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D51621

First Semester MA Degree Examination, November 2018.

(CUCSS-PG)

ECO 1C 04 - Quantitative Methods for Economic Analysis - I

(2015 Syllabus Year)

Time : 3 hrs

Maximum : 36 Weightage.

Part A

1. Evaluate $\begin{vmatrix} -1 & 2 & -3 \\ 2 & -3 & -1 \\ -3 & -1 & 2 \end{vmatrix}$

$= (-1)\begin{vmatrix} -3 & -1 \\ -1 & 2 \end{vmatrix} - (2)\begin{vmatrix} 2 & -1 \\ -3 & 2 \end{vmatrix} + (-3)\begin{vmatrix} 2 & -3 \\ -3 & -1 \end{vmatrix}$

$= (-1)[(-3\times 2) - (-1\times -1)] - 2[(2\times 2) - (-1\times -3)] + (-3)[(2\times -1) - (-3\times -3)]$

$= (-1)[-6 - 1] - 2[4 - 3] - 3[-2 - 6]$

$= -1\times(-7) - 2\times(1) - 3\times(-8)$

$= 7 - 2 + 24$

$= 7 + 24 - 2$

$= 31 - 2$

$= 29$.

2. Find the rank of $\begin{bmatrix} 4 & 1 & 2 \\ -3 & 2 & 4 \\ 8 & -1 & -2 \end{bmatrix}$.

Let $A = \begin{bmatrix} 4 & 1 & 2 \\ -3 & 2 & 4 \\ 8 & -1 & -2 \end{bmatrix}$

$|A| = \begin{vmatrix} 4 & 1 & 2 \\ -3 & 2 & 4 \\ 8 & -1 & -2 \end{vmatrix} = 4\begin{vmatrix} 2 & 4 \\ -1 & -2 \end{vmatrix} - 1\begin{vmatrix} -3 & 4 \\ 8 & -2 \end{vmatrix} + 2\begin{vmatrix} -3 & 2 \\ 8 & -1 \end{vmatrix}$

$= 4[-4 - (-4)] - 1[(-3\times -2) - (4\times 8)] + 2[(-3\times -1) - (2\times 8)]$

$= 4[-4 + 4] - [6 - 32] + 2[3 - 16]$

$= 4\times 0 - (-26) + 2(-13)$

$r(A) \neq 3$.

$r(A) < 3$.

Consider Evaluate next Lower order (order=2) minors.

$$\begin{vmatrix} 4 & 1 \\ 3 & -2 \end{vmatrix} = -8 - 3 = -11 \neq 0$$

$\therefore r(A) = \underline{\underline{2}}$.

rank of the given matrix = 2//.

3. Show that the characteristic equation of the square matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix} \text{ is } \lambda^3 - 3\lambda^2 - 7\lambda - 11 = 0.$$

Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$,

then $A - \lambda I = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$= \begin{bmatrix} 1-\lambda & 1 & 2 \\ 3 & 1-\lambda & 1 \\ 2 & 3 & 1-\lambda \end{bmatrix}$

Characteristic equation is

$|A - \lambda I| = 0$.

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 1 & 2 \\ 3 & 1-\lambda & 1 \\ 2 & 3 & 1-\lambda \end{vmatrix}$$

$= (1-\lambda)\begin{vmatrix} 1-\lambda & 1 \\ 3 & 1-\lambda \end{vmatrix} - (1)\begin{vmatrix} 3 & 1 \\ 2 & 1-\lambda \end{vmatrix} + 2\begin{vmatrix} 3 & 1-\lambda \\ 2 & 3 \end{vmatrix}$

$= (1-\lambda)\left[(1-\lambda)(1-\lambda) - (1\times 3)\right] - 1\left[3(1-\lambda) - (1\times 2)\right] + 2\left[(3\times 3) - (1-\lambda)2\right]$

$= (1-\lambda)\left[(1-\lambda)^2 - 3\right] - \left[3(1-\lambda) - 2\right] + 2\left[9 - 2(1-\lambda)\right]$

$$=(1-\lambda)[1-2\lambda+\lambda^2-3]-[3-3\lambda-2]+2[9-2+2\lambda]$$

$$=(1-\lambda)[\lambda^2-2\lambda+1-3]-[-3\lambda+3-2]+2[2\lambda+9-2]$$

$$=(1-\lambda)[\lambda^2-2\lambda-2]-[-3\lambda+1]+2[2\lambda+7]$$

$$=(1-\lambda)[\lambda^2-2\lambda-2]+3\lambda-1+4\lambda+14$$

$$=1(\lambda^2-2\lambda-2)-\lambda(\lambda^2-2\lambda-2)+3\lambda-1+4\lambda+14$$

$$=\lambda^2-2\lambda-2-\lambda^3+2\lambda^2+2\lambda+3\lambda-1+4\lambda+14$$

$$=-\lambda^3+\lambda^2+2\lambda^2-2\lambda+2\lambda+3\lambda+4\lambda-2-1+14$$

$$=-\lambda^3+3\lambda^2+0\lambda+7\lambda+14-3$$

$$=-\lambda^3+3\lambda^2+7\lambda+0\lambda+14-3$$

$$=-\lambda^3+3\lambda^2+7\lambda+11$$

i.e $|A-\lambda I| = -\lambda^3+3\lambda^2+7\lambda+11$

$|A-\lambda I|=0$

$\Rightarrow -\lambda^3+3\lambda^2+7\lambda+11=0$

$\Rightarrow -(-\lambda^3+3\lambda^2+7\lambda+11) = -0 = 0$

$\Rightarrow \lambda^3-3\lambda^2-7\lambda-11=0.$

i.e $|A-\lambda I|=0 \Rightarrow$

$\lambda^3-3\lambda^2-7\lambda-11=0.$

④ Let an exponential function be $y=a^x$, find $\frac{dy}{dx}$.

$$\frac{dy}{dx}=\frac{d}{dx}a^x = a^x \log_e a$$

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5) If the total cost of making x litres of an acid is $T = -30 + 80x^{1/2}$ rupees, Find the number of units at which the marginal cost is Rs 1.25.

Given $TC = -30 + 80x^{1/2}$

$= -30 + 80\sqrt{x}$

$MC = \frac{d}{dx}(TC)$

$= \frac{d}{dx}(-30 + 80\sqrt{x})$

$= \frac{d}{dx}(-30) + \frac{d}{dx}(80\sqrt{x})$

$= 0 + 80\frac{d}{dx}(\sqrt{x})$

$= 0 + 80 \frac{d}{dx}(x^{1/2})$

$= 80 \; \frac{1}{2} x^{\frac{1}{2}-1}$

$= 80 \frac{1}{2} x^{-1/2}$ $\left(\frac{1}{2} - 1 = -\frac{1}{2}\right)$

$= 80 \frac{1}{2x^{\frac{1}{2}}}$

$= \frac{80}{2} \times \frac{1}{x^{\frac{1}{2}}}$

$= \frac{40}{\sqrt{x}} \;||$

If $MC = 1.25$, then $\frac{40}{\sqrt{x}} = 1.25$

$\therefore 40 = 1.25\sqrt{x}$

$\therefore \frac{40}{1.25} = \sqrt{x}$

ie $x = \left(\frac{40}{1.25}\right)^2 = 32^2$

$= \underline{1024}$.

(6) Find the total derivative of $u = x^2y^3 + x^3y^2$

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy.$$

$$\frac{\partial u}{\partial x} = 2xy^3 + 3x^2y^2 \quad (y \text{ constant})$$

$$\frac{\partial u}{\partial y} = x^2 3y^2 + x^3 2y \quad (x \text{ constant})$$

$$\therefore du = (2xy^3 + 3x^2y^2)\,dx + (x^2 3y^2 + x^3 2y)\,dy$$

(7) The cost for a monopolist firm producing x mobile phones per week is given to be $4x^2 - 80x + 500$ rupees. To have a minimum cost, how many units should be produced per week?

$$\text{Cost/week} = 4x^2 - 80x + 500$$
$$= TC$$

$$\frac{d}{dx}(TC) = 4 \times 2x - 80 \times 1 + 0$$
$$= 8x - 80$$

$$\frac{d}{dx} TC = 0$$
$$\Rightarrow 8x - 80 = 0$$
$$\Rightarrow 8x = 80$$
$$\Rightarrow x = \frac{80}{8} = 10.$$

$\therefore$ To minimize cost, 10 units should be produced per week.

(8) Integrate $\dfrac{3x^3 - 5x^2 + 6x - 8}{x}$ with respect to x.

$$\int \frac{3x^3 - 5x^2 + 6x - 8}{x} dx = \int \frac{3x^3}{x} dx - \int \frac{5x^2}{x} dx + \int \frac{6x}{x} dx - \int \frac{8}{x} dx$$
$$= \int 3x^2 dx - \int 5x\, dx + \int 6\, dx - \int \frac{8}{x} dx$$
$$= 3\frac{x^3}{3} - 5\frac{x^2}{2} + 6x - 8\log x + C$$
$$= x^3 - \frac{5}{2}x^2 + 6x - 8\log x + C$$

9. Find $\int \frac{x-5}{x^2-10x+11}\,dx$

Let $u = x^2-10x+11$,

then $\frac{du}{dx} = 2x-10 = 2(x-5)$

$\therefore \quad du = 2(x-5)\,dx$

i.e $\quad \frac{du}{2} = (x-5)\,dx$

$\therefore \int \frac{x-5}{x^2-10x+11}\,dx = \int \frac{1}{u}\,\frac{du}{2} = \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\log u + C$

$= \frac{1}{2}\log(x^2-10x+11) + C$

10. Give the axiomatic defintion of probability.

Let 'S' - be the sample space of a random experiment. Let A be an event of the random experiment so that A is a subset of S. Then we can associate a real number P(A) to the event A. This number of P(A) will be called probability of A if it satisfies the following three axioms.

Axiom 1: P(A) is a real number such that $P(A) \geq 0$ for every A subset of S

Axiom 2: $P(S) = 1$, where 'S' is the sample space.

Axiom 3: $P(A \cup B) = P(A) + P(B)$ where A and B are two non intersecting subset of S, ie $A \cap B = \phi$.

11. State the addition theorem of probability.

Ans

(a) Addition rule for mutually exclusive events.
If A and B are two mutually exclusive events, then
$P(A \cup B) = P(A) + P(B)$

(b) Addition rule for any two events (need not be mutually exclusive)

If A and B are any two events, then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(12) If $E(x) = 2.5$, find $E(3X+7)$

$$E(3X+7) = 3EX + E(7)$$
$$= 3E(X) + 7$$
$$= 3 \times 2.5 + 7$$
$$= 7.5 + 7$$
$$= 14.5$$

Part B

(Answer any eight questions. Weightage 2 for each question)

(13) If $A = \begin{bmatrix} 1 & 1 & -1 \\ -2 & 3 & -4 \\ 3 & -2 & 3 \end{bmatrix}$, Show that $\frac{1}{2}(A - A^T)$ is skew-symmetric.

Ans

$$A = \begin{bmatrix} 1 & 1 & -1 \\ -2 & 3 & -4 \\ 3 & -2 & 3 \end{bmatrix}, \quad A^T = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 3 & -2 \\ -1 & -4 & 3 \end{bmatrix}$$

$$A - A^T = \begin{bmatrix} 1 & 1 & -1 \\ -2 & 3 & -4 \\ 3 & -2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & -2 & 3 \\ 1 & 3 & -2 \\ -1 & -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 3 & -4 \\ -3 & 0 & -2 \\ 4 & 2 & 0 \end{bmatrix}$$

$$\frac{1}{2}(A - A^T) = \frac{1}{2}\begin{bmatrix} 0 & 3 & -4 \\ -3 & 0 & -2 \\ 4 & 2 & 0 \end{bmatrix} = \begin{bmatrix} \frac{0}{2} & \frac{3}{2} & \frac{-4}{2} \\ \frac{-3}{2} & \frac{0}{2} & \frac{-2}{2} \\ \frac{4}{2} & \frac{2}{2} & \frac{0}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{3}{2} & -2 \\ -\frac{3}{2} & 0 & -1 \\ \ldots & 1 & 0 \end{bmatrix}$$

ie $\frac{1}{2}(A-A^T) = \begin{bmatrix} 0 & 1.5 & -2 \\ -1.5 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

Let $B = \frac{1}{2}(A-A^T)$

then $B^T = \begin{bmatrix} 0 & -1.5 & 2 \\ 1.5 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$

$\therefore -B^T = -\begin{bmatrix} 0 & -1.5 & 2 \\ 1.5 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1.5 & -2 \\ -1.5 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

$= B$

ie $B = -B^T$

therefore $B = \frac{1}{2}(A-A^T)$ is skew symmetric.

(14) $A = \begin{bmatrix} 2 & 3 & 6 \\ 3 & -6 & 2 \\ -6 & -2 & 3 \end{bmatrix}$, find A^TA. Hence or otherwise evaluate A^{-1}!

What is the peculiarity of the matrix A?.

Ans $A^T = \begin{bmatrix} 2 & 3 & -6 \\ 3 & -6 & -2 \\ 6 & 2 & 3 \end{bmatrix}$

$A^TA = \begin{bmatrix} 2 & 3 & -6 \\ 3 & -6 & -2 \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 6 \\ 3 & -6 & 2 \\ -6 & -2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 4+9+36 & 6-18+12 & 12+6-18 \\ 6-18+12 & 9+36+4 & 18-12-6 \\ 12+6-18 & 18-12-6 & 36+4+9 \end{bmatrix}$

$= \begin{bmatrix} 49 & 0 & 0 \\ 0 & 49 & 0 \\ 0 & 0 & 49 \end{bmatrix}$

$$= 49\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$= 49I$, where I is the identity matrix of order 3.

i.e., $A^TA = 49I$

$\therefore \frac{1}{49}(A^TA) = I$

i.e., $\left(\frac{1}{7}A^T\right)\left(\frac{1}{7}A\right) = I$

i.e. $\left(\frac{1}{7}A\right)^T\left(\frac{1}{7}A\right) = I$ —(1) $\left(\text{since } \frac{1}{7}A^T = \left(\frac{1}{7}A\right)^T\right)$

We have if A^{-1} is the inverse of A, then

$A^{-1}A = I$

In (1) $\left(\frac{1}{7}A\right)^T\left(\frac{1}{7}A\right) = I$

Therefore $\left(\frac{1}{7}A\right)^T$ is the inverse of $\frac{1}{7}A$

i.e., $\left(\frac{1}{7}A\right)^{-1} = \left(\frac{1}{7}A\right)^T$

i.e. $7A^{-1} = \left(\frac{1}{7}A\right)^T$ $\left(\text{since } (cA)^{-1} = c^{-1}A^{-1}\right)$

i.e., $7A^{-1} = \frac{1}{7}A^T$

$A^{-1} = \frac{1}{49}A^T$

The inverse of A is $\frac{1}{49}A^T$.

(15) A manufacturer produces three products A, B, C which are sold

in Delhi and Calcutta. Annual sales of these products are given below:

	Products A	B	C
Delhi	5000	7500	15000
Calcutta	9000	12000	8700

If the sale price of the products A, B, C per unit be Rs 20, Rs 30, Rs 40 respectively, calculate the total revenue in each centre by using matrices.

Let $A = \begin{bmatrix} 5000 & 7500 & 15000 \\ 9000 & 12000 & 8700 \end{bmatrix}$, $B = \begin{bmatrix} 20 \\ 30 \\ 40 \end{bmatrix}$

(The sales matrix) (Price matrix)

Find AB,

$$AB = \begin{bmatrix} 5000 & 7500 & 15000 \\ 9000 & 12000 & 8700 \end{bmatrix} \begin{bmatrix} 20 \\ 30 \\ 40 \end{bmatrix}.$$

$$= \begin{bmatrix} 5000 \times 20 + 7500 \times 30 + 15000 \times 40 \\ 9000 \times 20 + 12000 \times 30 + 8700 \times 40 \end{bmatrix}$$

$$= \begin{bmatrix} 835000 \\ 888000 \end{bmatrix}$$

Total revenue of Delhi Centre = 83500

" Calcutta " = 888000

(16) Using the function $f(x,y) = x^2 + y^2 - 2xy + 8x + 9y + 3$, show that $\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial^2 f}{\partial y \partial x}$.

Ans

$$f(x,y) = x^2+y^2-2xy+8x+9y+3.$$

$$\frac{\partial f}{\partial x} = 2x+0-2\times(1)y+8\times1+9\times0+0$$

$$= 2x-2y+8$$

$$\frac{\partial^2 f}{\partial y\partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}(2x-2y+8)$$

$$= 0-2+0$$

$$= -2 \quad -(1)$$

$$\frac{\partial f}{\partial y} = 0+2y-2x\times(1)+8\times0+9\times1+0$$

$$= 2y-2x+9$$

$$\frac{\partial^2 f}{\partial x\partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}(2y-2x+9)$$

$$= 0-2\times1+0$$

$$= -2 \quad -(2)$$

From (1) and (2).

$$\frac{\partial^2 f}{\partial y\partial x} = \frac{\partial^2 f}{\partial x\partial y}$$

(17) Given the production function $V=(\beta K^{-\rho}+\alpha L^{-\rho})^{-\frac{1}{\rho}}$, when V is the output, K is capital, L is labour, and α, β, ρ are constants. Find dV.

Ans

$$dV = \frac{\partial V}{\partial K}dK + \frac{\partial V}{\partial L}dL$$

$$\frac{\partial V}{\partial K} = -\frac{1}{\rho}(\beta K^{-\rho}+\alpha L^{-\rho})^{-\frac{1}{\rho}-1}\times(\beta\times(-\rho)K^{-\rho-1}+0)$$

$$= -\frac{1}{\rho}\times\beta\times-\rho\times K^{-\rho-1}(\beta K^{-\rho}+\alpha L^{-\rho})^{-\frac{1}{\rho}-1}$$

$$= \beta K^{-\rho-1}(\beta K^{-\rho}+\alpha L^{-\rho})^{-\frac{1}{\rho}-1}$$

$$\frac{\partial y}{\partial L} = -\frac{1}{\rho}\left(\beta k^{-\rho} + \alpha L^{-\rho}\right)^{-\frac{1}{\rho}-1} \times \left(0 + \alpha x - \rho L^{-\rho-1}\right)$$

$$= -\frac{1}{\rho}\left(\beta k^{-\rho} + \alpha L^{-\rho}\right)^{-\frac{1}{\rho}-1} \alpha x - \rho x L^{-\rho-1}$$

$$= -\frac{1}{\rho} \times \alpha x - \rho \times L^{-\rho-1}\left(\beta k^{-\rho} + \alpha L^{-\rho}\right)^{-\frac{1}{\rho}-1}$$

$$= \alpha L^{-\rho-1}\left(\beta k^{-\rho} + \alpha L^{-\rho}\right)^{-\frac{1}{\rho}-1}$$

$$\therefore dv = \beta k^{-\rho-1}\left(\beta k^{-\rho} + \alpha L^{-\rho}\right)^{-\frac{1}{\rho}-1} dk + \alpha L^{-\rho-1}\left(\beta k^{-\rho} + \alpha L^{-\rho}\right)^{-\frac{1}{\rho}-1} dL$$

$$= \left(\beta k^{-\rho} + \alpha L^{-\rho}\right)^{-\frac{1}{\rho}-1}\left[\beta k^{-\rho-1} dk + \alpha L^{-\rho-1} dv\right]$$

(19) Find the maximum and minimum values of the function

$$y = 2x^3 - 3x^2 - 36x + 12.$$

$$\frac{dy}{dx} = 2 \times 3x^2 - 3 \times 2x - 36 \times 1 + 0$$

$$= 6x^2 - 6x - 36$$

$$\frac{dy}{dx} = 0 \Rightarrow 6x^2 - 6x - 36 = 0$$

$$\Rightarrow 6(x^2 - x - 6) = 0$$

$$\Rightarrow x^2 - x - 6 = 0$$

$a = 1,\ b = -1,\ c = -6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1 \times -6}}{2 \times 1}$$

$$= \frac{1 \pm \sqrt{1 + 24}}{2}$$

$$= \frac{1 \pm \sqrt{25}}{2}$$