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Assignment 3.

Exercise 4.1

a) Find the covariance, cov (x, y).

$$cov(x,y) = \frac{\sum_{i=1}^{n} (x_i - \mu(x))(y_i - \mu(Y))}{n}$$

$$= E((X-\mu(x))(Y-\mu(Y)) = -184.69$$

	X	Y	$X-\mu(X)$	$Y-\mu(Y)$	$(X-\mu(X))(Y-\mu(Y))$
	45	5	17.1429	-17.1429	-293.8776
	10	30	-17.8571	7.8571	-140.3061
	25	20	-2.8571	-2.1429	6.1224
	40	10	12.1429	-12.1429	-147.4490
	15	40	-12.8571	17.8571	-229.5918
	55	15	27.1429	-7.1429	-193.8776
	5	33	-22.8571	12.8571	-293.8776
μ	27.857	22.143			-184.6900

b) Find the covariance around median $cov_e(X,Y)$.

$$cov_e(X,Y) = \frac{\sum_{i=1}^{n} (x_i - \mu_e(x))(y_i - \mu_e(Y))}{n}$$

$$= E(X-\mu_e(X))(Y-\mu_e(Y)) = -178.57$$

X	Y	$X - \mu_e(X)$	$Y - \mu_e(Y)$	$(X-\mu_e(X))(Y-\mu_e(Y))$
45	5	20	-15	-300
10	30	-15	10	-150
25	20	0	0	0
40	10	15	-10	-150
15	40	-10	20	-200
55	15	30	-5	-150
5	35	20	15	-300
μ 25	20			-178.57

c) Find the Pearson correlation coefficient $\rho(X,Y)$

$$\text{cov}(X,Y) = -184.69$$
$$\sigma(X) = 17.699$$
$$\sigma(Y) = 12.206$$
$$\rho(X,Y) = \frac{\text{cov}(X,Y)}{\sigma(X)\,\sigma(Y)}$$
$$= -\frac{184.69}{17.699 \times 12.206}$$
$$= -0.8549$$

d) Find the concordance correlation coefficient $\rho_c(X,Y)$

$$\text{cov}(X,Y) = -184.69$$
$$\mu(X) = 27.857$$
$$\mu(Y) = 22.143$$
$$\sigma^2(X) = 313.27$$
$$\sigma(Y) = 148.98$$

$$\rho_c(X,Y) = \frac{2\,\text{cov}(X,Y)}{\sigma^2(X) + \sigma^2(Y) + (\mu(X) - \mu(Y))^2}$$

$$= \frac{2X - 184.69}{313.27 + 148.98 + (27.857 - 22.143)^2}$$

$= -0.8599$

e) Find the Spearman rank correlation ~~method~~ coeffecient $\rho_s(X, Y)$

X	Y	R(X)	R(Y)
45	5	6	1
10	30	2	5
25	20	4	4
40	10	5	2
15	40	3	7
55	15	7	3
5	35	1	6

$$\rho_s(X, Y) = \rho(R(X), R(Y)) = -0.7857$$

$$= 1 - \frac{6}{n(n^2-1)} \sum_{i=1}^{n} (R(x_i) - R(y_i))^2$$

$$= 1 - \frac{6}{7 \times (7^2 - 1)} \times 100 = -0.7857$$

R(X)	R(Y)	$(R(X) - R(Y))^2$
6	1	25
2	5	9
4	4	0
5	2	9
3	7	16
7	3	16
1	6	25
		$\Sigma = 100$

f) Find the Kendall rank correlation coefficent, $\tau(X,Y)$

Kendall's tau, τ

X'	Y'	$R(X')$	$R(Y')$	
5	35	1	6	
10	30	2	5	d
15	40	3	7	c c
25	20	4	4	d d d
40	10	5	2	d d d d
45	5	6	1	d d d d d
55	15	7	3	d d d d c c

$C = 4$ and $D = 17$

$$\tau(X,Y) = \frac{C-D}{C+D} = \frac{4-17}{4+17} = 0.619$$

Exercise 4.2.

a) Find the variance values $\sigma^2(X_{1\sim 40})$ and $\sigma^2(Y_{1\sim 40})$.

$$\sigma^2(X_{1\sim n}) = \frac{(n-1)\left(\sigma^2(X_{1\sim n-1}) + \mu(X_{1\sim n-1})^2\right) + x_n^2}{n} - \mu(X_{1\sim n})^2$$

$$\sigma^2(X_{1\sim 40}) = \frac{39 \times (3871.5 + 282.179^2) + 255^2}{40}$$

$= 3792.5 \approx 3729.8$

$$\sigma^2(Y_{1\sim40}) = \frac{39 \times (140.6 + 28.744)^2 + 16^2}{40} - 28.425^2$$

$$= 141.07 \approx 141.04$$

b) Find the mean values, $\mu(X_{1\sim40})$ and $\mu(Y_{1\sim40})$

$$\mu(X_{1\sim n}) = \frac{(n-1)\mu(X_{1\sim n-1}) + x_n}{n}$$

$$\mu(X_{1\sim40}) = \frac{39 \times 282.179 + 255}{40} = 281.5$$

$$\mu(Y_{1\sim40}) = \frac{39 \times 28.744 + 16}{40} = 28.425$$

c) Find the covariance $cov(X_{1\sim40}, Y_{1\sim40})$

$$cov(X_{1\sim40}, Y_{1\sim40}) = \frac{(n-1)\left(cov(X_{1\sim n-1}, Y_{1\sim n-1}) + \mu(X_{1\sim n-1})\mu(Y_{1\sim n-1})\right) + x_n y_n}{n}$$

$$- \mu(X_{1\sim n})\mu(Y_{1\sim n})$$

$$cov(X_{1\sim40}, Y_{1\sim40}) =$$

$$\frac{39 \times (716.71 + 282.179 \times 28.744) + 255 \times 16}{40} - 281.5 \times 28.425$$

$$= 707.33 \approx 707.24.$$

d) Find the correlational coeffecient $\rho(X_{1\sim40}, Y_{1\sim40})$.

$$\rho(X,Y) = \frac{cov(X,Y)}{\sigma(X)\sigma(Y)}$$

$$= \frac{707.33}{\sqrt{3792.5 \times 141.07}} = 0.967$$

e) Find the concordance correlation coeffecient $\rho_c(X_{1\sim40}, Y_{1\sim40})$

$$\rho_c(X,Y) = \frac{2cov(X,Y)}{\sigma^2(X)+\sigma^2(Y)+(\mu(X)-\mu(Y))^2}$$

$$= \frac{2\times707.33}{3792.5+141.07+(281.5-28.425)^2}$$

$$= 0.02081.$$

Exercise 4.3.

a) Find the mean values, $\mu(X)$ and $\mu(Y)$.

$$\mu(X) = \frac{n_a\mu(X_a)+n_b\mu(X_b)}{n_a+n_b}$$

$$\mu(X) = \frac{100\times1.0579+152\times1.054}{252} = 1.0555 \approx 1.0556$$

$$\mu(Y) = \frac{100\times18.191+152\times19.7822}{252} = 19.1508 \approx 19.1508$$

b) Find the variance values, $\sigma^2(X)$ and $\sigma^2(Y)$.

$$\sigma^2(X) = \frac{n_a(\sigma^2(X_a) + \mu(X_a)^2) + n_b(\sigma^2(X_b) + \mu(X_b)^2)}{n_a+n_b} - \mu(X)^2$$

$$\sigma^2(X) = \frac{100\times(4.1948 + 10^{-4} + 1.0579^2) + 152\times(3.1603\times10^{-4} + 1.054^2)}{252} - 1.0555^2$$

$$= 3.6072\times10^{-4} \approx 3.0676\times10^{-4}$$

$$\sigma^2(Y) = \frac{100\times(80.763 + 18.191^2) + 152\times(61.593 + 19.7822^2)}{252} - 19.151^2$$

$$= 69.7579 \approx 69.7579.$$

c) Find the covariance, $\mathrm{cov}(X,Y)$.

$$\mathrm{cov}(X,Y) = \frac{n_a(\mathrm{cov}(X_a, Y_a) + \mu(X_a)\mu(Y_a)) + n_a(\mathrm{cov}(X_b, Y_b) + \mu(X_b)\mu(Y_b))}{n_a+n_b} - \mu(X)\mu(Y).$$

$$= \frac{100\times(-0.1795 + 1.0579\times18.91) + 152\times(-0.1392 + 1.054\times19.7822)}{252} - 1.0555\times19.1508$$

$$= -0.1567 \approx -0.1567$$

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d) Find the correlational coeffecient $\rho(X,Y)$

$$\rho(X,Y) = \frac{cov(X,Y)}{\sigma(X)\sigma(Y)}$$

$$\rho(X,Y) = \frac{-0.1567}{\sqrt{3.6072 \times 10^{-4}}\sqrt{69.7579}}$$

$$= -0.9877 \approx -0.9878$$

e) Find the concordance correlation coeffecient $\rho_c(X,Y)$

$$\rho_c(X,Y) = \frac{2cov(X,Y)}{\sigma^2(X)+\sigma^2(Y)+(\mu(X)-\mu(Y))^2}$$

$$= \frac{2 \times -0.1567}{3.6072 \times 10^{-4} + 69.7579 + (1.0555 - 19.1571)^2}$$

$$= -7.8901 \times 10^{-4}$$

$$\approx -7.8903 \times 10^{-4}$$

f) Find the following mean values:
$\mu(X_{la} - \{1.04843\})$, $\mu(Y_{la} - \{22.02\})$, $\mu(X_{ny} \cup \{1.04843\})$,
and $\mu(Y_{ny} \cup \{22.02\})$.

$$\mu(X_{1 \sim n}) = \frac{(n-1)\,\mu(X_{1 \sim n-1}) + x_n}{n}$$

$$\mu(X_{ny} \cup \{1.04843\}) = \frac{152 \times 1.054 + 1.0484}{153} = 1.0539634$$

$$\mu(Y_{ny} \cup \{22.2\}) = \frac{152 \times 19.7822 + 22.2}{153} = 19.798$$

$$\mu(X_{1\sim n-1}) = \frac{n\mu(X_{1\sim n}) - x_n}{n-1}$$

$$\mu(X_{la} - \{1.0484\}) = \frac{100 \times 1.0579 - 1.0484}{99} = 1.058$$

$$\mu(Y_{la} - \{22.2\}) = \frac{100 \times 18.91 - 22.2}{99} = 18.1505$$

g) Find the following variance values:

$\sigma^2(X_{la} - \{1.0484\})$, $\sigma^2(Y_{la} - \{22.2\})$, $\sigma^2(X_{ny} \cup \{1.0484\})$, and $\sigma^2(Y_{ny} \cup \{22.2\})$

$$\sigma^2(X_{1\sim n}) = \frac{(n-1)\left(\sigma^2(X_{1\sim n-1}) + \mu^2(X_{1\sim n-1})\right) + x_n^2}{n} - \mu^2(X_{1\sim n})$$

$$\sigma^2(X_{ny} \cup \{1.0484\}) = \frac{152 \times (3.1603 \times 10^{-4} + 1.054^2) + 1.0484^2}{153} - 1.0539634^2$$

$$= 3.1417 \times 10^{-4} \text{ or } 2.3701 \times 10^{-4} \leq \text{answer} \leq 3.1417 \times 10^{-4}$$

$$\sigma^2(Y_{ny} \cup \{22.2\}) = \frac{152 \times (61.513 + 19.7822^2) + 22.2^2}{153} - 19.798^2$$

$$= 61.149 \approx 61.1487$$

$$\sigma^2(X_{1\vee n-1}) = \frac{n(\sigma^2(X_{1\vee n}) + \mu^2(X_{1\vee n})) - x_n^2}{n-1} - \mu^2(X_{1\vee n-1})$$

$$\sigma^2(X_{1a} - \{1.04844\}) = \frac{100 \times (4.1948 \times 10^{-4} + 1.0579^2)}{99} - 1.058^2$$

$$= 4.1425 \times 10^{-4} \approx 4.2279 \times 10^{-4}$$

$$\sigma^2(Y_{1a} - \{22.02\}) = \frac{100 \times (80.763 + 18.0191^2) - 22.02^2}{99} - 18.1505^2$$

$$= 81.415$$

h) Find the following covariance values:

$cov(X_{1a} - \{1.04844\}, Y_{1a} - \{22.02\})$ and

$cov(X_{ny} \cup \{1.04844\}, Y_{ny} \cup \{22.02\})$

$cov(X_{1\vee n}, Y_{1\vee n}) =$

$$\frac{(n-1)(cov(X_{1\vee n-1}, Y_{1\vee n-1}) + \mu(X_{1\vee n-1})\mu(Y_{1\vee n-1})) + x_n y_n}{n}$$

$$- \mu(X_{1\vee n})\mu(Y_{1\vee n})$$

$cov(X_{ny} \cup \{1.04844\}, Y_{ny} \cup \{22.02\}) =$

$$\frac{152(-0.1392 + 1.054 \times 19.7822) + 1.0484 \times 22.02}{153} - 1.0539834 \times 19.798$$

$$= -0.1384$$

$cov(X_{1\vee n-1}, Y_{1\vee n-1}) =$

$$\frac{n(cov(X_{1\vee n}, Y_{1\vee n}) + \mu(X_{1\vee n})\mu(Y_{1\vee n})) - x_n y_n}{n-1} - \mu(X_{1\vee n-1})\mu(Y_{1\vee n-1})$$

$$\text{Cov}(X_{La} - \{1.0484\}, Y_{La} - \{22.2\}) =$$

$$\frac{100(-0.1795 + 1.0579 \times 18.191) - 1.0484 \times 22.2}{99} - 1.058 \times 18.1505$$

$$= 0.181 \approx 0.1809$$