CS660
Homework 4

Complete all **four** problems. Put your pages in order and scan your solutions and upload one PDF. I will not grade multiple files, jpegs, Mac Pages, or any other image files. Each problem is worth 4 points for a total of 20 points.

1. Compute the derivative $f'(x)$ for

$$f(x) = e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where μ and σ can be treated like constants.

2. Compute the derivatives df/dx of the following functions. Describe your steps in detail.

(a) Use the chain rule. Provide the dimensions of every single partial derivative.

$$f(z) = e^{-\frac{1}{2}z}$$

$$z = g(\boldsymbol{y}) = \boldsymbol{y}^\top \boldsymbol{S}^{-1}\boldsymbol{y}$$

$$y = h(\boldsymbol{x}) = \boldsymbol{x} - \boldsymbol{\mu}$$

where $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\mu} \in \mathbb{R}^{\boldsymbol{D}}$, $\mathbf{S} \in \mathbb{R}^{D\times D}$

(b) $f(\boldsymbol{x}) = tr(\boldsymbol{x}\boldsymbol{x}^\top + \sigma^2\boldsymbol{I})$, $\boldsymbol{x} \in \mathbb{R}^D$ (Hint: $\boldsymbol{x}\boldsymbol{x}^\top$ is the outer product, so you perform the outer product operation explicitly first to make it easier.)

(c) Use the chain rule. Provide the dimensions of every single partial derivative. You do not need to compute the product of the partial derivatives explicitly.

$\boldsymbol{f} = \sin(\boldsymbol{z}) \in \mathrm{R}^M$
$\boldsymbol{z} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}$, $\boldsymbol{x} \in \mathrm{R}^N$, $\boldsymbol{A} \in \mathrm{R}^{M\times N}$, $\boldsymbol{b} \in \mathrm{R}^M$

Here, sin is applied to every component of $\boldsymbol{z}$.

3. Consider the following functions:

$f_1(\boldsymbol{x}) = \sin(x_1)\cos(x_2), \boldsymbol{x} \in \mathrm{R}^2$

$f_2(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{x}^\top\boldsymbol{y}, \boldsymbol{x},\boldsymbol{y} \in \mathrm{R}^n$ $f_3(\boldsymbol{x}) = \boldsymbol{x}\boldsymbol{x}^\top, \boldsymbol{x} \in \mathrm{R}^n$

(a) What are the dimensions of $df\ /d\boldsymbol{x}$? [i]

(b) Compute the Jacobians.

4. Let $f(x) = e^x$.

(a) Find the Maclaurin series for f.
(b) Use the result from (a) to find the Maclaurin series for $g(x) = xe^x$.