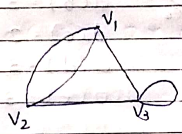
→ A graph which contain parallel edges is called multigraph.





* Pseudo graph:

→ A graph in which loops and parallel (multiple edge) edges are allowed is called Pseudo graph.

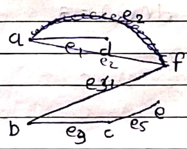


ex. Draw a diagram for each of the following G also calculate degree of all the vertices and identify the type of graph.

G(V, E)

1. V = {a, b, c, d, e, f}

E = {(a,b), (a,f), (b,c), (b,f), (c,e)}



d(a) = 2 d(d) = 1

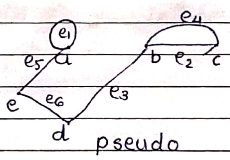
d(b) = 2 d(e) = 1

d(c) = 2 d(f) = 2

simple graph

2. V = {a, b, c, d, e}

E = {(a,a), (b,c), (b,d), (c,b), (e,a), (e,d)}



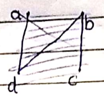
d(a) = 3 d(d) = 2

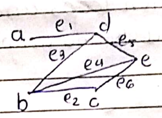
d(b) = 3 d(e) = 2

d(c) = 2

pseudo

3. V = {a, b, c, d, e}
E = {(a,d), (b,c), (b,d), (b,e), (d,e), (c,e)}

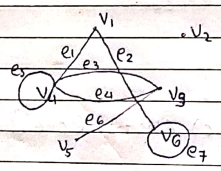




simple

d(a) = 1
d(b) = 3
d(c) = 2
d(d) = 3
d(e) = 3

ex. Find the degree of all the vertices of G. Identify ~~associa~~ isolated vertex, Pendent vertex, the type of graph.



$d(v_1) = 2$
$d(v_2) = 0$
$d(v_3) = 3$
$d(v_4) = 5$
$d(v_5) = 1$
$d(v_6) = 3$

V_2 is isolated
V_5 is Pendent
Pseudo graph.

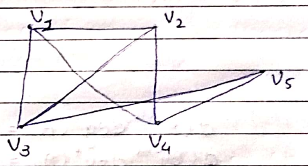
ex.

a) 4 4 4 3 2
b) 5 5 4 3 2 1
c) 3 3 3 3 2
d) 3 3 3 3 3 3

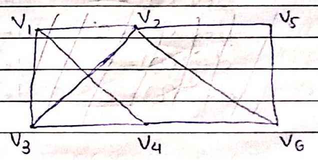
a) sum of degree of all the vertices = 17 which is an odd number. This is imposible. Hence the no graph exist with the following degree sequence.

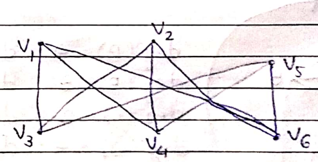
b) There are 6 vertices. Hence a vertex of degree 5 in the graph must be adjacent to all remaining vertex. As there are two vertex each of degree 5, all other vertecies should be a degree of atleast 2 but the given degree sequence contain 1 also. There is no graph exist with the following degree sequence.

c)

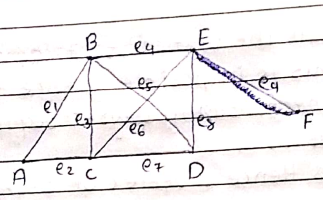


d)



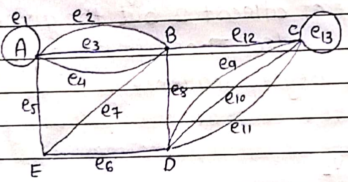


1.



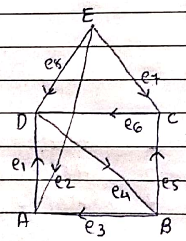
$d(A) = 2$ $\quad d(E) = 4$
$d(B) = 4$ $\quad d(F) = 1$
$d(C) = 4$
$d(D) = 3$

2.



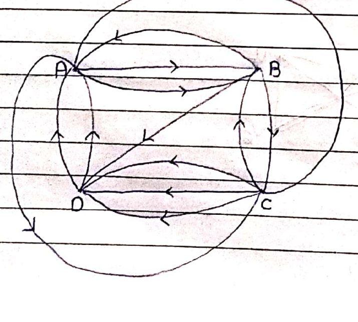
$d(A) = 6$ $\quad d(E) = 3$
$d(B) = 6$
$d(C) = 6$
$d(B) = 5$

3.



$d(A)^{in} = deg^-(A) = 2$
$d(A)^{out}\ deg^+(A) = 1$
$deg^-(B) = 1$ $\quad deg^+(B) = 2$
$deg^-(C) = 2$ $\quad deg^+(C) = 1$
$deg^-(D) = 2$ $\quad deg^+(D) = 1$
$deg^-(E) = 0$ $\quad deg^+(E) = 3$

4



$deg^-(A) = 4$ $\quad deg^+(A) = 3$
$deg^-(B) = 3$ $\quad deg^+(B) = 3$
$deg^-(C) = 2$ $\quad deg^+(C) = 5$
$deg^-(D) = 4$ $\quad deg^+(D) = 2$

ex. For each of the following graph
1. Calculate the degree of all vertex
2. Identify the type of graph
3. Identify isolated & pendent vertex
4. Isolve handshaking theorem for following graph
5. Graph notation

1. simple graph
F is pendent

$\sum_{i=1}^{n} d(v_i) = d(A) + d(B) + d(C) + d(D) + d(E) + d(F)$
$= 2+4+4+3+4+1$
$= 18$
$= 2 * 9$
$= 2 * e$

$G(V, E)$
$V = \{A, B, C, D, E, F\}$
$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$
$= \{(A,B), (A,C), (B,C), (B,E), (B,D), (E,C), (C,D), (D,E), (E,F)\}$

2. Pseudo graph
no isolated & pendent

$\sum_{i=1}^{n} d(v_i) = 6+6+6+5+3 = 26$
$= 2 * 13$
$= 2 * e$

$G(V, E)$
$V = \{A, B, C, D, E\}$
$E = \{(A,A), (A,B), (A,B), (B,A), (A,E), (E,D), (B,E), (B,D), (D,C), (D,C), (C,D), (B,C), (C,C)\}$