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TECHNISCHE HOCHSCHULE DEGGENDORF
EUROPEAN CAMPUS ROTTAL-INN

Sets, Subsets and Tuples

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Sets, Subsets and Tuples

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Sets

definition:
mathematical set: collection of unique elements

- no duplicates
- order irrelevant

Subsets

subset of a set:

- elements are unique, no repetitions
- order *does not* matter

Subsets

subset of a set:
- elements are unique, no repetitions
- order *does not* matter

examples:
- $\{A, B\} = \{B, A\} \subset \{A, B, C\}$
- $\{1, 2, 7, 206342\} = \{7, 1, 206342, 2\} \subset \mathbf{N}$
- {apple, pear, cherry} = {pear, cherry, apple}
  $\subset$ {apple, pear, cherry, banana, pineapple, passion fruit}
- {HI, MDH, GPH} $\subset$ {studies at ECRI}

Tuples

tuple:

- elements can repeat
- order *does* matter

Tuples

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tuple:
- elements can repeat
- order *does* matter

examples:
- (A, B) ≠ (B, A)
- (1, 2, 7, 206342, 2) ≠ (2, 7, 1, 206342, 2)
- (apple, pear, cherry) ≠ (apple, cherry, pear)
- (HI, MDH, GPH) ≠ (MDH, HI, GPH)

Tuples

tuple:
- elements can repeat
- order *does* matter

permutation:
a reordering of the elements in a tuple

Tuples

tuple:
- elements can repeat
- order *does* matter

permutation:
a reordering of the elements in a tuple

example:
- (B, A, C) is a permutation of (A, B, C)
- (3, 2, 1) is a permutation of (1, 2, 3)

Tuples

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How many permutations N_{perm} does a k-tuple have?

1st element:	k possibilities
2nd element:	k – 1 possibilities remaining
...	
2nd to last element:	2 possibilities remaining
last element:	1 possibility remaining

Tuples

How many permutations N_{perm} does a k-tuple have?

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altogether: $N_{perm} = k \cdot (k - 1) \cdot (k - 2) \cdot \ldots \cdot 2 \cdot 1$

$= k!$ possibilities

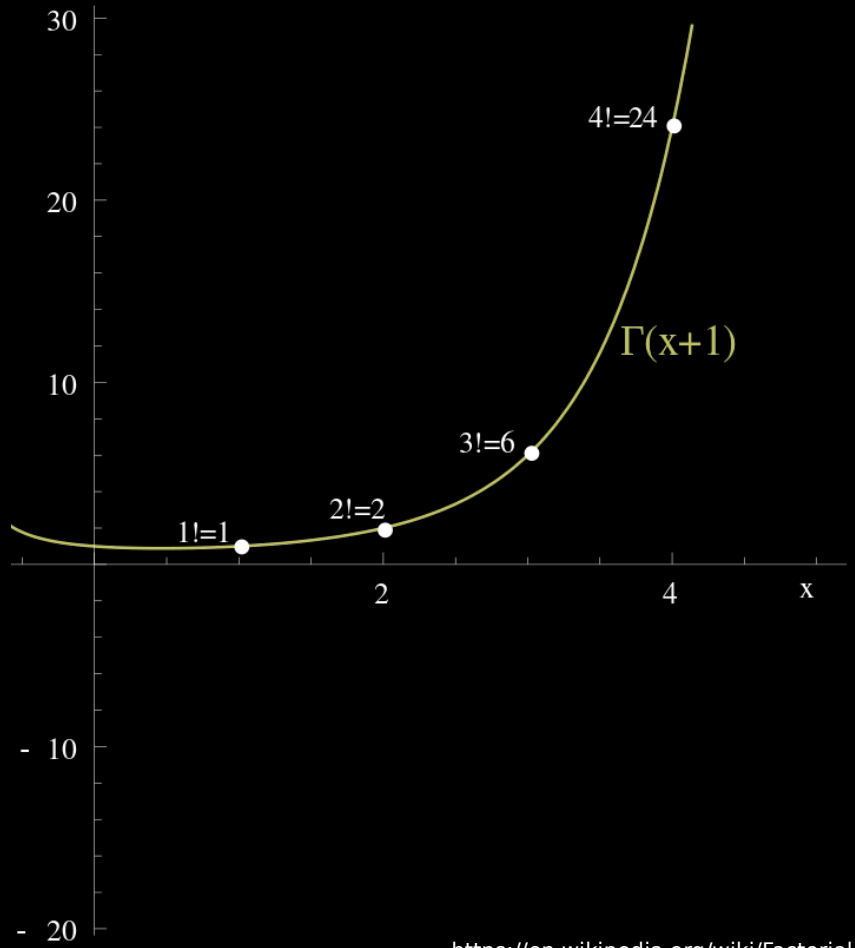
Tuples

recursive definition of the **factorial**:

- 0! = 1
- k! = k · (k – 1)!

explicitly:

- 0! = 1
- 1! = 1 · 1 = 1
- 2! = 2 · 1 = 2
- 3! = 3 · 2 = 6
- 4! = 4 · 6 = 24



https://en.wikipedia.org/wiki/Factorial

Summary

- subsets: no repetitions, order irrelevant
- tuples: repetitions allowed, order relevant
- tuple of length k: k-tuple
- permutations: putting a tuple into a different order
- number of permutations: factorial