BINARY SUBTRACTION

How to represent the Decimal Number 5 using 8 bits?
Ans: 00000101 = 5

How to represent the Decimal Number -5 using 8 bits?
Ans: We have to consider the first bit on the left side as sign bit. If it is one, the number is negative. If it is zero, the number is positive
Thus we have 10000101 = -5

How to represent the Decimal Number 5 using 4 bits?
Ans: 0101 = 5

How to represent the Decimal Number -5 using 4 bits?
Ans: 1101 = 5

CONCEPT OF SIGN BIT

Consider the Regular Counting in Binary using 4 bits for Positive Numbers and Negative Numbers. Let's consider the first bit as Sign bit and try to represent the Positive and Negative Numbers.

Sign bit				Decimal	
$-$	4	2	1	-8	
1	1	1	1	-7	
1	1	1	0	-6	
1	1	0	1	-5	
1	1	0	0	-4	
1	0	1	1	-3	
1	0	1	0	-2	
1	0	0	1	-1	
1	0	0	0	-0	Negative Zero
0	0	0	0	0	
0	0	0	1	1	
0	0	1	0	2	
0	0	1	1	3	
0	1	0	0	4	
0	1	0	1	5	
0	1	1	0	6	
0	1	1	1	7	

Let's Add two Numbers in this Representation

$$\begin{array}{r} 5 \\ +(-5) \\ \hline 0 \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ +1\,1\,0\,1 \\ \hline 1\,0\,0\,1\,0 \end{array} \longrightarrow \cancel{2}$$

Ignore

Wrong Answer

CONCEPT OF ONE'S COMPLEMENT

As we get wrong answer, let's try inverting the bits to represent the negative numbers and try a new method of represntation. This representation is known as 1's Complement.

Sign bit −	4	2	1	Decimal −8
1	0	0	0	−7
1	0	0	1	−6
1	0	1	0	−5
1	0	1	1	−4
1	1	0	0	−3
1	1	0	1	−2
1	1	1	0	−1
1	1	1	1	−0 Negative Zero
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7

Let's Add two Numbers in this Representation

$$\begin{array}{r} 5 \\ +(-5) \\ \hline 0 \end{array} \qquad \begin{array}{r} 0101 \\ +1010 \\ \hline 1111 \end{array} \longrightarrow -0$$

Answer is Close

$$\begin{array}{r} 5 \\ +(-3) \\ \hline 2 \end{array} \qquad \begin{array}{r} 0101 \\ +1100 \\ \hline 10001 \end{array} \longrightarrow 1$$

Ignore

Actual Answer is 2, but we get 1

$$\begin{array}{r} 6 \\ +(-2) \\ \hline 4 \end{array} \qquad \begin{array}{r} 0110 \\ +1101 \\ \hline 10011 \end{array} \longrightarrow 3$$

Ignore

Actual Answer is 4, but we get 3

If we closely observe , we can see a pattern. The answer is always "off" by one number up. So if we add one to the actual answer, we will get the correct answer in all the above cases.

CONCEPT OF TWO'S COMPLEMENT

Using this idea, let's create a new representation called 2's complement. Let's remove the Negative Zero Decimal from the above representation and move all the decimal numbers down by one position.

Sign bit				Decimal
−8	4	2	1	−8
1	0	0	0	−8
1	0	0	1	−7
1	0	1	0	−6
1	0	1	1	−5
1	1	0	0	−4
1	1	0	1	−3
1	1	1	0	−2
1	1	1	1	−1
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7

Let's Add two Numbers in this Representation

$$\begin{array}{r} 5 \\ +(-5) \\ \hline 0 \end{array} \qquad \begin{array}{r} 0101 \\ +1011 \\ \hline 10000 \end{array} \longrightarrow 0$$

Correct Answer!

$$\begin{array}{r} 5 \\ +(-3) \\ \hline 2 \end{array} \qquad \begin{array}{r} 0101 \\ +1101 \\ \hline 10010 \end{array} \longrightarrow 2$$

Ignore

Correct Answer!

$$\begin{array}{r} 6 \\ +(-2) \\ \hline 4 \end{array} \qquad \begin{array}{r} 0110 \\ +1110 \\ \hline 10100 \end{array} \longrightarrow 4$$

Ignore

Correct Answer!

In the case of Adding $+5$ and -5, each time we add bits at the same position, we get the result as 2, ie, 10 in binary. That is why it is called 2's complement.

Similary consider, the first bit as sign bit and assume a position value -8 to that number, we get an easy way to find the number in 2's complement representation. It has a mathematical meaning and that is why the number representation works correctly.

However, to find the 2's complement of a negative number, it is a two step process.
First Invert the number and then add one to the result.
Remember that the Positive numbers are represented as such.