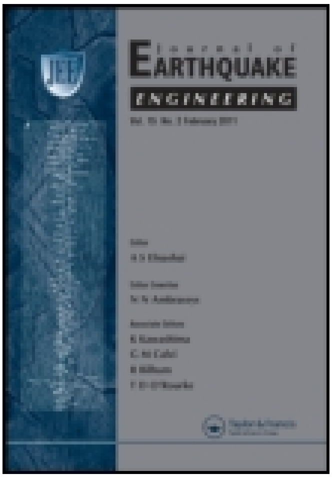

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A Methodology for the Direct Assessment of the Damping Ratio of Structures Equipped with Nonlinear Viscous Dampers

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The expressions proposed in literature for the supplemental damping ratio provided by nonlinear viscous dampers usually include a term related to the structural response, so that iterative procedures are required. This article proposes a simplified method for the direct assessment of the supplemental damping ratio on the basis of a new dimensionless parameter, called damper index. The methodology has been verified through numerical investigations, considering single- and multi-degree of freedom systems subjected to harmonic excitations and ground motions. A procedure is also proposed for obtaining design spectra of the damping ratio in terms of the damper index.

Keywords Nonlinear Viscous Dampers; Equivalent Damping Ratio; Damper Index; Design Spectra; Design Procedure

1. Introduction

The passive energy dissipation provided by supplemental viscous dampers is a recent technique used to mitigate the effects of seismic actions in civil structures. The introduction of these devices in building structures allows to reduce the amount of energy dissipated in the structural members, and consequently to limit their damage [Soong and Dargush, 1997; Constantinou *et al.*, 1998; Chopra, 2001; Christopoulos and Filiatrault, 2006]. Several researchers have studied the seismic response and the design criteria of structures equipped with viscous dampers [Constantinou and Symans, 1993; Gluck *et al.*, 1996; Hwang *et al.*, 2004; Silvestri *et al.*, 2010]. In particular, a lot of scientific contributions have been dedicated to the individuation of the optimal damper configurations [Zhang and Soong, 1992; Takewaki, 1997; Takewaki and Yoshitomi, 1998; Takewaki *et al.*, 1999; Agrawal and Yang, 1999; Lopez Garcia, 2001; Singh and Moreschi, 2001; Singh *et al.*, 2003; Liu *et al.*, 2005; Levy and Lavan, 2006; Lavan and Levy, 2006; Lavan and Dargush, 2009]. Most of the design methods for damping systems have been developed in the context of performance-based and displacement-based approaches [Priestley, 2000; Priestley *et al.*, 2007; Lin *et al.*, 2008], assuming the displacements, corresponding to specified performance levels, as fundamental design parameters. The viscous dampers may be used to reduce the seismic effects in new structures, but they are useful also for the retrofit of existing buildings [Miyamoto *et al.*, 2002, Dargush and Sant, 2005].

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Viscous dampers provide a velocity-dependent force and in general they can behave as linear or nonlinear elements. For nonlinear viscous dampers the exponent of velocity is characterized by values lower than unity [Makris *et al.*, 1993, 1995; Constantinou *et al.*, 1993; Constantinou and Symans, 1993]. The advantage of nonlinear viscous dampers is that the force in the damper can be controlled to avoid overloading the damper or the system to which is connected in presence of a large increase of velocity. Even if the forces in nonlinear viscous dampers remain out-of-phase with the displacements, the cyclic load-displacement response of nonlinear viscous dampers approaches the rectangular shape as the exponent of velocity decreases. In this case, a possible drawback could be that significant forces may be produced in the dampers also in presence of displacements close to the maximum ones.

The simplified procedures proposed in literature to evaluate the behavior of structures equipped with nonlinear viscous dampers are often based on the assessment of the effective damping ratio [Sadek *et al.*, 2000; Ramirez *et al.*, 2003]. By knowing this damping ratio it becomes easy to predict the response of the system through the damping reduction factor used to reduce the spectral ordinates. The values of the damping reduction factor proposed in recent research works [Ramirez *et al.*, 2002b; Lin and Chang, 2003] have been adopted in the FEMA 450 [BSSC, 2003]. The effective damping ratio can be derived as the sum of three terms [Ramirez *et al.*, 2002a]: the inherent damping ratio, the supplemental damping ratio provided by the dampers, and the hysteretic damping ratio, related to the nonlinear behavior of the structure. The last term is present only if the structure exceeds the elastic limit. The contribution due to the dampers may be obtained on the basis of the damping ratio provided for a linear elastic structural response. Therefore, it is clear that the main issue is to assess this damping ratio, which is the term really affected by the presence of the viscous dampers.

The supplemental damping ratio can be calculated by imposing the equivalence between a nonlinear damper and a linear one. According to the criteria presented in literature this equivalence is expressed in terms of energy dissipated [Soong and Constantinou, 1994; Seleemah and Constantinou, 1997; Lin and Chopra, 2002; Lin *et al.*, 2008] or power consumption [Peckan *et al.*, 1999]. In all these approaches the equivalent damping ratio is related to the maximum displacement of the system, so that iterative procedures have to be implemented. These criteria show the advantage to be easily extended to the multi-degree of freedom case [Ramirez *et al.*, 2000; Whittaker *et al.*, 2003].

This article proposes a simplified procedure aimed at avoiding iterative procedures for assessing the supplemental damping ratio. It should be noticed that in the design phase the iterations may be avoided. One approach, in fact, could be based on a first stage where linear dampers are designed, followed by a second stage where equivalent nonlinear dampers are determined. If the displacement, assumed as objective of the design, is maintained fixed, the second stage would not be iterative. For this reason the proposed procedure may be useful when nonlinear dampers exist and one would like to use simplified tools for the analysis or for the development of alternative design criteria. The proposed method is based on a new dimensionless parameter, called damper index, not related to the maximum displacement of the system, which has been defined both for single (SDF) and multi-degree of freedom (MDF) systems. This new dimensionless parameter, able to predict directly the response of structures equipped with nonlinear viscous dampers, could represent also a reference index for the various existing procedures aimed at determining the optimal distribution of dampers.

The methodology has been verified through numerical investigations regarding linear elastic structures equipped with nonlinear viscous dampers. Since the object of this article is the assessment of the damping ratio provided by dampers for a linear elastic structural response, the dissipation due to hysteretic structural response has not been considered. The application to inelastic structures will be the object of future developments of the research. The analyses have been performed considering single and multi-degree of freedom systems, in particular RC frames, subjected to harmonic and earthquake excitations.

2. Supplemental Damping Ratio

Some of the criteria presented in literature [Ramirez *et al.*, 2000; Lin and Chopra, 2002] for the calculation of the supplemental damping ratio due to nonlinear viscous dampers are recalled in the following as starting base of the proposed methodology. The supplemental damping ratio can be evaluated by imposing the equivalence between a nonlinear viscous damper and a linear one. According to this procedure, the supplemental damping ratio is obtained as the damping ratio of the equivalent linear damper.

2.1. SDF Systems

The more widespread criterion of equivalence, studied in literature and reported in seismic guidelines, is based on an energy approach. Let us consider a SDF system in which the displacement of the mass m, indicated with u, corresponds to the relative displacement between the ends of the damper, indicated with u_D. The properties of the fluid viscous devices allow the definition of the following force-velocity constitutive law:

$$F_D = c_\alpha \operatorname{sgn}(\dot{u}_D) \left|\dot{u}_D\right|^\alpha = c_\alpha \operatorname{sgn}(\dot{u}) \left|\dot{u}\right|^\alpha, \tag{1}$$

where F_D is the force in the damper, c_α is the damper coefficient, α is the damping exponent, and $\dot{u}_D = \dot{u}$ is the relative velocity between the ends of the device. Making reference to a displacement-controlled sinusoidal excitation $u(t) = u_0 \sin \Omega t$, the energy dissipated by a damper can be calculated as follows:

$$W_{D,\alpha} = \int F_D du = \int_0^{\frac{2\pi}{\Omega}} F_D \dot{u} dt = \int_0^{\frac{2\pi}{\Omega}} c_\alpha \left|\dot{u}\right|^{1+\alpha} dt. \tag{2}$$

From the integration of Eq. (2) it is obtained:

$$W_{D,\alpha} = \lambda c_\alpha \Omega^\alpha u_0^{1+\alpha}, \tag{3}$$

where the constant λ is equal to:

$$\lambda = \frac{2^{2+\alpha} \Gamma^2 (1 + \alpha/2)}{\Gamma (2 + \alpha)} \tag{4}$$

and Γ is the Euler Gamma function. The constant λ has been tabled in FEMA 450, for different values of the exponent α. For a linear damper $\alpha = 1.00$, $\lambda = \pi$ and Eq. (3) becomes:

$$W_{D,\alpha=1.00} = \pi c_1 \Omega u_0^2. \tag{5}$$

By imposing Eq. (3) equal to Eq. (5), that means that the energy dissipated in the nonlinear damper is equal to the energy dissipated in the linear one, it is possible to determine the damper coefficient of the equivalent linear damper, c_1. If the coefficient c_1 is divided by $2m\omega$, where ω is the natural frequency of the SDF system, the expression of the supplemental damping ratio is obtained:

$$\xi_{sd} = \frac{\lambda}{\pi}\frac{1}{2m\omega}\frac{c_\alpha}{(\Omega u_0)^{1-\alpha}}. \tag{6}$$

It should be noticed that in Eq. (6) the term u_0, which is the maximum displacement, is related, in usual problems, to the response. If the criterion based on the power consumption [Peckan *et al.*, 1999] had been followed, the expression of the supplemental damping ratio would have had the term $2/(1+\alpha)$ instead of λ/π.

2.2. MDF Systems

The supplemental damping ratio for a MDF system equipped with nonlinear viscous dampers can be evaluated using the concept of equivalent linear viscous damping ratio [Chopra, 2001]:

$$\xi_{sd} = \frac{W_D}{4\pi W_S}, \tag{7}$$

where W_D is the energy dissipated in the dampers and W_S is the elastic energy stored at the maximum displacement of the system. These two energy contributions can be expressed on the basis of the mode shapes. It should be noticed that for linear elastic structures with linear viscous dampers, the use of the mode shapes determined by eigenvalue analysis of the undamped system holds if the system is classically damped. Nevertheless, the use of the undamped frequencies and mode shapes together with energy-based calculation of the damping ratios may provide good estimates of the seismic response also for non classically damped systems. This is particularly true for buildings with complete vertical distribution of viscous dampers, but it may also be valid in some cases of incomplete vertical distribution [Costantinou and Symans, 1992]. The supplemental damping ratio can be evaluated directly for each mode of vibration of the MDF system if the dampers have a linear behavior; otherwise, this approach can be used properly for the first mode and not for the higher modes, unless a particular approximation [Ramirez *et al.*, 2000] is introduced. It is assumed that the MDF system undergoes a cycle of harmonic vibration such that:

$$\{u\} = D_{roof}\,\{\phi\}_1 \sin\left(\frac{2\pi t}{T_1}\right), \tag{8}$$

where D_{roof} is the amplitude of roof displacement, T_1 is the undamped first period of vibration, $\{\phi\}_1$ is the first undamped mode shape (normalized in order to have unit component at the roof). The amplitude of the relative displacement between the ends of a generic device j can be evaluated as follows:

$$u_{Dj0} = D_{roof}\, f_j \phi_{rj1}, \tag{9}$$

where f_j is a displacement magnification factor which depends on the geometrical arrangement of the damper and ϕ_{rj1} is the difference between the first modal ordinates associated with the degrees of freedom to which is connected the damper. The factor f_j equals unity for the chevron brace configuration and equals $\cos\theta_j$ for the diagonal configuration, where

θ_j is the angle of inclination of the device j. The energy dissipated by the dampers per cycle of motion in the first mode is:

$$W_D = \sum_{j=1}^{N_D} \left(\frac{2\pi}{T_1}\right)^{\alpha_j} c_{\alpha j}\lambda_j \left(D_{roof}\, f_j\phi_{rj1}\right)^{1+\alpha_j}, \tag{10}$$

where N_D is the number of dampers and λ_j is given by Eq. (4) as a function of α_j, which is the damper exponent of the device j. Considering that the maximum strain energy of the system is equal to the maximum kinetic energy, it is possible to obtain:

$$W_S = \frac{1}{2}\sum_{i=1}^{N} m_i\dot{u}_{i0}^2 = \frac{2\pi^2}{T_1^2}\sum_{i=1}^{N} m_i D_{roof}^2\phi_{i1}^2, \tag{11}$$

where m_i is the lumped mass at the degree of freedom i, ϕ_{i1} is the corresponding modal deformation, and N is the number of degrees of freedom. The mass m_i can be also expressed as the weight w_i divided by the gravity acceleration. By introducing Eqs. (10) and (11) into Eq. (7), it is possible to obtain the supplemental damping ratio for the first mode of vibration:

$$\xi_{sd1} = \frac{\sum\limits_{j=1}^{N_D} (2\pi)^{\alpha_j}\, T_1^{2-\alpha_j}\lambda_j c_{\alpha j} f_j^{1+\alpha_j} D_{roof}^{\alpha_j-1}\phi_{rj1}^{1+\alpha_j}}{8\pi^3\sum\limits_{i=1}^{N} m_i\phi_{i1}^2}. \tag{12}$$

As observed in Eq. (6) for SDF systems, also in Eq. (12) the supplemental damping ratio depends on D_{roof}, which is a term related to the response. The calculation of the damping ratio of the higher modes cannot be performed by directly applying Eq. (12). To bypass this difficulty, a particular procedure suggested by Seleemah and Constantinou [1997] can be used. They gave a physical interpretation of the higher mode response: small amplitude, high frequency motion centered around the first mode response. Therefore, it is possible to define a damping constant c_{effj} for each nonlinear viscous device. This constant is taken as the slope of the force-velocity curve of the device at the maximum device velocity in the first mode:

$$c_{effj} = \alpha_j c_{\alpha j}\dot{u}_{Dj0}^{\alpha_j-1} = \alpha_j c_{\alpha j}\left(\frac{2\pi}{T_1}\right)^{a_j-1}\left(f_j D_{roof}\phi_{rj1}\right)^{a_j-1}. \tag{13}$$

The damping ratio ξ_{sdn} of a mode n can then be calculated using Eq. (12) with the corresponding period T_n and mode shape $\{\phi\}_n$ and considering linear dampers ($\alpha_j = 1$) with damping coefficients equal to c_{effj}:

$$\xi_{sdn} = \left(\frac{T_n}{4\pi}\right)\frac{\sum\limits_{j=1}^{N_D} c_{effj} f_j^2\phi_{rjn}^2}{\sum\limits_{i=1}^{N} m_i\phi_{in}^2}. \tag{14}$$

If α is constant for all the dampers it is possible to derive $D_{roof}^{\alpha-1}$ from Eq. (12) as a function of ξ_{sd1} and other terms related to the properties of the system. By introducing in Eq. (13) the obtained expression of $D_{roof}^{\alpha-1}$ it is possible to express c_{effj}, and then ξ_{sdn}, by the product of ξ_{sd1} with terms not related to the response, but only related to the properties of the system. The expression is shown below:

$$\xi_{sdn} = \xi_{sd1} \frac{T_n}{T_1} \frac{\pi\alpha}{\lambda} \frac{\sum_{i=1}^{N} m_i \phi_{i1}^2}{\sum_{i=1}^{N} m_i \phi_{in}^2} \frac{\sum_{j=1}^{N_D} c_{\alpha j} f_j^{1+a} \phi_{rj1}^{a-1} \phi_{rjn}^2}{\sum_{j=1}^{N_D} c_{\alpha j} f_j^{1+a} \phi_{rj1}^{1+a}}. \tag{15}$$

3. Damper Index

As shown previously, the equivalent damping ratio for systems equipped with nonlinear viscous dampers is related to the response. For this reason, the determination of the response, through integration of the equations of motions or use of the response spectra, requires iterations to be performed. The simplified procedure proposed in this article to avoid these iterations is based on the introduction of a new dimensionless parameter, called damper index, which is defined in the following for SDF and MDF systems.

3.1. SDF Systems Subjected to Harmonic External Forces

By introducing the supplemental damping ratio, the equation of motion for a SDF system with mass m equipped with a nonlinear viscous damper, and subjected to an harmonic external force $ma_0 \sin\Omega t$, is:

$$\ddot{u} + 2\xi_0\omega\dot{u} + 2\xi_{sd}\omega\,(\lambda/\pi)^{-1}\,(\Omega u_0)^{1-\alpha}\,\mathrm{sgn}\,(\dot{u})\,|\dot{u}|^{\alpha} + \omega^2 u = a_0 \sin\Omega t, \tag{16}$$

where ξ_0 is the inherent damping. It should be noticed that a term related to the response, u_0, is included in the equation. Referring to Eq. (16), the response is affected only by the ratio of the frequencies and not by their single values. The response of the structure can be expressed in terms of displacement through a dimensionless parameter called deformation response factor. This parameter is equal to the ratio of the deformation amplitude u_0 to the maximum static deformation u_{st0}. In case of harmonic external force it can be expressed as $R_d = u_0/u_{st0} = u_0\omega^2/a_0$. If the deformation response factor is introduced into Eq. (16), the following equation of motion is derived:

$$\ddot{u} + 2\xi_0\omega\dot{u} + 2\xi_{sd}\frac{\Omega^{1-\alpha}}{\omega^{1-2\alpha}}\,(\lambda/\pi)^{-1}\,a_0^{1-\alpha}R_d^{1-\alpha}\,\mathrm{sgn}\,(\dot{u})\,|\dot{u}|^{\alpha} + \omega^2 u = a_0 \sin\Omega t. \tag{17}$$

For a SDF system subjected to a harmonic external force the damper index ε is defined as follows:

$$\varepsilon = \frac{\lambda}{\pi}\frac{c_\alpha}{2m}\frac{\omega^{1-2\alpha}}{\Omega^{1-\alpha}}\,(a_0)^{\alpha-1}\,. \tag{18}$$

This index does not depend from response parameters, but only from structural properties and from the intensity of the external action. The damping ratio ξ_{sd} is related to the damper index through the following relationship:

$$\xi_{sd} = \varepsilon\,(R_d)^{\alpha-1}\,. \tag{19}$$

It can be noticed that for $\alpha = 1.00$ we find out $\xi_{sd} = \varepsilon$. The damper index is then introduced into the equation of motion (Eq. (17)):

$$\ddot{u} + 2\xi_0\omega\dot{u} + 2\varepsilon\frac{\Omega^{1-\alpha}}{\omega^{1-2\alpha}}\,(\lambda/\pi)^{-1}\,a_0^{1-\alpha}\,\mathrm{sgn}\,(\dot{u})\,|\dot{u}|^{\alpha} + \omega^2 u = a_0\sin\Omega t. \tag{20}$$

With this operation the Eq. (20) does not include any terms related to the response. All the terms included can be calculated once the characteristics of the system and of the external force are known. The response can then be evaluated by solving the equation of motion only one time, without the need to repeat iteratively the calculation of the solution. By comparing Eqs. (16) and (20) we can say that the term related to the response (u_0) has been replaced by a term related to the input motion (a_0). This is possible because it is easy to verify that for a given input motion we have just a unique response. The advantage is that the input motion is known before performing the analysis whereas the response is not.

3.2. SDF Systems Subjected to Earthquake Motions

In case of ground acceleration $\ddot{u}_g(t)$, in the expression of ξ_{sd} (Eq. (6)) the frequency of the external force, Ω, can be taken equal to the natural frequency, ω. This assumption is frequent in literature [Lin and Chopra, 2002] since the response is more affected by the frequencies close to the resonance. Therefore, Eq. (16) becomes:

$$\ddot{u} + 2\xi_0\omega\dot{u} + 2\xi_{sd}\omega\,(\lambda/\pi)^{-1}\,(\omega u_0)^{1-\alpha}\,\mathrm{sgn}\,(\dot{u})\,|\dot{u}|^{\alpha} + \omega^2 u = -\ddot{u}_g\,(t)\,, \tag{21}$$

where u_0 is the peak displacement. The presence of terms related to the response in Eq. (21) requires one to obtain the solution through iterations. It has been demonstrated that in Eq. (21) the response u of the system varies linearly with the peak ground acceleration $\ddot{u}_{g0}$ [Lin and Chopra, 2002]. This result is really significant because it allows one to express the response in terms of displacement by the deformation response factor, which in this case is $R_d = u_0\omega^2/\ddot{u}_{g0}$.

The definition of the damper index, Eq. (18), can be extended for the case of ground acceleration by substituting the frequency of the external force, Ω, with the natural frequency of the system, ω and a_0 with $\ddot{u}_{g0}$:

$$\varepsilon = \frac{\lambda}{\pi}\frac{c_\alpha}{2m}\frac{1}{\omega^\alpha}\left(\ddot{u}_{g0}\right)^{\alpha-1}. \tag{22}$$

The relationship between the damping ratio and the damper index is still expressed by Eq. (19).

In the same way the Eq. (20) can be adapted as follows:

$$\ddot{u} + 2\xi_0\omega\dot{u} + 2\varepsilon\omega^{\alpha}\,(\lambda/\pi)^{-1}\,\ddot{u}_{g0}^{1-\alpha}\,\mathrm{sgn}\,(\dot{u})\,|\dot{u}|^{\alpha} + \omega^2 u = -\ddot{u}_g\,(t)\,. \tag{23}$$

The response of the SDF system may be obtained by the integration of the equation of motion or alternatively by the use of response spectra. In this case, it is possible to follow a procedure based on the calculation of the supplemental damping ratio in terms of the damper index.

3.3. MDF Systems

The damper index for MDF systems is defined on the basis of the Eq. (19):

$$\varepsilon_1 = \xi_{sd1} \left(R_{d1}\right)^{1-\alpha}, \tag{24}$$

where ξ_{sd1} is determined in Eq. (12), R_{d1} is the deformation response factor for the first mode of vibration, referred to the displacement of the top level, D_{roof}, and ε_1 is the damper index for the first mode. R_{d1} can be expressed as follows:

$$R_{d1} = \frac{D_{roof}}{\Gamma_1 \ddot{u}_{g0}} \omega_1^2, \tag{25}$$

where ω_1 is the frequency of the first mode of vibration and Γ_1 is the first mode participation factor. If we put the Eqs. (12) and (25) into Eq. (24), we finally obtain the subsequent relationship, considering α constant for all the dampers:

$$\varepsilon_1 = \frac{T_1^{\alpha}}{(2\pi)^{1+\alpha}} \frac{\lambda}{\left(\Gamma_1 \ddot{u}_{g0}\right)^{1-\alpha}} \frac{\sum_{j=1}^{N_D} c_{\alpha j} f_j^{1+\alpha} \phi_{rj1}^{1+\alpha}}{\sum_{i=1}^{N} m_i \phi_{i1}^2}. \tag{26}$$

Once again, in Eq. (26) any term related to the response does not appear.

4. Analysis of the Response in Terms of the Damper Index

The response of SDF systems has been studied with the purpose of determining the supplemental damping ratio in terms of the damper index. The analysis of the response has been carried out for SDF systems subjected to harmonic external forces and earthquake motions.

4.1. SDF Systems Subjected to Harmonic External Forces

By solving Eq. (20) numerically it is possible to obtain the response to harmonic external forces. The graphs of the deformation response factor versus the ratio of the frequencies Ω/ω, for $\xi_0 = 5\%$, are shown below for different values of ε and $\alpha = 0.50$ (Fig. 1) and for different values of α and $\varepsilon = 0.35$ (Fig. 2). Figure 1 shows that the damper index affects the response significantly for values of the ratio Ω/ω around the resonance. Figure 2 shows how much the exponent α affects the response for a fixed value of ε (that means for fixed characteristics of the system): we can individuate a first zone for Ω/ω less than 0.70, where for increasing values of α the response increases, a second zone from Ω/ω equal to 0.70 to Ω/ω equal to 1.40 where this trend is inverted and a last zone for Ω/ω larger than 1.40 where the behavior is similar to that of the first zone.

Once the response has been found by solving Eq. (20) numerically, we can assess the deformation response factor, R_d and, through the Eq. (19), the supplemental damping ratio, ξ_{sd}. The graphs of ξ_{sd} vs. the ratio of the frequencies Ω/ω are illustrated in Fig. 3 for different values of ε and $\alpha = 0.50$ and in Fig. 4 for different values of α and $\varepsilon = 0.35$.

Figure 4 shows that the spectrum of the supplemental damping ratio can be divided into three zones, as it has been noticed in the previous paragraph for the deformation response factor. In the two lateral zones, for a fixed value of ε, the supplemental damping ratio

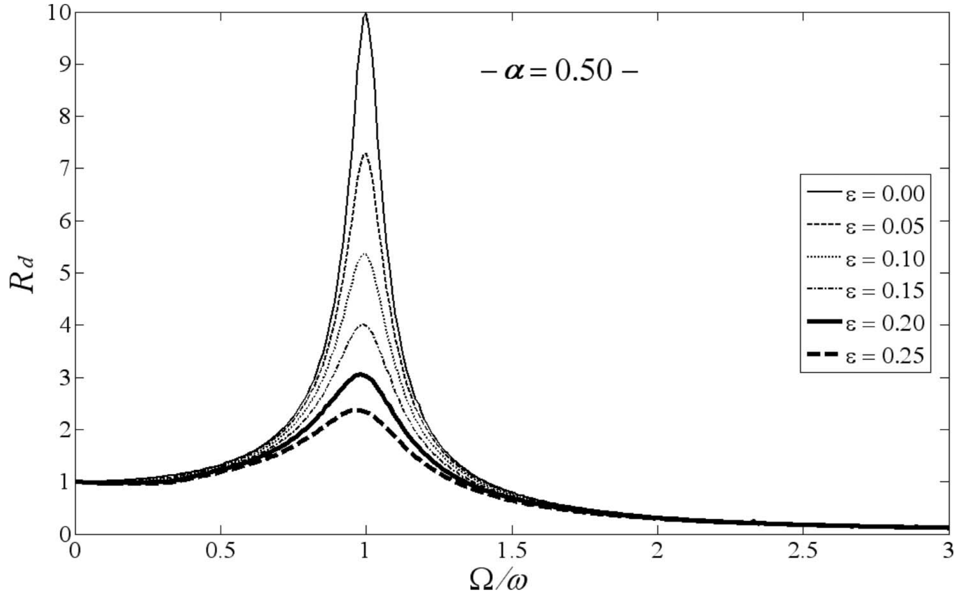

FIGURE 1 Deformation response factor for ε equal to 0.00, 0.05, 0.10, 0.15, 0.20, 0.25 and $\alpha = 0.50$.

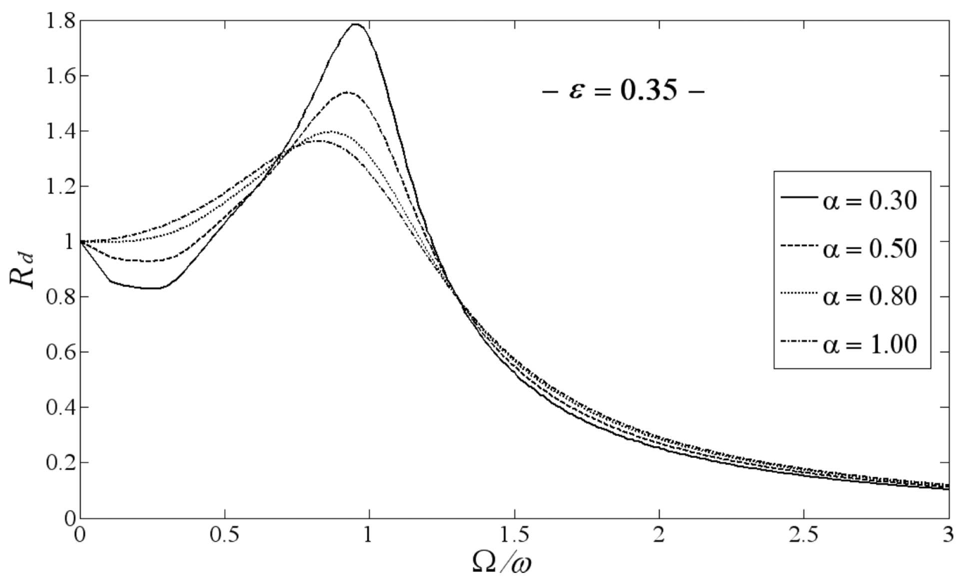

FIGURE 2 Deformation response factor for α equal to 0.30, 0.50, 0.80, 1.00 and $\varepsilon = 0.35$.

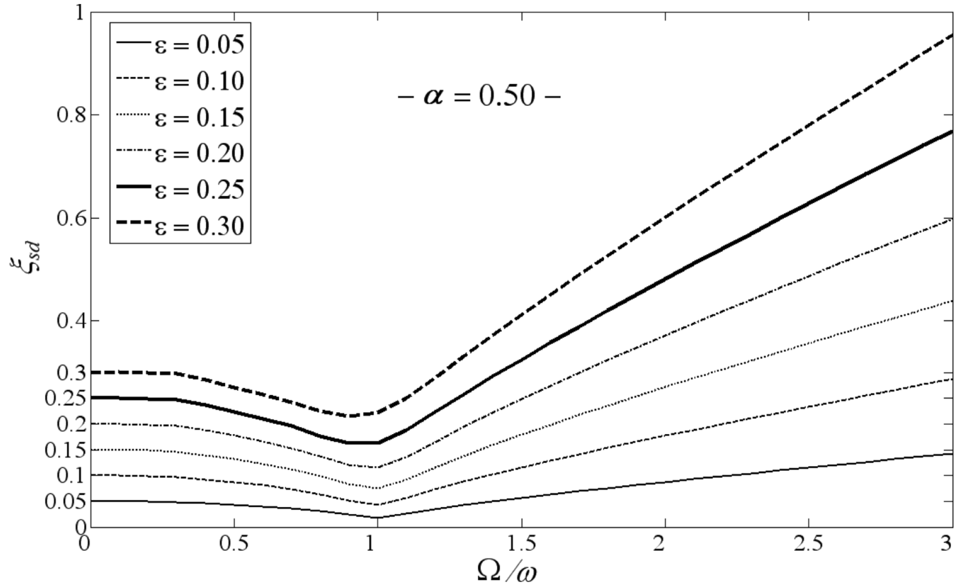

FIGURE 3 Supplemental damping ratio for ε equal to 0.05, 0.10, 0.15, 0.20, 0.25, 0.30 and $\alpha = 0.50$.

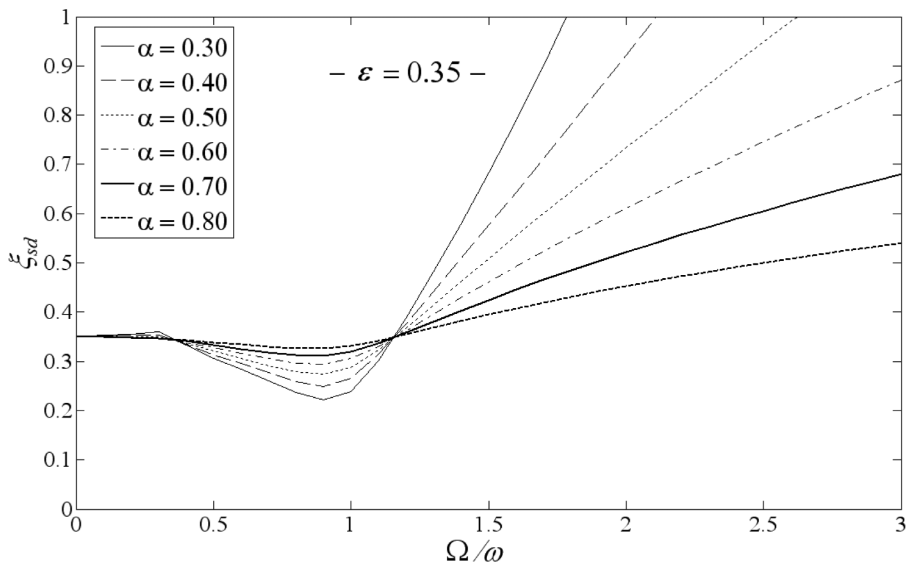

FIGURE 4 Supplemental damping ratio for α equal to 0.30, 0.40, 0.50, 0.60, 0.70, 0.80 and $\varepsilon = 0.30$.

increases for decreasing values of α: that means that for small values of α the system is able to dissipate a larger quantity of energy and to reduce more efficiently the response. This result is in accordance with the previous observations about Fig. 2. This trend changes for the values of the ratio Ω/ω close to the resonance.

Figure 3 shows that it is quite easy to approximate numerically the spectrum of the supplemental damping ratio. In this article we propose a linear approximation:

$$\begin{cases} \xi_{sd} = \varepsilon \quad \text{for } \frac{\Omega}{\omega} \leq 0.30 \\ \xi_{sd} = \varepsilon - k_1 \left(\frac{\Omega}{\omega} - 0.30\right) \quad \text{for } 0.30 \leq \frac{\Omega}{\omega} \leq 1.00 \\ \xi_{sd} = \varepsilon - 0.70k_1 + k_2 \left(\frac{\Omega}{\omega} - 1.00\right) \quad \text{for } \frac{\Omega}{\omega} \geq 1.00 \end{cases}, \tag{27}$$

where k_1 and k_2 are two constants which can be evaluated in terms of α and ε by a linear regression. The values of these two constants are shown in Tables 1 and 2.

Figures 5 and 6 show the results for $\alpha = 0.50$. The maximum error registered is equal to 15% for $\varepsilon = 0.05$, just close to the resonance condition, but for the major part of the spectrum it remains less than 5%. It is clear that the error decreases for increasing values of α. Since the deformation response factor is a function of the square root of the damping ratio (for linear viscous dampers), the error in terms of displacement tends to be much smaller than the one in terms of damping ratio.

The proposed spectrum of the supplemental damping ratio allows to assess directly the value of the supplemental damping ratio once the damper index and the exponent α are known, without iterative procedures and with good approximation.

TABLE 1 Values of k_1 in terms of α and ε

	ε					
α	0.05	0.10	0.15	0.20	0.25	0.30
0.15	0.06	0.11	0.16	0.21	0.25	0.27
0.30	0.05	0.10	0.14	0.18	0.20	0.21
0.45	0.04	0.08	0.11	0.14	0.15	0.15
0.60	0.03	0.06	0.08	0.10	0.10	0.10
0.75	0.02	0.04	0.05	0.06	0.06	0.06
0.90	0.01	0.02	0.02	0.02	0.02	0.02

TABLE 2 Values of k_2 in terms of α and ε

	ε					
α	0.05	0.10	0.15	0.20	0.25	0.30
0.15	0.14	0.29	0.47	0.64	0.89	1.15
0.30	0.10	0.20	0.32	0.46	0.60	0.76
0.45	0.07	0.14	0.21	0.29	0.37	0.46
0.60	0.05	0.09	0.14	0.18	0.22	0.26
0.75	0.03	0.05	0.08	0.10	0.12	0.14
0.90	0.01	0.02	0.03	0.04	0.04	0.05

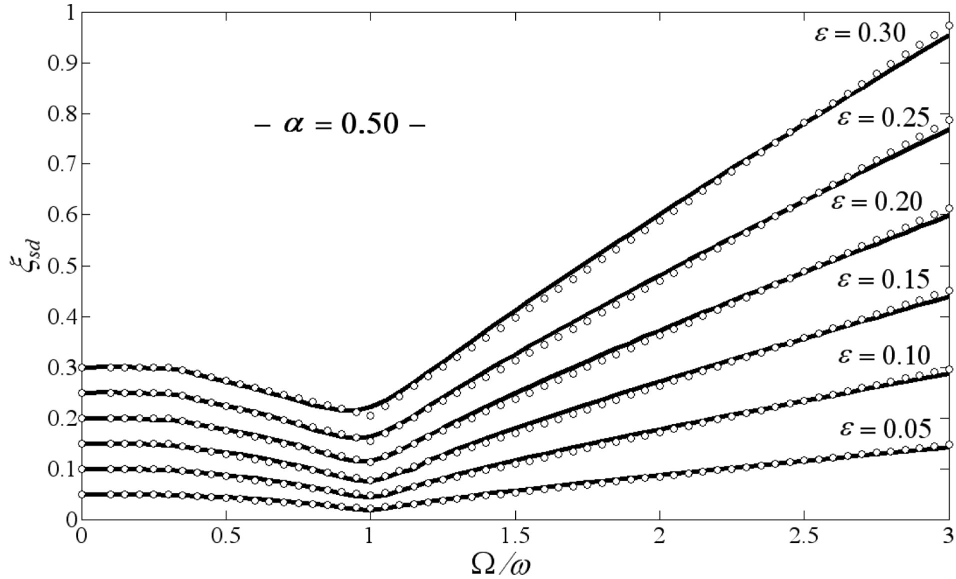

FIGURE 5 Spectrum of the supplemental damping ratio calculated using Eq. (25) (circles) and obtained from the solution of the equation of motion (continuous lines).

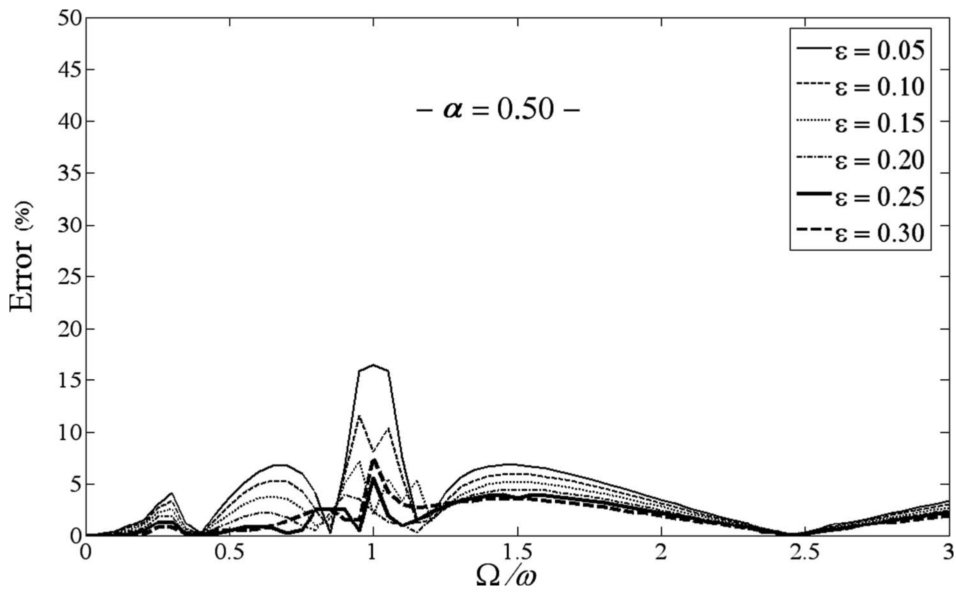

FIGURE 6 Error in percentage in the estimates provided by Eq. (25).

4.2. SDF Systems Subjected to Earthquake Motions

As in the case of harmonic external forces, the calculation of the supplemental damping ratio by the direct procedure requires the definition of the spectra of the supplemental damping ratio vs. the period in terms of ε and α. The procedure to determine the spectrum of the supplemental damping ratio for a given ground acceleration and for given values of ε and α is characterized by the following steps.

1. Solve Eq. (23) numerically for structures characterized by different values of the natural period T and obtain the diagram of R_d as a function of T.
2. Calculate the supplemental damping ratio ξ_{sd} using Eq. (19) and obtain the diagram of ξ_{sd} as a function of T.

The spectrum of the supplemental damping ratio has then to be determined for different values of the damper index ε and of the exponent α. Once these spectra are known it is possible to assess directly the supplemental damping ratio for known values of ε and α. Figure 7 shows the spectra obtained for the El Centro ground motion, for different values of ε and for $\alpha = 0.5$. In the same figure it is illustrated also an example of calculation of ξ_{sd}. First, the value of ε is computed using Eq. (22) for SDF systems or Eq. (26) for the first mode of MDF systems. Then the value of ξ_{sd} is calculated as a function of the fundamental period using the curve relative to the computed value of ε.

For design purposes, it is necessary to obtain envelopes of the spectra of the supplemental damping ratio of a sufficiently large number of earthquake motions. These envelopes should be derived for different values of ε and α. To do so, the methodology illustrated for the case of harmonic external forces could be used, which involves the execution of linear regressions on the results of different ground motions.

As an alternative, it is possible to take advantage of the well-known results presented in literature and adopted by seismic codes and guidelines regarding the damping reduction factor used to reduce the spectral ordinates as a function of the supplemental damping ratio. This factor, called B in the FEMA 450 [BSSC, 2003] and η in the Eurocode 8 [CEN, 2003],

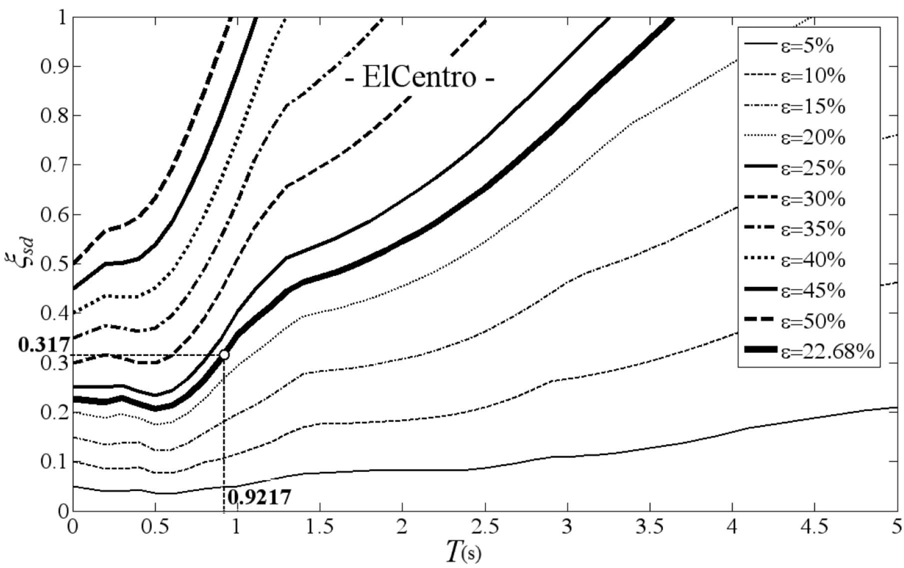

FIGURE 7 Spectra of the supplemental damping ratio for different values of ε (expressed in percentage) and $\alpha = 0.50$ for the El Centro ground motion.

allows one to obtain the acceleration response spectra associated to values of the damping ratio larger than the inherent one.

5. Design Spectra of the Supplemental Damping Ratio

As previously illustrated, the direct procedure requires the definition of the spectra of the supplemental damping ratio in terms of the damper index. Here, we describe a method for obtaining these spectra from the acceleration response spectra associated to different values of the damping ratio. The damper index, in fact, can be expressed as a function of the pseudo-acceleration, which can be derived from the design response spectra provided by seismic codes:

$$\varepsilon = \xi_{sd} \cdot \left[\bar{S}_a\left(T, \xi_{sd}\right)\right]^{1-\alpha}, \tag{28}$$

where $\bar{S}_a$ is the pseudo-acceleration normalized to the peak ground acceleration PGA $= \ddot{u}_{g0}$. The relationship between $\bar{S}_a$ and the damping ratio is usually provided by codes, so that once the damping ratio is known, the pseudo-acceleration is known.

Moving from this considerations, Eq. (28) can be used to determine the diagrams of the damper index as a function of the fundamental period T, for given values of the supplemental damping ratio and of the exponent α. These spectra are useful because if a horizontal line for a constant value of the damper index (such as 20%) is drawn, this line intersects the spectral curves associated to different values of the supplemental damping ratio. Hence, it is possible to find some points that can be plotted in a graph in order to create the spectra of the supplemental damping ratio. This procedure, shown in Figs. 8 and 9, has been realized by using the elastic response spectrum provided by Eurocode 8 for a category of soil A and an exponent α equal to 0.50.

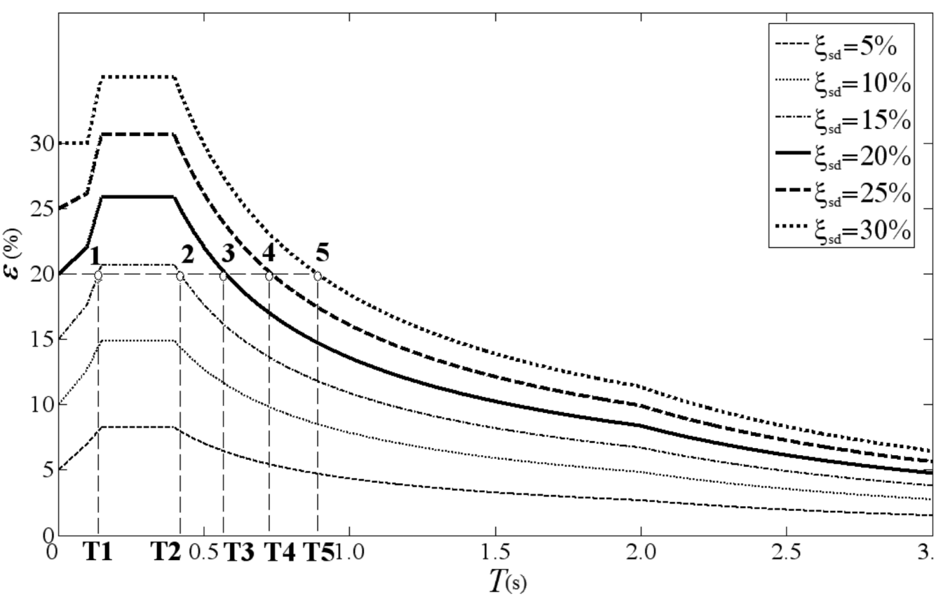

FIGURE 8 Spectra of the damper index.

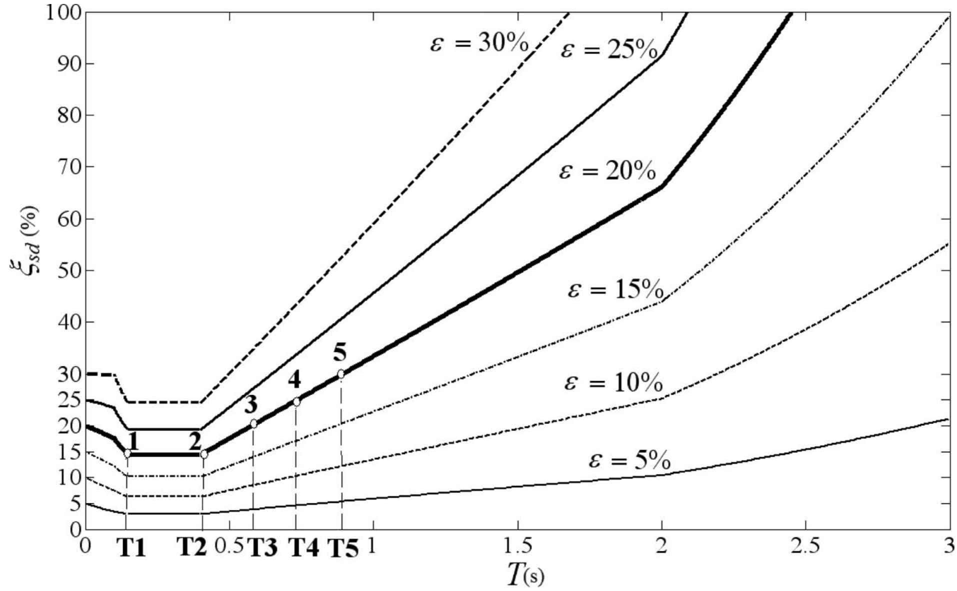

FIGURE 9 Spectra of the supplemental damping ratio.

6. Case Studies

A numerical investigation has been performed in order to validate the proposed direct methodology. Three reinforced concrete frames characterized by three bays and one (3B1F), three (3B3F), and six floors (3B6F) have been considered (Fig. 10). For all the frames the inter-story height is equal to 3.2 m and the bay width is equal to 6 m. The added floor weight (in addition to the self weight of the concrete frames) is equal to 593 kN at the roof and equal to 561 kN at all the intermediate levels. The adopted dimensions for all beams are: width equal to 35 cm and depth equal to 60 cm. The columns have square cross-sections with a side length variable from story to story. As shown in Fig. 10, this length varies from 60 cm for the interior base columns of the frame 3B6F to 35 cm for the top columns. The Young modulus of concrete has been assumed equal to $2.5 \cdot 10^4$ N/mm^2, the density of reinforced concrete equal to 25 kN/m^3 and all structures have been considered clamped at the base.

The seismic analyses have been performed considering a set of 11 recorded ground motions which are listed in Table 3. The records have been scaled to the design value of the peak ground acceleration, equal to 0.35 g. These records are characterized by an average response spectrum similar to the EC8 elastic response spectrum (Fig. 11). Figure 12 illustrates the displacement spectra of the selected ground motions and highlights the ordinates associated to the fundamental periods of the considered frames.

The frame 3B6F has been equipped with nonlinear dampers characterized by c_α equal to 32.00 kN (s/mm)$^{0.5}$ and α equal to 0.50 at each floor. With reference to horizontal translation, the periods of the first six modes are: $T_1 = 0.9217$ s, $T_2 = 0.3315$ s, $T_3 = 0.1992$ s, $T_4 = 0.1410$ s, $T_5 = 0.1074$ s, and $T_6 = 0.0888$ s. The first six mode shapes have been normalized to the roof displacement: $\{\phi_1\}^T = \{0.1029, 0.2879, 0.4861, 0.6936, 0.8755, 1.0000\}$, $\{\phi_2\}^T = \{-0.3080, -0.7430, -0.9197, -0.6236, 0.1541, 1.0000\}$, $\{\phi_3\}^T = \{0.5987, 1.0792, 0.4578, -0.9713, -1.0860, 1.0000\}$, $\{\phi_4\}^T = \{-1.3949, -1.5145, 1.1601, 1.8265, -2.5878, 1.0000\}$, $\{\phi_5\}^T = \{7.9191, 2.5665, -10.3110, 8.7665,$

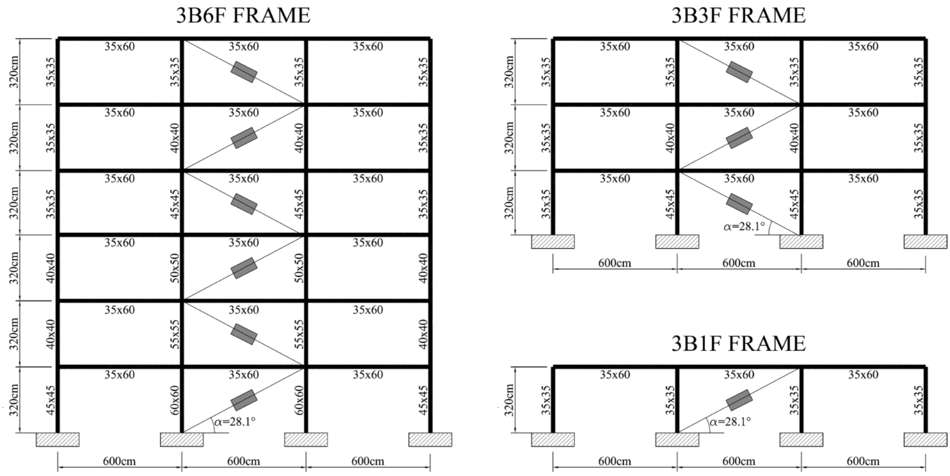

FIGURE 10 Configurations and geometric properties of the RC frames under study (cross-section dimensions in cm).

TABLE 3 List of the considered earthquake records

Location	Year	M_S	Station	Name	PGA (g)
Imperial Valley	1940	7	El Centro	ELCE	0.348
Kern County	1952	7.4	Taft	TAFT	0.178
Parkfield	1966	6.1	Cholame 2	PARK	0.476
Tabas	1978	7.4	Tabas	TABA	0.836
Imperial Valley	1979	6.5	El Centro Array	IMPV	0.463
Imperial Valley	1979	6.5	El Centro D. Array	IMVA	0.480
Montenegro	1966	–	Petrovac	PETR	0.438
Irpinia	1980	6.5	Sturno	STUR	0.358
Northridge	1994	6.7	Newhall	NEWH	0.583
Northridge	1994	6.7	Satiicoy	SATI	0.477
Kobe	1995	6.9	University	KBUN	0.290

-4.1679, $1.0000\}$, and $\{\phi_6\}^{\mathrm{T}} = \{-214.3178, 179.8424, -94.6873, 31.8295, -7.5685, 1.0000\}$. For this frame the first 3 modes activate about 94% of the total mass of the system, so that the contribution of the other modes to the total response can be considered less significant.

The frame 3B3F has been equipped with nonlinear dampers characterized by c_α equal to 24.00 kN (s/mm)$^{0.5}$ and α equal to 0.50 at each floor. The periods of the first three modes are: $T_1 = 0.5739$ s, $T_2 = 0.2041$ s, and $T_3 = 0.1262$ s. The first three mode shapes have been normalized to the roof displacement: $\{\phi_1\}^{\mathrm{T}} = \{0.2980, 0.6977, 1.0000\}$, $\{\phi_2\}^{\mathrm{T}} = \{-1.0420, -1.0045, 1.0000\}$, and $\{\phi_3\}^{\mathrm{T}} = \{4.0089, -3.1822, 1.0000\}$. For this frame the first two modes activate about 96% of the total mass of the system.

The frame 3B1F has been equipped with a nonlinear fluid viscous damper characterized by c_α equal to 12.00 kN (s/mm)$^{0.5}$ and α equal to 0.50. The fundamental period is $T_1 = 0.2729$ s.

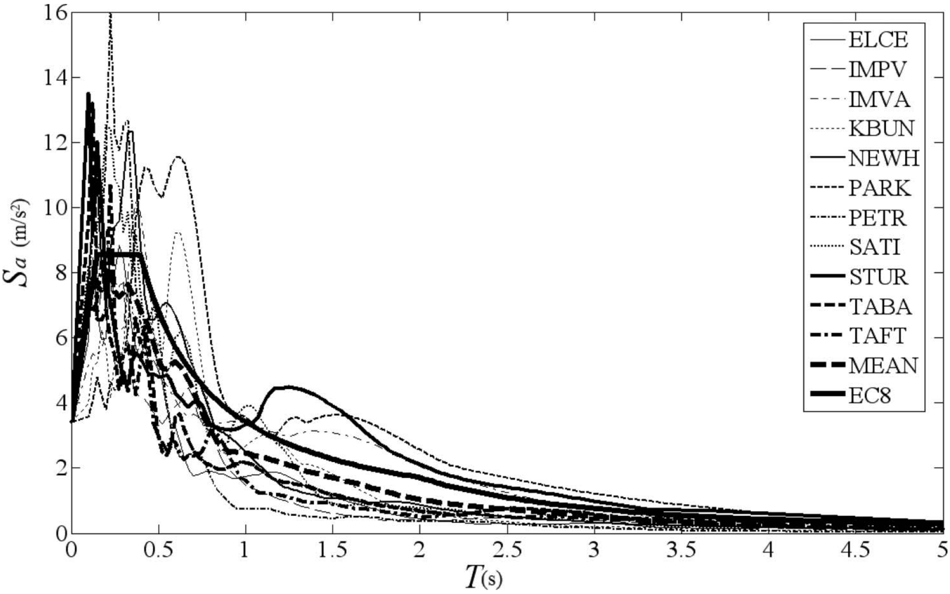

FIGURE 11 Pseudo-acceleration elastic response spectra of the considered ground motions.

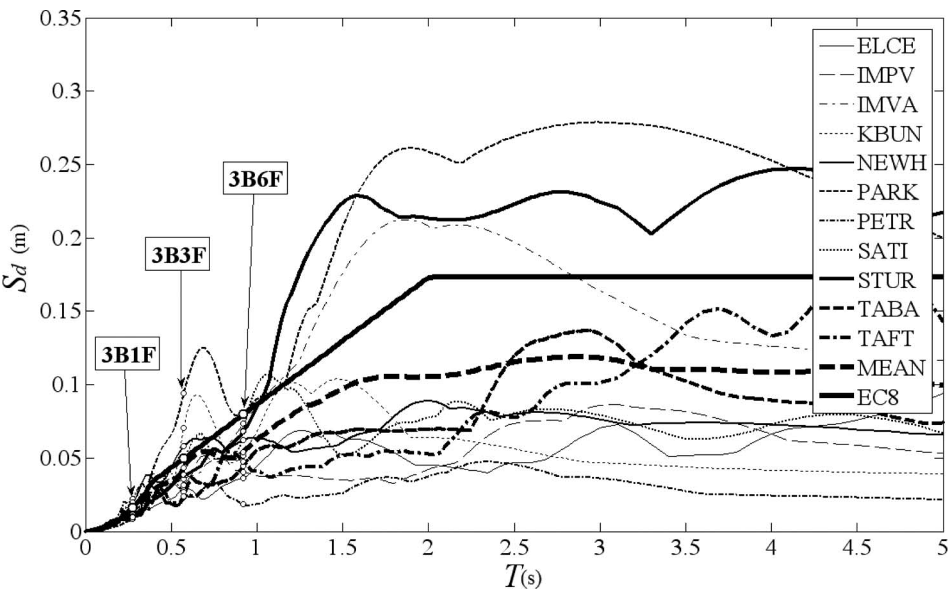

FIGURE 12 Displacement elastic response spectra of the considered ground motions.

For comparison purposes, the three frames, equipped with nonlinear viscous dampers and subjected to the considered ground motions, have been analyzed by performing nonlinear time-history analyses. This type of analysis has been carried out by direct integration of the equations of motion. The behavior of the dampers has been considered nonlinear while the response of the structure has been maintained linear elastic.

Subsequently, the structures under study have been analyzed using the iterative procedure proposed in literature, based on the equivalent energy approach [Lin and Chopra, 2002] and the direct methodology proposed in this article, based on the assessment of the damper index. Both the iterative procedure and the direct methodology arrive to define a value of the supplemental damping ratio for each considered vibration mode. With these two methods the dampers have been considered by assigning a supplemental damping ratio to each mode. The structures have been modelled without the dampers and the time-history analyses have been performed through modal superposition, by integration of the equations of motion relative to each mode. For these analyses, at each time instant the results have been derived by arithmetic summation of the results relative to each mode. Six modes have been considered in particular for the frame 3B6F and three modes for the frame 3B3F.

6.1. Assessment of the Supplemental Damping Ratio

The calculation of the supplemental damping ratio through the direct procedure has been carried out on the basis of the spectra of the supplemental damping ratio obtained for given values of ε and α for each single ground acceleration. For each frame the damper index of the first mode ε_I has been calculated using Eq. (26). It must be noticed that for a given structure the obtained value of ε_I is the same for all earthquake records, since these records have been scaled to the same PGA. For example, if the frame 3B6F is considered, the value $\varepsilon_1 = 22.68\%$ (Fig. 7) has been derived for each ground motion. From the knowledge of ε_I, the supplemental damping ratio of the first mode ξ_{sd1} has been calculated for each ground acceleration by entering the curve relative to ε_I with the period of the first mode (Fig. 7). However, if the spectra of the supplemental damping ratio are not available, it is not necessary to completely determine them by repeating the solution of Eq. (23) for different values of ε and T. It is sufficient to solve Eq. (23) only for the values of these parameters relative to the structure under study. As previously described, the supplemental damping ratio of the higher modes has been derived directly as the product of ξ_{sd1} with terms not related to the response, but only to the properties of the system (Eq. (15)).

The calculation of the supplemental damping ratio through the iterative procedure has been conducted according to the following steps.

1. Assume an initial value $\xi_{sd1}{}^{(0)} = 0$, determine the seismic response considering just the inherent damping ratio for all modes, and evaluate the roof displacement, $D_{roof}{}^{(0)}$.
2. For the ith iteration use Eq. (12) to calculate a new value $\xi_{sd1}{}^{(i)}$ for the supplemental damping ratio of the first mode on the basis of $D_{roof}{}^{(i-1)}$; the damping ratio of the higher modes can be determined directly as a function of the one of the first mode. Add the supplemental damping ratio to the inherent damping ratio. Determine the seismic response and a new value of roof displacement $D_{roof}{}^{(i)}$.
3. Repeat Step 2 until two successive values of ξ_{sd1} are sufficiently close (difference in percentage less than 1%).

It should be noticed that each iteration involves the execution of the time-history analysis of the structure under study. In Table 4 the assessed values of the supplemental damping ratio for each ground motion are shown with reference to the frame 3B6F. Three results are compared for the supplemental damping ratio: the one obtained with the iterative procedure ($\xi_{sd1,\mathrm{I}}$), the one obtained with the direct procedure ($\xi_{sd1,\varepsilon}$), and the one derived by introducing the roof displacement obtained from nonlinear time-history analysis into Eq. (12) (ξ_{sd1}). The latter is assumed as reference value in the calculation of the error of the estimates given by the iterative and the direct procedure.

TABLE 4 Supplemental damping ratio for the frame 3B6F

Record	$\xi_{sd1,\mathrm{I}}$ (%)	$\xi_{sd1,\varepsilon}$ (%)	ξ_{sd1} (%)	Δ_{I} (%)	Δ_{ε} (%)
ELCE	30.65	31.70	29.43	4.15	7.71
TAFT	28.07	30.38	27.19	3.24	11.73
PARK	24.74	26.80	24.96	−0.88	7.37
TABA	33.09	35.76	32.42	2.07	10.3
IMPV	30.43	33.00	31.15	−2.31	5.94
IMVA	20.89	23.21	22.12	−5.56	4.93
NEWH	18.14	19.48	18.37	−1.25	6.04
SATI	23.12	26.06	25.09	−7.85	3.87
KBUN	19.02	20.3	19.81	−3.99	2.47
PETR	25.47	26.86	24.59	3.58	9.23
STUR	26.22	29.24	28.09	−6.66	4.09

TABLE 5 Supplemental damping ratio for the frames 3B6F and 3B1F

	3B3F			3B1F		
Record	$\xi_{sd,\varepsilon}$ (%)	ξ_{sd1} (%)	Δ_{ε} (%)	$\xi_{sd,\varepsilon}$ (%)	ξ_{sd1} (%)	Δ_{ε} (%)
ELCE	32.59	30.90	5.47	33.62	33.00	1.88
TAFT	35.45	33.11	7.07	33.00	31.20	5.77
PARK	30.07	27.57	9.07	32.27	31.02	4.03
TABA	44.01	39.47	11.50	33.92	32.24	5.21
IMPV	42.47	38.19	11.21	35.06	33.71	4.00
IMVA	30.14	29.03	3.82	31.09	30.18	3.02
NEWH	26.18	24.94	4.97	29.62	28.81	2.81
SATI	33.05	31.10	6.27	29.99	24.76	21.10
KBUN	30.00	29.26	2.53	31.61	30.65	3.13
PETR	31.92	28.98	10.14	30.89	30.04	2.83
STUR	44.07	43.47	1.38	36.94	33.94	8.84

The average values of the error over all considered ground motions have been calculated. These values are equal to 3.46% and 6.14% for the iterative procedure and for the direct procedure, respectively. The performed iterations for the iterative procedure have been 4 or 5, depending on the different ground accelerations. It can be noticed that the difference between the two procedures in assessing the supplemental damping ratio is very low for all the ground motions and that the values obtained from the two procedures are very close to the one assumed as reference.

According to these results, Table 5 shows for the frames 3B3F and 3B1F just the comparison between the direct procedure and the procedure taken as reference.

The value of the damper index is equal to 31.13% for the frame 3B3F and to 32.03% for the frame 3B1F. The average errors are in these cases equal to 6.68% and 5.69% for the frames 3B3F and 3B1F, respectively.

6.2. Results

The response of the frames in terms of roof displacement has been calculated using the three methods described before. Table 6 shows the maximum values of roof displacement for the frame 3B6F expressed in cm. As for the supplemental damping ratio, the result of the iterative procedure ($D_{roof,\mathrm{I}}$) is compared with the ones of the direct procedure ($D_{roof,\varepsilon}$) and of nonlinear time-history analysis (D_{roof}). The results indicate that the direct method has provided an assessment of the response in terms of displacement very close to the iterative method. The average errors are equal to 7.26% and 6.67% for the iterative procedure and for the direct procedure, respectively.

It can be easily shown that a part of the total error illustrated in Table 6 is due to the linearization of the problem. It has been noticed that this part becomes more relevant for certain values of period and nearly zero for other values of period. In particular, Lin and Chopra [2002] showed that the damper nonlinearity has essentially no influence on the system response in the velocity-sensitive spectral region and small influence in the acceleration-sensitive and displacement-sensitive regions, with error up to 17%.

Table 7 illustrates the results for the frames 3B3F and 3B1F. The average errors are equal to 5.63% and 6.68% for the frames 3B3F and 3B1F, respectively. Figure 13 shows the spectra of the supplemental damping ratio for the examined cases. These spectra have been determined on the basis of the design response spectrum similar to the average spectrum of the considered earthquake motions. The same figure shows also, for each frame and for each earthquake motion, the values of the supplemental damping ratio derived by introducing the roof displacement obtained from nonlinear dynamic analysis into Eq. (12). It can be seen that the assessment of the supplemental damping ratio with the direct procedure applied to the spectra provided by seismic codes is more centered for the frame 3B3F; for the frame 3B1F, which lies in the range of periods characterized by constant acceleration, where the response is more dependent on the damping ratio, the proposed procedure seems to be conservative. A quite different trend has been observed for the frame 3B6F. It has to be noticed that the observed trends are influenced also by the differences between the spectra of the single earthquake records and the design response spectrum provided by the code. For example, considering the frame 3B6F, it is possible to notice in Fig. 12 that the spectral displacements of the selected ground motions at the fundamental period of the structure

TABLE 6 Maximum values of displacement for the frame 3B6F

Record	$D_{roof,\mathrm{I}}$ (cm)	$D_{roof,\varepsilon}$ (cm)	D_{roof} (cm)	Δ_{I} (%)	Δ_{ε} (%)
ELCE	5.35	5.13	5.81	−7.82	−11.68
TAFT	6.38	6.13	6.80	−6.13	−9.91
PARK	8.21	7.91	8.07	1.76	−1.96
TABA	4.59	4.37	4.78	−3.99	−8.63
IMPV	5.43	5.23	5.18	4.77	0.91
IMVA	11.52	11.07	10.28	12.06	7.68
NEWH	15.28	14.91	14.90	2.55	0.07
SATI	9.40	9.00	7.99	17.77	12.75
KBUN	13.92	13.70	12.81	8.67	6.95
PETR	7.75	7.47	8.32	−6.85	−10.24
STUR	7.31	6.97	6.37	14.75	9.30

TABLE 7 Maximum values of displacement for the frames 3B6F and 3B1F

	3B3F			3B1F		
Record	$D_{roof,\varepsilon}$ (cm)	D_{roof} (cm)	Δ_ε (%)	$D_{roof,\varepsilon}$ (cm)	D_{roof} (cm)	Δ_ε (%)
ELCE	4.04	3.79	6.81	0.67	0.61	10.88
TAFT	3.17	3.17	−0.03	0.76	0.68	12.02
PARK	4.57	4.57	−0.07	0.75	0.69	9.1
TABA	2.19	2.23	−1.88	0.7	0.64	9.7
IMPV	2.23	2.38	−6.51	0.64	0.58	9.54
IMVA	4.57	4.12	10.82	0.8	0.73	10.37
NEWH	6.05	5.59	8.36	0.83	0.8	3.69
SATI	3.76	3.59	4.65	1.07	1.08	−0.74
KBUN	4.5	4.06	10.87	0.76	0.7	7.69
PETR	4.13	4.14	−0.22	0.78	0.73	6.32
STUR	2.16	1.84	17.3	0.57	0.57	0.05

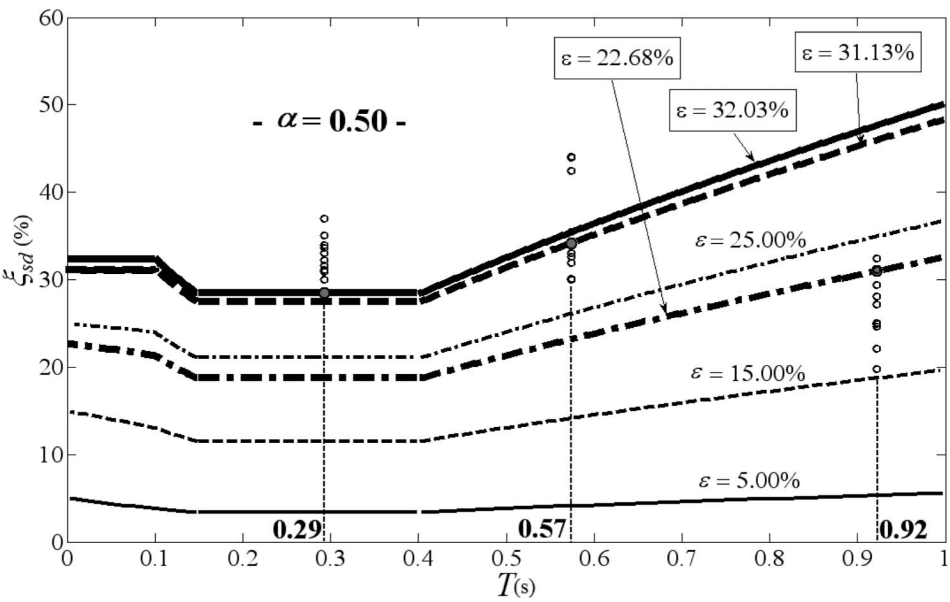

FIGURE 13 Spectra of the supplemental damping ratio for the examined cases.

are lower than the ordinate of the EC8 spectrum. Therefore, it is likely to have equivalent supplemental damping ratios lower than the ordinate of the corresponding design spectrum.

7. Conclusions

This article proposes a methodology to assess through a direct procedure the supplemental damping ratio provided by nonlinear viscous dampers inserted in a structure. The procedure is based on a new dimensionless parameter, called damper index, which has been introduced into the equations of motion. This parameter can be calculated once the properties of the structure and of the external actions are known. Initially the methodology has been studied with reference to the analysis of single degree of freedom systems subjected to harmonic excitations and earthquake motions. In order to get the direct procedure easily usable by structural designers, a method for defining the spectra of the supplemental damping ratio, for known values of the damper exponent and of the damper index, on the basis of the response spectra provided by seismic codes, has been proposed. The methodology has then been applied and verified through the seismic analysis of a group of multi-story RC frame structures equipped with nonlinear viscous dampers. From these analyses it has been possible to verify that the results of the direct procedure, in terms of supplemental damping ratio and of roof displacement, are very close to the ones obtained from the iterative procedure presented in literature and that the results of both procedures are close to the ones of nonlinear time-history analyses.

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References

Agrawal, A. K. and Yang, J. N. [1999] "Optimal placement of passive dampers on seismic and wind-excited buildings using combinatorial optimization," *Journal of Intelligent Material Systems and Structures* **10**(12), 997–1014.

BSSC [2003] *NEHRP Recommended Provisions for Seismic Regulations for New Buildings and other Structures*, FEMA 450, developed by BSSC for FEMA, Washington D.C.

CEN [2003] *Eurocode 8. Design of Structures for Earthquake Resistance – Part 1: General Rules, Seismic Actions and Rules for Buildings*, Brussels.

Chopra, A. K. [2001] *Dynamics Of Structures: Theory and Applications to Earthquake Engineering*, 2nd ed., Prentice-Hall, Upper Saddle River, NJ.

Christopoulos, C. and Filiatrault, A. [2006] *Principles of Passive Supplemental Damping and Seismic Isolation*, IUSS Press, Pavia, Italy.

Constantinou, M. C. and Symans, M. D. [1992] Experimental and analytical investigation of seismic response of structures with supplemental fluid viscous dampers, *Report No. NCEER-92-0032*, National Center for Earthquake Engineering Research, State University of New York at Buffalo, Buffalo, New York.

Constantinou, M. C. and Symans, M. D. [1993] "Seismic response of structures with supplemental damping," *The Structural Design of Tall Buildings* **2**, 77–92.

Constantinou, M. C., Symans, M. D., Tsopelas, P., and Taylor, D. P. [1993] "Fluid viscous dampers in applications of seismic energy dissipation and seismic isolation," *Proc. ATC 17-1 on Seismic Isolation, Energy Dissipation and Active Control* **2**, 581–591.

Constantinou, M. C., Soong, T. T., and Dargush, G. F. [1998] Passive energy dissipation systems for structural design and retrofit, Monograph no. 1, Multidisciplinary Center for Earthquake Engineering Research, State University of New York at Buffalo, Buffalo, New York.

Dargush, G. F. and Sant, R. S. [2005] "Evolutionary aseismic design and retrofit with passive energy dissipation," *Earthquake Engineering and Structural Dynamics* **34**, 1601–1626.

Gluck, N., Reinhorn, A. M., Gluck, J., and Levy, R. [1996] "Design of supplemental dampers for control for structures," *Journal of Structural Engineering* **122**(12), 1394–1399.

Hwang, J. S., Yi, S. L., and Ho, S. Y. [2004] "Equivalent damping ratios of structures with supplemental viscous dampers," *Engineering Structures* **26**, 346–365.

Lavan, O. and Dargush, G. F. [2009] "Multi-objective evolutionary seismic design with passive energy dissipation systems," *Journal of Earthquake Engineering* **13**, 758–790.

Lavan, O. and Levy, R. [2006] "Optimal peripheral drift control of 3D irregular framed structures using supplemental viscous dampers," *Journal of Earthquake Engineering* **10**(6), 903–923.

Levy, R. and Lavan, O. [2006] "Fully stressed design of passive controllers in framed structures for seismic loadings," *Structural and Multidisciplinary Optimization* **32**(6), 485–498.

Lin, Y. Y. and Chang, K. C. [2003] "A study on damping reduction factor for buildings under earthquake ground motions," *Journal of Structural Engineering* **129**(2), 206–214.

Lin, Y. Y., Chang, K. C., and Chen, C. Y. [2008] "Direct displacement-based design for seismic retrofit of existing buildings using nonlinear viscous dampers," *Bulletin of Earthquake Engineering* **6**, 535–552.

Lin, W. H. and Chopra, A. K. [2002] "Earthquake response of elastic SDF systems with non-linear fluid viscous dampers," *Earthquake Engineering and Structural Dynamics* **31**, 1623–1642.

Liu, W., Tong, M., and Lee, G. C. [2005] "Optimization methodology for damper configuration based on building performance indices," *Journal of Structural Engineering* **131**(11), 1746–1756.

Lopez Garcia, D. [2001] "A simple method for the design of optimal damper configurations in MDOF structures," *Earthquake Spectra* **17**(3), 387–398.

Makris, N., Constantinou, M. C., and Dargush, G. F. [1993] "Analytical model of viscoelastic fluid dampers," *Journal of Structural Engineering* **119**(11), 3310–3325.

Makris, N., Dargush, G. F., and Constantinou, M. C. [1995] "Dynamic analysis of viscoelastic fluid dampers," *Journal of Engineering Mechanics* **121**(10), 1114–1121.

Miyamoto, K., Taylor, D., and Duflot, F. [2002] "Seismic rehabilitation of a reinforced concrete hotel using fluid viscous dampers," *Proc. of the 12th European Conference on Earthquake Engineering*, London, U.K.

Peckan, G., Mander, J. B., and Chen, S. S. [1999] "Fundamental considerations for the design of non-linear viscous dampers," *Earthquake Engineering and Structural Dynamics* **28**, 1405–1425.

Priestley, M. J. N. [2000] "Performance based seismic design," *Proc. of the 12th World Conference on Earthquake Engineering*, Auckland, New Zealand.

Priestley, M. J. N., Calvi, G. M., and Kowalsky, M. J. [2007] *Displacement-Based Seismic Design of Structures*, IUSS Press, Pavia.

Ramirez, O. M., Constantinou, M. C., Gomez, J., Whittaker, A. S., and Chrysostomou, C. Z. [2002a] "Evaluation of simplified methods of analysis of yielding structures with damping systems," *Earthquake Spectra* **18**(3), 501–530.

Ramirez, O. M., Constantinou, M. C., Kirche, C. A., Whittaker, A. S., Johnson, M. W., Gomez, J. D., and Chrysostomou, C. Z. [2000] Development and evaluation of simplified procedures for analysis and design of buildings with passive energy dissipation systems, *Report MCEER-00-0010*, Multidisciplinary Center for Earthquake Engineering Research, State University of New York at Buffalo, Buffalo, New York.

Ramirez, O. M., Constantinou, M. C., Whittaker, A. S., Kircher, C. A., and Chrysostomou, C. Z. [2002b] "Elastic and inelastic seismic response of buildings with damping systems," *Earthquake Spectra* **18**(3), 531–547.

Ramirez, O. M., Constantinou, M. C., Whittaker, A. S., Kircher, C. A., Johnson, M. W., and Chrysostomou, C. Z. [2003] "Validation of 2000 NEHRP provisions equivalent lateral force and modal analysis procedures for buildings with damping systems," *Earthquake Spectra* **19**(4), 981–999.

Sadek, F., Mohraz, B., and Riley, M. A. [2000] "Linear procedures for structures with velocity-dependent dampers," *Journal of Structural Engineering* **126**(8), 887–895.

Seleemah, A. A. and Constantinou, M. C. [1997] Investigation of seismic response of buildings with linear and nonlinear fluid viscous dampers, *Report No. NCEER-97-0004*, National Center for Earthquake Engineering Research, State University of New York at Buffalo, Buffalo, New York.

Silvestri, S., Gasparini, G., and Trombetti, T. [2010] "A five-step procedure for the dimensioning of viscous dampers to be inserted in building structures," *Journal of Earthquake Engineering* **14**, 417–447.

Singh, M. P. and Moreschi, L. M. [2001] "Optimal seismic response control with dampers." *Earthquake Engineering and Structural Dynamics* **30**(4), 553–572.

Singh, M. P., Verma, N. P., and Moreschi, L. M. [2003] "Seismic analysis and design with Maxwell dampers," *Journal of Engineering Mechanics* **129**(3), 273–282.

Soong, T. T. and Constantinou, M. C. [1994] *Passive and Active Structural Vibration Control in Civil Engineering*, Springer-Verlag, Wien.

Soong, T. T. and Dargush, G. F. [1997] *Passive Energy Dissipation Systems in Structural Engineering*, John Wiley & Sons, Chichester, England.

Takewaki, I. [1997] "Optimal damper placement for minimum transfer functions," *Earthquake Engineering and Structural Dynamics* **26**(11), 1113–1124.

Takewaki, I. and Yoshitomi, S. [1998] "Effects of support stiffnesses on optimal damper placement for a planar building frame," *The Structural Design of Tall Buildings* **7**, 323–336.

Takewaki, I., Yoshitomi, S., Uetani, K., and Tsuji, M. [1999] "Non-monotonic optimal damper placement via steepest direction search," *Earthquake Engineering and Structural Dynamics* **28**(6), 655–670.

Whittaker, A. S., Constantinou, M. C., Ramirez, O. M., Johnson, M. W., and Chrysostomou, C. Z. [2003] "Equivalent lateral force and modal analysis procedures of the 2000 NEHRP provisions for buildings with damping systems," *Earthquake Spectra* **19**(4), 959–980.

Zhang, R. H. and Soong, T. T. [1992] "Seismic design of viscoelastic dampers for structural applications," *Journal of Structural Engineering* **118**(5), 1375–1392.