Latent Dirichlet Analysis for Document Topic Modelling

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Abstract

Latent Dirichlet Allocation (LDA) is a generative probabilistic model for collections of discrete data and text corpora proposed by Blei et al. in their 2003 *Journal of Machine Learning Research* paper. A common application of LDA is the unsupervised learning of document topic models. In this paper, we explore the LDA model according to Blei et al., 2003, and implement it using Gibbs sampling and variational inference methods. We evaluate our implementation using the topic modeling experiment done by Blei et al., 2003, applying it to the TREC AP corpus. We also compare LDA to the simple unigram topic model and discuss the subsequent conclusions gained from our experimentation.

1   Introduction

The problem of modeling text documents and extracting the topics within them given unsupervised training data is document topic modeling. Each topic is a short description of the members of a collection of documents it represents. Applications of this problem can range from extracting topics from lengthy journals and news articles to short social media posts or business reviews.

Latent Dirichlet Allocation (LDA) is a generative model proposed by Blei et al., 2003 [4], that can be used to solve this problem. It is a three-level hierarchical Bayesian model in which document is modeled as a mixture of underlying topics, and each topic is modeled by underlying probabilities of topics. LDA treats topics as the latent variables of the document model that are learned throughout the training process. There are other applications of LDA as well, such as in document classification and collaborative filtering; however, in this paper, we focus on the application of LDA to topic models.

We implement the model using both the variational inference method proposed in Blei et. al, 2003, as well as using the collapsed Gibbs Sampling method of Griffiths and Steyvers, 2004 [1]. Both methods offer a different perspective on how to train the LDA model using unsupervised training data, and each method has its advantages and disadvantages in terms of performance and implementation. To evaluate each method, we adapt the topic modeling experiment Blei et al., 2003, to compare the convergence of the training phase for each method using the TREC AP corpus [6] as our unlabeled training and testing dataset. We present our findings from this experiment and the subsequent conclusions gained from them.

1.1   Related Works

It is important to note that LDA is not the only probabilistic document model that exists. Other works in topic and document modeling using latent variables have been done using the unigram model,

mixture of unigrams, and probabilistic latent semantic indexing (pLSI). In the unigram model, words in each document are independently generated from a single multinomial probability distribution for the entire vocabulary of words. This model is highly limited in that each word, essentially, is its own topic.

In the mixture of unigrams model proposed by Nigam et al., 2000 [7], documents are generated by first choosing a topic from a distribution of all topics and then independently generating the words from a distribution of the vocabulary conditioned on the chosen topic. This model also highly over-fits to the training set. Each document is limited to being generated from one topic and overfitting to the training document set is a major issue.

The pLSI model proposed by Hofmann, 1999 [8], attempts to relax the assumption in the mixture of unigrams to allow documents to be generated from multiple topics with varying probabilities. However, it is not a well-defined model since it there is no intuitive way to assign probabilities to previously unseen documents in the testing set and it greatly overfits the model to the training set due to conditioning the model based only on the training set.

LDA overcomes both the problems these three other models encounter with overfitting and limited document models. It allows for documents to be generated from distribution of topics with varying probabilities from which the words are then generated. There is more flexibility in learning the hidden or latent topics, and the model generalizes better to new documents.

2 Notation

We used similar notation as those denoted in the paper:
K = number of topics
D = number of documents
V = vocabulary size
N = number of words in total.
z_j = the j-th topic
d_i = the i-th document
w_i = the i-th word in the document

3 LDA

The latent Dirichlet allocation (LDA) model explains the generation of text documents, which can be viewed as samples from a mixture of multinomial distributions over a vocabulary of words. Each multinomial mixture component is called a topic. The general process of the model to write a document is the following:

- The number of words N in document $\sim$ Poisson (ζ)

- The topic mixture θ for the document $\sim$ Dir (α) (with a fixed set of K topics)

- Then for each word w_i in the document:

  1. Choose a topic t_i based on multinomial distribution with parameter θ from step (2)
  2. Use this topic z_i to generate word itself by using the existing probability for each word in this topic (i.e. $p(w_i|z_i, \beta)$

This is the generative model for a collection of documents. LDA then tries to backtrack from the (training) documents to find a set of topics that are likely to have generated the collection. Now the question is how does LDA backtrack to find the parameters in this model. Suppose we have a set of documents D, and we set the number of topics to be K. What we want is to use LDA to learn two things 1) the topic representation of each document and 2) the words associated to each topic.

There are two methods to learn these two things: collapsed Gibbs sampling [1] and variational inference [2, 3]. We first discuss Gibbs sampling method here:

Figure 1: Graphic model of LDA. The boxes are "plates" representing replicates. The outer plate represents documents, while the inner plate represents the repeated choice of topics and words within a document.

3.1 Gibbs Sampling Method

- For each document, randomly assign each word in the document to one of the K topics.

    1. this step gives topic representation of all the documents
    2. this step gives word distributions of all the topics
    3. since randomly assign topics to each word is very native, so we need to improve it

- For each word w_k in document d_i

    1. For each topic t_j that this word belongs to, compute:
        (a) $p(z_j|d_i) = \frac{\text{number of words assigned to } z_j \text{ in } d_i}{\text{total number of words in } d_i}$
        (b) $p(w_k|z_j) = \frac{\text{number of words assigned to } z_j \text{ in } d_i}{\text{number of words assigned to } z_j \text{ for all docs}}$

- we compute the product of i) and ii) above which gives the new topics to assign to this word.

- repeating step 2 over and over until it reaches a steady state where the assignments make good sense.

- Use this model to estimate the topic mixtures of each document and words associated to each topic, which are the two things we want to learn.

3.2 Inference Method

With the same Dirichlet distribution model assumption shown in the previous section, variational parameters (γ and ϕ) are introduced. The way to do this is to place a distribution q over hidden variables (θ and Z) with free parameters which are the so called variational parameters. Then, an optimization process can be performed to make the placed distribution close to the posterior in KL divergence:

$$D(q(\theta, Z|\gamma, \phi)||p(\theta, Z|W, \alpha, \beta)) = \log p(W|\alpha, \beta) - L(\gamma, \phi; \alpha, \beta) \tag{1}$$

where

$$L(\gamma, \phi; \alpha, \beta) = E_q[\log p(\theta, Z, W|\alpha, \beta)] - E_q[\log q(\theta, Z)] \tag{2}$$

However, since the KL divergence is intractable, Jensen's inequality is applied to yield an evidence lower bound. Minimizing the KL divergence is equivalent to maximizing the evidence lower bound [5]. The posterior distribution can be obtained with those variational parameters using EM algorithm. In the maximization step, the gradient $g(\alpha^t)$ and Hessian $H(\alpha^t)$ are calculated using the evidence lower bound

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Algorithm (a) E-step:

1: **Estimation**
2: Initializatize: $\phi^0_{ni} = 1/K$ for all i and n
3: Initializatize: $\gamma^0_i = \alpha_i + N/K$ for all i
4: repeat
5: **for** $n = 1$ to N **do**
6:    **for** $i = 1$ to K **do**
7:       $\phi^{t+1}_{ni} = \beta_{iw_n} \exp\{\varphi(\gamma^t_i)$
8:       $-\varphi(\sum^K_{j=1} \gamma^t_j)\}$
9:    **end forend**
10: **end forend**
11: $\gamma^{t+1} := \alpha + \sum^N_{n=1} \phi^{t+1}_n$
12: $t \leftarrow t + 1$
13: until convergence

(a) E-step

Algorithm (b) M-step:

1: **Maximization**
2: Update β
3: **for** $i = 1$ to K **do**
4:    **for** $j = 1$ to V **do**
5:       $\beta_{ij} \propto \sum^D_{d=1} \sum^{N_d}_{n=1} \phi_{dni} w^j_{dn}$
6:    **end forend**
7: **end forend**
8: Update α
9: repeat
10: Caculate gradient $g(\alpha^t)$ and Hessian $H(\alpha^t)$
11: $\alpha^{t+1} = \alpha^t - H^{-1}(\alpha^t)g(\alpha^t)$
12: $t \leftarrow t + 1$
13: until convergence

(b) M-step

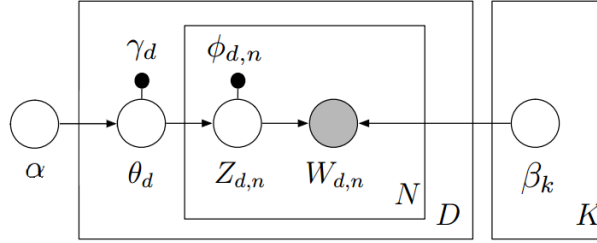</div>

Alg. 1: Two algorithms

Figure 2: Graphical model representation of the variational distribution used to approximate the posterior in LDA.

$$\frac{\partial L}{\partial \alpha_i} = D(\Psi(\sum_{j=1}^{K} \alpha_j) - \Psi(\alpha_i)) + \sum_{d=1}^{D}(\Psi(\gamma_{di}) - \Psi(\sum_{j=1}^{K} \gamma_{dj})) \tag{3}$$

$$\frac{\partial L}{\partial \alpha_i \alpha_j} = D\Psi'(\sum_{j=1}^{K} \alpha_j) - \delta(i,j)D\Psi'(\alpha_i) \tag{4}$$

When applying LDA to training documents, the estimation and maximization steps are iteratively applied until the convergence of the evidence lower bound (or equivalently, until the convergence of perplexity $\exp\{-L/N\}$).

In the maximization step, the gradient $g(\alpha^t)$ and Hessian $H(\alpha^t)$ are calculated using the evidence lower bound $L(\gamma, \phi; \alpha, \beta)$:
$g(\alpha^t)$

When applying LDA to training documents, the estimation and maximization steps are iteratively applied until the convergence of the evidence lower bound (or equivalently, until the convergence of perplexity $e^{-L/N}$).

4 Evaluations and Empirical Results

4.1 Dataset and Preprocessing

To evaluate our algorithm, we replicated one of the experiments implemented by Blei et al. on document modeling. We used AP dataset which contains 2246 (verify this number) news articles from the associated press [6]. The dataset was partitioned into 80 percent training set and 20 percent

test data. In preprocessing of the data, we removed a list of stop words from the text. We further removed words that appear less than 20 times (rare words), as well as words that appear in more than 90 percent of the documents (common words). We ended up with a vocabulary size of 3564.

4.2 Convergence of Algorithms

In AP dataset, each document is unlabeled. Therefore, here we are doing unsupervised learning with a purpose to estimate the likelihood of the test dataset. In natural language processing, the likelihood of a document is usually called *perplexity*, which is the inverse of the average per-word log likelihood. The perplexity of on the text corpus is defined by

$$perplexity(D_{test}) = \exp\left\{-\frac{\sum_{d=1}^{M}\log p(w_d)}{\sum_{d=1}^{M}N_d}\right\} \tag{5}$$

$$= \exp\left\{-\frac{1}{N}\sum_{d=1}^{D}\sum_{i=1}^{N_d}N_{di}\log\sum_{k=1}^{K}\theta_{w_{di},k}\phi_{d,k}\right\} \tag{6}$$

In Gibbs sampling LDA, $\theta_{v,k}$ and $\phi_{d,k}$ can be estimated by

$$\theta_{v,k} = \frac{C_{vk}+\beta}{\sum_{v'}C_{v'k}+\beta}, \phi_{d,k} = \frac{C_{dk+\alpha}}{\sum_{k'}C_{dk'}+K\alpha}$$

While implementing our algorithms, we first monitors the convergence of the algorithm. This was done by monitoring the perplexity of training data, as shown Figure 3. We can see that in both cases the algorithm converges after enough number of iterations, although the numbers of iterations required to reach convergence are very different. Variational inference LDA usually converges after about 10 iteration, while Gibbs sampling LDA needs several hundred of iterations. This is a primary reason that the variational inference algorithm is much faster and more commonly used in practice.

4.3 Extracted Subtopics

We trained a 10-topic LDA model on the training dataset using both Gibbs sampling and variational inference, and 5 of the generated subtopics in shown in Figure 4. For each topic, the 10 words that are most likely to occur is listed. As we expected, LDA automatically generated meaning full subtopics from the AP corpus, with similar words more likely to appear in the same topic. We also note that the choice and order of words in the topic generated by Gibbs sampling and variational inference are different, although the topics they represented are similar. We think this discrepancy is because the two algorithms we used might converge into different local opitma, and each algorithm used (different) random initialization.

5 Conclusion

Discuss the subsequent conclusions we gained from this reimplementation of LDA. Summarize advantages and disadvantages as well.

Author Contributions

All authors contributed to the overall study of LDA and its implementation. Xiang preprocessed AP dataset in Python to generate input matrices for LDA, and implemented Gibbs Sampling LDA in MATLAB. Zheng implemented variational inference LDA in MATLAB. Sajan implemented the Unigram model in MATLAB. Yan wrote MATLAB functions for calculating perplexity and extracting subtopics from LDA outputs. All authors participated in final data analysis and interpretation, as well as writing of the manuscript.
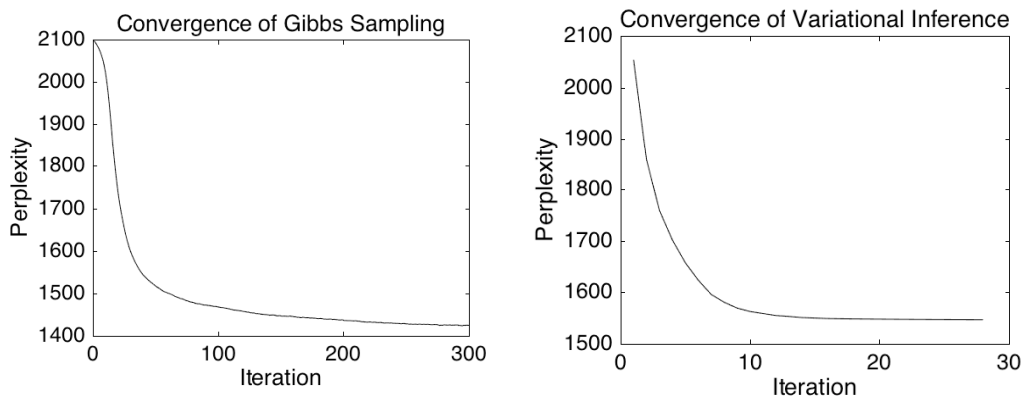
Figure 3: Convergence of LDA using Gibbs Sampling (*left*) and Variational Inference (*right*)

"Law"	"War"	"Trade"	"Politics"	"Company"
court	united	percent	government	million
state	states	market	soviet	year
federal	military	prices	party	billion
case	war	stock	union	company
department	president	dollar	south	new
law	american	trade	gorbachev	workers
attorney	iraq	year	political	based
judge	officials	late	west	last
office	aid	oil	country	corp
former	israel	higher	president	co

(a) List of topics generated by Gibbs sampling.

"Law"	"War"	"Trade"	"Politics"	"Company"
court	president	dollar	dukakis	percent
year	bush	late	bush	million
years	united	new	new	year
one	states	one	year	billion
two	government	yen	campaign	market
new	new	air	people	new
state	year	london	president	stock
people	soviet	two	state	prices
case	military	york	one	company
last	house	bid	democratic	last

(b) List of topics generated by variational inference.

Figure 4: List of topics generated by LDA using two difference algorithms.

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