16-720 B Assignment2

Sajal Maheshwari
sajalm@andrew.cmu.edu

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1 Keypoint Detector

1.1 Gaussian Pyramid

1.2 The DoG Pyramid

Figure 1: The figure shows outputs of DoG pyramids generated at five scales.

1.3

1.4

1.5 Putting it together

Figure 2: The figure shows positions of keypoints generated after edge suppression.

2 BRIEF Descriptor

2.1

2.2

2.3

2.4 Check Point: Descriptor Matching

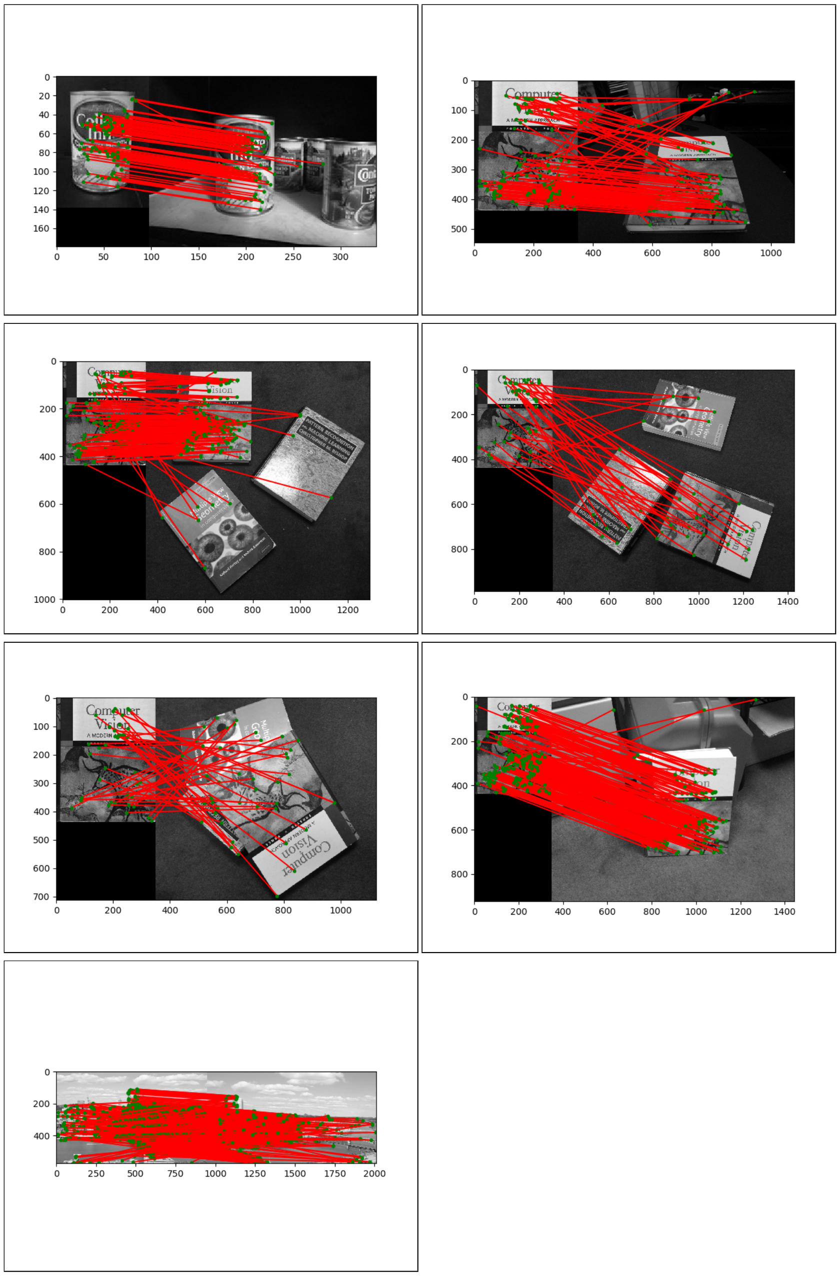

Figure 3: The figure shows example of brief descriptor matches

As can be seen from Fig. 3, BRIEF descriptor matches the keypoints in a reference image to corresponding keypoints in a different image quite accurately. However, there are some failure cases. For example, in Fig. 3, the left image of the third column fails, as BRIEF is not invariant to rotations.

2.5 BRIEF and rotations

Fig. 4 validates our observation that BRIEF is not invariant to rotations. As the angle of rotation increases, the number of matches decrease drastically.

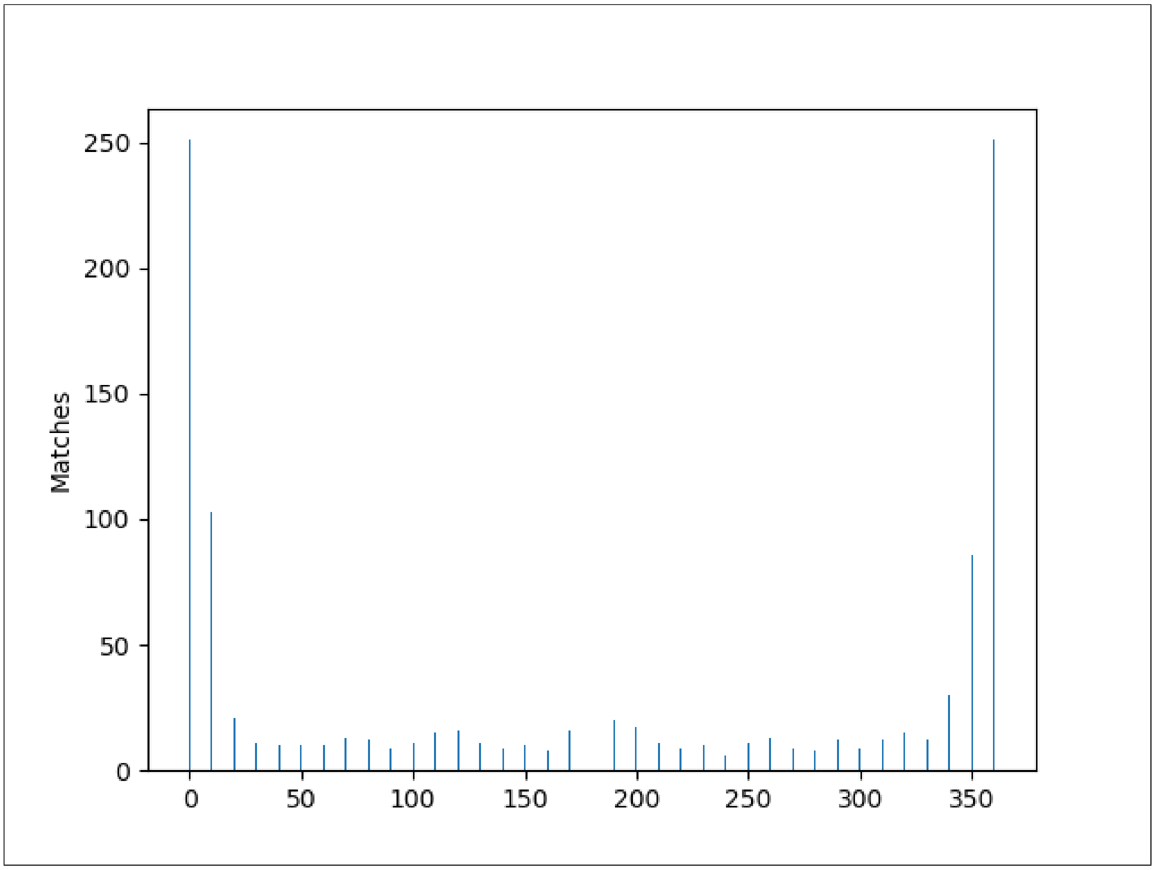

Figure 4: The figure shows effect of BRIEF matches with varying rotation angles. The angle is varied from 0 to 360 degrees with a least count of 10. The number of matches decrease dramatically as the angle changes, both in clockwise and counter-clockwise direction.

3 Planar Homographies: Theory

$$\lambda_n x_n = \mathbf{H} x_n \tag{1}$$

Here, x_n and y_n represent the homogeneous coordinates of two points from two cameras belonging to the same 3d world point. Since $\mathbf{H}$ is a 3×3 matrix, the above Equation 1 can be re-written as shown in Equation: 2

$$\lambda_n x_n = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \tag{2}$$

Breaking x_n and u_n into their respective dimensions,the Homography equation becomes:

$$\lambda_n \begin{bmatrix} x_n^{(1)} \\ x_n^{(2)} \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_n^{(1)} \\ u_n^{(2)} \\ 1 \end{bmatrix} \tag{3}$$

Now, since homogeneous coordinates represent a point in the 3D space, x_n and y_n will represent point towards the same direction, with a difference in the scaling. Therefore,

$$x_n \times u_n = 0 \tag{4}$$

Since, the cross-product is zero, every element of the resulting vector must be equal to zero. This gives us three equations of the form :

$$x_n^{(2)}(h_{31}u_n^{(1)} + h_{32}u_n^2 + h_{33}) - (h_{21}u_n^{(1)} + h_{22}u_n^{(2)} + h_{23}) = 0 \tag{5}$$

$$(h_{11}u_n^{(1)} + h_{12}u_n^{(2)} + h_{13}) - x_n^{(1)}(h_{31}u_n^{(1)} + h_{32}u_n^{(2)} + h_{33}) = 0 \tag{6}$$

$$x_n^{(1)}(h_{21}u_n^{(1)} + h_{22}u_n^2 + h_{33}) - x_n^2(h_{11}u_n^{(1)} + h_{12}u_n^{(2)} + h_{13}) = 0 \tag{7}$$

It should be noted that the three equations 5 6 and 7 are not independent. We have only two independent equations, which we use in the matrix form shown below :

$$\begin{bmatrix} 0 & 0 & 0 & -u_n^{(1)} & -u_n^{(2)} & -1 & x_n^{(2)}u_n^{(1)} & x_n^{(2)}u_n^{(2)} & x_n^{(2)} \\ u_n^{(1)} & u_n^{(2)} & 1 & 0 & 0 & 0 & -x_n^{(1)}u_n^{(1)} & -x_n^{(1)}u_n^{(2)} & -x_n^{(1)} \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = 0 \tag{8}$$

Equation 8 shows the relation for one pair of points. This can be extended for multiple pairs by extending the matrix for multiple points. In general for N points, the matrix equation is of the form :

$$
\begin{bmatrix}
0 & 0 & 0 & -u_1^{(1)} & -u_1^{(2)} & -1 & x_1^{(2)}u_1^{(1)} & x_1^{(2)}u_1^{(2)} & x_1^{(2)} \\
u_1^{(1)} & u_1^{(2)} & 1 & 0 & 0 & 0 & -x_1^{(1)}u_1^{(1)} & -x_1^{(1)}u_1^{(2)} & -x_1^{(1)} \\
0 & 0 & 0 & -u_2^{(1)} & -u_2^{(2)} & -1 & x_2^{(2)}u_2^{(1)} & x_2^{(2)}u_2^{(2)} & x_2^{(2)} \\
u_2^{(1)} & u_2^{(2)} & 1 & 0 & 0 & 0 & -x_2^{(1)}u_2^{(1)} & -x_2^{(1)}u_2^{(2)} & -x_2^{(1)} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & -u_N^{(1)} & -u_N^{(2)} & -1 & x_N^{(2)}u_N^{(1)} & x_N^{(2)}u_N^{(2)} & x_N^{(2)} \\
u_N^{(1)} & u_N^{(2)} & 1 & 0 & 0 & 0 & -x_N^{(1)}u_N^{(1)} & -x_N^{(1)}u_N^{(2)} & -x_N^{(1)}
\end{bmatrix}
\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = 0 \tag{9}
$$

Equation 9 is of the form $\mathbf{A}h = 0$. As can be seen, the number of variables is **9**, corresponding to each of the entries in the homography matrix.

Since the equation is only correct up to a scale, we only have 8 degrees of freedom, despite having 9 variables. As each point-to-point correspondence gives us two independent linear equations, the total number of point-to-point correspondence requires is 4 to solve this system.

If we set $||h||_2 = 1$, which is a choice since h only has 8DOF, this equation becomes a minimum direction problem. Also, we know that SVD provides us with a solution to project column spaces of a matrix(Equation **??**). Here, $\mathbf{V}^T$ is changing the matrix columns into the new bases, with Sigma values in $\mathbf{D}$ telling us the magnitude of each bases. Since we need the minimum direction, we pick the vector of $\mathbf{V}$ corresponding to the least Sigma value, which when reshaped appropriately gives us the matrix $\mathbf{H}$.

4 Planar Homographies: Implementation

5 RANSAC

6 Stitching it together: Panoramas

6.1

6.2

6.3

7 Augmented Reality