Statistics

Module - I Matrix

Types of Matrices - Operations of matrices Determinants - Properties of determinants - Minors and co-factors - Adjoint of a matrix - Inverse of a matrix - Rank of a matrix - solution of a system of linear equations using matrices - Crammer's rule - characteristics, equations - Characteristics roots - Applications in economics.

Matrices

$$\begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}_{2\times 2}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ -2 & 6 \end{bmatrix}_{3\times 2}$$

$m\times n$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 6 \end{bmatrix}_{2\times 3}$$

$$\begin{bmatrix} 1 & 3 & -2 \end{bmatrix}_{1\times 3}$$

$$\begin{bmatrix} 1+2i & 2-3i & 0 \\ -1 & 2 & 6 \end{bmatrix}_{2\times 3}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & -2 & 3 \\ 6 & -8 & 4 \end{bmatrix}_{3\times 3}$$

13 types

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Types of Matrices

1- Square matrix

An $m\times n$ matrix where, $n = m$ is called a square matrix or square matrix of order n,

eg:- $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$ $\begin{bmatrix} 2 & 4 & 6 \\ 8 & 9 & 3 \\ 5 & 7 & 2 \end{bmatrix}$

2- Raw Matrix

A $1 \times n$ matrix is called raw matrix.

Eg:- $\begin{bmatrix} 1 & -1 & 6 \end{bmatrix}_{1\times 3}$

$\begin{bmatrix} 1 \end{bmatrix}_{1\times 1}$ $\begin{bmatrix} 1 & -3 \end{bmatrix}_{1\times 2}$

3- Column Matrix

A $m \times n$ matrix is called column matrix

eg:- $\begin{bmatrix} 1 \\ -1 \\ -c \end{bmatrix}_{3\times 1}$ $\begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2\times 1}$ $\begin{bmatrix} -7 \end{bmatrix}_{1\times 1}$

4- Diognal Matrix

$a_{ij} = 0$

$i \neq j$

A matrix $A = [a_{ij}]$ is said to be a diagnel matrix. If A is a square matrix and each of its non diagnal element are zero. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Eg:- $\begin{bmatrix} 11 & 0 & 0 \\ 0 & 2i & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times 3}$

$\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}_{2\times 2}$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}_{4\times 4}$

5- Scalar Matrix :

A diagnal matrix whose diognal elements are all equal. is called a scalar matrix.

$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}_{3\times 3}$ $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}_{2\times 2}$

6- Triangular Matrix

If the element lies above or below the leading diognal ~~matrix~~ element are zero. It is known as triangular matrix.

eg:- $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ lower triangular

If the elements lies above the leading diagnal elements are zero. It is known as lower triangular matrix.

If the elements lies below leading diagnal elements are zero. It is known upper triangular matrix. $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ Upper triangular matrix

7 symmetric Matrix

If the given matrix equal to its transpose, it is called symmetric matrix.

Symmetric matrix → $A = A^{T} \rightarrow$ S.m

eg: $A \begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}$ $A^{T} = \begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}$

Here $A = A'$ so it is symmetric matrix

8 Unit Matrix : identity matrix

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

9 zero matrix (Null matrix)

All elements are zero.

A matrix rectangular / square each of whose elements are zero is called zero matrix.

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

10 Triangular Matrix :

A matrix $A = (a_{ij})$ is said to be upper triangular if $i > j = a_{ij} = 0$

eg:- $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

(5) A matrix $A = (a_{ij})$ is said to be lower triangular matrix if $i < j$. $a_{ij} = 0$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

Symmetric Matrix

A square matrix $A = (a_{ij})$ said to be symmetric $a_{ij} = a_{ji}$

eg - $i = 2$ $j = 3$

$$\begin{bmatrix} -2 & 3 & 6 \\ 3 & 4 & 100 \\ 6 & 100 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 6 & 8 \\ 2 & 5 & 10 & 0 \\ 6 & 10 & 1 & 2 \\ 8 & 0 & 2 & 1 \end{bmatrix}$$

10 * Skew symmetric

Transpose of symmetric matrix is a matrix

A square matrix $A = (a_{ij})$ is said to be skew symmetric, if $a_{ij} = -a_{ji}$

eg:- $\begin{bmatrix} 0 & 6 & 2 \\ -6 & 0 & 7 \\ -2 & -7 & 0 \end{bmatrix}$ It diagnal matrix 0

$c = x + iy$

$\bar{c} = \overline{x + iy} = x - iy$

eg of ~~conjugate of~~ Conjugate of c_1

$c_1 = 2 - 3i$

$c_2 = 2i$

$\bar{c_2} = -2i$

11 Hermition matrices

A square matrix $A = (a_{ij})$ is said to be hermitian if $a_{ij} = \overline{a_{ji}}$

(6) That is (i, j) element is the conjugate complex of the (j, i) element

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$a_{11} = \overline{a_{11}}$

$a_{12} = \overline{a_{12}}$

$a_{21} = \overline{a_{21}}$

$a_{22} = \overline{a_{22}}$

eg. $\begin{bmatrix} 3 & 2+3i \\ 2-3i & 6 \end{bmatrix}$

$$\begin{bmatrix} 2 & 2+3i \\ 2-3i & 4 \end{bmatrix}$$

$a_{21} = \overline{a_{12}}$

$2-3i = \overline{2+3i}$

$$Q = \begin{bmatrix} 2 & 2+3i & 1+2i \\ 2-3i & 4 & 3+4i \\ 1-2i & 3-4i & 6 \end{bmatrix} =$$

$$P = \begin{bmatrix} 7 & 2-3i & 3-4i \\ 2+3i & 2 & 4-5i \\ 3+4i & 4+5i & 6 \end{bmatrix}$$

12 → Skew Hermitian

$$A = \begin{bmatrix} 1 & 1+i \\ 1-i & 2 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 1+i \\ 1+i & 2 \end{bmatrix}$$

$$\overline{A^T} = \begin{bmatrix} 1 & 1-i \\ 1+i & 1 \end{bmatrix}$$

A square matrix $A = a(ij)$ is said to be skew Hermitian $\boxed{a_{ij} = -\overline{a_{ji}}}$

eg. $A = \begin{bmatrix} 1 & 2+i & 3+i \\ 2+i & 2 & 4+i \\ 3-i & -4+i & 5 \end{bmatrix}$

$-ai$

$2+i$

$2-i$

$-(3+i)$

$$\begin{bmatrix} 3i & -2+3i & 4+5i \\ 2+3i & -2i & 6-7i \\ -4+5i & -6-7i & 0 \end{bmatrix}$$

$(-2+3i)$

$-(2-3i)$

$-(-2-3i)$

$2+3i$

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⑦ eg:- $\begin{bmatrix} 2 & 5+8i & 2+6i \\ -5+8i & -2i & 3 \\ -2+6i & -3 & 3i \end{bmatrix}$

$a_{12} = -\overline{a_{21}}$

$= -\overline{(5+8i)}$

$= -(5-8i)$

$= -5+8i$

Operations of Matrices

1- Matrix Addition :-

$A = \begin{bmatrix} 1 & 2 \\ -2 & 4 \end{bmatrix}$

$B = \begin{bmatrix} 1 & -1 \\ 0 & 4 \end{bmatrix}$

$A + B = \begin{bmatrix} 2 & 1 \\ -2 & 8 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \end{bmatrix}$

$D = \begin{bmatrix} -1 & 0 & 2 \\ 4 & 5 & 6 \end{bmatrix}$

$C + D = \begin{bmatrix} 0 & 2 & 5 \\ 8 & 10 & \cancel{16}\,6 \end{bmatrix}$

2- Negative of a Matrix :-

Eg $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & -4 \end{bmatrix}$

$-A = \begin{bmatrix} -1 & -2 & 1 \\ 0 & -4 & 4 \end{bmatrix}$

$B = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$

$-B = \begin{bmatrix} 1 & -2 \\ -3 & -4 \end{bmatrix}$

3- Scalar Multiple of a Matrix :-

$A = \begin{bmatrix} 1 & -2 \\ 4 & 6 \end{bmatrix}$ if $k = 2$

$2A = 2\begin{bmatrix} 1 & -2 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 8 & 12 \end{bmatrix}$

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* $= (2+3i)(3)$

$= 6+9i$

* $(2+3i)(1+1i)$ $\quad i^2 = \sqrt{-1}^2$

$= -1$

$= 2+2i+3i+3i^2 \;\; 3\times -1$

$= 2+2i+3i-3$

$= -1+5i$

$A = \begin{bmatrix} 2 & -2i \\ 4 & 3 \end{bmatrix}$

$k = 3i$

$kA = 3i \begin{bmatrix} 2 & -2i \\ 4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3i(2) & 3i(-2i) \\ 3i(4) & 3i(3) \end{bmatrix}$

$= \begin{bmatrix} 6i & -6\times i^2 \\ 12i & 9i \end{bmatrix} = \begin{bmatrix} 6i & -6(-1) \\ 12i & 9i \end{bmatrix}$

$= \begin{bmatrix} 6i & 6 \\ 12i & 9i \end{bmatrix}$

1. If $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 \\ 0 & 5 \\ 3 & 1 \end{bmatrix}$

Find the matrix D such that

$A+B-D = 0$ zero matrix

Ans: $A+B = D$

$\begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ 0 & 5 \\ 3 & 1 \end{bmatrix}$

$D = \begin{bmatrix} 0 & 2 \\ 3 & 7 \\ 5 & 6 \end{bmatrix}$

2. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

Find $2A - 3B$

Ans. $2A = 2\begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 6 & 2 \\ 0 & -2 & 10 \end{bmatrix}$

$3B = 3\begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 6 & -3 \\ 0 & -3 & 9 \end{bmatrix}$

$2A - 2B = \begin{bmatrix} 4 & 6 & 2 \\ 0 & -2 & 10 \end{bmatrix} - \begin{bmatrix} 3 & 6 & -3 \\ 0 & -3 & 9 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 1 \end{bmatrix}$ //

Determinants

→ $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\boxed{|A| = (a_{11}a_{22}) - (a_{12}a_{21})}$

$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (1 \times 4) - (2 \times 3)$

$= 4 - 6 = -2$ //

2. $B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

$|B| = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6 - 0 = 6$ //

→ $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} = 3 - 0 = 3$ //

$B = \begin{bmatrix} 1 & 0 \\ 4 & 3 \end{bmatrix}$

$|B| = \begin{vmatrix} 1 & 0 \\ 4 & 3 \end{vmatrix} = 3 - 0 = 3$ //

* $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -3 \\ 4 & 1 & 5 \end{bmatrix}$

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$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 0 & 2 & -3 \\ 4 & 1 & 5 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix} - 2\begin{vmatrix} 0 & -3 \\ 4 & 5 \end{vmatrix} + 1\begin{vmatrix} 0 & 2 \\ 4 & 1 \end{vmatrix}$$

$$= 1(10 - -3) - 2(0 - -12) + 1(0 - 8)$$

$$= 1 \times 13 - 2 \times 12 + 1 \times -8$$

$$= 13 - 24 - 8$$

$$= -19$$

$$\rightarrow \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 6 \end{vmatrix}$$

$$= 1(-12 - 0) - 0(0 - 0) + 0(0 - 0)$$

$$= -12$$

$$\rightarrow \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 0 \\ 4 & 6 & -1 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 1 & 0 & 0 \\ 2 & -3 & 0 \\ 4 & 6 & -1 \end{vmatrix}$$

$$= 1(3 - 0) - 0(-2 - 0) + 0(12 - 12)$$

$$= 3 + 0 + \cancel{24}$$

$$= 3$$

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & -1 \\ 0 & 0 & -2 \end{bmatrix}$$

$$|C| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & -1 \\ 0 & 0 & -2 \end{vmatrix}$$

$$= 1(-8 - 0) - 2(0 - 0) + 3(0 - 0)$$

$$= -8$$

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \qquad C = \begin{bmatrix} a & -1 & 6 \\ 0 & b & -4 \\ 0 & 0 & c \end{bmatrix}$$

$$B = \begin{bmatrix} a & 0 & 0 \\ 2 & b & 0 \\ 1 & 2 & c \end{bmatrix}$$

$$|A| = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix}$$

$= a(bc-0) - 0(0-0) + 0(0-0)$

$= abc$ //

$$|B| = \begin{vmatrix} a & 0 & 0 \\ 2 & b & 0 \\ 1 & 2 & c \end{vmatrix}$$

$= a(bc-0) - 0(2c-0) + 0(4 \times b)$

$= abc$ //

$$C = \begin{vmatrix} a & -1 & 6 \\ 0 & b & -4 \\ 0 & 0 & c \end{vmatrix}$$

$= a(bc-0) - 01(0-0) + 6(0-0)$

$= abc$ //

$$D_{3\times3} = \begin{vmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{vmatrix} = diag(d_1, d_2, d_3)$$

Matrix Multiplication

$\rightarrow A_{m\times n} \quad B_{n\times p} = C_{m\times p}$

1- $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 4 \\ 0 & 2 \end{bmatrix}$

find AB

Ans: $AB =$ ~~$-1+0 \ -2 \ 0.$~~ $\begin{bmatrix} (1\times-1)+2\times0 & 1\times4+2\times2 \\ 3\times-1+4\times0 & 3\times4+4\times2 \end{bmatrix}$

$$AB = \begin{bmatrix} -1+0 & 4+4 \\ -3+0 & 12+8 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 8 \\ -3 & 20 \end{bmatrix}.$$

2. $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 4 \end{bmatrix}_{2\times 3}$ $B = \begin{bmatrix} -1 & 6 & -1 \\ 4 & 4 & 2 \\ 2 & 1 & 3 \end{bmatrix}_{3\times 3}$

Find AB.

Ans: $AB = \begin{bmatrix} -1+-4+0 & 6+-4+0 & -1+-2+0 \\ -2+4+8 & 12+4+4 & -2+2+12 \end{bmatrix}$

$$= \begin{bmatrix} -5 & 2 & -3 \\ 10 & 20 & 12 \end{bmatrix}_{2\times 3}$$

3. $A = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$

Find AB & BA. $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$

$$AB = \begin{bmatrix} 2+-4 & 3+-6 \\ 4+-12 & 6+-18 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -3 \\ -8 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -3 \\ 16 & 24 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2+6 & -2+9 \\ 4+12 & -4+18 \end{bmatrix}$$

$AB \neq BA$

$$BA = \begin{bmatrix} 8 & 7 \\ 16 & 14 \end{bmatrix}$$

4. $A = \begin{pmatrix} 1 & 2 \\ 4 & 6 \end{pmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Ans $AB = \begin{bmatrix} 1+0 & 0+2 \\ 4+0 & 0+6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$

$$BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 2+0 \\ 0+4 & 0+6 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$ //

$AB = BA$

5. $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ (1×1 2×2 3×1) (1×4 2×3 3×2) (1×6 4×4 3×3)

(4×1)

$B = \begin{bmatrix} 1 & 4 & 6 \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix}$ Find AB & BA

Ans: $AB = \begin{bmatrix} 1+4+3 & 4+6+6 & 6+8+9 \\ 4+10+6 & 16+15+12 & 24+20+18 \\ 7+16+9 & 28+24+18 & 42+32+27 \end{bmatrix}$

$$= \begin{bmatrix} 8 & 16 & 23 \\ 20 & 43 & 62 \\ 32 & 70 & 101 \end{bmatrix}$$ //

$$BA = \begin{bmatrix} 1 & 4 & 6 \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1+16+42 & 2+20+48 & 3+24+54 \\ 2+12+28 & 4+15+32 & 6+18+36 \\ 1+8+21 & 2+10+24 & 3+12+27 \end{bmatrix}$$

$$= \begin{bmatrix} 59 & 70 & 81 \\ 42 & 51 & 60 \\ 30 & 36 & 42 \end{bmatrix}$$ //

$AB \neq BA$

6. Calculate the following product ?

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}$$

Ans:- $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

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$$= \begin{bmatrix} 2+2+3 & 3+0+3 & 0+0+6 \\ 4+3+1 & 6+0+1 & 0+0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 6 & 6 \\ 8 & 7 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 6 & 6 \\ 8 & 7 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 7+30+36 \\ 8+35+12 \end{bmatrix} = \begin{bmatrix} 73 \\ 55 \end{bmatrix} //$$

H.W.

7. Show that the matrix A satisfies $A^3 - 6A^2 + 9A - 4I = 0$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

~~Ans.~~

8. Find AB and and BA.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} -4+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+12+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+12 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix} //$$

$$BA = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -4+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+12+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+12 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix} //$$

$AB = BA$

Minors and Cofactors

1. Find minor & factors of matrix

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

Ans:- $M_{11} = \begin{vmatrix} -1 & -1 \\ 1 & -1 \end{vmatrix} = 1+1 = 0$

$C_{11} = (-1)^{1+3} = (-1)^2 = 1 \quad C_{11} = -1(M_{11}) = -1 \times 0 = 0$

$M_{12} = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -2+1 = -3$

$C_{12} = (-1)^{1+2} = (-1)^3 = -1$

$= -1(M_{12}) = -1(-3) = 3$

$M_{13} = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 - -1 = 3$

$C_{13} = (-1)^4 = 1 \quad 1(M_{13}) = 1 \times 3 = 3$

$M_{21} = \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} = 1-2 = -1$

$C_{21} = (-1)^3 = -1 = -1(M_{21}) = -1 \cdot -1 = 1$

$M_{22} = \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} = -1-2 = -3$

$C_{22} = (-1)^4 = 1 \quad 1(-3) = -3$

$M_{23} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1 - -1 = 2$

$C_{23} = (-1)^5 = -1 = (-1) \cdot 2 = -2$

$M_{31} = \begin{vmatrix} -1 & 2 \\ -1 & 1 \end{vmatrix} = -1 - -2 = 1$

$C_{31} = (-1)^4 = 1 = 1 \times 1 = 1$

$M_{32} = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = 1-4 = -3$

$C_{32} = (-1)^5 = -1 = -1(-3) = 3$

$M_{33} = \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1 - -2 = 1$

$C_{33} = (-1)^6 = 1 = 1 \times 1 = 1$

7 Ans: $A^3 - 6A^2 + 9A - 4I = 0$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$\begin{matrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{matrix}$$

$$A^2 = \begin{bmatrix} 4+1+1 & -2+-2-1 & 2+1+2 \\ -2+2+1 & +1+4+1 & -1-2-2 \\ 2+1+2 & -1+-2-2 & 1+1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ 5 & & -3 \\ 2 & -5 & 6 \end{bmatrix} \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 8-5+4 & -4-10-4 & 4+5+8 \\ 10+4-3 & -5+8+3 & 5-4-6 \\ 4-5+6 & -2-10-6 & 2+5+2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -18 & 17 \\ 11 & 6 & -5 \\ 5 & -17 & 9 \end{bmatrix}$$

$$9A = \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix}$$

$$6A^2 = \begin{bmatrix} 24 & -30 & 24 \\ 30 & 24 & 12 \\ 12 & 30 & 36 \end{bmatrix} \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix}$$

$$4I = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

$$A^3-6A^2 = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 9 & -9 \\ 9 & -14 & 9 \\ -9 & 9 & -14 \end{bmatrix}$$

$$A^3-6A^2+9A = \begin{bmatrix} -14 & 9 & -9 \\ 9 & -14 & 9 \\ -9 & 9 & -14 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix}$$

$$A^3-6A^2+9A-4I = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} //$$

1. Find cofactor of matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & -3 \end{bmatrix}$

$m_{11} = -3$

$c_{11} = (-1)^2 \times -3 = -3$

$m_{12} = 3$

$c_{12} = (-1)^3 \times 3 = -3$

$m_{21} = 2$

$c_{21} = (-1)^3 \times 2 = -2$

$m_{22} = 1$ $\quad c_{22} = (-1)^4 \times 1 = 1 //$

-3 3
2 1

$$C = \begin{bmatrix} -3 & -3 \\ -2 & 1 \end{bmatrix}$$

Adjoint of a Matrix

$$adj(A) = C^T$$

Adjoint of a matrix denoted by $adj(A)$ is the transpose of co-factor matrix, That is, $adj(A) = C^T$ where, C is the co-factor matrix of A.

1. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ find the adjoint of the matrix.

Ans: $m_{11} = 4$ $\quad c_{11} = (-1)^2 \times 4 = 4$

$m_{12} = 3$ $\quad c_{12} = (-1)^3 \times 3 = -3$

$m_{21} = 2$ $\quad c_{21} = (-1)^3 \times 2 = -2$

$m_{22} = 1$ $\quad c_{22} = (-1)^4 \times 1 = 1$

Cofactor $= \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$

$C^T = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$\underline{\underline{adj(A) = C^T}}$

2. Find the adjoint of the matrix

$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

Ans: $m_{11} = \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} = +1 - 1 = 0$

$c_{11} = (-1)^2 \times 0 = 0$

$m_{12} = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -2 - 1 = -3$

$c_{12} = (-1)^3 \times -3 = 3$

$m_{13} = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 - -1 = 3$

$c_{13} = (-1)^4 \times 3 = 3$

$m_{21} = \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} = 1 - 2 = -1$

$c_{21} = (-1)^3 \times -1 = 1$

$m_{22} = \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} = -1 - 2 = -3$

$c_{22} = (-1)^4 \times -3 = -3$

$m_{23} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1 - -1 = 2$

$c_{23} = (-1)^5 \times 2 = -2$

$m_{31} = \begin{vmatrix} -1 & 2 \\ -1 & 1 \end{vmatrix} = -1 - -2 = 1$

$c_{31} = (-1)^4 \times 1 = 1$

$m_{32} = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = 1 - 4 = -3$

$c_{32} = (-1)^5 \times -3 = 3$

$m_{33} = \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1 - -2 = 1$

$c_{33} = (-1)^6 \times 1 = 1$

$$c = \begin{bmatrix} 0 & 3 & 3 \\ 1 & -3 & -2 \\ 1 & 3 & 1 \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} 0 & 1 & 1 \\ 3 & -3 & 3 \\ 3 & -2 & 1 \end{bmatrix}$$

Inverse of a Matrix

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

Inverse of a square matrix A is defined as $\boxed{A^{-1} = \frac{1}{|A|} \text{adj}(A)}$.

2 Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= 1\begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} - -1\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} + 2\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= 1(1-1) + 1(-2-1) + 2(2--1)$$

$$= 1(0) + 1(-3) + 2(3)$$

$$= 0 + -3 + 6 = \cancel{4}\ 3$$

$m_{11} = \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix} = \cancel{1 - 1} = 0$

$\cancel{c_{11} = (-1)^2}$ $\text{Adj}(A) = \begin{bmatrix} 0 & 1 & 1 \\ 3 & -3 & 3 \\ 3 & -2 & 1 \end{bmatrix}$

Inverse $A^{-1} = \frac{1}{|A|} \begin{bmatrix} 0 & 1 & 1 \\ 3 & -3 & 3 \\ 3 & -2 & 1 \end{bmatrix}$

$$= \frac{1}{3} \begin{bmatrix} 0 & 1 & 1 \\ 3 & -3 & 3 \\ 3 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0/3 & 1/3 & 1/3 \\ 1 & -1 & 1 \\ 1 & -2/3 & 1/3 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1/3 & 1/3 \\ 1 & -1 & 1 \\ 1 & 2/3 & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+2 & 1/3+1+4/3 & 1/3-1+2/3 \\ 0+1+1 & 2/3+1+2/3 & 2/3-1+1/3 \\ 0+1+1 & 1/3-1+2/3 & 1/3+1-1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

? Find the inverse of matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{vmatrix} = 1($$

$$= 1(18-16) - 2(12-12) + 3(8-9)$$

$$= 2 - 3 = -1$$

$$M_{11} = \begin{vmatrix} 3 & 4 \\ 4 & 6 \end{vmatrix} = 18 - 16 = 2$$

$$C_{11} = (-1)^2 \, 2 = 2$$

$$M_{12} = \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix} = 12 - 12 = 0$$

$$C_{12} = (-1)^3 \times 0 = 0$$

$$M_{13} = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = 8 - 9 = -1$$

$$C_{13} = (-1)^4 \times -1 = -1$$

$$M_{21} = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 12 - 12 = 0$$

$$C_{21} = (-1)^3 \times 0 = 0$$

$$M_{22} = \begin{vmatrix} 1 & 3 \\ 3 & 6 \end{vmatrix} = 6 - 9 = -3$$

$$C_{22} = (-1)^4 \times -3 = -3$$

$$M_{23} = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = -2$$

$$C_{23} = (-1)^5 \times -2 = 2$$

$$M_{31} = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = 8 - 9 = -1$$

$c_{31} = (-1)^4 \times -1 = -1$ //

$M_{32} = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = 4 - 6 = -2$

$c_{32} = (-1)^5 \times -2 = 2$ //

$M_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1$

$c_{33} = (-1)^6 \times -1 = -1$ //

$$C = \begin{bmatrix} 2 & 0 & -1 \\ 0 & -3 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

$$adj(A) = C^T = \begin{bmatrix} 2 & 0 & -1 \\ 0 & -3 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (adj(A))$$

$$= \frac{1}{-1} \begin{bmatrix} 2 & 0 & -1 \\ 0 & -3 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-2}{1} & 0 & 1 \\ 0 & 3 & \frac{-2}{1} \\ 1 & \frac{-2}{1} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$ //

$$AA^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+0+3 & 0+6-6 & 1-4+3 \\ -4+0+4 & 0+9+8 & 2+6+4 \\ -6+0+6 & 0+12-12 & 3-8+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ //

1

Find the inverse of the matrices.

$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj}(A)$

$|A| = 4 + 2 = 6$

$|B| = -1 - 2 = -3$

$\text{adj}(A) =$

$M_{11} = 4$

$C_{11} = (-1)^2 = 1(4) = 4$

$M_{12} = -1$

$C_{12} = (-1)^3 \times -1 = 1$

$M_{21} = 2$

$C_{21} = (-1)^3 = -1(2) = -2$

$M_{22} = 1$

$C_{22} = (-1)^4 = 1(1) = 1$

$C(A) = \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix}$

$\text{Adj}(A)\ C^T = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{6} \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4/6 & -2/6 \\ 1/6 & 1/6 \end{bmatrix}$

$|B| = -1 - 2 = -3$

$\text{adj}(B) =$

$M_{11} = 1$ $C_{11} = (-1)^2 \times 1 = 1$

$m_{12} = 2$

$c_{12} = (-1)^3 \times 2 = -2$

$m_{21} = 1$

$c_{21} = (-1)^3 \times 1 = -1$

$m_{22} = -1$

$c_{22} = (-1)^4 \times -1 = -1$

$$c(B) = \begin{bmatrix} 1 & -2 \\ -1 & -1 \end{bmatrix}$$

$$c^T = \begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}$$

$$B^{-1} \cancel{adj(B)} = \frac{1}{-3}\begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/3 & -1/3 \\ -2/-3 & -1/-3 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 4/6 & -2/6 \\ 1/6 & 1/6 \end{bmatrix}$$

$$= \begin{bmatrix} 4/6 + \frac{2}{6} & -2/6 + \frac{2}{6} \\ -4/6 + 4/6 & 2/6 + 4/6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BB^{-1} = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -1/3 & -1/3 \\ -2/-3 & -1/-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1/3 + \frac{-2}{3} & -1/3 + 1/3 \\ -2/3 + \frac{2}{3} & 2/3 + 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

HW Find the inverse of matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

Find inverse of the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Ans: $|A| = 2(12-0) - 0(0-0) + 0(0+0)$

$= -24$

adj(A) = $m_{11} = \begin{vmatrix} -4 & 0 \\ 0 & 3 \end{vmatrix} = -12-0 = -12$

$c_{11} = (-1)^2(0) = -12$

$m_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 3 \end{vmatrix} = 0-0 = 0$

$c_{12} = (-1)^3 \times 0 = 0$

$m_{13} = \begin{vmatrix} 0 & -4 \\ 0 & 0 \end{vmatrix} = 0-0 = 0$

$c_{13} = (-1)^4 \times 0 = 0$

$m_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 3 \end{vmatrix} = 0-0 = 0$

$c_{21} = (-1)^3 \times 0 = 0$

$m_{22} = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 6-0 = 6$

$c_{22} = (-1)^4 \times 6 = 6$

$m_{23} = \begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} = 0-0 = 0$

$c_{23} = (-1)^5 \times 0 = 0$

$m_{31} = \begin{vmatrix} 0 & 0 \\ -4 & 0 \end{vmatrix} = 0-0 = 0$

$c_{31} = (-1)^4 \times 0 = 0$

$m_{32} = \begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} = 0-0$ $\quad c_{32} = 0$

$m_{33} = \begin{vmatrix} 2 & 0 \\ 0 & -4 \end{vmatrix} = -8-0 = -8$

$c_{33} = (-1)^6 = -8$

$$C(A) = \begin{bmatrix} -12 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & -8 \end{bmatrix}$$

adj(A) =

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$$= \begin{vmatrix} -12 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & -8 \end{vmatrix}$$

$$A^{-1} = \frac{1}{|A|} (adj A)$$

$$= \frac{1}{-24} \begin{bmatrix} -12 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-12}{-24} & \frac{0}{-24} & \frac{0}{-24} \\ \frac{0}{-24} & \frac{6}{-24} & \frac{0}{-24} \\ 0/_{-24} & 0/_{-24} & -8/_{-24} \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & \frac{1}{-4} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{-1}{4} & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

//

$$A A^{-1} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & \frac{1}{-4} & 0 \\ 0 & 0 & -1/3 \end{bmatrix}$$

$$= \begin{bmatrix} 2/2+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+\frac{-4}{-4}+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+\frac{3}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A = diag(2 \quad -4 \quad 3)$

$A^{-1} = diag(\frac{1}{2} \quad \frac{-1}{4} \quad \frac{1}{3})$

Ans:

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

$$|A| = 1(-1-0) - 0 + 2(4 - -1)$$

$$= -1 + 10 = 9$$ //

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$M_{11} = \begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} = -1 - 0 = -1$

$C_{11} = (-1)^2 \times -1 = -1$

$M_{12} = \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2 - 0 = 2$

$C_{12} = (-1)^3 = -2$

$M_{13} = \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 4 - -1 = 5$

$C_{13} = (-1)^4 \times 5 = 5$

$M_{21} = \begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix} = 0 - 4 = -4$

$C_{21} = (-1)^3 \times -4 = 4$

$M_{22} = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1 - 2 = -1$

$C_{22} = (-1)^4 \times -1 = -1$

$M_{23} = \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} = 2 - 0 = 2$

$C_{23} = (-1)^5 \times 2 = -2$

$M_{31} = \begin{vmatrix} 0 & 2 \\ -1 & 0 \end{vmatrix} = 0 - -2 = 2$

$C_{31} = (-1)^4 \times 2 = 2$

$M_{32} = \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} = 0 - 4 = -4$

$C_{32} = (-1)^5 \times -4 = 4$

$M_{33} = \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} = -1 - 0 = -1$

$C_{33} = (-1)^6 \times -1 = -1$

$$C = \begin{bmatrix} -1 & -2 & 5 \\ 4 & -1 & -2 \\ 2 & 4 & -1 \end{bmatrix}$$

$$\operatorname{adj}(A) = C^T = \begin{bmatrix} -1 & 4 & 2 \\ -2 & -1 & 4 \\ 5 & -2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (\operatorname{adj}(A))$$

$$= \frac{1}{9} \begin{bmatrix} -1 & 4 & 2 \\ -2 & -1 & 4 \\ 5 & -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{9} & \frac{4}{9} & \frac{2}{9} \\ \frac{-2}{9} & \frac{-1}{9} & 4/9 \\ 5/9 & -2/9 & -1/9 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1/9 & 4/9 & 2/9 \\ -2/9 & \frac{-1}{9} & 4/9 \\ 5/9 & -2/9 & -1/9 \end{bmatrix}$$

$$= \begin{bmatrix} -1/9 + 0 + 10/9 & 4/9 + 0 + -4/9 & 2/9 + 0 + -2/9 \\ -2/9 + 2/9 + 0 & 8/9 + 1/9 + 0 & 4/9 + -4/9 + 0 \\ -1/9 + -4/9 + 5/9 & 4/9 + -2/9 + -2/9 & 2/9 + 8/9 - 1/9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rank of a Matrix

Rank is defined as the number of independent rows and columns.

The rank of a matrix is the largest collection of linearly independent columns (rows) of (A).

1. Find the rank of a matrix, $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

Ans $|A| = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$

$|A| \neq 0$

Rank $(A) = 2$ (order of matrix 2×2)

→ Let, A be an $m \times n$ matrix, the rank of A is the maximum order of a non-zero minor of A.

2. Find Rank of a matrix $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 2 & 4 & 2 \\ 0 & 2 & 2 & 1 \end{bmatrix}$

$|A| =$ 1 − 0   2 4 +

$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 4 \\ 0 & 2 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 4 \\ 0 & 2 & 2 \end{vmatrix}$

$= 1(4-8)+0(\cancel{0}\,0) - 2(0-0)$

$= \cancel{-8-2} = \cancel{-10}\ \cancel{8}$

$= -4$

$|A| \neq 0$

Rank of $A = 3$

3. Find the Rank of matrix

$$A = \begin{bmatrix} 2 & 4 & 3 & 2 \\ 3 & 3 & 1 & 4 \end{bmatrix}$$

Ans:

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 3 \end{bmatrix}$$ consider the 2×2 minor.

$$|A| = \begin{vmatrix} 2 & 4 \\ 3 & 3 \end{vmatrix} = 6 - 12 = -6$$

$|A| \neq 0$

Rank of $A = 2$

4. Find the Rank of a matrix?

i) $$A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & 1 & 0 & 1 \\ 4 & 1 & 4 & 7 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 4 & 1 & 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 4 & 1 & 4 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & 0 \\ 1 & 4 \end{vmatrix} + 0\begin{vmatrix} 2 & 0 \\ 4 & 4 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix}$$

$= 1(4-0) + 0(8-0) + 2(2-4)$

$= 4 + 4 = \cancel{8}\ 0$

~~Reorto~~ $$A = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & 4 & 7 \end{bmatrix}$$

$$A = \begin{vmatrix} 0 & 2 & 3 \\ 1 & 0 & 1 \\ 1 & 4 & 7 \end{vmatrix}$$

$= 0(0-4) - 2(7-1) + 3(4-0)$

$= -2(6) + 3(4)$

$= -12+12 = 0$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 4 & 4 & 7 \end{bmatrix}$$

$$|A| = \begin{vmatrix} \cancel{1} & 2 & 3 \\ 2 & 0 & 1 \\ 4 & 4 & 7 \end{vmatrix}$$

$$= 2(14-12) - 0 + 1(\cancel{4}-8)$$

$$= 4 \cancel{0} 0 - 4 = 0$$

$$A\cancel{\beta} = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 4 & 1 & 7 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 4 & 1 & 7 \end{vmatrix}$$

$$= 1(7-1) - 0(14-4) + 3(2-4)$$

$$|A| = 6 + 6 = 0$$

Rank is less than 3

$$[A] = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix}$$

$$= 1 - 0 = 1$$

$$|A| \neq 0$$

Rank $(A) = 2$

5. Rank the matrix

$$A = \begin{bmatrix} -1 & 0 & -2 & 1 \\ 0 & 2 & 4 & 1 \\ -3 & -2 & -10 & 2 \end{bmatrix}$$

$$A\cancel{\$} = \begin{bmatrix} -1 & 0 & -2 \\ 0 & 2 & 4 \\ 3 & -2 & -10 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -1 & 0 & -2 \\ 0 & 2 & 4 \\ 3 & -2 & -10 \end{vmatrix}$$

$= -1(-20 - -8) + -2(0-6)$

$= -1(-12) - 12$

$= 12 - 2 = 0$ //

$A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 2 & 1 \\ -3 & -2 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & 0 & 1 \\ 0 & 2 & 1 \\ -3 & -2 & 2 \end{vmatrix}$

$= -1(4+2)\ 0 + 1(0 - -6)$

$= \cancel{8}\ -6 + 6 = 0$ //

$A = \begin{bmatrix} -1 & -2 & 1 \\ 0 & 4 & 1 \\ -3 & 10 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & -2 & 1 \\ 0 & 4 & 1 \\ -3 & 10 & 2 \end{vmatrix}$

$= -1(8 - -10) + 2(0+3) + 1(0+12)$

$= -1(-2) + 2(+3) + 1(+12)$

$= -18 + 6 + 12)$

$= -12 + 12 = 0$ //

$|A| = \begin{vmatrix} 0 & -2 & 1 \\ 2 & 4 & 1 \\ -2 & -10 & 2 \end{vmatrix}$

$= - -2(4 - -2) + 1(-20 - -8$

$= 2(6) + 1(-12)$

$= 12 - 12 = 0$ //

Rank is less than 3

$A = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$ $|A| = \begin{vmatrix} -1 & 0 \\ 0 & 2 \end{vmatrix}$

$= -2 - 0) = -2$ //

$|A| \neq 0$

Rank is 2 //

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Crammer's Rule

Finding solution of system of linear equation. suppose we have the system of linear equations.

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

define,

$$D = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \quad D_x = \begin{bmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{bmatrix}$$

$$D_y = \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix} \quad D_z = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}$$

Crammer's rule state that,

$$x = \frac{|D_x|}{|D|}, \quad y = \frac{|D_y|}{|D|} \quad z = \frac{|D_z|}{|D|}$$

1. Find solution of the system of equation

$$x + y = 2$$
$$x - y = 0$$

$$D = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$$

$$|D_1| = \begin{vmatrix} 2 & 1 \\ 0 & -1 \end{vmatrix} = -2 - 0 = -2$$

$$D_2 = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$|D_2| = \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = 0 - 2 = -2$$

$$|D| = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1 - 1 = -2$$

$$x = \frac{|D_1|}{|D|} = \frac{-2}{-2} = 1$$

$$y = \frac{|D_2|}{|D|} = \frac{-2}{-2} = 1$$

$$\cancel{z = \frac{|D_3|}{|D|} = \frac{-2}{-2} = 1}$$

$$x = 1 \qquad y = 1$$

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2. $2x - y = -1$

$x + y = 4$

$D = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 \\ 4 \end{bmatrix}$

$D_1 = \begin{bmatrix} -1 & -1 \\ 4 & 1 \end{bmatrix}$

$|D_1| = \begin{vmatrix} -1 & -1 \\ 4 & 1 \end{vmatrix} = -1 - -4 = 3$ //

$D_2 = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}$

$|D_2| = \begin{vmatrix} 2 & -1 \\ 1 & 4 \end{vmatrix} = 8 - -1 = 9$

$|D| = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 - -1 = 3$ //

$x = \frac{|D_1|}{|D|} = \frac{3}{3} = 1$ //

$y = \frac{|D_2|}{|D|} = \frac{9}{3} = 3$ //

$\underline{\underline{x = 1}}, \quad \underline{\underline{y = 3}}$

3. $2x - y = 3$

$x + 3y = -1$

Ans: $D = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \quad \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$|D| = \begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} = 6 - -1 = 7,$

$D_1 = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$

$|D_1| = \begin{vmatrix} 3 & -1 \\ -1 & 3 \end{vmatrix} = 9 - 1 = 8$

$D_2 = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$

$D_2 = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -2 - 3 = -5$

$x = \frac{|D_1|}{|D|} = \frac{8}{7}$ //

$y = \frac{|D_2|}{|D|} = \frac{-5}{7}$ //

4. $2x - 4y = 3$

$x + 3y = -1$

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$$D = \begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ -18 \end{bmatrix}$$

$$|D| = \begin{vmatrix} 2 & -4 \\ 1 & 3 \end{vmatrix} = 6 - -4 = 10$$

$$D_1 = \begin{bmatrix} 3 & -4 \\ -1 & 3 \end{bmatrix}$$

$$|D_1| = \begin{vmatrix} 3 & -4 \\ -1 & 3 \end{vmatrix} = 9 - +4 = \cancel{13}\ 5$$

$$D_2 = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$$

$$D_2 = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -2 - 3 = -5$$

$$x = \frac{|D_1|}{|D|} = \frac{5}{10} = 0.5$$

$$y = \frac{|D_2|}{|D|} = \frac{-5}{10} = -0.5$$

$x = 0.5$ // $\quad y = -0.5$ //

5

$$8x - 7y + 10z = 15$$
$$2x + 3y + 8z = 7$$
$$-4x + 5y - 2z = -9$$

$$D = \begin{bmatrix} 8 & -7 & 10 \\ 2 & 3 & 8 \\ -4 & 5 & -2 \end{bmatrix} \begin{bmatrix} 15 \\ 7 \\ -9 \end{bmatrix}$$

$$|D| = \begin{vmatrix} 8 & -7 & 10 \\ 2 & 3 & 8 \\ -4 & 5 & -2 \end{vmatrix}$$

$$D = 8\begin{vmatrix} 3 & 8 \\ 5 & -2 \end{vmatrix} = 8(6-40) - -7(-4 - -32) + 10(10 - -12)$$

$$= 8(-46) + 7(28) + 10(22)$$

$$= -368 + 196 + 220 = 48 //$$

$$D_1 = \begin{bmatrix} 15 & -7 & 10 \\ 7 & 3 & 8 \\ -9 & 5 & -2 \end{bmatrix}$$

$$|D_1| = \begin{vmatrix} 15 & -7 & 10 \\ 7 & 3 & 8 \\ -9 & 5 & -2 \end{vmatrix} = 15(6-40) - -7(14 - -72) + 10(35 - -27)$$

$$= 15(-46) + 7(58) + 10(62)$$

$$= -690 + 406 + 620 = 336 //$$

$$D_3 = \begin{bmatrix} 8 & -7 & -15 \\ 2 & 3 & 7 \\ -4 & 5 & -9 \end{bmatrix}$$

$$|D_3| = \begin{vmatrix} 8 & -7 & 15 \\ 2 & 3 & 7 \\ -4 & 5 & -9 \end{vmatrix} = \begin{matrix} 8(-62) + 7(10) + 15(10-4) \\ -496 + 70 + 330 = -96 \end{matrix}$$

$$D_2 = \begin{bmatrix} 8 & -5 & 10 \\ 2 & 7 & 8 \\ -4 & 9 & -2 \end{bmatrix}$$

$$|D_2| = \begin{vmatrix} 8 & 15 & 10 \\ 2 & 7 & 8 \\ -4 & -9 & -2 \end{vmatrix} = 8(-56) - 15(-28) + 10(10)$$

$$464 - 420 + 100 = 144$$

$$x = \frac{|D_1|}{|D|} = \frac{336}{48} = 7$$

$$y = \frac{|D_2|}{|D|} = \frac{144}{48} = 3$$

$$z = \frac{|D_3|}{|D|} = \frac{-96}{48} = -2$$

$$\underline{\underline{x = 7}}, \quad \underline{\underline{y = 3}}, \quad \underline{\underline{z = -2}}$$

find solution of the equation?

$$10x + 2y + 3z = 20$$

$$4x + 5y + 6z = 47$$

$$7x + 8y + 9z = 74$$

Ans:

$$\begin{bmatrix} 10 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \begin{bmatrix} 20 \\ 47 \\ 74 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 20 & 2 & 3 \\ 47 & 5 & 6 \\ 74 & 8 & 9 \end{bmatrix}$$

$$|D_1| = \begin{vmatrix} 20 & 2 & 3 \\ 47 & 5 & 6 \\ 74 & 8 & 9 \end{vmatrix}$$

$$= 20(45 - 48) - 2(423 - \cancel{444}) + 3(376 - 370)$$

$$= 20(-3) - 2(-\cancel{243}\,21) + 3(6)$$

$$= -60 + \overset{42}{\cancel{456}} + 18$$

$$= 0$$

$$D_2 = \begin{bmatrix} 10 & 20 & 3 \\ 4 & 47 & 6 \\ 7 & 74 & 9 \end{bmatrix}$$

$$D_2 = \begin{vmatrix} 10 & 20 & 3 \\ 4 & 47 & 6 \\ 7 & 74 & 9 \end{vmatrix}$$

$$= 10(423 - 444) - 20(\overset{36-42}{\cancel{42-36}}) + 3(296 - 329)$$

$$= 10(-21) - 20(-6) + 3(-33)$$

$$= -210 + 120 - 99$$

$$= -189$$

$$D_3 = \begin{bmatrix} 10 & 2 & 20 \\ 4 & 5 & 47 \\ 7 & 8 & 74 \end{bmatrix}$$

$$D_3 = \begin{vmatrix} 10 & 2 & 20 \\ 4 & 5 & 47 \\ 7 & 8 & 74 \end{vmatrix}$$

$$= 10(370 - 376) - 2(296 - 329) + 20(32 - 35)$$

$$= 10(-6) - 2(-33) + 20(-3)$$

$$= -60 + 66 - 60$$

$$= -54$$

$$|D| = \begin{vmatrix} 10 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$$= 10(45 - 48) - 2(36 - 42) + 3(32 - 35)$$

$$= 10(-3) - 2(-6) + 3(-3)$$

$$= -30 + 12 - 9$$

$$= -27$$

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$$D_x = \frac{|Dx|}{|D|}$$

$$= \frac{0}{-27} = 0$$

$$D_y = \frac{|Dy|}{|D|} = \frac{-189}{-27} = 7$$

$$D_z = \frac{|Dz|}{|D|} = \frac{-54}{-27} = 2$$

$10 \times 0 + 2 \times 7 + 3 \times 2 = 20$

$0 + 14 + 6 = 20$

? 1) $3x + 2y = 13$

$-2x + 3y = 0$

2). $x - y = 2$

$2x + y = 1$

3) $-3x + 9y = 0$

$x + 3y = -6$

Ans : 1 $\begin{bmatrix} 3 & 2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 13 \\ 0 \end{bmatrix}$

$$|D| = \begin{vmatrix} 3 & 2 \\ -2 & 3 \end{vmatrix}$$

$$= 9 - -4 = 13$$

$$D_1 = \begin{bmatrix} 13 & 2 \\ 0 & 3 \end{bmatrix}$$

$$|D_1| = 39 - 0 = 39$$

$$D_2 = \begin{bmatrix} 3 & 13 \\ -2 & 0 \end{bmatrix}$$

$$|D_2| = 0 - -26 = 26$$

$$D_x = \frac{|Dx|}{|D|} = \frac{39}{13} = 3$$

$$D_y = \frac{|Dy|}{|D|} = \frac{26}{13} = 2$$

$x = 3$ $y = 2$

$2 = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$|D| = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1 - -2 = 3$

$D_x = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$

$|D_x| = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 - -1 = 3$

$D_y = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

$D_y = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = 1 - 4 = -3$

$D_x = \frac{|D_x|}{|D|} = \frac{3}{3} = 1$

$D_y = \frac{|D_y|}{|D|} = \frac{-3}{3} = -1$

$x = 1$ $\quad y = -1$

3 Ans: $-3x + 9y = 0$

$x + 3y = -6$

$D = \begin{bmatrix} -3 & 9 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ -6 \end{bmatrix}$

$|D| = \begin{vmatrix} -3 & 9 \\ 1 & -3 \end{vmatrix} = -9 - 9 = \cancel{-18}\ 0$

$D_1 = \begin{bmatrix} 0 & 9 \\ -6 & -3 \end{bmatrix}$

$D_1 = \begin{vmatrix} 0 & 9 \\ -6 & 3 \end{vmatrix} = 0 - -54 = 54$

$D_2 = \begin{bmatrix} -3 & 0 \\ 1 & -6 \end{bmatrix}$

$|D_2| = \begin{vmatrix} -3 & 0 \\ 1 & -6 \end{vmatrix} = 18 - 0 = 18$

$D_x = \frac{|D_x|}{|D|} = \frac{54}{\cancel{-18}\ 0} = 0$

$D_y = \frac{18}{0} = 0$

? Use crammer's rule to solve this equation?

$$2x - 3(y+1) = -3$$

$$2y = 3x - 5$$

Ans.

$$2x - 3(y+1) = -3$$

$$\cancel{2y} \rightarrow 2x - 3y - 3 = -3$$

$$2x - 3y = -3 + 3 = 0$$

$$2x - 3y = 0 \quad -(1)$$

$$2y = 3x - 5$$

$$-3x + 2y = -5 \quad -(2)$$

$$2x - 3y = 0$$

$$-3x + 2y = -5$$

$$a = \begin{bmatrix} 2 & -3 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ -5 \end{bmatrix}$$

$$|D| = \begin{vmatrix} 2 & -3 \\ -3 & 2 \end{vmatrix} = 4 - +9 = \cancel{13} -5$$

$$\cancel{|D| = 4}$$

$$D_1 = \begin{bmatrix} 0 & -3 \\ -5 & 2 \end{bmatrix}$$

$$|D_1| = \begin{vmatrix} 0 & -3 \\ -5 & 2 \end{vmatrix} = 0 - +15 = -15$$

$$D_2 = \begin{bmatrix} 2 & 0 \\ -3 & -5 \end{bmatrix}$$

$$|D_2| = \begin{vmatrix} 2 & 0 \\ -3 & -5 \end{vmatrix}$$

$$= -10 - 0 = -10$$

$$Dx = \frac{|Dx|}{|D|} = \frac{-15}{-5} = 3$$

$$Dy = \frac{|Dy|}{|D|} = \frac{-10}{-5} = 2$$

$$x = 3 \qquad y = 2$$

$$2 \times 3 - 3 \times 2 = 0$$

$$6 - 6 = 0$$

HW?

$y = x - 7$

$y = -2x + 5$

Ans:

$y - x = \cancel{0} - 7$

$-x + y = -7 \quad - (1)$

$y = -2x + 5$

$y + 2x = 5$

$2x + y = 5 \quad - (2)$

$D = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -7 \\ 5 \end{bmatrix}$

$|D| = \begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} = -1 - 2 = -3$

$D_1 = \begin{bmatrix} -7 & 1 \\ 5 & 1 \end{bmatrix}$

$D_1 = \begin{vmatrix} -7 & 1 \\ 5 & 1 \end{vmatrix} = -7 - 5 = -12$

$D_2 = \begin{bmatrix} -1 & -7 \\ 2 & 5 \end{bmatrix}$

$D_2 = \begin{vmatrix} -1 & -7 \\ 2 & 5 \end{vmatrix} = -5 - -14$

$= 9$

$D_x = \frac{|D_x|}{|D|} = \frac{-12}{-3} = 4$

$y = \frac{|D_y|}{|D|} = \frac{9}{-3} = -3$

$2 \times 4 + -3 = 5$

$x = 4 \quad y = -3$

? $x - 6 = y + 1$

$y = -3x + 1$

Ans.

$x - 6 = y + 1$

$x - y = 6 + 1 = 7$

$x - y = 7$

$y = -3x + 1$

$y + 3x = 1$

$3x + y = 1 \quad - (2)$

$x - y = 7$
$3x + y = 1$

$$\begin{bmatrix} 1 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 1 \end{bmatrix}$$

$$|D| = \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} = 1 - -3 = 4$$

$$D_1 = \begin{bmatrix} 7 & -1 \\ 1 & 1 \end{bmatrix}$$

$$|D_1| = \begin{vmatrix} 7 & -1 \\ 1 & 1 \end{vmatrix} = 7 - -1 = 8$$

$$D_2 = \begin{bmatrix} 1 & 7 \\ 3 & 1 \end{bmatrix}$$

$$|D_2| = \begin{vmatrix} 1 & 7 \\ 3 & 1 \end{vmatrix} = 1 - 21 = -20$$

$$x = \frac{|D_1|}{|D|} = \frac{8}{4} = 2$$

$$y = \frac{|D_2|}{|D|} = \frac{-20}{4} = -5$$

$x = 2$ $y = -5$

?

$1 - x = 6 + y$
$5 - 2x = 6 - y$

Ans:-

$1 - x = 6 + y$
$-x - y = 6 - 1 = 5$ —(1)

$5 - 2x = 6 - y$
$-2x + y = 6 - 5 = 1$

$-x - y = 5$
$-2x + y = 1$

$$D = \begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$|D| = \begin{vmatrix} -1 & -1 \\ -2 & 1 \end{vmatrix} = -1 - 2 = -3$$

$$D_1 = \begin{bmatrix} 5 & -1 \\ 1 & 1 \end{bmatrix}$$

$$|D_1| = \begin{vmatrix} 5 & -1 \\ 1 & 1 \end{vmatrix} = 5 - -1 = 6$$

$D_2 = \begin{bmatrix} -1 & 5 \\ -2 & 1 \end{bmatrix}$

$|D_2| = \begin{vmatrix} -1 & 5 \\ -2 & 1 \end{vmatrix}$

$= -1 - 10 = 9$

$x = \frac{|D_x|}{|D|} = \frac{6}{-3} = -2$

$y = \frac{|D_y|}{|D|} = \frac{9}{-3} = -3$

$\underline{\underline{x = -2}}$ $\quad$ $\underline{\underline{y = -3}}$

7 $\quad y = 0.05x$

$x + y = 504$

Ans:- $\quad y = 0.05x$

$y - 0.05x = 0$

$\frac{y}{x} = \cancel{0.053}$

$-0.05x + y = 0 \quad - (1)$

$x + y = 504 \quad - (2)$

$D = \begin{bmatrix} -0.05 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 504 \end{bmatrix}$

$|D| = \begin{vmatrix} 0.05 & 1 \\ 1 & 1 \end{vmatrix} = -0.05 - 1 = -1.05$

$D_x = \begin{bmatrix} 0 & 1 \\ 504 & 1 \end{bmatrix}$

$|D_x| = \begin{vmatrix} 0 & 1 \\ 504 & 1 \end{vmatrix} = 0 - 504 = -504$

$D_y = \begin{bmatrix} -0.05 & 0 \\ 1 & 504 \end{bmatrix}$

$|D_y| = \begin{vmatrix} -0.05 & 0 \\ 1 & 504 \end{vmatrix} = -25.2 - 0$

$= -25.2$

$x = \frac{|D_x|}{|D|} = \frac{-504}{-1.05} = 480$

$y = \frac{|D_y|}{|D|} = \frac{-25.2}{-1.05} = 24$

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? $0.05x + 0.10y = 6$

$y = 98 - 6x$

Ans $0.05x + 0.10y = 6$ —(1)

$y + x = 98$

$x + y = 98$ —(2)

$$D = \begin{bmatrix} 0.05 & 0.10 \\ 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 6 \\ 98 \end{bmatrix}$$

$$|D| = \begin{vmatrix} 0.05 & 0.10 \\ 1 & 1 \end{vmatrix} = 0.05 - 0.10 = -0.05$$

$$D_1 = \begin{bmatrix} 6 & 0.10 \\ 98 & 1 \end{bmatrix}$$

$$|D_1| = \begin{vmatrix} 6 & 0.10 \\ 98 & 1 \end{vmatrix} = 6 - 9.8 = -3.8$$

$$D_2 = \begin{bmatrix} 0.05 & 6 \\ 1 & 98 \end{bmatrix} = \cancel{4.9 - 6 = -1.1}$$

$$|D_2| = \begin{vmatrix} 0.05 & 6 \\ 1 & 98 \end{vmatrix} = 4.9 - 6 = -1.1$$

$$x = \frac{|D_x|}{|D|} = \frac{-3.8}{-0.05} = 76$$

$$y = \frac{|D_y|}{|D|} = \frac{-1.1}{-0.05} = 22$$

$x = 76$ $\quad y = 22$

Characteristic equations and roots (eigen values)

Let A be an $n \times n$ matrix, then the equation $\boxed{|A - \lambda I| = 0}$ is called the characteristic equation.

? Let $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

Find the eigen value of the matrix ?

Let $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

$|A - \lambda I| = 0$

$$A - \lambda I = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{bmatrix}$$

the characteristic eqt is,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix}$$

$$= (3-\lambda)(3-\lambda) - 1$$

$$= 9 - 3\lambda - 3\lambda + \lambda^2 - 1$$

$$= \lambda^2 - 6\lambda + 8$$

Therefor, the characteristic eqt is,

$$\lambda^2 - 6\lambda + 8 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$ $b = -6$ $c = 8$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 1 \times 8}}{2 \times 1}$$

$$= \frac{6 \pm \sqrt{36 - 32}}{2} = \frac{6 \pm \sqrt{4}}{2}$$

$$= \frac{6 \pm 2}{2} = \frac{8}{2} = 4$$

$$= \frac{6-2}{2} = \frac{4}{2} = 2$$

$x = 2$ or 4

eigonal values $= 2$ or 4

2. Find the eigonal values of the matrix.

$$A = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$$

Ans. Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} =$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & -1-\lambda \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$= \begin{vmatrix} 2-\lambda & 1 \\ 0 & -1-\lambda \end{vmatrix}$$

$$= (2-\lambda)(-1-\lambda) - 0$$

$$= -2 - 2\lambda + \lambda + \lambda^2$$

$$-2-\lambda+\lambda^2$$

$$= \cancel{\lambda^3 - 2\lambda - 2}$$

$$= \cancel{\lambda^3 - 2\lambda} - 2$$

$$= \cancel{\lambda^2 + \lambda}$$

$$= \lambda^2 - \lambda - 2$$

$$|A - \lambda I| = 0$$

$$\lambda^2 - \lambda - 2 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a = 1 \quad b = -1 \quad c = -2$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4\times1\times(-2)}}{2\times1}$$

$$= \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm \sqrt{9}}{2}$$

$$= \frac{1+3}{2} = \frac{4}{2} = 2$$

$$\text{or} = \frac{1-3}{2} = \frac{-2}{2} = -1$$

eigonal values = 2 or -1

3. $A = \begin{bmatrix} 3 & -2 \\ 5 & -3 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 3 & -2 \\ 5 & -3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 \\ 5 & -3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 3-\lambda & -2 \\ 5 & -3-\lambda \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 3-\lambda & -2 \\ 5 & -3-\lambda \end{vmatrix}$$

$$= (3-\lambda)(-3-\lambda) - (-2)(5)$$

$$= (3-\lambda)(-3-\lambda) + 10$$

$$= -9 - 3\lambda + 3\lambda + \lambda^2 + 10$$

$$= \lambda^2 + 10 - 9$$

$$= \lambda^2 + 1$$

$\cancel{\lambda^2 = -1}$

$\cancel{\lambda = \sqrt{-1}}$

Therefore, the ~~eigonal~~ characteristic eqt is

$\lambda^2 + 1 = 0$

$\sqrt{-4} = \sqrt{-1} \times \sqrt{4}$

$\pm i 2$

$2i$

$\lambda^2 = -1$

$\lambda = \pm\sqrt{-1}$

$\lambda = \pm i$

? Find the eigonal values of the matrix

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

Ans.

$$A - \lambda I = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 3-\lambda & 0 & 0 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & -1-\lambda \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 3-\lambda & 0 & 0 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & -1-\lambda \end{vmatrix}$$

$$= 3-\lambda \begin{vmatrix} 2-\lambda & 1 \\ 0 & -1-\lambda \end{vmatrix} + 0 - 0$$

$$= 3-\lambda(2-\lambda(-1-\lambda) - 0)$$

$$= (3-\lambda)(2-\lambda)(-1-\lambda)$$

$$\cancel{= (3-\lambda)(2-\lambda) + (3-\lambda)(1-\lambda)}$$

The characteristic eqt is $|A - \lambda I| = 0$

$$\cancel{= 6 - \lambda^2 + 3}$$

$$\cancel{= 6 - 3\lambda - 2\lambda + \lambda^2 + 3 - 3\lambda - \lambda + \lambda^2}$$

$$= (3-\lambda)(2-\lambda)(-1-\lambda) = 0$$

Roots of the above eqt.

$\cancel{a\lambda^3 + b\lambda^2 + c\lambda}$

$\lambda = 3, \quad \lambda = 2, \quad \lambda = -1$

So, the eigonal values,

3, 2, -1

? Find the eigenal values of the matrix?

$$A = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & -1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} -\lambda & -1 & -1 \\ 1 & 2-\lambda & 1 \\ 1 & 1 & 2-\lambda \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} -\lambda & -1 & -1 \\ 1 & 2-\lambda & 1 \\ 1 & 1 & 2-\lambda \end{vmatrix}$$

$$= -\lambda \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} + 1 \begin{vmatrix} 1 & 1 \\ 1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} 1 & 2-\lambda \\ 1 & 1 \end{vmatrix}$$

$$= -\lambda(2-\lambda(2-\lambda)-1) + 1((2-\lambda)-1) - 1(1-(2-\lambda))$$

$$= -\lambda((2-\lambda)(2-\lambda)-1) + 1(2-\lambda-1)$$

$$(-1+2-\lambda)$$

$$= -\lambda(4-2\lambda-2\lambda+\lambda^2)-1) \quad 2-\lambda-1-1+2-\lambda$$

$$= -\lambda(4-4\lambda+\lambda^2)-1)$$

$$= -4\lambda+4\lambda^2-\lambda^3+\lambda+2-\lambda-1-1+2-\lambda$$

$$= -\lambda^3+4\lambda^2-3\lambda+2-\lambda-1-1+2-\lambda$$

$$= -\lambda^3+4\lambda^2-5\lambda+2$$

$$|A - \lambda I| = -\lambda^3+4\lambda^2-5\lambda+2$$

$$= -\lambda^3+4\lambda^2-5\lambda+2 = 0$$

$$= -1^3+4\times1^2-5\times1+2 = 0$$

$$= -1+4-5+2 = 0$$

$$\lambda = 1$$

$= -2^3 + 4\times 2^2 - 5\times 2 + 2 = 0$

$= -8 + 16 - 10 + 2 = 0$

$\lambda = 2$

$\boxed{-\lambda^3 + 4\lambda^2 - 5\lambda + 2}$

expand the eqt to,

$= (\lambda - a)(\lambda - b)(\lambda - c)$ devide the eqt by $\lambda - a$

$= (\lambda - 1)(\lambda - b)(\lambda - c) / \lambda - 1$

$= (\lambda - b)(\lambda - c)$

$$
\begin{array}{r}
-\lambda^2 + 3\lambda - 2 \\
(\lambda - 1)\overline{)\,-\lambda^3 + 4\lambda^2 - 5\lambda + 2} \\
-\lambda^3 + \lambda^2 \qquad\qquad \\
\hline
0 + 3\lambda^2 - 5\lambda \qquad \\
+3\lambda^2 - 3\lambda \qquad \\
\hline
0 - 2\lambda + 2 \\
-2\lambda + 2 \\
\hline
0 + 0
\end{array}
$$

$\cancel{2 + (-\lambda^2)}$

$\lambda \times -\lambda^2$

$\cancel{-\lambda^2 + 3\lambda - 2} =$

$-\lambda^3 + 4\lambda^2 - 5\lambda + 2 = -\lambda^2 + 3\lambda - 2$

Therefore, $-\lambda^3 + 4\lambda^2 - 5\lambda + 2 = 0$

$(\lambda - 1)(-\lambda^2 + 3\lambda - 2)$

$\lambda - 1 = 0$ or $-\lambda^2 + 3\lambda - 2 = 0$

$\lambda = 1$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4\times -1\times -2}}{2\times -1}$$

$$= \frac{-3 \pm \sqrt{9-8}}{-2} = \frac{-3 \pm \sqrt{1}}{-2}$$

$$= \frac{-3+1}{-2} \text{ or } \frac{-3-1}{-2}$$

$$= \frac{-2}{-2} \text{ or } \frac{-4}{-2}$$

$= 1$ or 2

$\lambda = 1, 1, 2$

? Find eignol values of matrix,

$$A = \begin{bmatrix} 3 & -18 \\ 2 & -9 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 3 & -18 \\ 2 & -9 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -18 \\ 2 & -9 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 3-\lambda & -18 \\ 2 & -9-\lambda \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 3-\lambda & -18 \\ 2 & -9-\lambda \end{vmatrix}$$

$$= (3-\lambda)(-9-\lambda) - -36$$

$$= (3-\lambda)(-9-\lambda) + 36$$

$$= -27 - \cancel{3\lambda} + 9\lambda + \lambda^2 + 36$$

$$= \lambda^2 + 6\lambda + 9$$

$$|A - \lambda I| = 0$$

$$\lambda^2 + 6\lambda + 9 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 9}}{2 \times 1}$$

$$= \frac{-6 \pm \sqrt{36 - 36}}{2} = -3$$

$$\lambda = -3, -3$$

? find eigonal values of the matrix?

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

Ans:

$$A - \lambda I = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & 2 & 1 \\ 1 & -1-\lambda & 1 \\ 2 & 0 & 1-\lambda \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 & 1 \\ 1 & -1-\lambda & 1 \\ 2 & 0 & 1-\lambda \end{vmatrix}$$

$$= 1-\lambda((-1-\lambda)(1-\lambda) - 0) - 2(1-\lambda - 2) + 1(0 - 2(-1-\lambda))$$

$$= (1-\lambda)(-1+\lambda-\lambda+\lambda^2) - 2(-1-\lambda) + 1(2+2\lambda)$$

$= \cancel{-1+\lambda-\lambda+\lambda^2+\lambda-\lambda^2+\lambda^2-\lambda^3}$

$= (1-\lambda)(-1+\lambda^2)+2(-1-\lambda)+(2+2\lambda)$

$= -1+\lambda^2+\lambda-\lambda^3+2+2\lambda+2+2\lambda$

$= -\lambda^3+\lambda^2+5\lambda+3$

$|A-\lambda I| = 0$

$-\lambda^3+\lambda^2+5\lambda+3 = 0$

If $\lambda = 1$

$-1^3+1^2+5\times1+3 = 0$

$-1+1+5+3 = 8$

If $\lambda = -1$

$(+1)^3+(-1)^2+5\times-1+3 =$

$1+1-5+3 = 2-2 = 0$

$\lambda = -1$ is one of the eigonal value.

$$\begin{array}{r|l} & -\lambda^2+2\lambda+3 \\ \hline (\lambda+1) & -\lambda^3+\lambda^2+5\lambda+3 \\ & -\lambda^3-\lambda^2 \\ \hline & 0+2\lambda^2+5\lambda \\ & +2\lambda^2+2\lambda \\ \hline & 0+3\lambda+3 \\ & +3\lambda+3 \\ \hline & 0 \end{array}$$

Therefore, $-\lambda^3+\lambda^2+5\lambda+3$

$(\lambda+1)(-\lambda^2+2\lambda+3)$

$\lambda = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \frac{-2\pm\sqrt{2^2-4\times-1\times3}}{2\times-1}$

$= \frac{-2\pm\sqrt{4+12}}{-2}$

$= \frac{-2\pm\sqrt{16}}{-2} =$

$= \frac{-2+4}{-2}$ or $\frac{-2-4}{-2}$

$= \frac{2}{-2}$ or $\frac{-6}{-2}$

$= -1$ or 3

$\lambda = -1, -1, 3$

eigonal values, $-1, -1, 3$

Idempotent Matrix

A square matrix A is said to be idempotent if $\boxed{A^2 = A}$

eg. $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 1+0 \\ 0+0 & 0+0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ //

$\underline{\underline{A^2 = A}}$ A is a idempotent matrix.

? $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$= \begin{bmatrix} 4+2-4 & -4-6+8 & -8-8+12 \\ -2-3+4 & 2+9-8 & 4+12-12 \\ 2+2-3 & -2-6+6 & -4-8+9 \end{bmatrix}$

$= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$\underline{\underline{A^2 = A}}$, A is a idempotent matrix

? Find eigonal values ?

$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix} \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$= \begin{bmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{bmatrix}$

$|A - \lambda I| = \begin{vmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{vmatrix}$

$= 1-\lambda\left((-5-\lambda)(4-\lambda)+18\right)+3\left(3(4-\lambda)-18\right)$

$+3\left(-18-(6)(-5-\lambda)\right)$

$= 1-\lambda\left(-20+5\lambda-4\lambda+\lambda^2+18\right)+3\left(12-3\lambda-18\right)$

$+3\left(-18+30+6\lambda\right)$

$= -20+5\lambda-4\lambda+\lambda^2+18+20\lambda-5\lambda^2+$

$4\lambda^2+\lambda^3-18\lambda+36-9\lambda-54-$

$54+90+18\lambda$

$= -\lambda^3+\lambda^2-5\lambda^2+4\lambda^2+5\lambda^2-4\lambda+20\lambda-$

$18\lambda-9\lambda+18\lambda-20+18+36-54$

$-54+90$

$= -\lambda^3+0+12\lambda+16$

$= -\lambda^3+12\lambda+16$

If $\lambda = 1$

$(-1)^3+12\times1+16 = 27$

If $\lambda = -1$

$(+1)^3+12\times-1+16 = 4$

If $\lambda=-2$ $= (+2)^3+12\times2+16 =$

$+8-24+16 = 0$

Therefore, $\lambda^3+12\lambda+16 = 0$

$\lambda = -2$

$$\begin{array}{r|l} & -\lambda^2 \\ \lambda+2 & -\lambda^3+12\lambda+16 \\ & \lambda^3 \\ \hline & 0-\lambda^2+0\lambda+8 \end{array}$$

$$\begin{array}{r|l} \lambda+2 & -\lambda^3+0\lambda+12\lambda+16 \\ & \lambda^3+2\lambda^2 \\ \hline & 0+2\lambda^2+12\lambda \\ & 2\lambda^2+4\lambda \\ \hline & 0+8\lambda+16 \\ & 8\lambda+16 \\ \hline & 0 \quad 0 \end{array}$$

Therefore, $\lambda^3-12\lambda+16$

$(\lambda+2)(-\lambda^2+2\lambda+8)$

$\therefore \lambda = \dfrac{-b\pm\sqrt{b^2+4ac}}{2a} = \dfrac{-2\pm\sqrt{2^2-4\times-1\times8}}{2\times-1}$

$= \dfrac{-2\pm\sqrt{36}}{-2} = \dfrac{-2\pm6}{-2}$

$= \dfrac{-2+6}{-2}$ $\qquad \dfrac{-2-6}{-2} = \dfrac{-8}{-2}$

$= \dfrac{4}{-2} = -2$ $\qquad = 4$

eigonal values, $-2, -2, 4$

? $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -2 \\ 2 & 2 & -1 \end{bmatrix}$ find eigonal values?

Ans $A - \lambda I = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -2 \\ 2 & 2 & -1 \end{bmatrix} \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$= \begin{bmatrix} 1-\lambda & 2 & 0 \\ 0 & 1-\lambda & -2 \\ 2 & 2 & -1-\lambda \end{bmatrix}$

$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 & 0 \\ 0 & 1-\lambda & -2 \\ 2 & 2 & -1-\lambda \end{vmatrix} = 0$

$= 1-\lambda((1-\lambda)(-1-\lambda)+4) - 2(0+4)+0$

$= 1-\lambda(-1-\lambda+\lambda+\lambda^2+4) - 8$

$= -1+\lambda^2+4+\lambda+\lambda^3-4\lambda-8$

$|A-\lambda I| = -\lambda^3+\lambda^2-3\lambda-5 = 0$

If $\lambda = 1$ $\quad -\lambda^3+\lambda^2-\lambda-3$

$(-1)^3+1^2-3(1)-5 =$

$-1+1-3-5 = -8$

If $\lambda = -1$ $\quad -\lambda^3+\lambda^2-3\lambda-5$

$-(-1)^3+1^2-3(-1)-5 =$

$1+1+3-5 = 0$

$\lambda = -1$ one of the eigonal value.

$(\cancel{\lambda+1}) \overline{) \cancel{-\lambda^3+\lambda^2-3\lambda-5}}$

$-\lambda^2+2x-5$

$(\lambda+1) \overline{) -\lambda^3+\lambda^2-3x-5}$

$-\lambda^3-\lambda^2$

$0 \quad 2\lambda^2-3\lambda$

$2\lambda^2+2\lambda$

$0 \quad -5\lambda-5$

$-5x-5$

$0+0$

Therefore, $-\lambda^3+\lambda^2-3\lambda-5 = -\lambda^2+2x-5$

$(-\lambda^2+2x-5)(\lambda+1)$

$\lambda+1=0, \quad \lambda = 1$

$-\lambda^2+2\lambda-5 = 0$

$\lambda = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$= \frac{-2 \pm \sqrt{2^2 - 4 \times -1 \times 5}}{2 \times -1}$$

$$= \frac{-2 \pm \sqrt{4+20}}{-2}$$

$$= \frac{-2 \pm \sqrt{-16}}{-2}$$

$$= \frac{-2+4}{-2} \text{ or } \frac{-2-4}{-2}$$

$$= \frac{2}{-2} \text{ or } \frac{-6}{-2}$$

$$= -1 \text{ or } 3$$

So the eigonal values,

$\lambda = -1, -1, 3$

? Find eigonal values of the matrix $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

Ans. Let $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = 3-\lambda(3-\lambda) - 1$$

$$= 9 - 3\lambda - 3\lambda + \lambda^2 - 1$$

$$= \lambda^2 - 6\lambda + 8$$

$|A - \lambda I| = 0 \; : \; \lambda^2 - 6\lambda + 8 = 0$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times 8}}{2 \times 1}$$

$$= \frac{6 \pm \sqrt{36-32}}{2} = \frac{6 \pm \sqrt{4}}{2}$$

$$= \frac{6+2}{2} \text{ or } \frac{6-2}{2}$$

$$= \frac{8}{2} \text{ or } \frac{4}{2}$$

$$= 4 \text{ or } 2$$

eigonal values, 4, 2