

Module 2

Normal Distribution

Normal distribution is perhaps the most important and useful distribution in statistics. Many statistical data concerning business and economic problems can be displayed in the form of normal distribution.

Normal distribution was first discovered by De Moivre in 1733 as limiting form of the binomial distribution.

Numerous phenomena such as the age distribution of a given species of animals, height of adult persons, the intelligent test scores of school children and many other distributions are considered to be normally distributed. Normal distribution is related to the distributions of errors, made by chance or experimental measurement.

Consider the following ex:- A company manufactures screws of a particular diameter.

There may be some variations in the measurement. But most of the screws have diameter very near to what is required. Few of them are either much larger or much smaller. If we draw histograms of the frequency distributions of measurements, we see that the histogram has maximum height at the centre and the height decreased on either side on almost same rate. So if the class and class interval of the frequency distribution is made smaller and smaller we get a curve of the shape of the bell.

It is the "normal curve".

Definition :- A continuous random variable x is said to follow normal distribution if the its probability function is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\text{---} < \mu < \infty \text{ and } \sigma > 0$$

$$x \sim N(\mu, \sigma^2)$$

Normal distribution as a limiting case of binomial distribution

Binomial distribution is an important theoretical distribution for discrete variables. Binomial distribution tends to Normal distribution under the following conditions:

- 1) No. of trials (n) is very large
- 2) P and q (probability for success in a single trial) and the probability for its failure are almost equal.

Then the binomial distribution can be approximated to normal.

Properties of Normal Distribution

- 1) Normal curve is a continuous curve.
- 2) Normal curve is bell shaped.
- 3) Normal curve is symmetric about mean.
- 4) Mean, median, mode are equal for a normal distribution. ($\text{mean} = \text{median} = \text{mode}$)

5) The height of normal curve is maximum at its mean.

6) Coefficient of skewness is zero ($S_3 = 0$).

7) Normal curve is unique. It has only one model.

8) The points of inflection occur at $M \pm \sigma$. At the point of inflection the curve changes from concavity to convexity.

9. Q_1 and Q_3 are equidistant from median.

10. Mean deviation, $MD = \frac{4}{5}\sigma$

11. Standard deviation, $SD = \sqrt{\frac{4}{5}}\sigma$

12. Kurtosis ($K = 3$) (i.e., $\left(\frac{\sigma^2}{\mu^2} = 3\right)$)

13. Odd order central moments are zero.

14. Area under the normal curve is distributed among three standard deviations as follows:

→ $M \pm \sigma$ covers 68.27% area.

→ $M \pm 2\sigma$ covers 95.45% area.

→ $M \pm 3\sigma$ covers 99.73% area.

(23) If x and y are two independent normal variables, then their sum is also a variable

$$\text{Additive property} \quad x_1 \sim N(\mu_1, \sigma_1^2) \quad x_2 \sim N(\mu_2, \sigma_2^2)$$

Importance of Normal Distribution

1- Most of the decrease probability distributions

tend to normal distribution, as 'n' becomes larger.

2- The various tests of significance like, t-test, F-test etc. are based on the assumption that the parent population from which the samples have been drawn follows Normal distribution.

3- It is extensively used in large sampling theory to find the estimates of parameters, confidence limits etc.

4- Normal distributions has the following property stated in the central limit theorem. As per this theorem, when the sample size increased, the sample means will tend to be normally distributed.

5- Normal distributions finds application in

statistical quality control, and industrial experiments. Many distributions in social and economic data are approximately normal.

E.g.: birth, deaths etc. are normally distributed

In psychological and educational data many distributions are of normal type.

Properties and Demerits of ND

Merits:

1- Normal distribution is the mostly used distributions in inferential statistics

2- Most of our measurements and a large variety of physical observations have approximately normal distributions.

3- The normal distribution has a no. of mathematical properties.

4- Most of the distributions in nature are either normal or that can be approximated

- Demerits:**
- 1- The variables which are not continuous cannot be normally distributed. Therefore many distributions in Economics like distribution no. of children per family, cannot be studied under Normal distribution.
 - 2- It cannot be applied to situations where distribution is highly skewed. For eg:- distribution of income is very much skewed. Therefore here normal distribution will not be appropriate.

Standard Normal Distribution

If x is a random variable following normal distribution with mean (m) and standard deviation σ . Then the variable,

$$z = \frac{x-m}{\sigma}$$

is known as standard normal variable. The distribution of z is known as Standard Normal Distribution. The probability density function of z is

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

$$E(ax+b) = aE(x) + b$$

$$V(ax+b) = a^2 V(x)$$

$$E(z) = \frac{m - m}{\sigma} = 0$$

$$E(z) = E\left(\frac{x-m}{\sigma}\right)$$

$$= E\left(\frac{x}{\sigma} - \frac{m}{\sigma}\right)$$

$$= \frac{1}{\sigma} E(x) - \frac{m}{\sigma}$$

$$\text{i.e., } E\left(\frac{x}{\sigma} - \frac{m}{\sigma}\right) = E\left(\frac{1}{\sigma} x + -\frac{m}{\sigma}\right)$$

$$= \frac{1}{\sigma} E(x) + -\frac{m}{\sigma}$$

$$= \frac{1}{\sigma} m + -\frac{m}{\sigma}$$

$$(1) \therefore E(z) = \frac{m}{\sigma} - \frac{m}{\sigma} = 0$$

$$V(z) = V\left(\frac{x-m}{\sigma}\right)$$

$$\text{Probability and variance } = V\left(\frac{x-m}{\sigma}\right)$$

$$= V\left(\frac{1}{\sigma} x + -\frac{m}{\sigma}\right)$$

$$= \left(\frac{1}{\sigma}\right)^2 V(x)$$

$$= \left(\frac{1}{\sigma}\right)^2 \times \sigma^2$$

$$\text{Var of } z = \frac{1}{\sigma^2} \times \sigma^2 = 1$$

$$Ex = M$$

$$Vx = \sigma^2$$

standard normal

$$\textcircled{1} \quad f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

$$\textcircled{2} \quad E(z) = 0, \quad V(z) = 1$$

$$z \sim N(0, 1)$$

$$\textcircled{3} \quad x \sim N(\mu, \sigma)$$

$$z = \frac{x-\mu}{\sigma} \sim N(0, 1)$$

Ex: $x \sim N(30, 4)$

thus, $\frac{x-30}{2} \sim N(0, 1)$

$$\text{If } x_2 \sim N(40, 16)$$

$$\text{Then, } \frac{x_2 - 40}{4} \sim N(0, 1)$$

The Standard Normal distribution Table.

This is a table showing the probability for z taking values below zero and gives value. The probability thus obtained is the area of the standard normal curve between the ordinates or ordinates at $z=0$ and at the given value. For eg:- when $z = 1.17$ table value equal to 0.3790

This table value is the area between zeros and

1.17. That is,

$$P(0 < z < 1.17) = 0.3790$$

Q. Find

$$\text{Find } \textcircled{1} \quad P(0 < z < 1.44)$$

$$\textcircled{2} \quad P(0 < z < 1.5)$$

$$\textcircled{3} \quad P(0 < z < 2.6)$$

$$\textcircled{4} \quad P(0 < z < 0.45)$$

$$\textcircled{5} \quad \text{where } z \sim N(0, 1)$$

Ans. $\textcircled{1} \quad P(0 < z < 1.44) = 0.4251$

$$\textcircled{2} \quad P(0 < z < 1.5) = 0.4332$$

$$\textcircled{3} \quad P(0 < z < 2.6) = 0.4953$$

$$\textcircled{4} \quad P(0 < z < 0.45) = 0.1736$$

Q. Find,

$$\textcircled{1} \quad P(-1.78 < z < 1.78)$$

$$\textcircled{2} \quad P(1.52 < z < 2.01)$$

$$3) P(-1.52 < Z < 0.75)$$

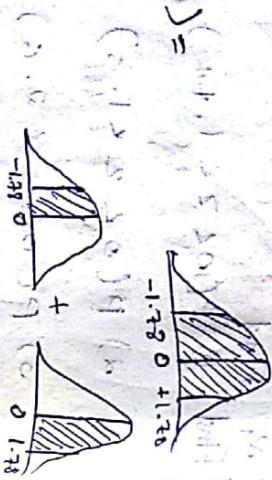
$$= P(0 < Z < 0.01) - P(0 < Z < 1.52)$$

$$4) P(Z > 1.8)$$

$$5) P(Z < -1.5)$$

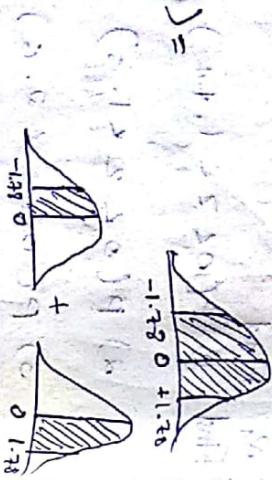
Ans:-

$$1) P(-1.78 < Z < 1.78)$$



$$= 0.0421$$

$$3) P(-1.52 < Z < 0.75)$$



$$= 0.4357 - 0.2734$$

$$= P(-1.78 < Z < 0) + P(0 < Z < 1.78)$$

$$= P(0.4241; 1.78) + P(0 < Z < 1.48)$$

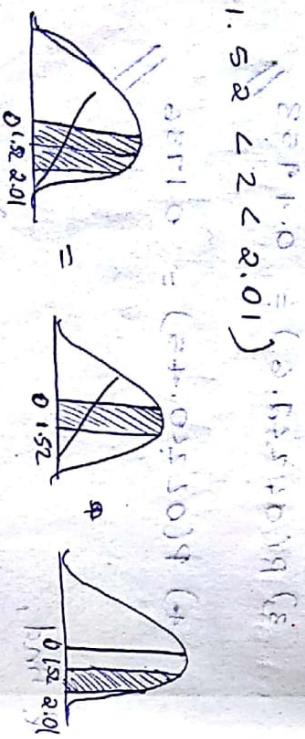
$$= 2P(0 < Z < 1.78)$$

$$= 2 \times 0.4625$$

$$= 0.9250 \quad (0.4625 \times 2)$$

$$= 0.9250$$

$$2) P(1.52 < Z < 2.01)$$

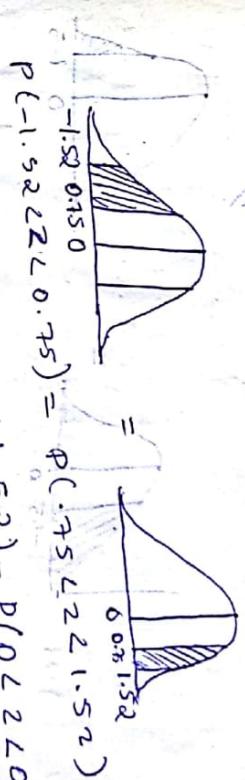


$$= P(0.4241; 2.01)$$

$$= 0.4641 - 0.2734$$

$$= 0.1907$$

$$4) P(Z > 1.8)$$

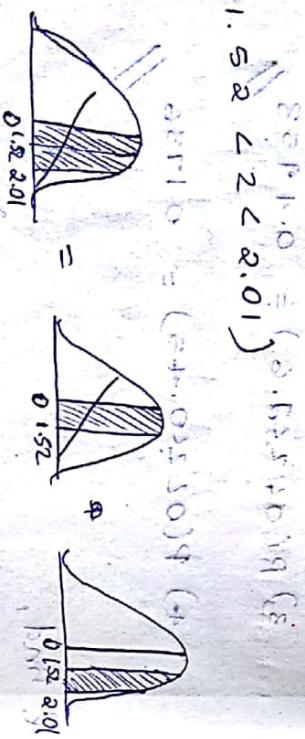


$$P(-1.52 < Z < 0.75) = P(0.75 < Z < 1.52)$$

$$= P(0 < Z < 1.52) - P(0 < Z < 0.75)$$

$$= 0.4357 - 0.2734$$

$$5) P(Z < -1.5)$$

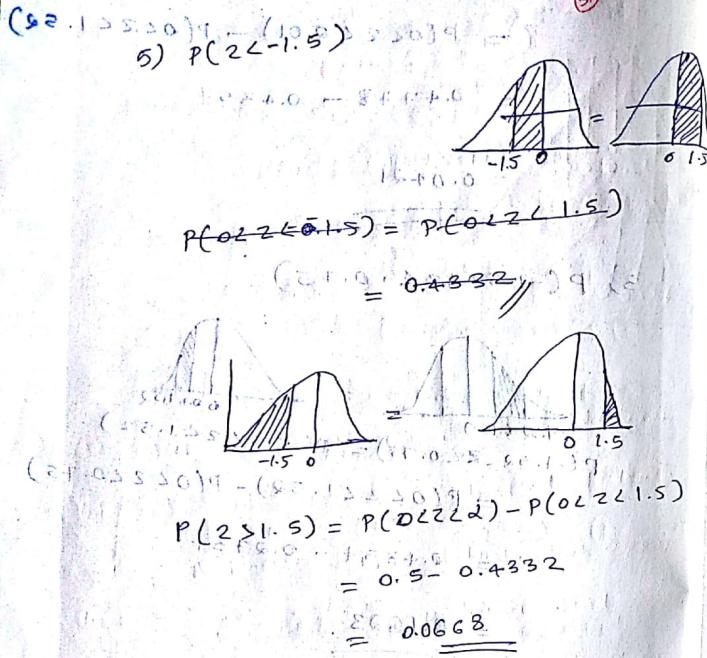


$$= P(0 < Z < -1.5)$$

$$= 0.5 - 0.4641$$

$$= 0.1641$$

$$= 0.1641 \quad 0.0359$$



Ques. 2) The variable X follows a normal distribution with mean 45 and standard deviation = 10. Find the probability that

$$\begin{aligned} 1) & P(X > 60) \\ 2) & P(40 < X \leq 56) \end{aligned}$$

Ans) Here x is normal distribution $x \sim N(45, 10)$

1) $P(X > 60)$

$$= P\left(\frac{x-45}{10} > \frac{60-45}{10}\right)$$

$$= P\left(Z > \frac{15}{10}\right) \quad \left| \begin{array}{l} P(Z = \frac{x-45}{10}) \\ x \sim N(45, 10) \end{array} \right.$$

$$= P(Z > 1.5) = 0.0668$$

2) $P(40 < X \leq 56)$

$$= P\left(\frac{40-45}{10} < \frac{x-45}{10} \leq \frac{56-45}{10}\right)$$

$$= P\left(-0.5 < \frac{x-45}{10} \leq 1.1\right)$$

$$= P\left(\frac{40-45}{10} < \frac{x-45}{10} \leq \frac{56-45}{10}\right)$$

$$= P\left(\frac{-5}{10} < Z \leq \frac{11}{10}\right)$$

$$\begin{aligned}
 & P(-0.5 \leq Z \leq 1.1) \\
 & = P(0 \leq Z \leq 0.5) + P(0 \leq Z \leq 1.1) \\
 & = P(0 \leq Z \leq 0.5) + P(Z \geq 0.5) \\
 & = 0.1915 + 0.3643 \\
 & = 0.5558
 \end{aligned}$$

Q. The scores of students in a test follow normal distribution with mean = 80 and standard deviation = 15. A sample of 1000 students has been drawn from the population. Find,

1) Appropriate number of student scores in box

65 u 95

(2) The probability that random choose the

Students has scores greater than 100.

$$\text{Ans: } P(61 \leq X \leq 95) = \frac{x - M}{\sigma} = \frac{95 - 80}{15} = 1$$

$$\begin{aligned}
 P(65 < x < 95) &= P(65 - 80 < x - 80 < 95 - 80) \\
 &= P\left(\frac{65-80}{15} < \frac{x-80}{15} < \frac{95-80}{15}\right) \\
 &= P\left(-\frac{15}{15} < Z < \frac{15}{15}\right) \\
 &= P(-1 < Z < 1) \\
 &= P(-1 < Z_1) + P(Z_2 < 1) \\
 &= 2P(Z < 1) \\
 &= 2 \times 0.3413 \\
 &= 0.6826
 \end{aligned}$$

~~For 2nd part~~

(a) No. of students scores b/w 65 and 95

$$\begin{aligned}
 P(65 < x < 95) &= 1000 \times 0.6826 \\
 &= 682.6 \\
 &= 683
 \end{aligned}$$

(b) $P(x > 100)$

$$\begin{aligned}
 P(x > 100) &= P\left(\frac{x-80}{15} > \frac{100-80}{15}\right) \\
 &= P(Z > \frac{20}{15}) \\
 &= P(Z > 1.33)
 \end{aligned}$$

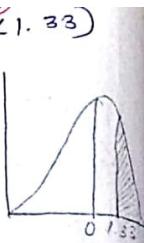
$$P(Z > 1.33) = 0.5000 - P(Z \leq 1.33)$$

$$\left(\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \right) dx = 0.5000 - 0.4082$$

$$\left(\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \right) dx = 0.0918$$

No. of students greater than 100

$$(1 - 0.0918)q = 1 - 0.0918 = 0.9082$$



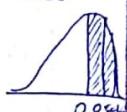
$$= P(Z \geq 0.66) + P(Z \geq -0.66)$$

$$= 0.2454 + 0.2454 = 0.4908$$

$$= 0.309 q - (0.309 q)$$

$$= 0.309 q - (0.309 q)$$

$$= 0.309 q - (0.309 q)$$



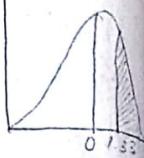
Q. The height of the school children of a institution is normally distributed with mean 54.5 and $\sigma = 12$. What percentage of students are height below 46 or 56.

$$\text{Ans: } 1) \quad z = \frac{x - \mu}{\sigma} = \frac{x - 54}{12}$$

$$P(46 < X < 56) = P(46 - 54 < X - 54 < 56 - 54)$$

$$= P\left(\frac{-8}{12} < \frac{X - 54}{12} < \frac{12}{12}\right)$$

$$= P(-0.66 < Z < 1.33) = 0.16$$



$$= \text{percentage of students are height below 46 or 56} = 0.309 \times 100$$

$$= 30.9\%$$

$$= 30.9\%$$

Locate value of 'z' when the area(p) is given

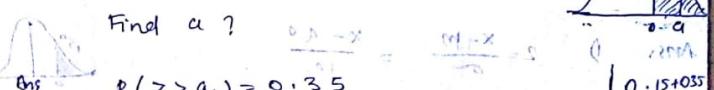
Q. If $P(Z > a) = 0.4332$ find 'a'

$$a = 1.5$$

Ans: 1) $P(Z > a) = 0.4332$

Q. If $P(Z > a) = 0.35$ find 'a'

Find 'a'?

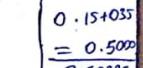
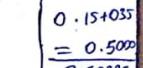
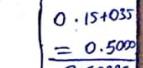
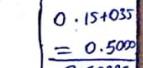
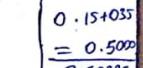
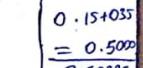
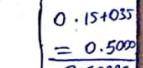
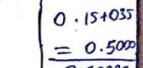
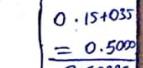
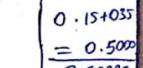
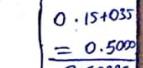
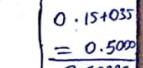
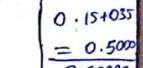
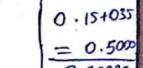
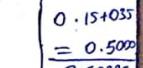
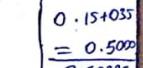
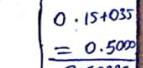
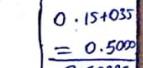
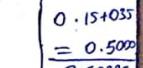
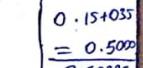
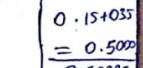
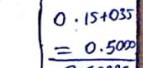
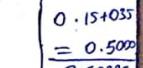
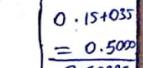
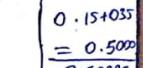
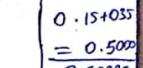
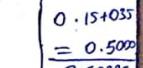
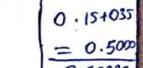
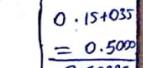
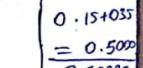
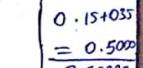
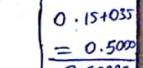
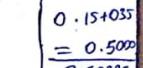
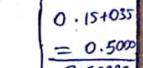
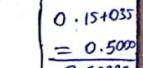
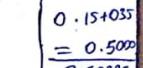
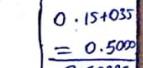
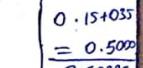
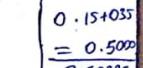
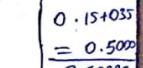
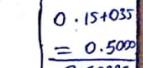
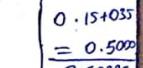
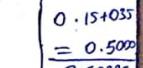
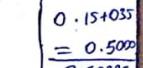
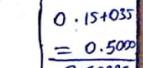
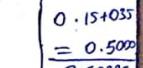
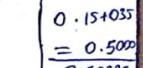
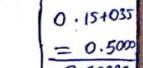
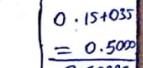
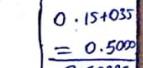
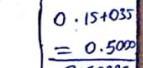
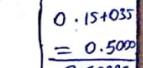
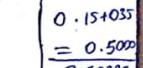
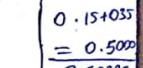
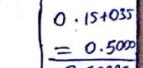
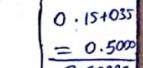
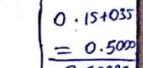
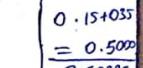
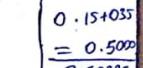
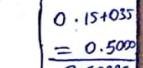
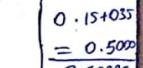
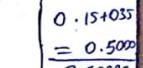
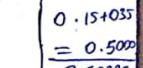
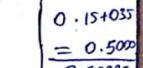
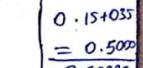
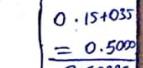
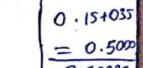
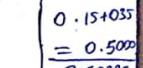
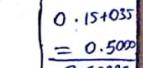
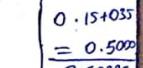
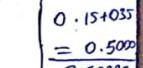
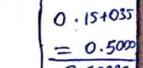
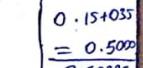
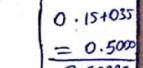
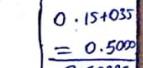
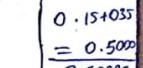
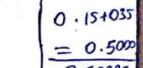
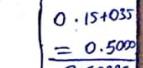
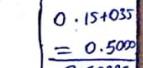
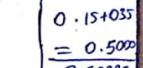
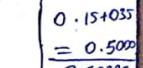
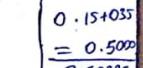
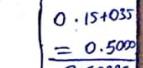
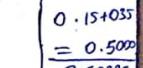
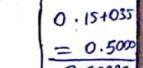
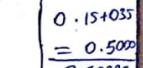
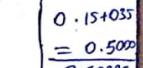
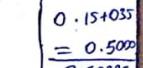
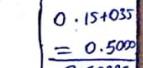
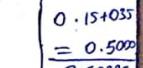
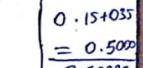
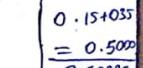
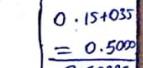
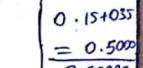
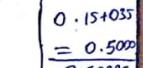
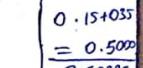
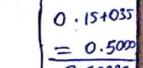
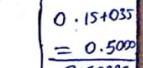
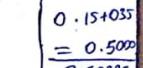
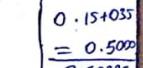
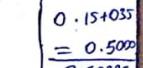
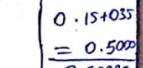
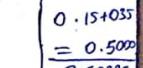
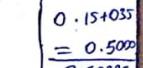
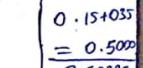
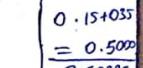
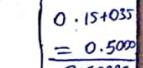
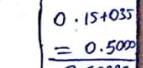
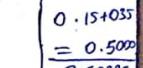
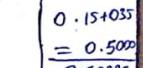
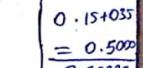
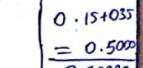
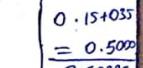
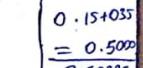
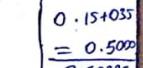
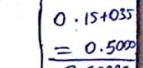
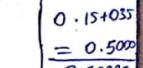
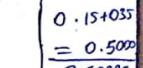
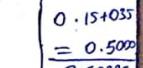
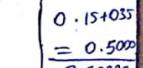
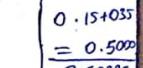
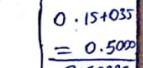
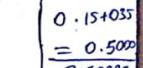


$$P(Z > a) = 0.35$$

$$P(Z \leq a) = 0.65$$

$$P(Z \leq a) = 0.15$$

$$\therefore \text{from standard table } a = 0.392$$



12) $\log_{10} 100 = 2$ The value of the answer is ⁽¹²⁾ the right

$$P(X > a) = 20\% = 0.2$$

$$P(X > a) = P\left(\frac{X - 40}{10} > \frac{a - 40}{10}\right) = 0.2$$

$$= \varphi(2 > \frac{10 - 4\sigma}{1.6}) = 0.2$$

$$= P(2 > b_2) = 0.2$$



$$P(2 > b_2) = 0.2$$

$$P(O \subset Z \subset b_2) = 0.5 - 0.2$$

On 3rd April

from the standard table.

$$b_2 = 0.804 \quad 0.84$$

$$\frac{1}{10} = 0.804 \text{ or } 0.84$$

$$L_{\text{eff}} = 10 \times 0.804 = 0.84$$

$$\frac{30 - m}{m} = -0.95$$

$$a = -0.95$$

$$30 \times 0.95 = 28.5$$

$$\frac{(\theta \rho_{\text{air}})^2 - 3\rho - M}{\rho} = -0.95$$

From the Standards table, 1

0.33

०.७५

$$\rho(r^2 < a) = 0.17$$

$$\begin{array}{r} 0 \\ -0.81 \\ \hline 0.81 \end{array}$$

$$P\left(\frac{x-m}{\sigma} \leq \frac{30-m}{\sigma}\right) = 0.17$$



shows that, $\mathbb{P}(\mathcal{E}_2 \leq a) = 1 - e^{-a}$

such that, $\mathbb{P}(G_2 \leq g) = 1$

Heseltine, Conven., $P(x \geq 30) = 17\% = 0.17$

above 60. Find the mean & SD?

In a normal distribution 2.7% of the stems (2 P. O. - 2) - 0.02 are below 30 and 17% of the area

Now we have to find standard deviation

$$m - 0.95\sigma = 30 \Rightarrow m = 30 + 0.95\sigma$$

On other hand, given,

$$P(Z > b) = P(X > 60) = 17\% = 0.17$$

Now let's find corresponding value of Z

$$\text{i.e., } P(Z > b) = 0.17, \text{ just draw}$$

$$\text{i.e., } P\left(\frac{X-m}{\sigma} > \frac{60-m}{\sigma}\right) = 0.17$$

$$\text{i.e., } P\left(Z > \frac{60-m}{\sigma}\right) = 0.17$$

$$P(Z > b) = 0.17$$



$$P(Z > b) = 0.5 - P(Z \leq b)$$

$$0.5 - 0.33 = 0.17$$

From the Standard table,

$$\text{Value of } Z \text{ is } 0.95 \text{ from}$$

$$\text{i.e., } \frac{60-m}{\sigma} = 0.95$$

$$60 - m = 0.95\sigma$$

$$m = 60 - 0.95\sigma$$

$$60 = m + 0.95\sigma$$

$$m - 0.95\sigma = 30 \Rightarrow m = 30 + 0.95\sigma$$

$$m + 0.95\sigma = 60 \Rightarrow m = 60 - 0.95\sigma$$

$$m = \frac{30 + 60}{2} = 45$$

$$\text{Substituting } m = 45 \text{ in Eqn (1)}$$

$$45 - 0.95\sigma = 30$$

$$45 - 30 = 0.95\sigma$$

$$15 = 0.95\sigma$$

$$\frac{15}{0.95} = \sigma$$

$$\sigma = 15.79$$

$$\text{Thus, } m = 45$$

$$\sigma = 15.79$$

$$\sigma = 15.79$$

$$15.79 + (1.96 \times 15.79) = 45.8$$

$$45.8 + (1.96 \times 15.79) = 61.5$$

Q. The following table is frequencies of occurrence of variant X between certain limits.

X	f	$\frac{f}{N}$
40	30	0.3
40 or more but less than 50	33	0.33
50 and more	37	0.37

The distribution is exactly normal. Find the average & SD of X .

$$P(40 < X < 50) = \frac{f}{N} = \frac{f}{100}$$

Ans:

Here answer,

$$P(X < 40) = \frac{30}{100} = 0.3$$

$$P(X > 50) =$$

$$P(40 < X < 50) = \frac{33}{100} = 0.33$$

$$P(X > 50) = \frac{37}{100} = 0.37$$

That is,

$$P(X < 40) = 0.3$$

$$P\left(\frac{X - m}{\sigma} < \frac{40 - m}{\sigma}\right) = 0.3$$

$$P\left(\frac{X - m}{\sigma} < \frac{40 - m}{\sigma}\right) = 0.3$$



$$P\left(Z > -\left(\frac{40-m}{\sigma}\right)\right) = 0.3$$

$$\text{From standard table, } -0.5 - 0.3 = 0.2$$

$$-\left(\frac{40-m}{\sigma}\right) = 0.52$$

$$\frac{40-m}{\sigma} = -0.52$$

$$\text{i.e., } \frac{40-m}{\sigma} = -0.52$$

$$40 - m = -0.52\sigma$$

$$m - 40 = 0.52\sigma \quad \text{--- (1)}$$

On the other hand,

$$P(X > 50) = 0.37$$

$$P\left(\frac{X - m}{\sigma} > \frac{50 - m}{\sigma}\right) = 0.37$$



$$P\left(Z > \frac{50-m}{\sigma}\right) = 0.37$$

$$\frac{50-m}{\sigma} = 0.52$$

$$0.52$$

$$0.52 = 0.5 - 0.37$$

$$= 0.13$$

$$0.13$$

From the standard table,

$$\frac{50-m}{\sigma} = 0.62 \cdot 0.34$$

$$\text{i.e. } 50-m = 0.62 \cdot 0.34 \sigma$$

$$-(2)$$

$$50 = m + 0.34 \sigma$$

From eqt ① & ②

$$4 \sigma = 0.52 \sigma$$

$$50 = m + 0.34 \sigma$$

$$90 = m + 0$$

$$\begin{aligned} ① - ② \\ m - 0.52 \sigma = 40 \end{aligned} \quad -(1)$$

$$m + 0.34 \sigma = 50$$

$$0.52 \sigma = 10$$

A random variable x is said to follow log normal distribution.

This is, if $\log x$ follows normal distribution, then x follows log normal distribution.

The probability density function is,

$$\begin{aligned} \sigma &= \frac{10}{-0.86} \\ &= \frac{10}{-0.86} \\ &= 11.63 \end{aligned}$$

$$\text{Substituting values in part (1)} \\ 0.52 \times 11.63 = 40$$

$$\sigma = 46.0476$$

$$\text{Then, i.e., } \sigma = 11.63$$

$$m = 46.0476$$

Log Normal Distribution

$$\boxed{f(x) = \frac{1}{x\sqrt{2\pi}} e^{-\frac{1}{2}(\log x - \mu)^2}}$$

Properties of Lognormal distribution

(3)

- 1 - Lognormal variant is a continuous variable
 - 2 - The range of variate values of the variable are $x > 0$
 - 3 - Mean of the lognormal distribution is
- $$\text{mean} = e^m + \frac{1}{2}\sigma^2$$
- and SD is, $SP = e^m + \frac{1}{2}\sigma^2 \sqrt{e^{2\sigma^2} - 1}$
- 4 - Lognormal distribution is positively skewed, $\beta_1 > 0$, $\gamma_1 > 0$
 - 5 - Lognormal distribution is unimodal.

$$\begin{cases} \gamma_1 = \text{gamma} \\ \gamma_2 = \text{gamma} \end{cases}$$

Uses (Applications) of Logarithmic Normal Distribution

- a - The Lognormal distribution possesses moments of any moment like $1, 2, 3, \dots$ order while putting $\alpha = 1, 2, 3, \dots$ etc., we have
- b - Lognormal distribution is $e^{2m + \frac{1}{2}\sigma^2}$ skewed.
- c - Most of the economic data are not suitable to the field of Economics, it is of very great use.
- d - Approximating to normal distribution.
- e - Lognormal distribution is the appropriate distribution for approximating them. The economic distributions like distribution of income, price, expenditure on particular commodities etc. are skewed in nature. They all follow Lognormal distribution.
- f - Further, these variables take only positive values, which is a restriction required as per the Lognormal distribution. Thus in the field of economics, the movements of many variables can be studied by applying Lognormal Distribution. Lognormal

Distributions can be used for the study of income distribution, similarly Bank Deposits and total wealth possessed by individual persons can also be studied with the help of Lognormal Distributions. For the study of the relation b/w the demand for a commodity and consumer income.

Lognormal distribution can be used. Prices paid for commodities, Expenditure on particular commodities, Savings, behaviours etc., approximate to Lognormal Distribution.

In other fields;

In Biology the Lognormal Distribution is extensively used. It is seen that the size of a growing organism follows Lognormal Distribution.

The study of heights or weights of human beings can be made with the help of Lognormal distribution.

In physical and industrial fields also the

Lognormal distribution is used. It can be seen that the ages of men and women at their first marriage follow Lognormal Distribution.

control also, the Lognormal Distribution is used.

Merits & Demerits

Lognormal Distribution can be applied to situations where a no. of independent factors influence a variable in a multiplicative model.

The Lognormal Distribution is the best approximation to skewed distributions. Many of the economic variables have skewed distribution. Thus the study of these economic variables can be made by using Lognormal Distribution.

The Lognormal Distribution can be used for the study of many variables where Normal Distribution is not a good approximation. The variables like income, price, land owning etc.. are examples. Frequency curves drawn to those variables come in no way close to normal distribution. But the logarithm values of these variables give a normal curve. So they obey Lognormal law.

Distinguish b/w Normal & Lognormal

1- Normal Distribution is symmetric while LD is positively skewed.

LD is positively skewed.

2- When a distribution drawn on a graph, if it gives a symmetrical curve then it is normal distribution. When the logarithmic values of the variable are plotted on a graph if it gives a symmetrical distribution then it follows a LD.

3- The ND arises from a theory of elementary errors in an additive fashion, while LD arises from a theory of elementary errors in multiplication fashion.

4- The ND is symmetric and therefore mean, median, and mode are equal. If a population follows LD its mean, median & mode are not equal.

5- The Arithmetic Mean of 'n' independent normal variates follows a NID while Geometric Mean of 'n' independent lognormal variates follows a LD.

Module 3

Sampling Distributions

In any statistical distributions, we are interested in studying the various characteristics of individuals (items) of a particular group. This group of individuals under study is known as population. Any population can be considered as the set of admissible value of a random variable. The distribution of this random variable is called the distribution of population.

Eg: If we want to study the expenditure habit of the family in a city, then the population will consist of all the households in that city.

A population contains finite no. of objects or items & is known as finite population.

Eg: Students in a college \rightarrow population in a city