1 1.4	Page							
4/10/24	CS 660 Homework 2							
0	Consider 123 with the inner product							
	$\langle n, y \rangle = \pi \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix} y$							
	12-19							
	0 71 2							
	Furthermore, we define e, e, and ez as the standard/ Canonical bases on 123.							
(0)	Determine the outhoponal perojection #4 (ex) of ez onto &							
(3)	U= span [e, e3].							
(b)	Compute the distance d (ez, U)							
	Given,							
	$\langle n, y \rangle = n^{T} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$							
	1 2 -1							
1-11	6 8 2 (100) [2 1 2 2 (2)]							
	·(e, e3) = (1,0,0) 2 1 0 ) (0)							
	0-12 1							
	Law Assessment Alexander							
	$= (100) \left(2x0+1x0+0x1\right)$							
	1x0+2x0+(-1)x1							
	0x0+(-1)x0+2x1							
	Voltanol +							
	2 (100) (0)							
	2 1x 0 + 0 x ( 1) + 0 x 2							
	2 1x0f0x (-1) f 0x2							



Therefore  $e_1$  and  $e_2$  are ofthogonal to each other.  $\Rightarrow \langle e_1, e_1 \rangle = \langle 1, 0, 0 \rangle \langle 2 | 0 \rangle \langle 1 \rangle \langle 1$ 

 $\frac{2(100)(2x1+1x0+0x0)}{(1x1+2x0+(-1)x0)}$   $\frac{2x1+1x0+0x0}{(1x1+2x0+(-1)x0)}$ 

= (100) (2)

= |X2+0X1+0X0

 $\Rightarrow \langle e_3, e_3 \rangle^2 = (0, 0, 1) \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ 

 $\frac{2(001)(2x0+1x0+0x1)}{1x0+2x0+(-1)x1}$   $\frac{0x0+(-1)x0+2x1}{0x0+(-1)x0+2x1}$ 

<sup>2</sup> (001) (0) (-1) (2)

= 0x0+ (-1) x0+1x2

Therefore \\ \frac{1}{52} \, \

and U= span {e, e, y = span { 1 e, d e2}



Now, 
$$\langle e_2, e_1 \rangle = (0 10) \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 10 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

$$\langle e_2, e_3 \rangle = (0 \mid 0) \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \mid 0 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix}$$

b) Required distance d (e2, u) = distance between e2
and tu(er)

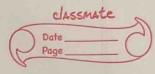
= \( (-1/2, 1, 1/2) g (-1/2, 1, 1/2) >

(omputing < (-1/2, 1, 1/2)>

$$= (-1/2) 1/2 ) (2x-1/2+1x1+0x/2) 1x-1/2+2x1+(-1)x1)+2x1/2 0x+1/2+(-1)x1)+2x1/2 )$$

$$= \left(-\frac{1}{2} \right) \left(-\frac{1}{1} + 0\right) \\ -\frac{1}{2} + 2 - \frac{1}{2} \\ 0 - |+|$$

2



Therefore the grequired dectance is = V(1/2,1,1/2), (1/2,1,1/2) 2 Let w be the sequence of 12 spanned by U, = [1] and

12 [-2]

[1] Find the vector (s) than span(s) the osuthogonal complement who fw:

> we need to find vectors that are outhogonal to both up and us. Let v= (n, y, z) be a vector en W. To be orthonogonal component, v' must satisfy e'  $(u_1, v) = 0$  and  $(u_2, v) = 0$ .  $(u_1, v) = 0$  and  $(u_2, v) = 0$ .  $(u_1, v) = 0$  and  $(u_2, v) = 0$ . <u, v> = ant by + cz & n 123, where uz {a, b, c} and v= {n, y, z}. Solving equations (1) and (ii) 2 + 2y + 3z = 0 72 - 2y + 11z = 0 8n+1/2=0. 2=-4 n



Semplifying = & n eq. (9) n+2y+3(-4)n 20.

n - 12n + 2y = 0

-5n+2y=0.

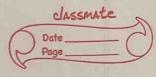
y= 5 n.

So, any vector of the form  $v = \{n, 5, n, -4, n\}$ will be orthogonal to both u, and us and thus span
the orthogonal complement who five.

We can choose any nonzero value for n to obtain a

If n=14, then v=[14,5,-8] span W.

.. One wector that spans W- 95 [14,5, -8]



Let w be the Rubspace of R spanned by u= Fonda basis of the osthogonal complement W of w a and v, 9. e (u. 2) = 0) and (v. n = 0), where n= From (u n=0):-[1n,+2n2+3n3-1ny+2ny-0] : n, + 2n2 + 3n2 - ny + 2ng=0 (9) From condition (von-0):-2n, +4n2 + 7n3 + 2ny - n5 = 0 From eq (?): n, z -2n2 -3n3+ny-2nj-puting it in eq (i) = 2 2n2 - 3n3+ny -2x5]+4n2+7n3+2ny-n5-=0. 4n2 +-6n3 +2ny -4ng +4n2 +7n3 +2ny -ng =0. [(4n2-4n2)+(-6n3+7n3)+(2ny+2ny)+(-4ng-2g)=0] :n3+4ny -5n-=0 - (ii) so, n3 = 5n5 - 4ng

Substituting n3 m m: n=-2n2-3 (5nj-4ny) + my-2mg

-2n2-15n5+1284+7ny-2ns

n12-2n2+13my-17n5-

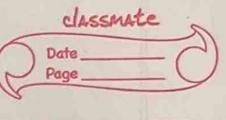
: The averangement of h is:

-2n2+13ny-17n5 57g-4ny

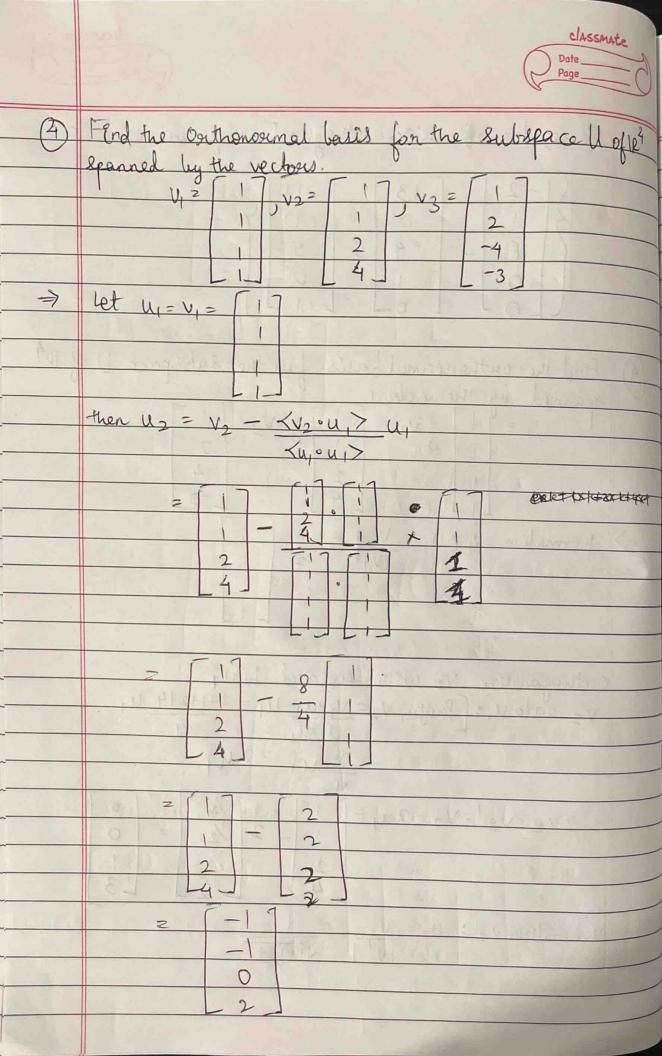
The perensise vectors are e-(For (n<sub>2</sub> = 1), (n<sub>y</sub>-0), (n<sub>5</sub>-=0):

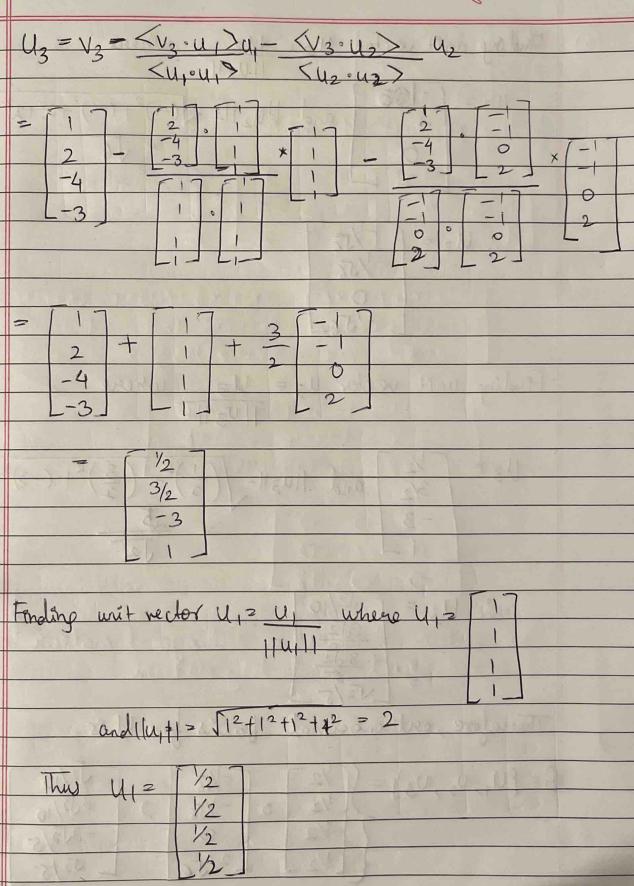
For (n2=0), (ny=1), (nj=0)=

For (n = 0), (ny=0), (ng=1)=



1,1	Hence	, the	Reca	midda	onth	oumal	6	mponent W + 95:
N.		Andi	B	1. G		8		
		-27		13		[-17]	7	
	3	110		0		0	4	
		0	3	-4	9	5		
		0			17	0		
		LOJ		0				LL VILLE FAIR C
				1		W B		





	Finding unit voctor U2 = U2 where
	$u_2 = \sqrt{-1000}$ and $  u_2  _2 = \sqrt{(-1)^2 + (-1)^2 + 0^2 + 2^2}$
	2
0	2
	: U2 = -156 -156
	2/56
	The state of the s
	Finding unit ve dor Uz = Uz where
	1/43/1
	1/2 = [1/2] . [0.120 (0.20)
	$\frac{1}{3} = \frac{1}{2}$ and $\frac{1}{2} = \frac{3}{2} = $
	-3
	12
	: U3 = 52/10
	352
	-3JZ -3JZ -72/5
	B= {U, U2, U3} = { /2 } -/56 } 52/10 }
	1/2 5 -1/56 5 3/2/10
	3,75
	[ 1/2 ] L2/56 ] _ J2/5

(5)	Let A=	TILL	-1
		1 3	4
		1 - 5	2

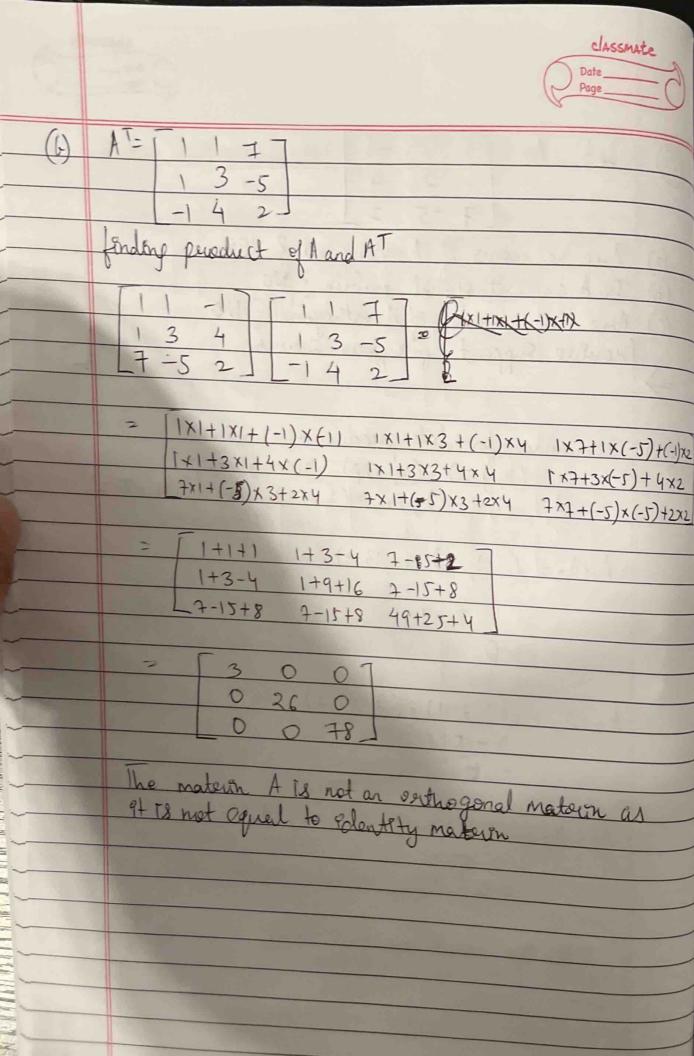
- (a) Asse the seaws of A orthogonal? YES
  (b) Is A an orthogonal matrin? No
  (c) Here the columns of A orthogonal? No
  Poroulde Suppost for your answers.

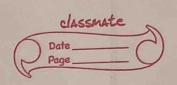
$$A_1 \circ A_2 = (1, 1, -1) \circ (1, 3, 4)$$

$$A_1 \cdot A_3 = (1, 1, -1) \cdot (7, -5, 2)$$

$$= 7 - 5 - 2$$

... All the nows are orthogonal





(c)	The Column vectors	ano	Acı =	(1,1	(+)	Ac2 = (	1,3,	-5),
	A(3=(-1,4,2)			( ) .				

$$A_{1} \cdot A_{12} = (1,1,7) \cdot (1,3,-5)$$

$$= 1+3-35$$

$$= -31$$

$$A c_3 \circ A_{(2} = (-1, 4, 2) \circ (1, 3, -5)$$
  
= -1+12-10

The columns are not outhogonal to each other