Bayes' Law

Bayes' law for conditional probabilities: $p(a|b) = \frac{p(b|a)p(a)}{p(b)}$ So in our case:

$$\begin{split} \hat{e}_1^{\hat{I}} &= \operatorname*{argmax} p(e_1^I|f_1^J) & \text{Apply Bayes' law} \\ &= \operatorname*{argmax} \frac{p(f_1^J|e_1^I)p(e_1^I)}{p(f_1^J)} & p(f_1^J) & \text{constant} \\ &= \operatorname*{argmax} p(f_1^J|e_1^J)p(e_1^I) & \Rightarrow \text{irrelevant in maximization} \\ &= \operatorname*{argmax} p(f_1^J|e_1^I)p(e_1^I) & \end{split}$$

Also called "Noisy Channel" model.