Unit-1 : Introduction to Proposation

* Introduction :

→ DM is a part of mathematics, dedicated to the study of descrete / distinct / separated objects.

→ It is used whenever objects are countables & when process involving finite no. of steps are analyzed.

* Applications of DM in computer science :

→ Algorithm
→ Programming language
→ Cryptography (securing info transfering from one remote area to...)
→ Automatic theory prooving
→ Software Development
→ Applications for task management
→ Topological sorting
→ Traveling salesman problem
→ Graph theory

* Proposation :

→ A declarative sentence which is true or false but not both is known as Proposation.

→ If a proposation is true, it's truth value will be denoted as 1 or T.

→ If a proposation is false, it's truth value will be denoted as 0 or F.

Proposation:
ex. 2+2 = 5 (F)
India is a country. (T)
Programe will be compiled at runtime. (T)

Not proposation:
He is an animal.
Good Morning!

* Connectives:

1. Conjuction (right AND operator ∧)

→ when p and q are proposations, the 'p and q' denoted by and it's called as conjunction of p and q.
→ it's defined as the decompound proposation that is true when p and q both will be 1 (true) and is false otherwise.

p	q	p∧q
F	F	F
F	T	F
T	F	F
T	T	T

ex. India is a country and today is Friday (T)

2. Disjunction (or operator ∨)

→ It's defined as the compound proposation is false when both p and q are false and true is otherwise.

p	q	p∨q
F	F	F
F	T	T
T	F	T
T	T	T

3. Negation (NOT operator ¬)

→ If p is true negation p is false and if p is false than negation p is true.

p	¬p(~p)
F	T
T	F

ex. p (proposation): Newdelhi is on India.

Statement connector: ¬p ¬p
1. it's not the case that newdelhi is it india.
2. Newdelhi is not in india - F - ¬p

* Order of precedence for logical connectives
¬, ⊕, ∧, ∨, →, ↔

ex. p∨q∧p   ex. (p∨q)∧q

* Conditional and biconditional proposation :

→ If p and q are proposations the compound proposation. If p then q ($p \to q$) is called conditional proposations which is false when p is true and q is false and true otherwise.

→ In this conditional proposation p is called as hypothesis or premicis and q is called as conclusion or consequence.

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

ex p: I ~~will~~ get up at 5 am.
q: I will go for a walk.
$p \to q$: If I get up at 5 am then I will go for a walk.

⇒ Biconditional proposation:

→ If p and q are proposations the compound proposation p if and only if q ($p \to q$) is true when both p and q are true. have the same truth value and is false otherwise

p	q	$p \to q$
F	F	T
F	T	F
T	F	F
T	T	T

* Truthtable

→ It's a table that display the relationship betⁿ the truth values of some sub proposations and that of compound proposations constructed from them.

* Tautology and Contradiction :

→ A compound proposation $P = P(p_1, p_2, \dots p_n)$ where $p_1, p_2 \dots p_n$ are variables / elemental proposations is called a tautology. If it's true for every truth assignment for $p_1, p_2 \dots p_n$.

→ P is a contradiction if it's false for every truth assignment for $p_1, p_2 \dots p_n$

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

→ If a proposation is neither a tautology nor a contradiction than it's called as contingency.

ex $A \Leftrightarrow B$ $\quad A \equiv B$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$ (contingency) (D'morgns)

proof:

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

equvivalent

2. $\overset{A}{p \to q}$ is equuivalent to $\overset{B}{\neg p \vee q}$ $A \leftrightarrow B$

equuivalent find tautology ly extra the

p	q	$p \to q$	$\neg p$	$\neg p \vee q$	$A \leftrightarrow B$
F	F	T	T	T	T
F	T	T	T	T	T
T	F	F	F	F	T
T	T	T	F	T	T

→ A is equuivalent to B when A if and only B is tautology. $(A \equiv B)(A \leftrightarrow B)$

3. $p \vee q \to p \wedge q$

p	q	$p \vee q$	$p \wedge q$	$p \vee q \to p \wedge q$
F	F	F	F	T
F	T	T	F	F
T	F	T	F	F
T	T	T	T	T

4. $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to (q \to r))$ Ans	$(p \to q) \to (p \to r)$	Ans
F	F	F	T	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T	T
F	T	T	T	T	T	T	T	T
T	F	F	F	T	F	T	T	T
T	F	T	F	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	T	T	T	T	T	T	T	T

ex. 1. There are four proposations

1. p : I cheat.
q : I will get coat
r : I write an exam
s : I will fail.

$(r \wedge p) \to (q \wedge s)$

If I write an exam and I cheat then I will get coat and I will fail.

2. If James does not die then merry will not get any money and Jam's family will be happy.

p : James dies.
q : Merry will get any money.
r : James family will be happy.

$\neg p \to (\neg q \wedge r)$

3. Check $\overset{p \wedge q}{p \,\&\, q}$ if and only if negation of $\overset{\neg p \vee q}{p \text{ or } q}$ for checking $A \leftrightarrow B$ see the truth value for A & B and if they both are false either true or false then A & B said bicondional.

p	q	A: $p \wedge q$	$p \vee q$	B: $\neg(p \vee q)$	
T	T	T	T	F	A↔B there
T	F	F	T	F	are not
F	T	F	T	F	bicondional
F	F	F	F	T	

4. Negation of p & q $(\neg p \wedge \neg q)$ biconditional $\neg p \vee \neg q$

$(\neg(p \wedge q)) \leftrightarrow (\neg p \vee \neg q)$

p	q	$\neg p$	$\neg q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	F	F
T	F	F	T	F	T
F	T	T	F	T	T
F	F	T	T	T	T

$(\neg(p \wedge q)) \leftrightarrow (\neg p \vee \neg q)$ are ~~not~~ bicondional

5. write a truth table & draw a truth table for $((p \to q) \to r) \to s$

p	q	r	s	$p \to q$	$(p \to q) \to r)$	$((p \to q) \to r) \to s$
T	T	T	T	T	T	T
T	T	T	F	T	T	F
T	T	F	T	T	F	T
T	T	F	F	T	F	T
T	F	T	T	F	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	T
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	T	F	T	T	F
F	T	F	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	T	T
F	F	T	F	T	T	F
F	F	F	T	T	F	T
F	F	F	F	T	F	T

6. Determine which of the following compound proposation are tautology and contradition.

→ $((p \to q) \wedge (q \to r)) \to (p \to r)$ Tautology

→ $\neg(q \to r) \wedge r \wedge (p \to q)$ contradition

p	q	r	$p \to q$	$q \to r$		$p \to r$	1st ans
T	F	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$\neg(q \to r)$	$\wedge r$	2nd ans
F	F	F
T	F	F
F	F	F
F	F	F
F	F	F
T	F	F
F	F	F
F	F	F

Algebra of Proposation:

→ A proposation in a compound proposation can we replaced by 1 that is equauivalent to it without changing the truth table of the compound proposation

⇒ loss of Algebra of proposation:

1. ~~A~~ Idempotent law:

$p \vee p \equiv p$

dual: $p \wedge p \equiv p$

2. Identity law

$p \vee f \equiv p$ dual: $p \wedge t \equiv p$

3. Dominate law

$p \vee t \equiv t$ $p \wedge f \equiv f$

4. Compliment law

$p \vee \neg p \equiv 1/T$ $p \wedge \neg p \equiv F/0$

5. Commutative law

$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$

6. Associative law:

$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

7. distributive law:

$\cancel{(p \vee q)}\ p \vee (q \wedge r) \equiv (p \vee q) \wedge (q \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (q \wedge r)$

8. Absorption law:

$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$

9. D'morgans law:

$\neg(p \vee q) \equiv \neg p \wedge \neg q$ $\neg(p \wedge q) \equiv \neg p \vee \neg q$

⇒10. equuivalent involving conditionals

1. $p \rightarrow q \equiv \cancel{\neg(p \vee \neg q)}\ \neg p \vee q$
2. $p \rightarrow q \equiv \neg q \rightarrow \neg p$
3. $p \vee q \equiv \neg p \rightarrow q$
4. $p \wedge q \equiv \neg(p \rightarrow \neg q)$
5. $\neg(p \rightarrow q) \equiv p \wedge \neg q$
6. $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
7. $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$
8. $p \rightarrow (q \vee r) \equiv (p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
9. $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \vee q) \rightarrow r$

⇒ equivalencing involving bicondions:

1. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
2. $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
3. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
4. $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

ex. $(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q$

$\equiv (\neg p \vee q) \wedge ((p \wedge p) \wedge q)$ associative
$\equiv (\neg p \vee q) \wedge (p \wedge q)$
$\equiv (p \wedge q) \wedge (\neg p \vee q)$ comutative
$\equiv ((p \wedge q) \wedge \neg p) \vee ((p \wedge q) \wedge q)$ distribute
$\equiv ((p \wedge \neg p) \wedge q) \vee ((p \wedge (q \wedge q))$
$\equiv (F \wedge q) \vee (p \wedge q)$ Dominate
$\equiv F \vee (p \wedge q)$ Identify
$\equiv p \wedge q$

ex. $\neg(p \leftrightarrow q) \equiv (p \vee q) \wedge \neg(p \wedge q) \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$

$\equiv \neg((p \rightarrow q) \wedge (q \rightarrow p))$
$\equiv \neg((\neg p \vee q) \wedge (\neg q \vee p))$
$\equiv \neg[((\neg p \vee q) \wedge \neg q) \vee ((\neg p \vee q) \wedge p)]$
$\equiv \neg[((\neg p \wedge \neg q) \vee (q \wedge \neg q)) \vee ((\neg p \wedge p) \vee (q \wedge p))]$
$\equiv \neg[((\neg p \wedge \neg q) \vee F) \vee ((F \vee (q \wedge p))]$
$\equiv \neg[(\neg p \wedge \neg q) \vee (q \wedge p)]$
$\equiv \neg[\neg(p \vee q) \vee (q \wedge p)]$
$\equiv (p \vee q) \wedge \neg(q \wedge p) \rightarrow ①$
$\equiv$ ~~$(p \wedge q) \vee \neg((p \wedge q)$~~
$\equiv (p \wedge \neg q) \vee (q \wedge \neg p)$

$\equiv (p \vee q) \wedge (\neg q \vee \neg p)$
$\equiv ((p \vee q) \wedge \neg q) \vee ((p \vee q) \wedge \neg p)$
$\equiv [(p \wedge \neg q) \vee (q \wedge \neg q)] \vee [(p \wedge \neg p) \vee (q \wedge \neg p)]$
$\equiv [(p \wedge \neg q) \vee F] \vee [F \vee (q \wedge \neg p)]$
$\equiv (p \wedge \neg q) \vee (q \wedge \neg p)$
$\equiv (p \wedge \neg q) \vee (\neg p \wedge q) \rightarrow ②$

ex. $p \rightarrow (q \rightarrow r) \equiv p \rightarrow (\neg q \vee r) \equiv (p \wedge q) \rightarrow r$

$\equiv p \rightarrow (\neg q \vee r) \rightarrow ①$

$\equiv \neg p \vee (\neg q \vee r)$
$\equiv (\neg p \vee \neg q) \vee r$
$\equiv \neg(p \wedge q) \vee r$
$\equiv \neg(\neg p \vee \neg q) \rightarrow r$
$\equiv (p \wedge q) \rightarrow r \rightarrow ②$

ex. $((p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))) \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$

$\equiv ((p \vee q) \wedge \neg(\neg p \wedge \neg(q \wedge r))) \vee \neg(p \vee q) \vee \neg(p \vee r)$
$\equiv ((p \vee q) \wedge (p \vee (q \wedge r)) \vee \neg(p \vee q) \vee \neg(p \vee r)$
$\equiv ((p \vee q) \wedge (p \vee q) \wedge (q \wedge r)) \vee \neg(p \vee q) \vee \neg(p \vee r)$
$[((p \vee q) \wedge p) \vee (p \wedge (q \wedge r))] \vee$

$\equiv ((p \vee q) \wedge ((p \vee q) \wedge (p \vee r)) \vee (\neg((p \vee q) \wedge (p \vee r)))$

$\equiv (((p \vee q) \wedge (p \vee q)) \wedge (p \vee r)) \vee (\neg((p \vee q) \wedge (p \vee r)))$

$\equiv ((p \vee q) \wedge (p \vee r)) \vee \{\neg((p \vee q) \wedge (p \vee r)) \equiv T$
$\equiv$ Tautology

* Duality law:

→ The dual of a compound proposation that contains only the logical operators OR, AND, NEgation is the proposation obtain by replacing each or by and each and by or, each T by F and each F by T. The dual of a proposation A is denoted by A^*

Que. Prove the following equivalance by proving the equivalance of dual

or

Prove the dual form of following compound proposation.

$$(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$$

$$\equiv \neg(p \vee q) \vee r \equiv (\neg p \vee r) \wedge (\neg q \vee r)$$

$$\not\equiv \cancel{(\neg p \vee r)}$$

$$\equiv \neg(p \wedge q) \wedge r \equiv (\neg p \wedge r) \vee (\neg q \wedge r)$$

Que $(p \wedge (p \leftrightarrow q)) \rightarrow q \equiv T$

$(p \wedge ((p \rightarrow q) \wedge (q \rightarrow p))) \rightarrow q \equiv T$

$(p \wedge ((\neg p \vee q) \wedge (\neg q \vee p))) \rightarrow q \equiv T$

$\neg[(p \wedge ((\neg p \vee q) \wedge (\neg q \vee p)))] \vee q \equiv T$

$\neg((p \vee ((\neg p \wedge q) \vee (\neg q \wedge p)))] \wedge q \equiv F$

Que. $\neg(p \vee q) \rightarrow (\neg p \vee (\neg p \vee q)) \equiv (\neg p \vee q)$

$\neg[\neg(p \vee q) \vee (\neg p \vee (\neg p \vee q))] \equiv (\neg p \vee q)$

* Converse, Inverse and contra-positive statement.

ex. p : You are guilty
q : You are punished

→ converse: for $p \to q$ the converse will be $q \to p$

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

→ Inverse: for $p \to q$ the inverse will be $\neg p \to \neg q$

p	q	$p \to q$	$\neg p \to \neg q$
T	T	T	~~F~~ T
T	F	F	T
F	T	T	F
F	F	T	T

→ contrapositive : for $p \to q$ the contrapositive will be $\neg q \to \neg p$

p	q	$p \to q$	$\neg q \to \neg p$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

$p \to q \equiv \neg q \to \neg p$ $q \to p \equiv \neg p \to \neg q$

ex p: Freezing water expands.

→ if water freezing then expand.

ex A Racer win the ~~win~~ race only if he ran fast

- if A racer run fast then he win the race.

ex Roses are vegatable if carrot are flowers.

- if carrot are flowers then Roses are vegatables

* Predicates & Contifires

ex $x > 10$
subject is x & predicate >10