Module 4

Probability Theory

→ Factorial

Eg:- $4! = 1\times2\times3\times4 = 24$

$3! = 1\times2\times3 = 6$

$0! = 1$

$1! = 1$

positions (A, B, C)

$$\boxed{n! = 1\times2\times3 \dots (n-1)\times n}$$

→ Permutations

in terms of factorial

$$nP_k = \frac{n!}{(n-k)!}$$

subgroups (A, B, C)

Eg:- $n = 3 \quad k = 2$

$$3P_2 = \frac{3!}{(3-2)!} = \frac{1\times2\times3}{1!}$$

$$= \frac{6}{1} = 6$$

(A, B)
(B, A)
(B, C)
(C, B)
(A C)
(C A)

$\}$ 6

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Probability :

The probability of a given event may be defined as the numerical value given to ~~beyond~~ the likelyhood of the occurance of that event. It is a number lying between 0 and 1.

→ Random experiment :

An experiment that has 2 or more possible outcomes which vary in an ~~unp~~ unpredictable manner from trial to trial when conducted under unkown conditions, is called a random experiment.

1 - It has more than 1 result
2 - The outcome unpredictable
3 - The experiment is repeatable

Eg, Tossing of a coin is a random experiment

it has two specified outcomes, Head or Tail.

But ~~to~~ we are uncertain whether head will turn on tail when the coin is tossed.

→ Throwing a die is an experiment. when we throwing a die, possible outcomes are 1, 2, 3, 4, 5 & 6.

→ Sample point :

Every indecomposable outcome of a random experiment is called sample point of that random experiment..

Eg :- 1 - when a coin is tossed "getting Head" is sample point. when two dice are thrown getting (2, 3) is a sample point.

→ Sample space :

The sample space of a random experiment is the set of all possible outcomes (sample points) of that random experiment.

Eg, 1- when a coin is tossed the sample space is

S {Head, Tail}

when 2 coins are tossed the sample space is

$$S = \{HH, HT, TH, TT\}$$

S = sure event
null = impossible event

→ Event :

An event is a subset of the sample space of a random experiment.

Eg :- Tossing 2 coins at a time, then the sample space is, $S = \{HH, HT, TH, TT\}$ then, the $\{HH, HT\}$ is an event of the random experiment. $\{TT\}$ and $\{HT, TH\}$

Sure Events (sample space)

An event whose occurance is inevitable is called sure events.

Impossible events

If an event can't occure, when the random experiment is conducted, then that event is an impossible event. An empty set (non set) can be represented by ϕ.

Uncertain Events

An event is said to be uncertain if its occurance is neither sure nor impossible.

Let, $S = \{1, 2, 3, 4, 5, 6\}$ be the sample space and Let $A = \{1, 2, 3\}$ then 'A' is an uncertain event.

Equally likely Events

Two events are said to be equally likely, if any one of them can't be expected to occure in preference to the others.

Consider the random experiment of tossing a coin.

Then $S = \{H, T\}$

$A = \{H\}$ $B = \{T\}$

Then 'A' and 'B' are equally likely events.

Let $S = \{1, 2, 3, 4, 5, 6\}$

$A = \{2, 3\}$ $B = \{4, 5\}$

'A' and 'B' are equally likely events.

Mutually Exclusive Events

A set of events are said to be mutually exclusive if the occurance of one of them excludes (prevents) the possibility of the occurance of the others.

If A, B. and C are mutually exclusive

to $A \cap B = \phi$

$A \cap C = \phi$

$B \cap C = \phi$

Eg:- Let $S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 5\}$, $B = \{2, 4\}$, $C = \{3\}$, $D = \{6\}$

Here, $A \cap B = \phi$

$A \cap C = \phi$

$A \cap D = \phi$

$B \cap C = \phi$

$B \cap D = \phi$

$C \cap D = \phi$

Therefore, A, B, C and D are mutually exclusive events.

Mutually exhaustive events

A group of events is said to be exhaustive when it includes all possible outcomes of the random experiments. That is, if a set of events are exhaustive, atleast one of them will occur in any trial of the random experiment.

Let $S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2\}$, $B = \{3, 4, 5\}$ $C = \{5, 6\}$

Then A, B & C are mutually exhaustive events

Here, $A \cup B \cup C = \{1, 2, 3, 4, 5, 6\} = S$

→ A group of events $A_1, A_2, A_3, \ldots A_K$ are said to be exhaustive events if,

$A_1 \cup A_2 \cup \ldots \cup A_K = S$ (sample space).

Mutually Exclusive and Exhaustive Events

A group of events are said to be mutually exclusive and exhaustive, if they are mutually exclusive and exhaustive.

For eg:-

I $S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2\}$ $B = \{3, 4, 5\}$ $C = \{6\}$

mutually exclusive

$A \cap B = \phi$

$A \cap C = \phi$

$B \cap C = \phi$

Therefore, A, B & C are mutually exclusive.

Mutually exhaustive

$A \cup B \cup C = \{1, 2\} \cup \{3, 4, 5\} \cup \{6\}$

$= \{1, 2, 3, 4, 5, 6\}$

ie, $A \cup B \cup C = S$

II $S = \{HH, HT, TH, TT\}$

$A = \{HH, TT\}$

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$B = \{TH, TH\}$

$A \cap B = \phi$

$A \cup B = \{HH, HT, TH, TT\}$

ie, $A \cup B = S$ //

Alegebra of Events

If A & B are 2 events,

1- A^c not A

2- $A \overset{or}{\cup} B$ — atleast one

3- $A \overset{and}{\cap} B$ = Both   Eg: $A = \{1, 2\}$

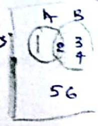
$B = \{2, 3, 4\}$

$A \cap B = \{2\}$

4- $A \cap B^c$ = Exactly A $B^c = \{1, 5, 6\}$

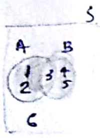
$A = \{1, 2\}$

$A \cap B^c = \{1\}$



5 - ~~$(A \cap B^c) \cup (A^c \cap B)$~~

$(A \cap B^c) \cup (A^c \cap B)$ = Exactly one



$= \{1, 2, 4, 5\}$

Classical definition of Probability

Let a random experiment produce only a pre-finite number of outcomes, say n. Let all these outcomes be equally likey and mutually exclusive. Let 'f' of these outcomes be favourable to an event 'A', then the probability of the event A is defined as,

$$\boxed{P(A) = \frac{f}{n}}$$

$n(A) = f$

$n(S) = n$

for eg:- $S = \{H, T\}$

$P\{H\} = \frac{1}{2}$

? · Two coins are tossed, what is the probability of getting ?

→ Both heads

→ one head

→ Atleast one head

→ No head.

1, 2   3, 4, 5

Ans:- $S = \{HH, HT, TH, TT\}$

1- ~~both heads~~ $P\{\text{both head}\} = \{HH\}$ $\quad f=1$

$= \frac{1}{4}$ $\quad n=4$

2- $\{\text{One head}\} = \{HT, TH\}$

$P[\{\text{one head}\}] = \frac{2}{4}$

3- Atleast one head $= \{HH, HT, TH\}$

$P\{\text{Atleast one head}\} = \frac{3}{4}$

4- $\{\text{No Head}\} = \{TT\}$

$P(\{\text{No head}\}) = \frac{1}{4}$

HW

2. Three unbaised coins are tossed. What is the probability of obtaining, ~~all heads~~

1- All heads

2- two heads

3- One head

4- Atleast one head

5- Atleast 2 heads

6- At the most one head

Ans: $S = \{HHH, HTH, HHT, HTT, TTH, THH, THT, TTT\}$

1- $\{\text{All heads}\} = \{HHH\}$

$P(\{\text{All heads}\}) = \frac{1}{8}$

2- $\{\text{Two heads}\} = \{HTH, HHT, THH\}$

$P(\{\text{Two heads}\}) = \frac{3}{8}$

3- $\{\text{one head}\} = \{HTT, TTH, THT\}$

$P(\{\text{one head}\}) = \frac{3}{8}$

4- $\{\text{Atleast one head}\} = \{HHH, HTH, HHT, HTT, TTH, THH, THT\}$

$P(\{\text{Atleast one head}\}) = \frac{7}{8}$

5- $\{\text{At least two heads}\} = \{HHH, HTH, HHT, THH\}$

$P(\{\text{Atleast two heads}\}) = \frac{4}{8}$

6- $\{\text{At the most, one head}\} = \{HTT, TTH, THT, TTT\}$

$P(\{\text{At most one head}\}) = \frac{4}{8}$

3. A ball is drawn from bag containing 4 white, 6 black & 5 green balls. Find the probability that a ball drawn is

1- white

2- Green

3- black

4- Not green

5- Green or white

Ans:- $n = 15$

$S =$

1- {white} $= \frac{4}{15}$

$P(\text{white}) = \frac{4}{15}$

2- {Green} $=$ $P(\text{Green}) = \frac{5}{15}$

3- {Black} $=$ $P(\text{black}) = \frac{6}{15}$

4- {Not green} $=$ $P(\text{not green}) = \frac{10}{15}$

5- {Green or white} $= \frac{9}{15}$

7. A card is drawn from a packet of card. What is the probability that it is ?

1- A black card

2- king

3- Queen

4- A Spade

5- A spade king

6- A king or a queen

$\frac{13 \times 4 = 52}{52}$

3:3

Spade (Black) Clubs (Black) Hearts (Red) Diamond (Red)

Jack

King

Queen 2 to 10

Ace

2

3

4

5

6

7

Ans:-

1- {A black card} $= \frac{26}{52}$

2- {King} $=$ $P(\text{King}) = \frac{4}{52}$

3- {Queen} $= \frac{4}{52}$

4- {A spade} $= \frac{13}{52}$

5- {A spade king} = $\frac{1}{52}$

6- {king or queen} = $\frac{8}{52}$

5. A bag containing 6 white and 4 black balls what is the probability of (6 W, 4 B)

1- Drawing a white ball

2- Drawing 2 white balls

3- Drawing 4 white balls

4- Drawing 2 white & 2 Black balls

5- Drawing 1 white & 3 black balls.

6- Drawing 3 white & 1 black balls.

Ans. 1- P(Drawing a white ball) = $\frac{6C_1}{10C_1} = \frac{6}{10}$ //

2- P(Drawing 2 white balls) =

$n(s) = 10C_2$

$n(A) = 6C_2$

$= \frac{6C_2}{10C_2}$ //

3- P(Drawing 3 white balls) = $\frac{6C_3}{10C_3}$ //

4- P(Drawing 4 white balls) = $\frac{6C_4}{10C_4}$ //

5- P(Drawing 2 white & 2 black balls) =

$= \frac{6C_2 \times 4C_2}{10C_4}$ //

6- P(Drawing 1 white & 3 black balls) =

$= \frac{6C_1 \times 4C_3}{10C_4}$ //

7- P(Drawing 3 white & 1 black balls) =

$= \frac{6C_3 \times 4C_1}{10C_4}$ //

HW 6. A bag contains 7 white and 9 black balls. 3 balls are drawn together. what is the probability that

1- All are black

2- All are white

3- 1 white & 2 black

4- 2 white & 1 black

Frequency definition of Probability

If we repeate a random experiment a great numbers of times. under essentially the same condition, the limit of the ratio of the number of times that an event occured to the total number of trials as the no. of trials increase indefinitely is called the probability of the occurance of the event.

That is, $P(A) = \lim_{n \to \alpha} \frac{f}{n}$

$$P(A) = f/n$$

where, n = no. of repetations of the experiment

f = no. of times A occured.

Ans.

1- $P(\text{All are black}) = \frac{{}^{9}C_3}{{}^{16}C_3}$

2 $P(\text{All are white}) = \frac{{}^{7}C_3}{{}^{16}C_3}$

3- $P(1 \text{ white } \& \ 2 \text{ black}) = \frac{{}^{7}C_1 \times {}^{9}C_2}{{}^{16}C_3}$

4- $P(2 \text{ white } \& \ 1 \text{ black} = \frac{{}^{7}C_2 \times {}^{9}C_1}{{}^{16}C_3}$

Axiomatic Definition of probability

Let 'S' be the sample space of a random experiment. Let 'A' be an event of the random experiment so that 'A' is the subset of ~~the~~ 'S'. Then we can associate a real number $P(A)$ to the event A. This number $P(A)$ will be call probability of A if it ~~keeping~~ satisfies the following 3 axioms :-

Axiom I : $P(A) \geq 0$, for every $A \subset S$

Axiom II ; $P(S) = 1$

Axiom III : : $P(A \cup B) = P(A) + P(B)$ if $A \cap B = \Phi$

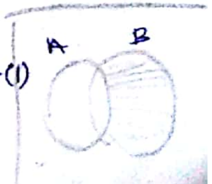
Result (Addition Rule for any two events)

1- If A & B are any 2 events then ;

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

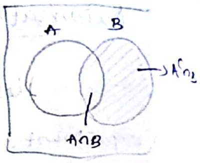
Proof, Since $A \cup B = A \cup (A^c \cap B)$

$P(A \cup B) = P(A) + P(A^c \cap B)$ -(1) by axiom III



Since,

$B = (A \cap B) \cup (A^c \cap B)$



$P(B) = P[(A \cap B) \cup (A^c \cap B)]$

$P(B) = P(A \cap B) + P(A^c \cap B)$

$P(B) - P(A \cap B) = P(A^c \cap B)$

$P(A^c \cap B) = P(B) - P(A \cap B) \quad = (2)$

substitute eqt (2) in eqt (1)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2-	$P(A^c) = 1 - P(A)$
3-	$P(\phi) = 0$

$A \cup A^c = S$

$S \cup \phi = S$

1. If A & B are two mutually exclusive events and $P(A) = 0.45$ and $P(B) = 0.35$. Find $P(A \text{ or } B)$

Ans.

$P(A \text{ or } B) = P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.45 + 0.35 - P(A \cap B)$

$P(A \cap B) = 0$

$\cancel{P(A^c) = 1 - P(A)}$

$= \cancel{1 - 0.45} = \cancel{0.55}$

$= 0.80 - \phi$

$= \cancel{0.80} = 0.80$ //

2. If $P(A) = 1/5$ $P(B) = 1/4$ and $P(A \cap B) = 1/10$ Find $P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 1/5 + 1/4 - 1/10$

$= 7/20$ //

$\frac{9}{20} - \frac{1}{10}$

$\frac{90-20}{200}$

$\frac{70}{20}$

3. A bag contains 4 white, 2 black, 3 yellow & 3 red balls. What is the probability of getting. A white or A red ball at random in a single draw of one.

Ans. $P(\text{a white or a red}) = P(A \cup B)$

$(A \cap B) = \phi$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 4/12 + 3/12 - P(A \cap B)$

$= 7/12$ //

4. Find the probability of drawing an Ace or a spade from a pack of cards ?

$P(\text{An Ace or a spade}) = P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$

$= \frac{17}{52} - \frac{1}{52}$

$= \frac{16}{52} = \frac{4}{13}$ //

5. $P(A) = \frac{1}{13}$, $P(B) = \frac{1}{4}$ & $P(A \cup B) = \frac{4}{13}$

Find $P(A \cap B)$

Ans. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{4}{13} = \frac{1}{13} + \frac{1}{4} - P(A \cap B)$

$P(A \cap B) = \frac{1}{13} + \frac{1}{4} - \frac{4}{13}$

$= \frac{17}{52} - \frac{4 \times 4}{13 \times 4}$

$= -\frac{1}{52}$ //

6. $P(A) = \frac{1}{4}$ $P(B) = \frac{1}{3}$ $P(A \cup B) = \frac{1}{2}$

Find the values of

1) $P(A \cap B)$

2) $P(A \cap B^c)$

3) $P(A^c \cap B^c)$

Ans. 1) $P(A \cap B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{1}{2} = \frac{1}{4} + \frac{1}{3} - P(A \cap B)$

$P(A \cap B) = \frac{1}{4} + \frac{1}{3} - \frac{1}{2}$

$= \frac{7}{12} - \frac{1}{2} = \frac{1}{12}$ //

2) $P(A \cap B^c) = P(A) - P(A \cap B)$

$= \frac{1}{4} - \frac{1}{12}$ $\quad \frac{12-4}{48}$

$= \frac{8}{48} = \frac{1}{6}$ //

3) $P(A^c \cap B^c) = P$

$\boxed{A^c \cap B^c = (A \cup B)^c}$

$P(A\cup B)^c = 1 - P(A\cup B)$

$= 1 - 1/2$

$= 1/2$ //

$$\boxed{\begin{aligned} P(A\cap B^c) &= P(A) - P(A\cap B) \\ P(A^c\cap B) &= P(B) - P(A\cap B) \\ P(A^c\cap B^c) &= P(A\cup B)^c \\ &= P(A\cup B)^c = 1 - P(A\cup B) \end{aligned}}$$

HW 7. $P(A) = 0.5$, $P(B) = 0.7$, $P(A\cap B) = 0.3$

Find $P(A\cup B)$

Ans.. $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= 0.5 + 0.7 - 0.3$

$= 1.2 - 0.3 = 0.9$ //

8. $P(A) = 0.4$ $P(B) = 0.3$ $P(A\cup B) = 0.6$

Find $P(A\cap B)$

Ans:- $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$0.6 = 0.4 + 0.3 - P(A\cap B)$

$P(A\cap B) = 0.4 + 0.3 - 0.6$

$= 0.1$ //

Conditional Probability

Probability of an event 'A' given that the event 'B' has happened is called the conditional probability A given B and is denoted by $P(A/B)$

Eg:- I Consider a family with 2 children. The different outcomes are (Boy, Boy), (Boy, Girl), (Girl, Boy) (Girl, Girl). If it is note that the first is a boy, the outcomes are,

(Boy, Boy) (Boy, Girl) so that, probability for both boys equal to $1/2$. This is under the condition, 1st is a boy.

P(both boys / first is a boy) $= 1/2$

If the condition is not given probability of both boys is equal to $1/4$.

$$\boxed{\begin{aligned} P(A/B) &= \frac{P(A\cap B)}{P(B)} \\ P(B/A) &= \frac{P(A\cap B)}{P(A)} \end{aligned}}$$

1. If $P(A) = \frac{1}{13}$, $P(B) = \frac{1}{4}$ and $P(A\cap B) = \frac{1}{52}$

Find ?

1) $P(A/B)$
2) $P(B/A)$

Ans:- 1) $P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1}{52}}{(\frac{1}{4})}$

$= \frac{1}{52} \times \frac{4}{1} = \frac{4}{52} = \frac{1}{13}$

2) $P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{\frac{1}{52}}{(\frac{1}{13})}$

$= \frac{1}{52} \times \frac{13}{1} = \frac{13}{52} =$

$= \frac{1}{4}$

Independence of two events

If A & B are two events, such that $P(A/B) = P(A)$ and $P(B/A) = P(B)$ we say that 'A' & 'B' are independent.

Eg:- Consider a random experiment of tossing 2 coins at a time. The sample space is $S = \{HH, HT, TH, TT\}$

Define the event A = {getting first head} and B = {getting second ~~event~~ head}

$A = \{HH, HT\}$

$B = \{TH, HH\}$

$A\cap B = \{HH\}$.

$P(A) = \frac{2}{4} = \frac{1}{2}$ $P(B) = \frac{2}{4} = \frac{1}{2}$

$P(A\cap B) = \frac{1}{4}$

$P(A)\,P(B) = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4} = P(A\cap B)$

$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2} = P(A)$

$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2} = P(B)$

If 2 events are independent,

$$P(A\cap B) = P(A)P(B)$$

Multiplication Rule of Probability

a) Multiplication Rule for any two events :-

If A & B are any two events then probability for both A & B to take place together, ~~P(A∩B)~~

$$\boxed{P(A\cap B) = P(A|B)\,P(B) = P(B|A)\,P(A)}$$

b) Multiplication rule for two independent events :-

$$\boxed{P(A\cap B) = P(A)\,P(B)}$$

1. If $P(A) = 4/5$; $P(B) = 3/5$. Find $P(A\cap B)$ if A & B are independent.

$P(A\cap B) = P(A)\,P(B)$

$= 4/5 \times \frac{3}{5} = 12/25$

2. If $P(A) = 2/5$ & $P(B) = 3/8$. $P(A\cap B) = 1/20$ Examine whether A & B are independent.

$P(A\cap B) = P(A)\,P(B)$

$= \frac{2}{5} \times \frac{3}{8} = 6/40$

$= 3/20$

This not equal to $1/20$ $\therefore$ A & B are not independent.

3. $P(A) = 2/3$, $P(B) = 4/9$ $P(A\cap B) = 8/27$. Examine whether A & B are independent ?

$P(A\cap B) = P(A)\,P(B)$

$= 2/3 \times 4/9$

$= 8/27$

$P(A\cap B) = P(A)\cdot P(B) = 8/27$. so, A & B are independent

4. $P(A) = 0.5$ $P(B) = 0.2$ $P(A\cap B) = 0.1$ find $P(A|B)$ and $P(B|A)$?

$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.1}{0.2} = 0.5$

$P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{0.1}{0.5} = 0.2$

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5. $P(A) = 0.4$ $P(B) = 0.3$ $P(A \cup B) = 0.6$ Find ?

1- $P(A \cap B)$ -

2- $P(A|B)$

3- $P(B|A)$.

Ans:- 1- $P(A \cap B) = \cancel{P(A).P(B)}$

$= \cancel{0.4 \times 0.3} = \cancel{0.12}$

2 ~~P(A|B)~~ =

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.6 = 0.4 + 0.3 - P(A \cap B)$

$P(A \cap B) = 0.4 + 0.3 - 0.6$

$= 0.1$ //

2- $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.3} = 0.3$

3- $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.1}{0.4} = 0.25$

6. If A, B & C are mutually exclusive & exhaustive &

$P(A) = \frac{1}{2} \times P(B) = \frac{1}{3} \times P(C)$.

Find $P(A)$ $P(B)$, $P(C)$

A, B, C are mutually exclusive,

$P(A \cap B \cap C) = 0$

Exhaustive events,

$P(A \cup B \cup C) = P(S)$

$= 1$

$\cancel{P(A \cup B) + P(C) = 1}$

$P(A) + P(B) + P(C) = 1$

$P(A) = \frac{1}{2} P(B) = \frac{1}{3} P(C)$

$P(A) = \frac{1}{2} P(B)$

$2P(A) = P(B)$

$P(A) = \frac{1}{3} P(C)$

$3P(A) = P(C)$

$P(A) + P(B) + P(C) = 1$

$P(A) + 2P(A) + 3P(A) = 1$

$6P(A) = 1$

$P(A) = \frac{1}{6}$ //

$P(B) = 2 \times P(A)$

$= 2 \times \frac{1}{6} = \frac{2}{6}$

$= \frac{1}{3}$ //

$P(C) = 3P(A)$

$= 3 \times 1/6 = 3/6$

$P(C) = 1/2$

7. Probability that A, solve the problem is 0.5, Probability that B solve ~~the~~ p. is 0.4. what is the probability that ~~that~~ the problem is solve by atleast one of them.

Here A & B are Independent,

so. $A \cap B = P(A) \cdot P(B)$

$= 0.5 \times 0.4$

$= .20$

P(atleast one) = $P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.5 + 0.4 - .20$

$= 0.7$

Q.9. Q20

~~Baye Theor~~ Baye's Theorm

Let $B_1, B_2 \ldots B_n$ be a set of mutually exclusive & exhaustive events. and Let A be an ~~arab.~~ arbitrary event. Then by baye's theorm,

$$\boxed{P(B_k/A) = \frac{P(B_k)P(A|B_k)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + \ldots P(B_n)P(A|B_n)} = \frac{P(B_k)P(A|B_k)}{\sum_{n-1}^{n} P(B_i)P(A|B)}}$$

Proof,

Since $B_1, B_2 \ldots B_n$ are mutually exclusive & exhaustive. That is, $B_1 \cap B_2 \ldots \cap B_n = \phi$ & $B_1 \cup B_2 \ldots \cup B_n = S$

We know that,

$P(B_k|A) = \frac{P(A \cap B)}{P(A)}$

$P = \frac{P(B_k)P(A|B_k)}{P(A)}$ (conditional probabl) —(1)

$B_1, B_2 \ldots B_n$ are exhaustive. Therefore,

$B_1 \cup B_2 \cup \ldots \cup B_n = S$, sample space;

Therefore, $A \cap (B_1 \cup B_2 \ldots \cup B_n) = A \cap S$

$A \cap S = A$
$B \cap S = B$

$= A$ //

That is, $(A \cap B_1) \cup (A \cap B_2) \cup \ldots (A \cap B_n) = A$

$P[(A \cap B_1) \cup (A \cap B_2) \ldots \cup (A \cap B_n)] = P(A)$

$P(A \cap B_1) + P(A \cap B_2) + \ldots P(A \cap B_n) = P(A)$

$A \cap B_1, A \cap B_2, A \cap B_n$ are disjoint.

$P(B_1) P(A/B_1) + P(B_2) P(A/B_2) + \ldots P(B_n) P(A/B_n) = P(A)$ —(2)

Substitute eqt (2) in eqt (1)

we get, $P(B_k/A) = \dfrac{P(B_k) P(A/B_k)}{P(B_1) P(A/B_1) + P(B_2) P(A/B_2) + \ldots P(B_n) P(A/B_n)}$

1. There are 2 urns. One containing 5 white & 4 black balls. and other containing 6 white & 5 black balls. One urn is choosen and one ball is drawn. If it is white, what is the probability that the urn selected is the first.

Ans:
$P(A/B_1) = 5/4$

$P(A/B_2) = 6/11$

$P(B_1) = 1/2$

$P(B_2) = 1/2$

$$P(B_1/A) = \frac{P(B_1) P(A/B_1)}{P(B_1) P(A/B_1) + P(B_2) P(A/B_2)}$$

$$= \frac{\frac{1}{2} \times \frac{5}{9}}{\frac{1}{2} \times \frac{5}{9} + \frac{1}{2} \times \frac{6}{11}}$$

$$= \frac{\frac{5}{18}}{\frac{5}{18} + \frac{6}{22}}$$

$$= \frac{5/18}{\frac{110 + 108}{396}} = 5/18 \times 396/218$$

2. $P(B_1) = P(B_2) = P(B_3) = 1/3$

$P(A/B_1) = 1/2$, $P(A/B_2) = 1/4$, $P(A/B_3) = 1/5$

Find : $P(B_2/A)$

$$P(B_2/A) = \frac{P(B_2) P(A/B_2)}{P(B_1) P(A/B_1) + P(B_2) P(A/B_2) + P(B_3) P(A/B_3)}$$

$$= \frac{1/3 \times 1/4}{1/3 \times 1/2 + 1/3 \times \frac{1}{4} + 1/3 \times 1/5}$$

$$= \frac{1/12}{1/6 + 1/12 + 1/15}$$

$$= \frac{1}{6} + \frac{1}{15} = \frac{15+6}{90} = \frac{21}{90} + 1/12$$

Or $= \dfrac{1/12}{10/60 + 5/60 + 4/60}$

$$= \frac{1/12}{\frac{10+5+4}{60}} = \frac{1/12}{19/60}$$

$$= 1/12 \times 60/19 = \frac{60}{12 \times 19}$$

$$= 5/19$$

3. The probability that a doctor will diagnose a particular disease correctly 0.6. The probability that a patient will die by his treatment after correct diagnosis is 0.4, And the probability of death by wrong diagnosis is 0.7. A patient of the doctor whose ~~are~~ had the disease dead. What is the probability that his disease has not correctly diagnosed?

Ans:- A stands for patient died

B_1 stands for diagnosis correctly

B_2 stands for diagnosis wrongly

$P(B_1) = 0.6$

$P(B_2) = 0.4$

$P(A|B_1) = 0.4$ patient die after correct diagnosis

$P(A|B_2) = 0.7$ Patient die after wrong diagnosis

~~And~~

$$P(B_2|A) = \frac{P(B_2)\,P(A|B_2)}{P(B_1)\,P(A|B_1) + P(B_2)\,P(A|B_2)}$$

$$= \frac{0.4 \times 0.7}{(0.6 \times 0.4) + (0.4 \times 0.7)}$$

$$= \frac{0.28}{.24 + 0.28}$$

$$= \frac{0.28}{0.52} = 0.54$$

0.28
.24
.52

4. A bag contains 2 white & 3 black balls another contains 3 white & 2 black balls. A ball is drawn from one of the bags to be white. What is the probability that its from 1^{st} bag.

B_1 = selecting 1st bag I

B_2 = selecting 2nd bage II

~~AB_1 -- 2 whi~~ A stands for drawing a white ball

$P(B_1) = 1/2$

$P(B_2) = 1/2$

$P(A|B_1) = 2/5$

$P(A|B_2) = 3/5$

$$P(B_1/A) = \frac{P(B_1)P(A|B_1)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2)}$$

$$= \frac{1/2 \times 2/5}{1/2 \times 2/5 + 1/2 \times 3/5}$$

$$= \frac{\frac{2}{10}}{2/10 + 3/10} = \frac{2/10}{5/10} = \frac{2}{10} \times \frac{10}{5} = 2$$

$$= 4/10 = 2/5$$

5. In a bolt factory machine A, B, C manufactur[e] respectively 25%, 35% & 40% of the total of of their output. 5, 4, 2 percentages are defective bolts. A bolt is draw at random from the product is ~~find~~ found to be defective. What is the probability that it was manufactured by machine A?

Ans: B_1 = Machine A

B_2 = Machine B

B_3 = Machine C

A = Defective bolt.

~~B_1~~ $P(B_1) = 25 = \frac{25}{100} = 0.25$

$P(B_2) = 35 = 35/100 = 0.35$

$P(B_3) = 40\% = \frac{40}{100} = 0.40$

$P(A|B_1) = 5 = 5/100 = 0.05$

$P(A|B_2) = 4 = 4/100 = 0.04$

$P(A|B_3) = 2. = 2/100 = 0.02$

$$P(B_1/A) = \frac{P(B_1)P(A|B_1)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)}$$

$$= \frac{0.25 \times .05}{(0.25 \times .05) + (0.35 \times .04) + 0.40 \times .02}$$

$$= \frac{.0125}{.0125 + .0140 + .0080}$$

$$= \frac{0.0125}{0.0345}$$

$$= 0.362$$

6. A factory produces a certain type of outputs by 3 types of machines. The respective daily production figures are; machine I : 3000 units, machine II: 2500 & machine III : 4500 units. Past experiance shows that 1% of the output produced by machine I is defective. The corresponding fraction of the defective for the other 2 machines are respectively 1.2% and 2%. And item is drawn at random from the days production run and is found to be defective. What is the probability that it comes from the output of

A) – Machines I

B) = Machine II

C = Machine III

Ans:-

B_1 = Machine I

B_2 - Machine II

B_3 - Machine III

A = defective

$P(B_1) = \frac{3000}{10000} = 0.3$

$P(B_2) = \frac{2500}{10000} = \cancel{0.25}$ 0.25

$P(B_3) = \frac{4500}{10000} = 0.45$

$P(A|B_1) = 1\% = 0.01$

$P(A|B_2) = 1.2\% = 0.012$

$P(A|B_3) = 2\% = 0.02$

$$P(B_1/A) = \frac{P(B_1)P(A|B_1)}{P(B_1)P(A|B_1) P(B_2) P(A|B_2) P(B_3) P(A|B_3)}$$

$$= \frac{0.3 \times 0.01}{0.3 \times 0.01 + 0.25 \times 0.012 + 0.45 \times 0.02}$$

$$= \frac{.003}{0.003 + .003 + 0.009}$$

$$= \frac{.003}{0.015} = \cancel{0.02} \quad 0.2$$

$$P(B_2|A) = \frac{P(B_2)P(A|B_2)}{P(B_1)P(A|B_1)P(B_2)P(A|B_2)P(B_3)P(A|B_3)}$$

$$= \frac{0.25 \times 0.012}{0.3\times 0.01 + 0.25\times 0.012 + 0.45\times 0.02}$$

$$= \frac{0.003}{0.003+0.003+0.009}$$

0.003
0.003
0.009
15

$$= \frac{0.003}{0.015} = 0.02$$

$$P(B_3|A) = \frac{P(B_3)P(A|B_3)}{P(B_1)P(A|B_1)+P(B_2)P(A|B_2)+P(B_3)P(A|B_3)}$$

$$= \frac{0.45\times 0.02}{0.3\times 0.01 + 0.25\times 0.012 + 0.45\times 0.02}$$

$$= \frac{0.009}{0.003+0.003+0.009}$$

$$= \frac{0.09}{0.015} = 0.6$$

Random Variable

A real ~~variable~~ valued function, defined over the sample space of a random experiment is called the random variable. associated to that random experiment.

Descrete & continous random.

Discreate & Continous Random Variable

A ranmod variable is said to be descrete if it assumes only specified values in an interval. when x takes values 1, 2, 3, 4, 5, 6 it is a descreate variables.

A random variable is said to be continous if it can assume any value in a given interval.

Probability distribution (Probability mass function) (Pmf)

Let 'x' be a random variable assuming values x_1, x_2 etc. Then probality that, the random variable 'x' tends the value 'x' is defined as probability function of 'x'. And this denoted by $p(x)$. Therefore,

$$P(x) = P(X = x)$$

when x takes the values $x_1, x_2, \dots$ etc. ~~the~~ we have the corresponding probabilities $p(x_1)$, $p(x_2)$, etc such that,

1- $P(x_i) \geq 0$ for every x_i

2- $\sum P(x_i) = 1$

Exam X - be a random variable (r, v) can takes values 0, 1, 2 with probability 1/4, 1/4 & 1/2

Then we can write

X	0	1	2
P(x)	1/4	1/4	1/2

1.- Examine whether the following is a probability distribution.

$f(x) = 0.2$ for $x = -1$

$f(x) = 0.3$ for $x = 0$

$f(x) = 0.1$ for $x = 1$

$f(x) = 0.2$ for $x = 2$

$f(x) = 0.2$ for $x = 3$

$f(x) = 0$ x = otherwise

Ans:- I - $f(x) \geq 0$ for every $x = -1, 0, 1, 2, 3$

II = $\Sigma f(x) = 1$

$= f(-1) + f(0) + f(1) + f(2) + f(3)$

$= 0.2 + 0.3 + 0.1 + 0.2$

$= 1$

Therefore,

$f(x)$ is probability distribution.

2. Evaluate K if the following is a probability density function. Also obtain $P(1 \leq X \leq 3)$

X	0	1	2	3
P(x)	1/8	1/2	K/10	1/30

Ans:

$\Sigma P(x) = 1$

$\frac{1}{8} + \frac{1}{2} + \frac{K}{10} + \frac{1}{30} = 1$

$1 = \frac{1}{8} + \frac{1}{2} + \frac{K}{10} + \frac{1}{30}$

$1 - \frac{1}{8} - \frac{1}{2} - \frac{1}{30} = \frac{K}{10}$

$\frac{8-1}{8} - \frac{1}{2} - \frac{1}{30} = \frac{K}{10}$

$\frac{7}{8} - \frac{1}{2} - \frac{1}{30} = \frac{K}{10}$

$\frac{7-4}{8} - \frac{1}{30} = \frac{K}{10}$

$\frac{3}{8} - \frac{1}{30} = \frac{K}{10}$

$\frac{90-8}{240} = \frac{K}{10}$

$\frac{82}{240} = \frac{K}{10}$

$\frac{K}{10} = \frac{82}{240}$

$K = \frac{82}{240} \times 10$

ie = $\frac{5}{30} + \frac{15}{30} + \frac{3k}{30} + \frac{1}{30} = 1$

ie = $\frac{5+15+3k+1}{30} = 1$

$= \frac{21+3k}{30} = 1$

$= 21+3k = 1 \times 30 = 30$

$3k = 30 - 21$

$3k = 9$

$k = \frac{9}{3} = 3$ //

x	0	1	2	3
$P(x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

$P(1 \le x \le 3) = P(x = 1 \text{ or } 2 \text{ or } 3)$

$= P(1) + P(2) + P(3)$

$= \frac{1}{2} + \frac{3}{10} + \frac{1}{30}$

$= \cancel{\frac{2}{4} + \frac{6}{20} + \frac{2}{60}}$

$= \frac{15}{30} + \frac{9}{30} + \frac{1}{30}$

$= \frac{25}{30} = \frac{5}{6}$ //

3. Evaluate k if $f(x) = k$, $x = 1, 2, 3, 4, 5, 6$ is a probability mass function?

Ans:-

x	1	2	3	4	5	6
$P(x)$	k	k	k	k	k	k

$\Sigma P(x) = 1$

$k+k+k+k+k+k = 1$

$6k = 1$

$k = \frac{1}{6}$ //

Therfore the is a probability mass function

HW

4. A random variable X has the following probability function.

Values of X: 0, 1, 2, 3, 4, 5, 6, 7

$f(x)$: 0, 2k, 3k, k, 2k, k^2, $7k^2$, $2k^2+k$

1- Find the k

2- $P(x \not< 6)$

3- $x \ge 6$

4- $P(0 \le x < 5)$ 0.8

Ans:-

X	0	1	2	3	4	5	6	7
$P(x)$	0	2k	3k	k	2k	k^2	$7k^2$	$2k^2+k$

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1) $\Sigma P(x) = 1$

$= 0 + 2k + 3k + k + 2k + k^2 + 7k^2 + 2k^2 + k = 1$

$= 9k + 10k^2 = 1$

$= 10k^2 + 9k = 1$

$10k^2 + 9k - 1 = 0$

$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-9 \pm \sqrt{9^2 - 4 \times 10 \times -1}}{2 \times 10}$

$= \frac{-9 \pm \sqrt{81 + 40}}{20} = \frac{-9 \pm \sqrt{121}}{20}$

$= \frac{-9 \pm \sqrt{11}}{20} =$

$= \frac{-9 + 11}{20}$ or $\frac{-9 - 11}{20}$

$= \frac{2}{20}$ or $\frac{-20}{20}$

$k = 1/10$ or -1

$\cancel{10k^2}\ 10 \times \frac{1}{10} + 9 \times \frac{1}{10} - 1 = 0$

$f(2k) = 2 \times 1/10 = 2/10 = 1/5$

$f(3k) = 3 \times 1/10 = 3/10$

$f(k) = 1/10$

$f(2k) = 2 \times 1/10 = 2/10$

$f(k^2) = (1/10)^2 = 1/100$

$f(7k^2) = 7 \times (\frac{1}{10})^2 = 7/100$

$f(2k^2 + k) = 2 \times (1/10)^2 + 1/10 = 2/100 + 1/10$

$= \cancel{20} + 12/100$

$\Sigma P(x) = 1$

$2/10 + 3/10 + 1/10 + 2/10 + 1/100 + 7/100 + 12/100$

$= \frac{20}{100} + \frac{30}{100} + \frac{10}{100} + \frac{20}{100} + \frac{10}{100} + \frac{70}{100} + \frac{120}{100}$

2) $P(x < 6) = P(X = 0$ or 1 or 2 or 3 or 4 or $5)$

$= P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

$= 0 + 2/10 + 3/10 + 1/10 + 2/10 + 1/100$

$= 8/10 + 1/100 = 81/100$

$= 0.81$

3) $x \geq 6 = x = 6$ or 7

$= 1 - 0.81$

$= 0.19$

$\frac{121}{121}$ 0.81 0.19

4) $P(0 < X < 5) = P(X = 1 \text{ or } 2 \text{ or } 3 \text{ or } 4)$

$= P(1) + P(2) + P(3) + P(4)$

$= 2/10 + 3/10 + 1/10 + 2/10$

$= \dfrac{2+3+1+2}{10}$

$= 8/10 = 0.8$

Expectation of X — E(x) (Mean)

Let the random variable X assume the values x_1, x_2, etc.. with corresponding probabilities $P(x_1)$, $P(x_2)$, etc.... Then the expected value of the random ~~experiment~~ variable X, denoted by $E(x)$ is defined as,

$$E(x) = x_1 P(x_1) + x_2 (P x_2) + etc....$$

$$\boxed{\begin{aligned} E(x) &= \cancel{\Sigma x_i P} \\ &= \Sigma x_i P(x_i) \end{aligned}}$$

for eg:-

1. A random variable 'X' takes values 0, 1, 2, 3, 4 with corresponding probabilities $1/4$, $1/5$, $2/5$, $1/8$, $1/40$. find the expectation of x (E(x))

X	0	1	2	3	4
P(x)	1/4	1/5	2/5	1/8	1/40
x.P(x)	0	1/5	4/5	3/8	4/40

$E(x) = \Sigma x_i P(x_i)$

$= 0 \times \frac{1}{4} + \cancel{0}1 \times 1/5 + \cancel{2/5}\, 2 \times 2/5 + 3 \times 1/8 + 4 \times 1/40$

$= \cancel{0 + 1/5 + 4/5 +}$

$= 0 + 1/5 + 4/5 + 3/8 + 4/40$

$= 8/40 + 32/40 + \frac{15}{40} + 4/40$

$= 59/40 = 1.47$ //

HW 2. If $f(x) = 0.2$ when $x = 1$, $f(x) = 0.3$ when $x = 2$, $f(x) = 0.5$ when $x = 3$. Find the Expectation of X ?

Ans:

X	1	2	3
f(x)	0.2	0.3	0.5
x.P(x)	0.2	0.6	1.5

$E(x) = \Sigma x_i P(x_i)$

$= 1 \times 0.2 + 2 \times 0.3 + 3 \times 0.5$

$= 0.2 + 0.6 + 1.5$

$= 2.3$ //

0.2
0.6
0.5

Varience of X

Variance of a random variable x whose expectation denoted by $E(x)$ is defined as,

$$\boxed{\begin{aligned} V(X) &= E[X - EX]^2 \\ &= EX^2 - (Ex)^2 \end{aligned}}$$

where, $EX^2 = x_1^2 P(x_1) + x_2^2 P(x_2) + \dots$ etc.

1. A random variable X takes values 1 & 2 with corresponding probabilities $1/3$ & $2/3$ find the $E(X)$ & $V(X)$? (mean & variance) ! $5/3$ $2/9$

2. A random variable X follows a probability distribution as given below.

x	0	1	2	3
$f(x)$	$K/2$	$K/3$	$\frac{K+1}{3}$	$\frac{2K-1}{6}$

find the value of k. Also find the mean & variance of the variable ?

k
$5/9$
E(x) 1.28
E(x^2) 2.46
V(x) = 0.788

Answers

1.

X	1	2
$P(x)$	$1/3$	$2/3$

$E(x) = \Sigma x_i P(x_i)$

$= 1 \times 1/3 + 2 \times 2/3$

$= 1/3 + 4/3 = \frac{1+4}{3}$

$= 5/3$

$V(x) = EX^2 - (EX)^2$

$EX^2 = 1^2 \times 1/3 + 2^2 \times 2/3$

$= 1 \times 1/3 + 4 \times 2/3$

$= 1/3 + 8/3 = \frac{1+8}{3}$

$= 9/3 = 3$

$Ex^2 = (5/3)^2 = 25/9$

$V(x) = EX^2 - (EX)^2$

$= 3 - 25/9$

$= \frac{27-25}{9} = 2/9$

2.

X	0	1	2	3
f(x	$k/2$	$k/3$	$\frac{k+1}{3}$	$\frac{2k-1}{6}$

$\Sigma P(x) = 1$

$= k/2 + k/3 + \frac{k+1}{3} + \frac{2k-1}{6} = 1$

$= k/2 + k/3 + k/3 + 1/3 + 2k/6 - 1/6 = 1$

$= \cancel{k/2} + \cancel{\frac{2k}{3}} + \cancel{\frac{2k}{6}} + \cancel{1/3} - \cancel{1/6} = 1$

$= \frac{3k}{6} + \frac{2k}{6} + \frac{2k}{6} + \frac{2}{6} + \frac{2k-1}{6}$

$= \frac{3k+2k+2k+2+2k-1}{6} = 1$

$= \frac{9k+1}{6} = 1$

$\Rightarrow 9k+1 = 1\times 6 = 6$

$9k = 6-1 = 5$

$k = 5/9$

X	0	1	2	3
f(x	$\frac{5/9}{18}$	$5/27$	$14/27$	$1/54$

$f(k/2) = \frac{5/9}{2} = \frac{5}{9} \times \frac{1}{2} = \frac{5}{18}$

$f(k/3) = \frac{5/9}{3} = 5/9 \times \frac{1}{3} = \frac{5}{27}$

$f\left(\frac{k+1}{3}\right) = \frac{5/9+1}{3} = \frac{\frac{5+9}{9}}{3} = \frac{\frac{14}{9}}{3} = 14/9 \times \frac{1}{3} = \frac{14}{27}$

$f\left(\frac{2k-1}{6}\right) = \frac{2(5/9)-1}{6} = \frac{\frac{10}{9}-1}{6} = \frac{\frac{10-9}{9}}{6}$

$= \cancel{\frac{-8}{18} \times \frac{1}{6} = \frac{-8}{108}}$

$= \frac{1}{9} \times \frac{1}{6} = \frac{1}{54}$

$E(x) = \Sigma x_i P(x_i)$

$= 0\times 5/18 + 1\times 5/27 + 2\times 14/27 + 3\times 1/54$

$= 0 + 5/27 + 28/27 + 3/54$

$= \frac{10}{54} + \frac{56}{54} + 3/54$

$= 69/54 = 1.28$

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$V(x) = Ex^2 - (Ex)^2$

$Ex^2 = 1^2 \times 5/27 + 2^2 \times 14/27 + 3^2 \times 1/54$

$= 1 \times 5/27 + 4 \times 14/27 + 9 \times 1/54$

$= 5/27 + 56/27 + 9/54$

$= 10/54 + \frac{112}{54} + 9/54$

$= \frac{131}{54} = 2.426$

$(Ex)^2 = (69/54)^2 = \frac{4761}{2916} =$ 1.634

$= 1.639$

$V(x) = Ex^2 - (Ex)^2$

$= 2.426 - 1.639$

$= 0.788$

Moments

1- Raw Moments :

Let x be a random variable assuming values x_1, x_2, x_3 etc. with corresponding probabilities $p(x_1), p(x_2), \ldots$ Then the r^{th} order raw moments about a constant 'a' is defined as,

$M'_r(a) = E(x-a)^r$

$= \sum_x (x-a)^r p(x)$

Let $a = 0$

$M'_r(0) = M'_r$

$M'_1 = Ex$

$M'_2 = Ex^2$

$M'_3 = Ex^3$

That is, $M'_r(a) = (x_1 - a)^r p(x_1) + (x_2 - a)^r p(x_2) + \ldots\ldots$

2- Central Moments :

r^{th} order central moment for the 'x' is defined as,

$M_r = E(x - Ex)^r$

$= \sum (x - Ex)^2 p(x)$

Example :

- $\mu_1 = E(x-Ex)^1$

$V(x) = \mu_2 = E(x - E\{x\})^2$

$\mu_3 = E(x-Ex)^3$

Distribution Function

Let 'x' be a random variable and 'x' by be any value of 'x' then the distribution function defined as;

$F(x) = P(x \leq x)$

$= \sum_{x \leq x} p(x)$

1. A random variable x as the following probability function. $f(0) = \frac{1}{6}$, $f(1) = \frac{2}{6}$, $f(2) = \frac{3}{6}$ & $f(x) = 0$ otherwise,

a) write down the density function

b) write down the distribution function

Ans: a)

x	0	1	2
$P(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$

b) $F(x) = P(X \leq x)$

$= \sum_{X \leq x} P(x)$

Case I : $x < 0$

$f(x) = 0$

Case II : $0 \leq x < 1$

$F(x) = P(x \leq x)$

$F(x) = P(x = 0)$

$= \frac{1}{6}$

Case III : $1 \leq x < 2$

$F(x) = P(X \leq x)$

$= P(x=0) + P(x=1)$

$= \frac{1}{6} + \frac{2}{6} = \frac{3}{6}$

Case IV :

$2 \leq x < \alpha$

$F(x) = P(x=0) + P(x=1) + P(x=2)$

$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6}$

$= \frac{6}{6} = 1$

Therefore,

$$F(x) = \begin{cases} 0 & x < 0 \\ 1/6 & 0 \le x < 1 \\ 3/6 & 1 \le x < 2 \\ 1 & 2 \le x < \infty \end{cases}$$

2. Find the distribution function of the following:

$f(x) = 1/8$ for $x = -2$

$f(x) = 2/8$ for $x = -1$

$f(x) = 3/8$ for $x = 0$

$f(x) = 2/8$ for $x = 2$

$f(x) = 0$ elsewhere,

Ans:-

x	-2	-1	0	2
$f(x)$	$1/8$	$2/8$	$3/8$	$2/8$

Case I : $x < -2$

$F(x) = 0$

Case II : $-2 \le x < 1$

$F(x) = P(X \le x)$

$P(X = -2)$

$= 1/8$

Case III : $-1 \le x < 0$

$F(x) = P(X = -2) + P(X = -1)$

$= 1/8 + 2/8 = 3/8$

Case IV : $0 \le x < 2$

$F(x) = P(X = -2) + P(X = -1) + P(X = 0)$

$= 1/8 + 2/8 + 3/8 =$

$= 6/8$

Case V : $2 \le x < \infty$

$F(x) = P(X = -2) + P(X = -1) + P(X = 0) + P(X = 2)$

$= 1/8 + 2/8 + 3/8 + 2/8$

$= 8/8 = 1$

Therefore,

$$F(x) = \begin{cases} 0 & x < -2 \\ 1/8 & -2 \le x < -1 \\ 3/8 & -1 \le x < 0 \\ 6/8 & 0 \le x < 2 \\ 1 & 2 \le x < \infty \end{cases}$$

Properties of distribution Function (step function)

1- $F(x) \geq 0$

2- $F(-\infty) = 0$, $F(\infty) = 1$

3- $F(x)$ is non decreasing.