📘 NSE Options Data Analysis – Formula Reference

This document explains the mathematical formulas and statistical concepts used throughout the data processing pipeline for NSE Options analysis, including:

- Black-Scholes pricing & Greeks
- Implied Volatility estimation
- Yang-Zhang Realized Volatility
- IV Percentile & Rank
- Interest rate fallback logic
- Expiry and TTE calculation

1️⃣ Black-Scholes Option Pricing

Formula (European Option):

For call options: $C = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2)$

For put options: $P = Ke^{-rT}N(-d_2) - Se^{-qT}N(-d_1)$

Where:

- (S): Underlying asset price
- (K): Strike price
- (T): Time to expiry (in years)
- (r): Risk-free rate
- (q): Continuous dividend yield
- ($N(\cdot)$): Cumulative standard normal distribution
- ($d_1 = \dfrac{\ln(S/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}$)
- ($d_2 = d_1 - \sigma\sqrt{T}$)

2️⃣ Black-Scholes Greeks

Delta

$$\Delta = \begin{cases} e^{-qT}N(d_1), & \text{for Call} \\ -e^{-qT}N(-d_1), & \text{for Put} \end{cases}$$

Gamma (common to both):

$$\Gamma = \frac{e^{-qT}\phi(d_1)}{S\sigma\sqrt{T}} \text{ where } \phi(d_1) = \frac{1}{\sqrt{2\pi}}e^{-d_1^2/2}$$

Theta

For call: $\Theta = \dfrac{-S\phi(d_1)\sigma e^{-qT}}{2\sqrt{T}} - rKe^{-rT}N(d_2) + qSe^{-qT}N(d_1)$

For put: $\Theta = \dfrac{-S\phi(d_1)\sigma e^{-qT}}{2\sqrt{T}} + rKe^{-rT}N(-d_2) - qSe^{-qT}N(-d_1)$

(*Divided by 365 in code to annualize per day*)

Vega

$$\text{Vega} = \frac{Se^{-qT}\phi(d_1)\sqrt{T}}{100}$$

Rho

Call: $\text{Rho} = \dfrac{KTe^{-rT}N(d_2)}{100}$

Put: $\text{Rho} = \dfrac{-KTe^{-rT}N(-d_2)}{100}$

3️⃣ Implied Volatility (IV)

Using **Bisection Method** to find the implied volatility (σ) that makes the theoretical price match the market price.

Objective: $\text{Find } \sigma \text{ such that } BS(S, K, T, r, \sigma) \approx \text{Market Price}$

Iteratively:

- Evaluate price using current sigma.
- Adjust upper/lower bounds based on error.
- Stop when tolerance is reached.

4️⃣ Yang-Zhang Realized Volatility (YZ RV)

It combines the benefits of:

- **Close-to-Close volatility**
- **Overnight (Open-to-Previous Close) volatility**

- **Open-to-Open volatility**

$$\sigma^2_{\text{YZ}} = \sigma^2_{cc} + k \cdot \sigma^2_{on} - (1-k) \cdot \sigma^2_{oo}$$

Where:

- (σ^2_{cc}): Variance of log returns of Close prices
- (σ^2_{on}): Variance of overnight returns
- (σ^2_{oo}): Variance of Open-to-Open returns
- ($k = \dfrac{0.34}{1.34 + \frac{n+1}{n-1}}$)

Annualized: $\sigma_{\text{YZ}} = \sqrt{\sigma^2_{\text{YZ}} \cdot 252}$

5️⃣ IV Percentile & Rank (30-Day Rolling)

Given a rolling window of last 30 IV values:

- **Percentile**: Relative percentage of days with IV ≤ current day
  $\text{Percentile} = \dfrac{\text{count}(IV_i \leq IV_{today})}{30}$
- **Rank (Min-Max Scaled)**: $\text{Rank} = \dfrac{IV_{today} - IV_{min}}{IV_{max} - IV_{min}}$

6️⃣ Interest Rate Fallback Logic

For any given date `D`, if `D` is not in the map:

- Use the closest **previous** date's interest rate.
- If no earlier date exists, use the **next** available date.

Uses `bisect_right` on a sorted date list for efficiency.

7️⃣ Time to Expiry (T in Years)

Using:

- Current date = bhavcopy date
- Expiry date = F&O expiry

$$T = \frac{\text{days to expiry}}{252}$$

Assumes 252 trading days in a year.

8️⃣ Strike Selection Logic

Select a strike price (K) that:

- Is **closest to spot price**
- Exists in all 3 nearest expiry series
- Has both CE and PE options

Ensures comparability across expiries for volatility and Greek analysis.

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