

# PROBABILITY DISTRIBUTION

## Discrete and Continuous Random Variable

A random variable may be discrete or continuous. A random variable is said to be discrete if it assumes only specified values in an interval.

When  $X$  takes values  $1, 2, 3, 4, 5, 6, \dots$  it is a discrete variable.

A random variable is said to be continuous if it can assume any value in a given interval. When  $X$  takes any value in a given interval  $\star, (a, b)$  it is a continuous variable in that interval.

### Continuous

Probability function of a continuous random variable  
(Probability Density function)

Let  $X$  be a continuous random variable.

Let the possible range of values of  $X$  be the interval  $(a, b)$ . Consider a small interval within the interval  $(a, b)$  say  $(x, x+dx)$ . where,  $x$  is any value of  $X$ .

Let, probability for  $x$  lying between  $x$  and  $x+dx$ .

$P(b \leq x \leq b+dx)$  be denoted by  $f(x)dx$ .

Then  $f(x)dx$  is called probability density function of  $x$ .

### Properties

1)  $f(x) \geq 0$ ,  $\forall x$

2)  $\int f(x) dx = 1$

3)  $f(x)dx$  is probability for  $x$  lies between  $x$  and  $x+dx$ .

1- Show that  $f(x) = e^{-x}$ ,  $x > 0$  is a probability density function.

Ans:  $e^{-x} \geq 0$

$$\begin{aligned} 2) \int f(x) dx &= \int_0^\infty e^{-x} dx \\ &= \left[ -e^{-x} \right]_0^\infty \\ &= 1 \end{aligned}$$

Probability density function  $\neq 0$  for all  $x$ .

Therefore,  $f(x) = \frac{e^{-x}}{\int_0^\infty e^{-x} dx} = \frac{e^{-x}}{1}$

$$\begin{aligned} \text{Ans: } f(x) &= \frac{e^{-x}}{\int_0^\infty e^{-t} dt} \\ &= \frac{e^{-x}}{1} \\ &= e^{-x} \end{aligned}$$

2. If prove that,  $f(x) = \frac{12}{37} (10-x)x^2$  is a probability density function for  $0 \leq x \leq 1$ .

$$f(x) = \frac{12}{37} (10-x)x^2, \quad 0 \leq x \leq 1$$

$$\int f(x) dx = \int_0^1 \frac{12}{37} (10-x)x^2 dx$$

$$= \frac{12}{37} \int_0^1 (10-x)x^2 dx$$

$$= \frac{12}{37} \int_0^1 (10x^2 - x^3) dx$$

$$= \frac{12}{37} \left[ \int_0^1 10x^2 dx - \int_0^1 x^3 dx \right]$$

$$= \frac{12}{37} \left[ 10 \left[ \frac{x^3}{3} \right]_0^1 - \left[ \frac{x^4}{4} \right]_0^1 \right]$$

$$= \frac{12}{37} \left\{ 10 \left( \frac{1^3}{3} - 0 \right) - \left( \frac{1^4}{4} - 0 \right) \right\}$$

$$= \frac{12}{37} \left\{ 10 \left( \frac{1}{3} - 0 \right) - \frac{1}{4} \right\}$$

$$= \frac{12}{37} \left( 10 \times \frac{1}{3} - \frac{1}{4} \right)$$

$$= \frac{12}{37} \left( \frac{10}{3} - \frac{1}{4} \right)$$

$$= \frac{12}{37} \left( \frac{40 - 3}{12} \right)$$

$$= \frac{12}{37} \left( \frac{37}{12} \right)$$

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$$P(X=0) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

↓

$$P(X=1) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

↓

$$P(X=2) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

### Discrete probability distribution

#### ① Binomial distribution

A random variable  $X$  is said to follow

Binomial distribution with parameters  $n$  &  $p$ , if

its probability func's,

$$P(x) = f(x) = \begin{cases} \binom{n}{x} p^x q^{n-x}, & x = 0, 1, 2, \dots, n \\ 0, & \text{otherwise} \end{cases}$$

$0 < p < 1$

$q = 1-p$

$p+q=1$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \binom{n}{3} a^{n-3} b^3 + \dots + b^n$$

$X \sim B(n, p)$  for eg.  $X \sim B(10, \frac{1}{2})$

$$f(x) = \binom{10}{x} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{10-x} \quad \text{for } x = 0, 1, 2, 3, 4, 5, 6, \dots, 10$$

### Total Probability

$$\begin{aligned} P(x) &= P(x=0) + P(x=1) + P(x=2) + \dots + P(x=n) \\ &= \sum_{x=0}^n P(x) = \binom{n}{0} p^0 q^{n-0} + \binom{n}{1} p^1 q^{n-1} + \binom{n}{2} p^2 q^{n-2} + \dots + \binom{n}{n} p^n q^{n-n} \\ &= \sum_{x=0}^n \binom{n}{x} p^x q^{n-x} \end{aligned}$$

$$\begin{aligned} &= q^n + \binom{n}{1} p^1 q^{n-1} + \binom{n}{2} p^2 q^{n-2} + \dots + p^n \\ &= (q+p)^n \end{aligned}$$

Applying binomial theorem  $(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^k b^{n-k}$

In these,  $a = p$  = probability of success

$b = q$  = probability of failure

$n$  = no. of trials

$\rightarrow$   $f(x) = P(x)$  is the probability function of the binomial distribution  $X$  is the no. of success out of  $n$  trials

$\rightarrow$   $p$  represent probability for success in a single trial

Binomial distribution can be applied;

- 1) The random experiment has  $n$  outcomes, which can be called "success" & "failure".

2) Probability for success in a single trial remains constant from trial to trial of the experiment.

3) The experiment is repeated finite no. of times.

a) Trials are independent.

1- 4 coins are tossed simultaneously. What is the probability of getting 2 Heads.

Let,  $n = \text{no. of coins tossed} = 4$

$p = \text{Probability of getting head in a single}$

toss =  $\frac{1}{2}$

The distribution follows binomial with  $n=4$

$P = \frac{1}{2}$  therefore,  $P(x)$

$$P(x) = \binom{n}{x} p^x q^{n-x}$$

$$= \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$$

$$= \left(\frac{1}{2}\right)^4 \cdot \frac{1}{2}^x \cdot \frac{1}{2}^{4-x}$$

$P(\text{getting two heads}) = P(x=2)$

$$= P(2) \text{ binomial}$$

$$= \binom{4}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 \cdot \frac{4x^2}{16}$$

$$= 6 \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}$$

$$= \frac{6}{16} = \frac{3}{8} = \frac{3}{4} //$$

Probability that two batsmen score a century in a cricket match is  $\frac{1}{3}$ . Find the probability that out of 5 matches he may score century.

Ans:-

(i) exactly 2 matches.

(ii) No matches.

Let,  $n = \text{no. of matches} = 5$

$P = \text{Probability of getting century} = \frac{1}{3}$

The distribution follows binomial with  $n=5$

$P = \frac{1}{3}^x \cdot \frac{2}{3}^{5-x}$  therefore,

$$P(x) = \binom{5}{x} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{5-x}, x=0, 1, 2, 3, 4, 5$$

$$= \binom{5}{x} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{5-x}$$

(i)  $P(\text{getting century in 2 matches}) = P(x=2)$

$$= P(2) \text{ binomial}$$

$$= \binom{2}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^0$$

$$= 10 \times \frac{1}{9} \times \left(\frac{2}{3}\right)^3$$

$$= 10 \times \frac{1}{9} \times \frac{8}{27}$$

$$= \frac{10 \times 4}{81} = \frac{40}{81}$$

$$= 10 \times \frac{1}{9} \times \frac{8}{27}$$

$$= 10 \times \frac{8}{243}$$

$$= \frac{80}{243}$$

$\Rightarrow P(\text{getting exactly } n \text{ no matches}) = P(x=0)$

$$E = \text{number of } 0 = \binom{5}{0} \left(\frac{1}{3}\right) \left(\frac{2}{3}\right)^5$$

$$= 5 \times 0$$

$$P(x=0) \text{ or variance coefficient} = 1 \times \frac{1}{3} \times \frac{1}{3}$$

$$= 1 \times 1 \times \frac{32}{243}$$

$$= \frac{32}{243}$$

### Mean & Variance of Binomial Distribution

$$\text{Mean} = np$$

$$\text{Variance} = npq$$

$$\frac{5x+10}{(27)^2}$$

$$3 \times 3 \times 5$$

$$\frac{6}{27}$$

$$\frac{9}{27}$$

$$\frac{243}{27}$$

$$\frac{81}{27}$$

$$\frac{27}{27}$$

$$\frac{9}{27}$$

$$\frac{3}{27}$$

$$\frac{1}{27}$$

Q. For a Binomial distribution mean = 4 & variance equal to  $12/9$ . write down the probability distribution?

Ans:-

$$P(x) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, 3, 4, \dots, n$$

Given, mean = 4

$$\text{variance} = \frac{12}{9}$$

$$np = 4$$

$$npq = \frac{12}{9}$$

$$4q = \frac{12}{9}$$

$$q = \frac{12}{4} = \frac{3}{9}$$

$$q = \frac{1}{3}$$

&lt;math

Since  $n=4$   
 $n \times 2/3 = 4$

$$n \times 2/3 = 4$$

$$n = \frac{4}{(2/3)} = 4 \times \frac{3}{2}$$

$$= \frac{12}{2} = 6$$

$$P(x) = {}^6C_x P^x Q^{6-x}$$

$$= 20 \times \frac{8}{27} \times \left(\frac{1}{3}\right)^3$$

$$P(0) = {}^6C_0 P^0 Q^6 = \left(\frac{1}{3}\right)^6$$

$$= 1 \times 1 \times \frac{1}{729}$$

$$\frac{2^3}{3^3} \times \frac{8^3}{27}$$

$$P(1) = {}^6C_1 P^1 Q^5 = P$$

$$\frac{6 \times 5}{1 \times 2} =$$

$$= 6 \times \frac{2}{3} \left(\frac{1}{3}\right)^5$$

$$= 6 \times \frac{2}{3} \times \frac{1}{243}$$

$$= \frac{12}{729}$$

$$P(5) = {}^6C_5 P^5 Q^1 = \left(\frac{1}{3}\right)^6$$

$$P(2) = {}^6C_2 P^2 Q^4 = \left(\frac{1}{3}\right)^6$$

$$= 15 \times \frac{4}{9} \times \left(\frac{1}{3}\right)^4$$

$$= 15 \times \frac{4}{9} \times \frac{1}{81} = \frac{112}{729}$$

$$P(3) = {}^6C_3 P^3 Q^3 = \left(\frac{1}{3}\right)^6$$

$$= 20 \times \frac{8}{27} \times \frac{1}{27}$$

$$= \frac{160}{729}$$

$$P(4) = {}^6C_4 P^4 Q^2 = \left(\frac{1}{3}\right)^6$$

$$= 15 \times \frac{16}{81} \times \left(\frac{1}{3}\right)^2$$

$$= 15 \times \frac{16}{81} \times \frac{1}{9}$$

$$= \frac{240}{729}$$

$$\frac{6 \times 5 \times 4}{1 \times 2 \times 3} =$$

$$\frac{120}{6} =$$

$$\frac{20}{3} =$$

$$\frac{5}{6} =$$



Ans:

Gives, mean  $NP = 2$

& variance  $NPq = 2 \cdot \frac{1}{4}$

$$NPq = 2 \cdot \frac{1}{4}$$

$$\frac{1}{2} \cdot \frac{1}{4} = p \cdot q$$

$$q = \frac{1}{2} = \frac{2}{4}$$

$$P(X=2) = \frac{2}{4}$$

$$1-p = \frac{2}{4}$$

$$1-\frac{2}{4} = p$$

$$\frac{2}{4} = p$$

$$P(X=2) = \frac{2}{4} \cdot \frac{2}{4}$$

Therefore,  $NP = 2$

$$n \times p = 2$$

$$n \times p = 2 \quad \text{and} \quad n \times q = 2 \quad \text{and} \quad p + q = 1$$

$$n \times p = 2 \quad \text{and} \quad n \times q = 2 \quad \text{and} \quad p + q = 1$$

$$P(X=x) = \binom{n}{x} p^x q^{n-x}$$

$$P(X=x) = \binom{8}{x} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{8-x}$$

1) Probability of 2 successes.

$$P(2 \text{ successes}) = P(X=2)$$

$$= P(2)$$

$$= \binom{8}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{8-2}$$

$$= 28 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^6$$

$$= 28 \times \frac{1}{4} \times \frac{1}{64} \times \left(\frac{1}{2}\right)^8$$

$$= \frac{28}{256} = \frac{1}{64} //$$

2) Probability of less than 2 success.

$$P(\text{less than 2 successes}) = P(X=0 \text{ or } X=1)$$

$$= P(0) + P(1)$$

$$P(0) = \binom{8}{0} (0.5)^0 (0.5)^8$$

$$= 1 \times \left(\frac{1}{2}\right)^8$$

$$= \left(\frac{1}{2}\right)^8 = \frac{1}{256} //$$

$$P(1) = \binom{8}{1} \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^7$$

$$= \binom{8}{1} \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^7$$

$$P(2 \leq 2) = \left(\frac{1}{2}\right)^8 + 8 \times \left(\frac{1}{2}\right)^8$$

$$= 8 \times (0.5)^8 = \frac{8}{256}$$

$$= \frac{1}{32} //$$



Expected number of sets of 12 tossed giving

$$8 heads equal to \(\frac{256}{2} \times P(X=8)\)$$

$$\frac{256}{2} \times 0.1208 = \underline{\underline{30.93}}$$

$$M_3 = npq (q-p)$$

$$M_2 = npq$$

$$P_1 = \frac{M_3^2}{M_2^3}$$

Want

$$\rightarrow P_1 = \frac{M_3^2}{M_2^3}$$

+ know about  $M_2 = npq$

$$\gamma_1 = \sqrt{P_1}$$

$$\gamma_2 = P_2 - 3$$

central moments

raw moments

$$M_1 = E[X] = np$$

$$M_2 = E[X^2]$$

$$M_2 = E[X^2]$$

$$= V(X) = npq$$

$$M_3 = E[(X - EX)^3] = E[X^3] - (EX)^3 = \frac{npq}{n^3 p^2 q^3} = \frac{(q-p)}{npq}$$

$\Rightarrow$  A binomial distribution is positively skewed if  $q > p$ , symmetric if  $q = p$  and negatively skewed if  $p > q$ .

Skewness and kurtosis

in Binomial Distribution

- Skewness

$$\frac{q-p}{\sqrt{npq}} = 0 \Rightarrow q = p$$

skewness is symmetric if  $q = p$ ,  $p = 1/2$

$\rightarrow \gamma_1 = 0$  distribution is symmetric

$\rightarrow$  heavily skewed  $\gamma_1 \geq 0$ ,  $q > p$ ,  $q-p > 0$ .

$\rightarrow$  very skewed  $\gamma_1 \leq 0$ ,  $q < p$ ,  $q-p < 0$ .

## 2- Kurtosis

$$\beta_2 = 3 + \frac{1 - 6pq}{npq}$$

$$\gamma_2 = \beta_2 - 3 = \frac{1 - 6pq}{npq}$$

→ A distribution is leptokurtic if  $\gamma_2 > 0$ , and

mesokurtic if  $\gamma_2 = 0$  and platykurtic if  $\gamma_2 < 0$ .

→ A binomial distribution is leptokurtic if  $pq > \frac{1}{6}$ ,

mesokurtic if  $pq = \frac{1}{6}$  and platykurtic if  $pq < \frac{1}{6}$ .

$$\rightarrow \gamma_2 = \frac{1 - 6pq}{npq} = 0$$

$$1 - 6pq = 0 \times npq$$

because  $1 - 6pq = 0$  corresponds to mesokurtic binomial

distribution. If  $1 - 6pq < 0$  then the distribution is leptokurtic.

$$\gamma_2 = \beta_2$$

## Fitting of Binomial Distribution

By fitting a binomial distribution we mean

determined the expected or theoretical binomial frequencies against the given observed frequencies.

The theoretical frequencies are obtained by multiplying the known probabilities by the total frequency.

If  $f(x)$  denotes the binomial frequency function,

$$f(x) = np^x q^{n-x}$$

$$= \binom{n}{x} p^x q^{n-x}$$

$$x = 0, 1, 2, \dots, n$$

It is denoted by,

$$f(x) = \binom{n}{x} p^x q^{n-x}$$

To calculate the binomial probabilities we require  $n$  &  $p$ .  $n$  can be determined from the values of  $x$

In the data, if  $p$  is not available in the data, find the mean of frequency distribution and equate it to  $np$ , mean of the binomial distribution.

Let the litters of a male mice. The no. of litters with contained zero, 1, 2, 3, 4 female samples were noted. The figures are given the table below:-

No. of females mice : 0, 1, 2, 3, 4 total

No. of litters : 8, 32, 34, 24, 5 (03

16 chance of obtaining female in a single trial assumed constant, estimate the constant or unknown probability. Find also the expected frequencies?

Ans: Mean =  $\bar{x} = \frac{\sum x p(x)}{n}$

(12)

$$P(X) = \binom{4}{x} (0.466)^x (0.534)^{4-x}$$

$x = 0, 1, 2, 3, 4$

$x$	$p(x)$	frequency ( $f$ )	$x f$	$p(x)$	$N.P(x)$
0	0.0813	8	0	0.0813	8.375
1	0.283	32	32	0.283	29.149
2	0.371	68	136	0.371	38.123
3	0.216	24	72	0.216	22.248
4	0.0471	5	20	0.0471	4.857
					$\frac{22.248}{N} = 10.3$

$$P(0) = \binom{4}{0} (0.466)^0 (0.534)^4$$

$$= 1 \times 1 \times (0.534)^4$$

$$P(1) = \binom{4}{1} (0.466)^1 (0.534)^{4-1}$$

$$= 4 \times 0.466 \times (0.534)^3$$

$$= 4 \times 0.466 \times 0.152$$

$$= 0.283$$

$$= 0.283$$

$$\text{Mean} = \bar{x} = \frac{10.3}{10.3}$$

$$P(2) = \binom{4}{2} (0.466)^2 (0.534)^{4-2}$$

$$= 24 \times 0.217 \times (0.534)^2$$

$$= 24 \times 0.217 \times 0.285$$

$$= 0.371$$

$$P(3) = \binom{4}{3} (0.466)^3 (0.534)^{4-3}$$

$$= 4 \times 0.101 \times 0.534$$

$$= 0.216$$

$$p(x) = \binom{n}{x} p^x q^{n-x}$$

$$P(X=4) = \binom{4}{4} (0.466)^4 (0.534)^{4-4}$$

$$\text{mean} = \bar{x} = \frac{\sum x_i f_i}{\sum f_i}$$

$$0.466^4 (0.534)^0 = 0.0471 \times 1$$

Thus, Expected frequency:

$$\begin{aligned}
 0.813 \times 10^3 &= 8.375. \\
 0.883 \times 10^3 &= 8.813 = 9 \\
 0.371 \times 10^3 &= 3.71 = 3 \\
 0.216 \times 10^3 &= 2.16 = 2 \\
 0.0471 \times 10^3 &= 0.471 = 1
 \end{aligned}$$

$x$	Observed frequency( $f$ )	$x f$	$P(x)$	N. P( $x$ )
0	7	0	0.0048	1.2533
1	6	6	0.064	3.192
2	38	76	0.1805	23.104
3	19	57	0.2813	36.0064
4	35	140	0.261	33.408
5	30	150	0.147	18.816
6	23	138	0.045	5.76
7	42	294	0.00613	0.7846
	128	433		

$$\text{Mean } \bar{x} = \frac{433}{128}$$

$$= 3.383 - 3.382 = 0.001$$

2. The screws produced by certain machines were checked by examining sample. The following table shows the distribution of 128 sample according to the no. of defective items. It contains

$$\begin{array}{ccccccc}
 \text{No. of defect} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
 \text{No. of samples} & 7 & 6 & 19 & 35 & 30 & 23 & 7 & 1
 \end{array}$$

Fit a binomial distribution to find the mean and variance of the distribution?

$$\text{Therefore } np = 3.6083 \times 3.382$$

$$\text{i.e., } P = \frac{3.6083}{128} = 0.0283$$

$$= 0.515$$

$$115.69251 = 0.4831$$

Therefore,  $\frac{1}{2} = 1 - p \cdot 2^x = 0.09$

$$= 1 - 0.483$$

$$= 0.517$$

(14)

$$\text{Ques. } P(X) = \binom{n}{x} p^x q^{n-x}$$

$$P(X) = \binom{7}{x} (0.483)^x (0.517)^{7-x}$$

$$P(X) = \binom{7}{0} (0.483)^0 (0.517)^7$$

$$= 1 \times 1 \times 0.00987$$

$$= 0.00987$$

$$P(X) = \binom{7}{1} (0.483)^1 (0.517)^6$$

$$= 7 \times 0.483 \times 0.01909$$

$$= 0.064$$

$$P(X) = \binom{7}{2} (0.483)^2 (0.517)^5$$

$$= 21 \times 0.233 \times 0.0369$$

$$= 0.1805$$

$$P(X) = \binom{7}{3} (0.483)^3 (0.517)^4$$

$$= 35 \times 0.1126 \times 0.0714$$

$$\frac{7 \times 6}{1 \times 2} = \frac{42}{2} = 21$$

$$\frac{7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 2} = 35$$

$$P(X) = 0.2813$$

$$P(+)=\binom{7}{4} (0.483)^4 (0.517)^3$$

$$= 35 \times 0.054 \times 0.138$$

$$= 0.261$$

$$P(X) = \binom{7}{5} (0.483)^5 (0.517)^2$$

$$= 21 \times 0.0262 \times 0.067$$

$$= 0.147$$

$$P(X) = \binom{7}{6} (0.483)^6 (0.517)^1$$

$$= 7 \times 0.0126 \times 0.517$$

$$= 0.045$$

$$P(X) = \binom{7}{7} (0.483)^7 (0.517)^0$$

$$= 1 \times 0.00613 \times 1$$

$$= 0.00613$$

$$nsp = 3.382 \times 0.483 \times 0.517$$

$$\text{variance.} = 1.748$$

Thus, expected frequency,

$$X \quad f \quad (x, f(x)) \quad P(x) \quad N.P(x) \quad N$$

$$0 \quad 6 \quad 0.0575 \quad 0.0575 \quad 4.6 = 5$$

$$1 \quad 20 \quad 0.2016 \quad 0.2016 \quad 17.7306 = 18$$

$$2 \quad 28 \quad 0.3408 \quad 0.3408 \quad 27.27 = 27$$

$$3 \quad 12 \quad 0.2627 \quad 0.2627 \quad 21.017 = 21$$

$$4 \quad 8 \quad 0.1011 \quad 0.1011 \quad 8.088 = 8$$

$$5 \quad \frac{6}{80} \quad \frac{0.0155}{174} \quad 0.0155 \quad 1.246 = 1$$

$$P(X=0) = 0.0575 \times 128 = 0.192 = 3$$

$$P(X=1) = 0.2016 \times 128 = 23.104 = 23$$

$$P(X=2) = 0.3408 \times 128 = 36.0064 = 36$$

$$P(X=3) = 0.2627 \times 128 = 32.408 = 33$$

$$P(X=4) = 0.1011 \times 128 = 14.736 = 15$$

$$P(X=5) = 0.0155 \times 128 = 2.016 = 2$$

$$\therefore \text{Mean} = \frac{\sum x_i f_i}{\sum f} = \frac{174}{80} = 2.175$$

The following data show the number seeds germinating out of 5 rib damp filter for 80 sets of seeds. Fit a binomial distribution of the data and find the expected frequencies.

$$x : 10, 12, 10, 2, 3, 4, 5$$

$$f : 6, 20, 23, 12, 8, 6$$

Since 'p' is not given, find mean of the given data.

$$P(X) = \binom{n}{x} p^x q^{n-x}$$

$$P(X) = \binom{5}{x} (0.435)^x (0.565)^{5-x}$$

for  $x = 0, 1, 2, 3, 4, 5$

where,  $n=80$

Ans.

ANSWER

$$(x)_{9,4}^M = \binom{5}{0} (0.435)^5 (0.565)^{5-0}$$

$$= 1 \times 1 \times 0.0575$$

$$= 0.0575$$

$$= 0.0575 \times 80$$

$$\text{Expected frequency} = 0.0575 \times 80$$

$$= 4.6 \quad \ddot{\Sigma} = 5 //$$

$$= 4.6 \quad \ddot{\Sigma} = 5 //$$

$$\binom{P}{1} = \binom{5}{1} \times (0.435)^4 \times (0.565)^{5-1}$$

$$= 5 \times 0.435 \times 0.1019$$

$$= 0.2216$$

$$\binom{P}{4} = \binom{5}{4} (0.435)^4 \times (0.565)^{5-4}$$

$$= 5 \times 0.0358 \times 0.565$$

$$\text{Expected frequency} = 0.1011 \times 80$$

$$= 8.088 \quad \ddot{\Sigma} = 8 //$$

$$(P_2) = \binom{5}{2} \times (0.435)^2 \times (0.565)^{5-2}$$

$$= 10 \times 0.189 \times 0.1803$$

$$= 0.3408 \quad \ddot{\Sigma} = 0 \quad \text{Ans}$$

$$\text{Expected frequency} = 0.0155 \times 80$$

$$= 0.0155 \quad \ddot{\Sigma} = 0 \quad \text{Ans}$$

$$\text{Expected frequency} = 0.0155 \times 80$$

$$= 1.240 \quad \ddot{\Sigma} = 1 //$$

$$x - 2 \times 2.0 \times 2.0 = 20 \text{ not}$$

$$0.8 = 1.1 \text{ Ans}$$

$$(P_3) = \binom{5}{3} (0.435)^3 \times (0.565)^{5-3}$$

$$= 10 \times 0.0823 \times 0.319$$

$$= 0.2627$$

$$\text{Expected frequency} = 0.2627 \times 80$$

$$= 21.017 \quad \ddot{\Sigma} = 21 //$$

$$= 21.017 \quad \ddot{\Sigma} = 21 //$$

$$= 21.017 \quad \ddot{\Sigma} = 21 //$$

$$\binom{P}{5} = \binom{5}{5} (0.435)^5 \times (0.565)^{5-5}$$

$$= 1 \times 1 \times 0.0001$$

$$= 0.0001$$

$$\text{Expected frequency} = 0.0001 \times 80$$

$$= 0.008 \quad \ddot{\Sigma} = 0 //$$

$$\text{Expected frequency} = 0.008 \times 80$$

$$= 0.064 \quad \ddot{\Sigma} = 0 //$$

$$(P_4) = \binom{5}{4} (0.435)^4 \times (0.565)^{5-4}$$

$$= 5 \times 0.0155 \times 1$$

$$= 0.0775 \quad \ddot{\Sigma} = 0 //$$

$$\text{Expected frequency} = 0.0775 \times 80$$

$$= 6.200 \quad \ddot{\Sigma} = 6 //$$

$$\text{Expected frequency} = 6.200 \times 80$$

$$= 496 \quad \ddot{\Sigma} = 496 //$$

$$x - 2 \times 2.0 \times 2.0 = 20 \text{ not}$$

$$0.8 = 1.1 \text{ Ans}$$

$$= 0.0001$$

$$= 0.0001$$

## Poisson Distribution

is said to

A discrete random variable  $X$  is said to follow poisson distribution if its probability density function is

$$P(X=x) = \begin{cases} \frac{e^{-\lambda} \lambda^x}{x!}, & x=0, 1, 2, \dots \\ 0, & \text{otherwise} \end{cases}$$

arg(m)

Poisson distribution as a limiting case of binomial distribution. Poisson distribution may be obtained as a limiting case of binomial distribution

under the following conditions :-

1 - No. of trials is very large ( $n \rightarrow \infty$ ).

2 - The probability of success for each trial is very small ( $P \rightarrow 0$ ).

3 -  $np$  is finite ( $\approx 3$ ).   
  $3+0.1 =$

Practical situations where poisson distribution can be drawn, to cover

manufacturing product.

- To count the no. of defects per unit of a
- To count the no. of accidents taking place in a day on a city road.
- To count the no. of casualties due to same diseases.

- Q. If 3% of electric bulbs manufactured by a company are defective. Find the probability that in a sample of 100 bulbs, exactly 5 bulbs are defective.

Ans. Here,  $n=100$

$P \rightarrow$  Probability of defective bulb  $= 3/100 = 0.03$

Since,  $P$  is small and  $n$  is large ( $> 30$ ). We may assume poisson distribution. Therefore,  $\lambda = np$

But  $np = 100 \times 0.03 = 3$ . Since  $\lambda = 3$ , therefore,  $P(X=3) = 3/100 = 0.03$

$$P(x) = \frac{e^{-3} 3^x}{x!}$$

Q. A firm has 3 defective parts. The probability of getting 5 defective parts =  $\frac{e^{-3} 3^5}{5!}$

$$= 0.0498 \times 243,$$

$$\text{No. of defective parts} = \frac{120}{120} = 1 \text{ part}$$

$$P(x) = \frac{12.1014}{120}$$

No. of breakdowns within 0.100845

Ans. Considering all parts surviving new processes

Mean and Variance and no breakdowns in

$$E\{x\} = x$$

$$V(x) = x$$

$$E\{x\} = 12$$

$$V(x) = 12$$

Q. It is noted known. If 80000 part exposure, that

In a certain plant, there are on the average

4 industrial accidents per month. Find the

probability that no. of given year will be less than

3 accidents. Assume Poisson distribution?

Ans. Here,  $\lambda = 4$   
Hence,  $P(x)$  follows Poisson distribution

$$P(x) = \frac{e^{-4} 4^x}{x!}$$

$$P(\text{less than 3 accidents}) = P(x=0) + P(x=1) + P(x=2)$$

$$(e^{-4} + 4e^{-4} + 16e^{-4}) = \frac{e^{-4}}{0!} + \frac{4e^{-4}}{1!} + \frac{16e^{-4}}{2!}$$

$$(1.0 + 1.0 - 0.01832) = 0.01832$$

$$P(1) = \frac{4e^{-4}}{1!} = \frac{0.01832 \times 4}{1} = 0.07328$$

$$P(2) = \frac{16e^{-4}}{2!} = \frac{0.01832 \times 16}{2} = 0.14656$$

$$1.0 \times 0.01832 = \frac{0.01832}{2} = 0.00916$$

$$P(\text{less than 3 accidents}) = 0.01832 + 0.07328 + 0.14656 = 0.23816$$

In a town 10 accidents took place in a span of 100 days. Assume that the number of accidents follows Poisson, find the probability that there will be 3 or more accident in a day.

Ans:

$$\text{Average no. of trials} = \frac{10}{100} = 0.1$$

Therefore,  $\lambda = 0.1$

$$P(x=0) = \frac{e^{-0.1}}{x!} = \text{Probability of having 0 accident}$$

$$P(\text{no. of accidents} \geq 3) = 1 - P(x=0) - P(x=1) - P(x=2)$$

$$P(x=0) = \frac{e^{-0.1}}{0!} = 0.9048$$

$$P(x=1) = \frac{e^{-0.1} \times 0.1}{1!} = 0.9048 \times 0.1$$

$$P(x=2) = \frac{e^{-0.1} \times (0.1)^2}{2!} = 0.9048 \times 0.01$$

$$= \frac{0.9048 \times 0.01}{2} = 0.009048$$

### Fitting of Poisson Distribution

If we want to fit a Poisson distribution to a frequency distribution, we have calculate the mean of given distribution and take it as  $\lambda$ . Once  $\lambda$  is known the poisson distribution is obtained by putting their value of  $\lambda$  in  $\frac{e^{-\lambda} \lambda^x}{x!}$ . The expected (theoretical) frequencies can be obtained by

$$\begin{aligned} P(x=0) &= \frac{e^{-0.1} \times (0.1)^0}{0!} = 0.9048 \\ P(x=1) &= \frac{e^{-0.1} \times 0.1^1}{1!} = 0.9048 \times 0.1 \\ P(x=2) &= \frac{e^{-0.1} \times (0.1)^2}{2!} = 0.9048 \times 0.01 \\ P(x=3) &= \frac{e^{-0.1} \times (0.1)^3}{3!} = 0.9048 \times 0.001 \\ P(x=4) &= \frac{e^{-0.1} \times (0.1)^4}{4!} = 0.9048 \times 0.0001 \\ P(x=5) &= \frac{e^{-0.1} \times (0.1)^5}{5!} = 0.9048 \times 0.00001 \\ P(x=6) &= \frac{e^{-0.1} \times (0.1)^6}{6!} = 0.9048 \times 0.000001 \\ P(x=7) &= \frac{e^{-0.1} \times (0.1)^7}{7!} = 0.9048 \times 0.0000001 \\ P(x=8) &= \frac{e^{-0.1} \times (0.1)^8}{8!} = 0.9048 \times 0.00000001 \\ P(x=9) &= \frac{e^{-0.1} \times (0.1)^9}{9!} = 0.9048 \times 0.000000001 \\ P(x=10) &= \frac{e^{-0.1} \times (0.1)^{10}}{10!} = 0.9048 \times 0.0000000001 \end{aligned}$$

$$\text{Putting } \alpha = 0, \quad \text{Ansatz: } f(x) = N \times \frac{e^{-\lambda} \lambda^x}{x!}$$

Q. Fit a Poisson distribution to the following data & calculate the theoretical frequencies.

$x$	0	1	2	3	4
$f$	123	59	14	3	1

$X$	$f$	$xf$	$P(x)$	$N(P(x))$
0	123	0	0.6065	80.0121
1	59	59	0.30325	61
2	14	28	0.0768	15
3	3	9	0.0128	3
4	1	4	0.001516	0
	200	100		200

or calculated by finding the total success  $N = 200$

$$\text{Mean } \bar{x} = \frac{\sum xf}{\sum f} = \frac{100}{200} = 0.5$$

standard deviation  $s_x = \sqrt{\frac{\sum (x - \bar{x})^2 f}{\sum f}} = \sqrt{\frac{(0-0.5)^2 \cdot 123 + (1-0.5)^2 \cdot 59 + (2-0.5)^2 \cdot 14 + (3-0.5)^2 \cdot 3 + (4-0.5)^2 \cdot 1}{200}} = \sqrt{0.25 \cdot 123 + 0.25 \cdot 59 + 0.25 \cdot 14 + 0.25 \cdot 3 + 0.25 \cdot 1} = \sqrt{0.25 \cdot 198} = 0.5$

Therefore  $\lambda = 0.5$

for binomial distribution with probability  $p = 0.5$  & number of trials  $n = 200$

$$\text{but since } P(x) = \frac{e^{-0.5} \cdot 0.5^x}{x!}, \text{ for } x=0, 1, 2, 3, 4$$

$$P(0) = \frac{e^{-0.5} \cdot 0.5^0}{0!} = 1$$

$$= \frac{0.6065 \times 1}{1!} = 0.6065$$

$$= 0.6065$$

$$P(1) = \frac{e^{-0.5} \times 0.5^1}{1!} = 0.30325$$

$$= 0.30325$$

$$P(2) = \frac{e^{-0.5} \times 0.5^2}{2!} = 0.0768$$

$$= 0.0768$$

$$P(3) = \frac{e^{-0.5} \times 0.5^3}{3!} = 0.0128$$

$$= 0.0128$$

$$P(4) = \frac{e^{-0.5} \times 0.5^4}{4!} = 0.001516$$

$$= 0.001516$$

$$P(5) = \frac{e^{-0.5} \times 0.5^5}{5!} = 0.000196$$

$$= 0.000196$$

$$P(6) = \frac{e^{-0.5} \times 0.5^6}{6!} = 0.000196$$

$$= 0.000196$$

$$P(7) = \frac{e^{-0.5} \times 0.5^7}{7!} = 0.000196$$

$$= 0.000196$$

$$P(8) = \frac{e^{-0.5} \times 0.5^8}{8!} = 0.000196$$

$$= 0.000196$$

$$P(9) = \frac{e^{-0.5} \times 0.5^9}{9!} = 0.000196$$

$$= 0.000196$$

$$P(10) = \frac{e^{-0.5} \times 0.5^{10}}{10!} = 0.000196$$

$$= 0.000196$$

$$\begin{aligned}
 & \text{Ans} \quad X \quad f \quad xf \quad P(x) \quad N(P(x)) \\
 & 0 \quad 48 \quad 0 \quad 0.3716 \quad 37.16 = 37 \\
 & 1 \quad 27 \quad 27 \quad 0.3678 \quad 36.78 \\
 & 2 \quad 12 \quad 24 \quad 0.1821 \quad 18.21 = 18 \\
 & 3 \quad 7 \quad 21 \quad 0.0600 \quad 6.00 \\
 & 4 \quad 4 \quad 16 \quad 0.0148 \quad 1.487 = 2 \\
 & 5 \quad 1 \quad 5 \quad 0.0029 \quad 0.29 \\
 & 6 \quad 1 \quad 6 \quad 0.0004 \quad 0.48 \\
 & \hline
 & \text{Mean} = \frac{\sum xf}{\sum f} = \frac{99}{100} = 0.99
 \end{aligned}$$

Q. A systematic sample of 100 pages was taken from the dictionary and the observed frequency distribution of foreign words per page was found to be as follows.

No. of foreign words per page (x)	0	1	2	3	4	5	6
Frequency	48	27	12	7	4	1	1

Calculate the expected frequencies using Poisson distribution. Also compute the value of fitting distribution?

$$\begin{aligned}
 P(x) &= \frac{e^{-0.99} x^0}{x!} \\
 &= \frac{e^{-0.99} x^0}{0!} \quad x = 0, 1, 2, 3, 4, 5, 6 \\
 P(0) &= \frac{e^{-0.99} x^0}{0!} \\
 &= \frac{0.3716 \times 1}{1} \\
 &= 0.3716 //
 \end{aligned}$$

$$\begin{aligned}
 e^{-0.90} &= 0.4066 * \\
 e^{-0.91} &= 0.9139 \\
 e^{-0.99} &= 0.3716
 \end{aligned}$$

$$\begin{aligned}
 P(1) &= \frac{e^{0.99}}{6!} = 0.3716 \times 0.99 \\
 S &= 51.68 \\
 E &= 37.16 \\
 P(2) &= \frac{0.3716 \times 0.99}{2!} = 0.3642 \\
 F(2) &= 0.3642 \\
 P(3) &= \frac{0.3716 \times 0.99}{3!} = 0.06008 \\
 P(4) &= \frac{0.3716 \times 0.99}{4!} = 0.01487 \\
 P(5) &= \frac{0.3716 \times 0.99}{5!} = 0.003188
 \end{aligned}$$

$NXP(0) = 100 \times 0.3716 = 37.16$	$\approx 37$
$NXP(1) = 100 \times 0.3678 = 36.78$	$\approx 37$
$NXP(2) = 100 \times 0.1821 = 18.21$	$\approx 18$
$NXP(3) = 100 \times 0.0608 = 6.008$	$\approx 6$
$NXP(4) = 100 \times 0.01487 = 1.487$	$\approx 2$
$NXP(5) = 100 \times 0.0029 = 0.29$	$\approx 0$
$NXP(6) = 100 \times 0.0048 = 0.48$	$\approx 0$

Expected frequencies:

$\alpha = 0$	$1$	$2$	$3$	$4$	$5$	$6$
$\alpha = 37$	$37$	$18$	$6$	$2$	$0$	$0$

HQ  
Q. Following mistakes per page where observed  
in a book.

No. of mistakes : 0 1 2 3 4

No. of pages : 211 90 19 5 0

Fit a Poisson distribution to the above table?

Ans:-

x	f	xf	P(x)	N&P(x)
0	242	0	0.64404	209.313 = 212
1	90	90	0.2833	92.0.0725 = 92
2	19	38	0.06024	20.2605 = 21
3	5	15	0.00914	2.9703 = 3
4	0	0	0.001	0.325 = 0
	325	143		325

$$\text{Mean} = \frac{\sum xf}{\sum f} = \frac{143}{325} = 0.44$$

$$P(x=0) = \frac{0.64404 \times 0}{325} = 0.409 \cdot 0.44$$

$$P(x=1) = \frac{0.64404 \times 0.44}{325} = (0.1936 \times 0.44)$$

$$\text{ie: } x = 0.409 \cdot 0.44 //$$

$$P(x) = \frac{e^{-0.44} \cdot 0.44^x}{x!}$$

$$= \frac{e^{-0.44} \cdot 0.44^x}{x!}$$

$$P(0) = \frac{e^{-0.44} \cdot 0.44^0}{0!}$$

$$= \frac{0.64404}{1} = 0.64404 //$$

$$P(1) = \frac{e^{-0.44} \cdot 0.44^1}{1!}$$

$$= \frac{0.64404 \cdot 0.44}{1} = 0.2833 //$$

$$P(2) = \frac{e^{-0.44} \cdot 0.44^2}{2!}$$

$$= \frac{0.64404 \times 0.1936}{2} = 0.06234 //$$

$$P(3) = \frac{e^{-0.44} \cdot 0.44^3}{3!}$$

$$= \frac{0.64404 \times 0.0852}{6} = 0.00852 //$$

