

Dynamic Programming

- D.P. { ① Recursion
 ② Memoisation
 ③ Tabulation

Recursion → Repetition of some problem,

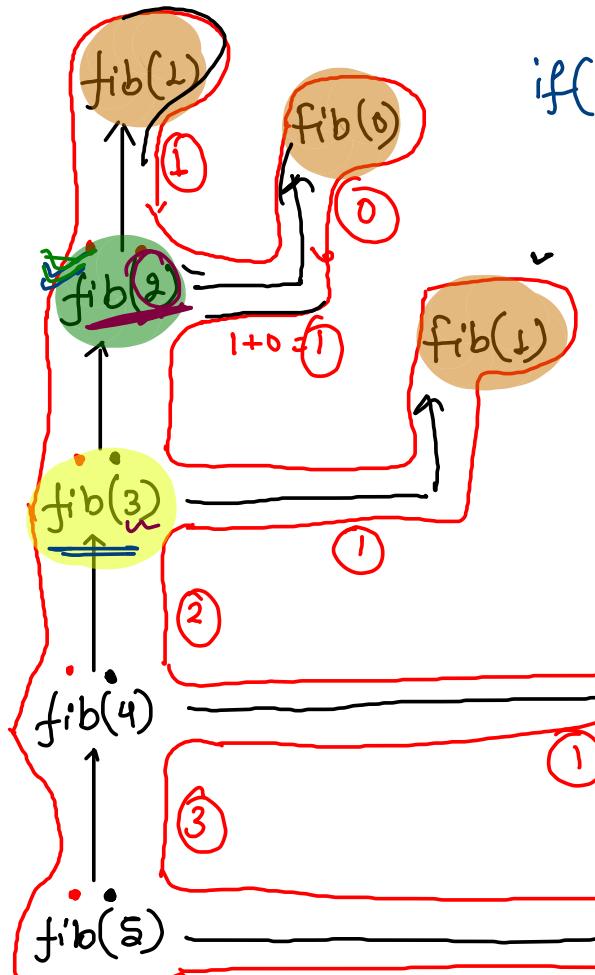
→ In the recursion we are trying to solve a same problem again and again

→ Sacrifice with space and save time. } Store the repetition of problem

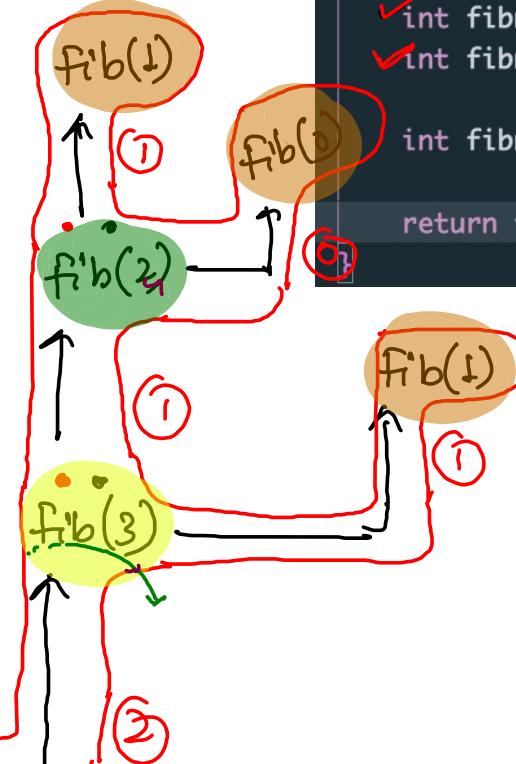
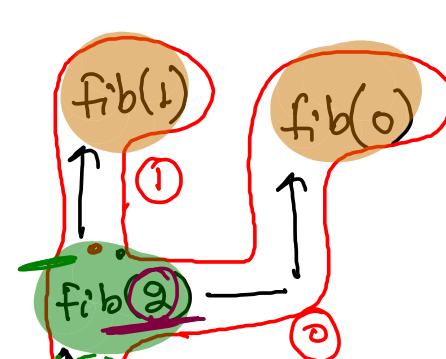
- ① Fibonacci - Series -

] D.P.
Memoisation
Tabulation

Fibonacci - with Recursion



$\text{if}(n==0 \text{ || } n==1) \text{ return } n;$



```
public static int fib_rec(int n) {
    ✓ int fibnm1 = fib_rec(n - 1); order
    ✓ int fibnm2 = fib_rec(n - 2); solving
    int fibn = fibnm1 + fibnm2;
    return fibn;
}
```

Diagram illustrating the recursive computation of $\text{fib}(5)$ with a call stack. The stack shows multiple entries for fib_rec at different levels of recursion. The stack grows as the function calls itself, and it shrinks as it returns values. The base case is reached when $n=0$ or $n=1$.

Repetition point

- Base case
- $\text{fib}(2)$
- $\text{fib}(3)$

$$2T(n-2) \leq T(n) \leq 2T(n-1)$$

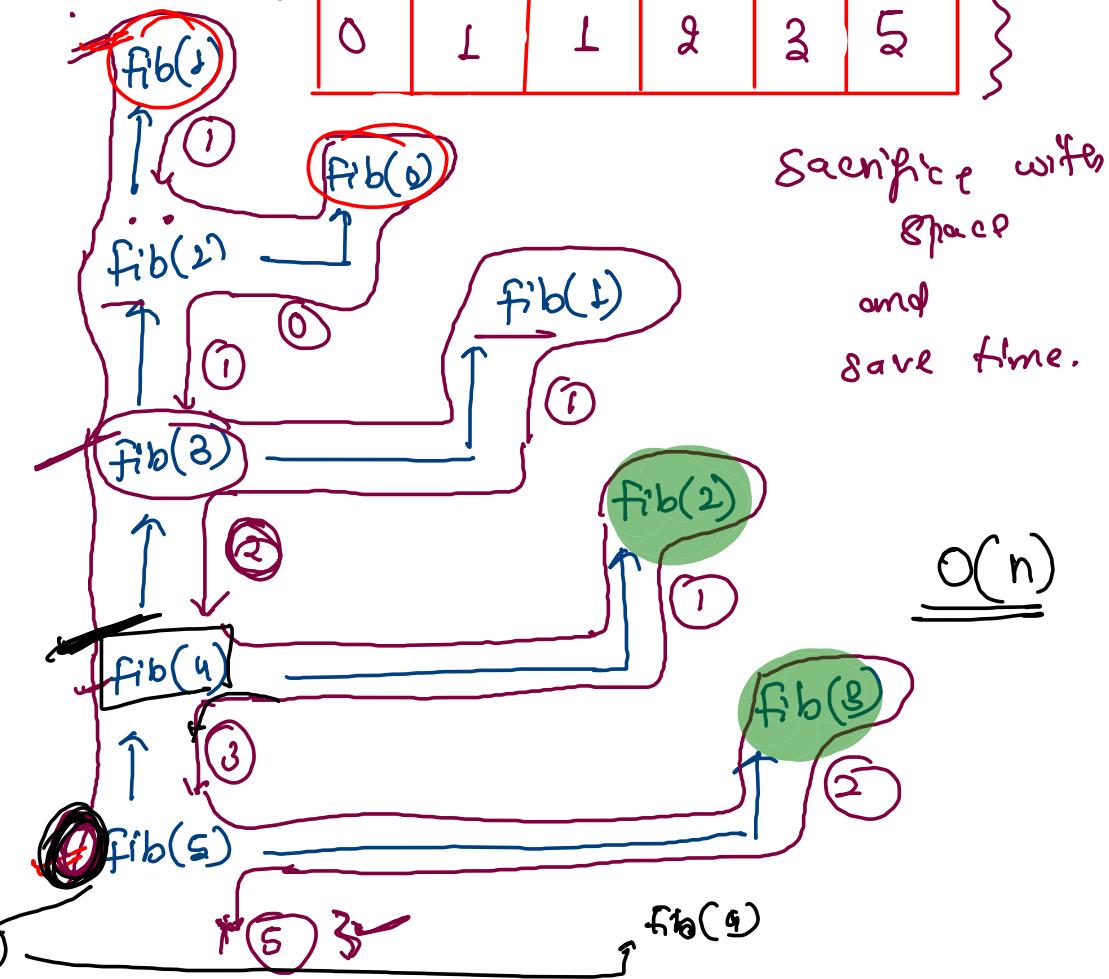
Time: $\mathcal{O}(2^n)$

Fibonacci - Memoisation

$n=5$.

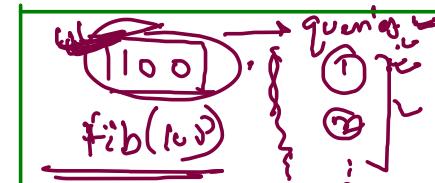
Size = $n+1$

dp →	0	1	1	2	3	4	5
	0	1	1	2	3	2	5



① DP-Storage to store the problem.

→ size of storage ($n+1$)



```

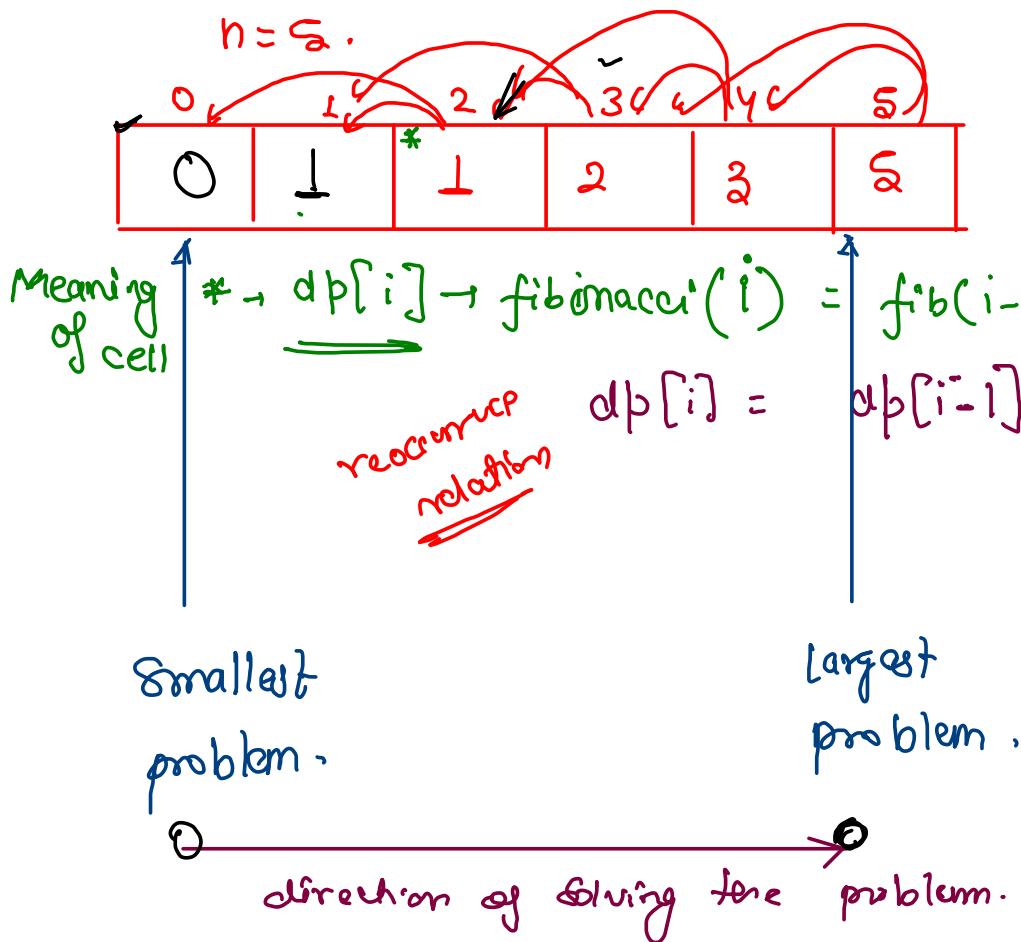
public static int fib_memo(int n, int[] dp) {
    if(n == 0 || n == 1) {
        return dp[n] = n; // Smallest problem
    }
    // 1. if Problem is already solved, then return
    if(dp[n] != 0) {
        return dp[n];
    }
    int fibnm1 = fib_memo(n - 1, dp);
    int fibnm2 = fib_memo(n - 2, dp);

    int fibn = fibnm1 + fibnm2;
    // 2. If not solved, then solve the problem
    return dp[n] = fibn;
}

```

Fibonacci - with Tabulation

Repetition is on single variable.



- Steps:
 - ① figure out repetition.
 - ② Make a storage of n+1 size
 - ③ assign meaning to cell
 - ④ figure out the direction of smallest to largest problem
 - ⑤ Pre requisite marking on dp.

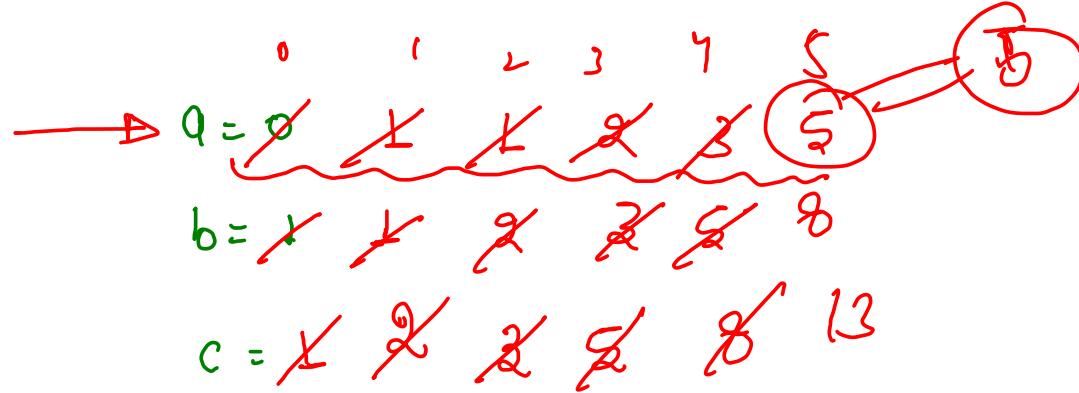
Base case of recursion

$$\underline{\underline{dp[0] = 0}}$$

$$\underline{\underline{dp[1] = 1}}$$

final result $\rightarrow dp[n]$

Fibonacci - btr : \rightarrow $n = 5$.

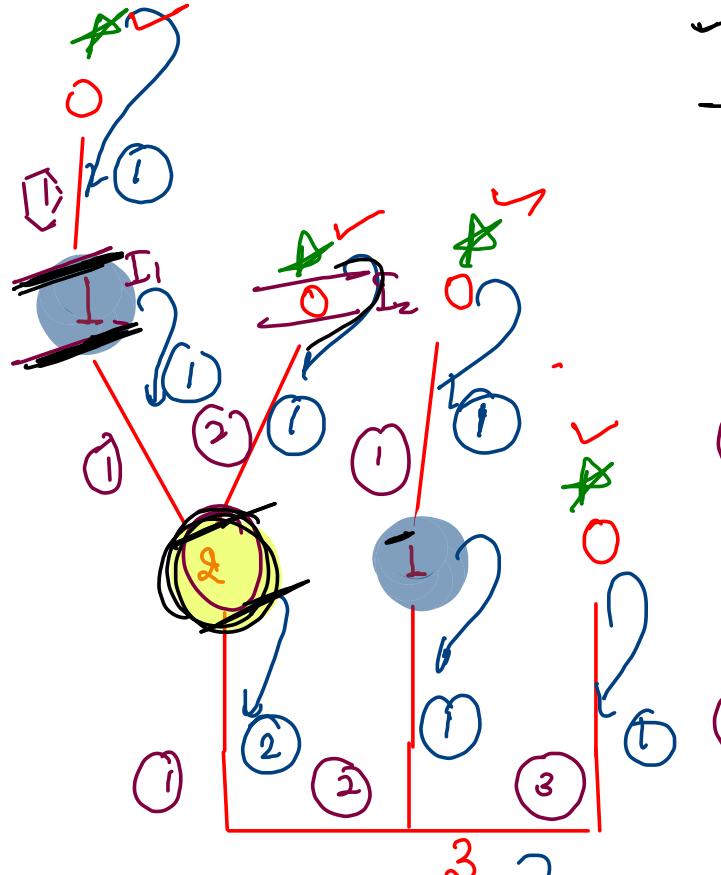


$$c = a + b;$$

$$a = b;$$

$$b = c;$$

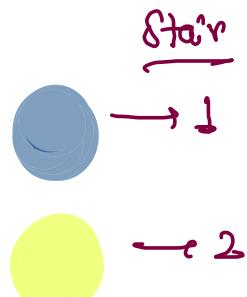
Climb Stair - Recursion.



$0 \xrightarrow{*} n$ $n \rightarrow 0$] both are same. \rightarrow Two variable \rightarrow fix at every level
 no. of jump is top \rightarrow Bottom
 Bottom \rightarrow top

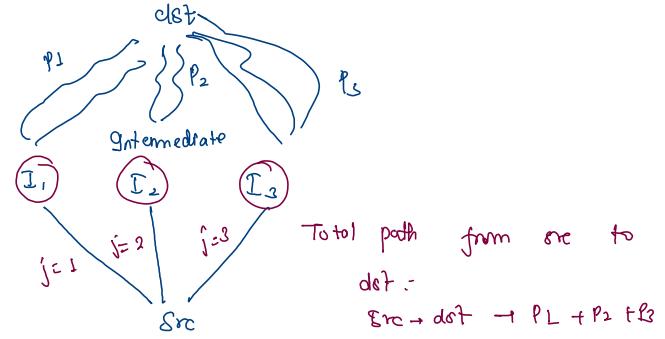
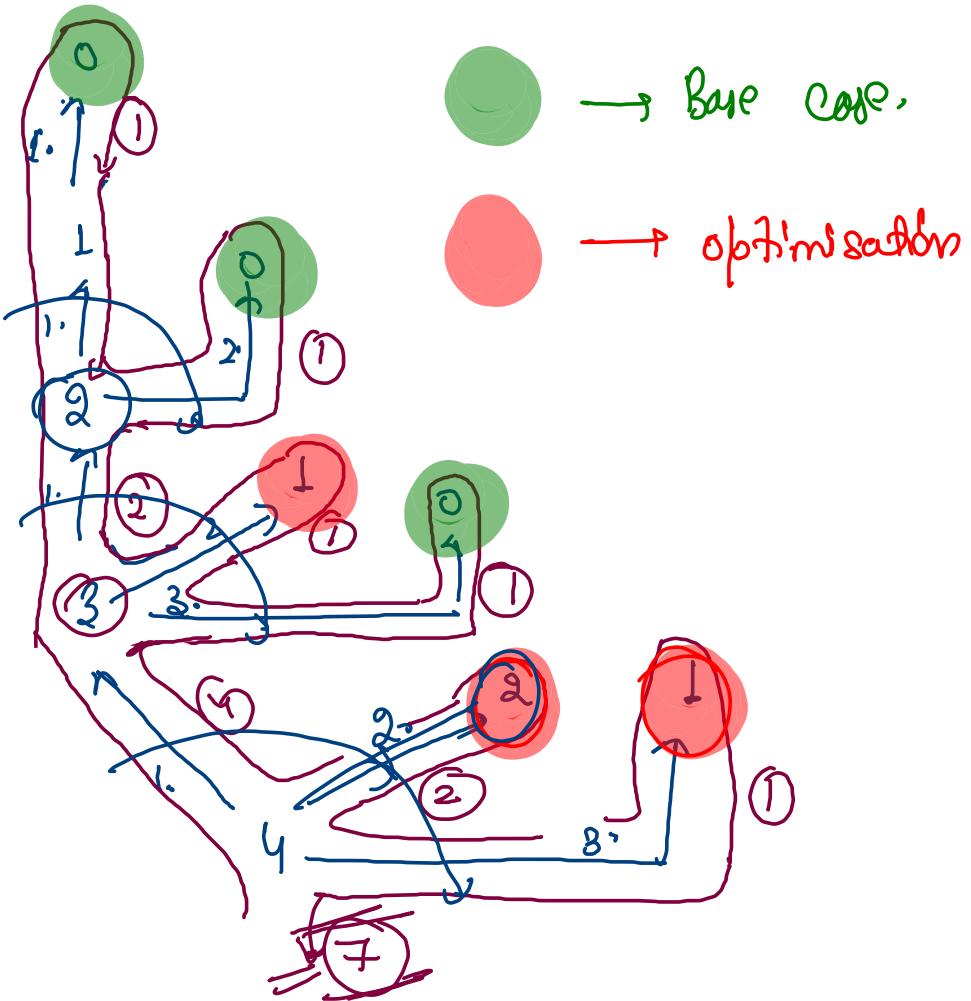
path from $0 \rightarrow 0$] Don't move

paths	0	1	1	1	1
0	0	1	1	1	1
1	1	1	1	1	1
2	1	2	1	1	1
3	1	3	1	1	1
4	2	1	1	1	1
5	2	2	1	1	1
6	3	1	1	1	1
7	3	2	1	1	1



0	1	2	3	4
1	1	2	4	7

order of problem-
0 →



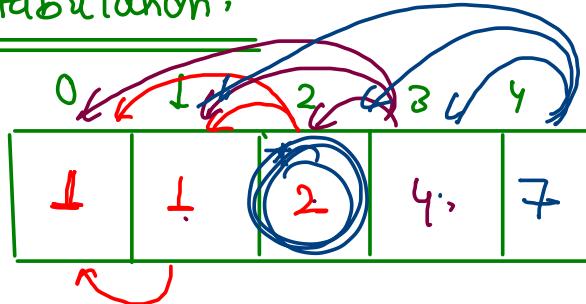
$n=4$

```
public static int climbStair_memo(int n, int[] dp) {
    if(n == 0) {
        return dp[0] = 1;
    }

    if(dp[n] != 0) {
        return dp[n];
    }

    int count = 0;
    for(int j = 1; j <= 3; j++) {
        if(n - j >= 0) {
            count += climbStair_memo(n - j, dp);
        }
    }
    return dp[n] = count;
}
```

ClimbStair - tabulation



result at - $dp[4]$
 Go $\rightarrow 7$

* $dp[i] \rightarrow$ count of paths from i^{th} step to destination(0)

jump / path

① Storage.

② Meaning of cell

③ Direction of traversal → smallest problem is at 0

④ Prerequisite.

$dp[0] = 1 \rightarrow$ there is one path

from $0 \rightarrow 0$
 i.e., don't move.

{ } { }

{ } { }

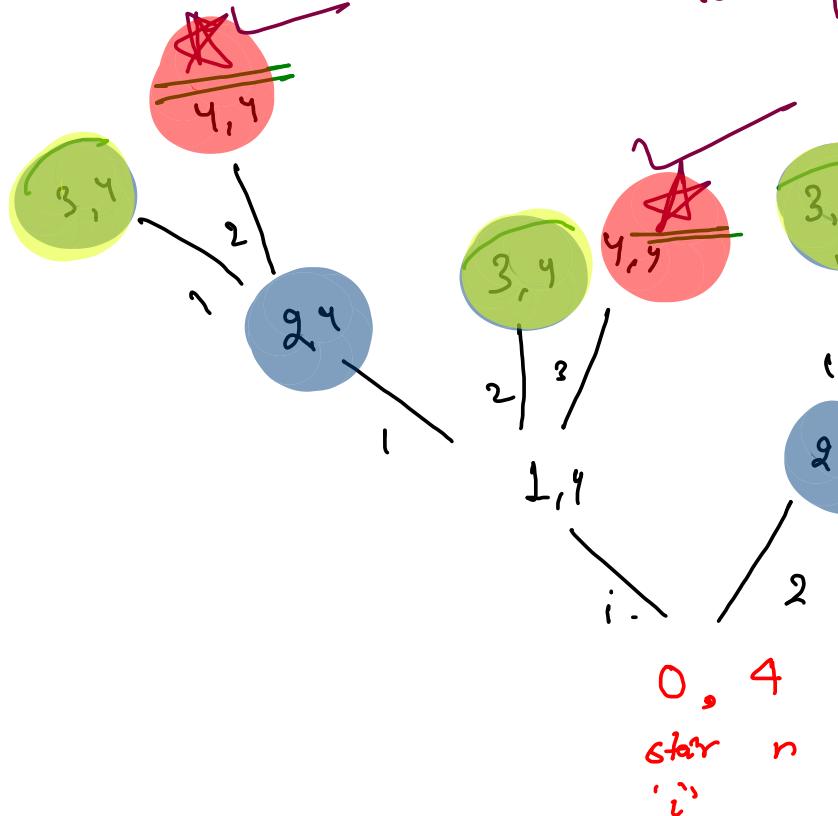
Climb Stairs with Variable Jumps.

0	↓	2	3	4.
2	3	2	0	1

0 → 4. Total no. of path.

variables which participate in repetition is basically number of stair

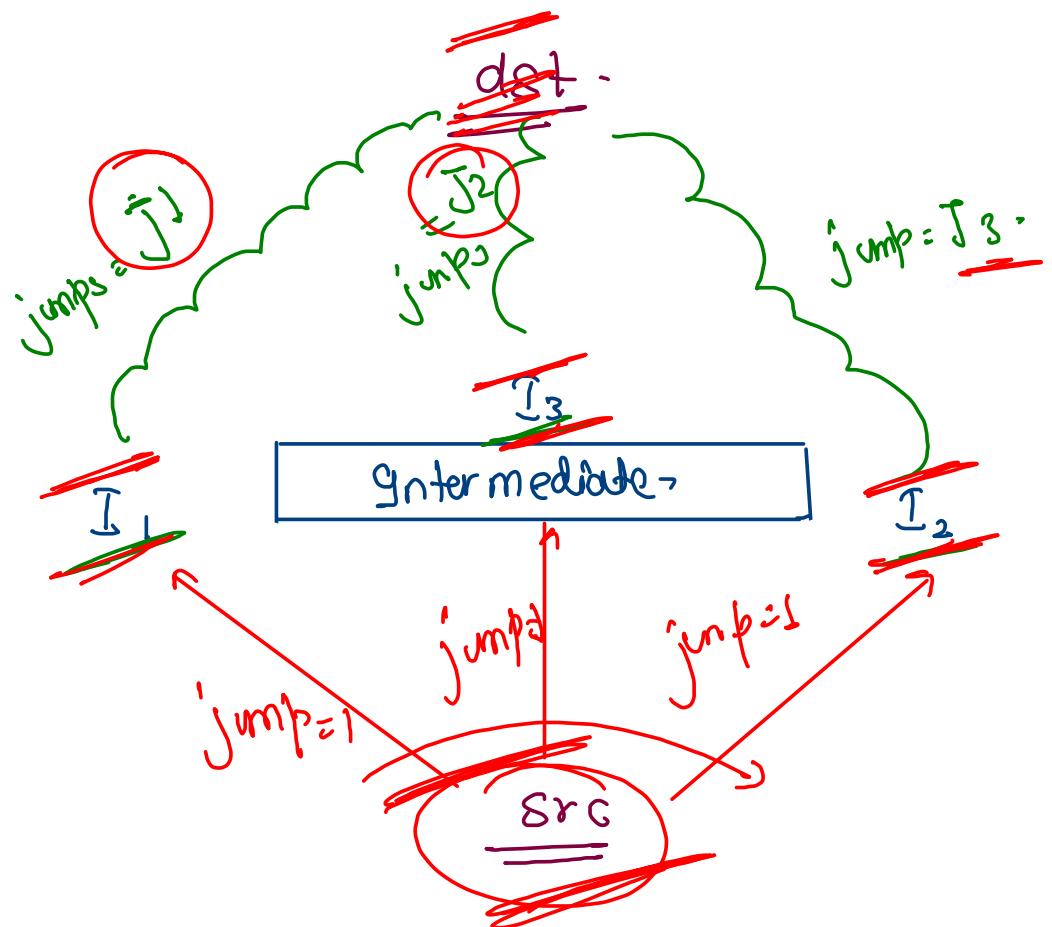
Total path ↗ 3



→ Base case

→ 3rd stair

→ 2nd stair



src → dst (-min jumps)

