Compositionality of Vector Representations

Karlgren and Kanerva (2019) show "Holographic Reduced Reprs.":

- Addition: Preserves similarity, useful to represent bag-of-…
- Hadamard product (elem-wise multiplication),
  - Invertible; product dissimilar to its operands: $A * B \nsim A$.
  - Bipolar vectors ($\{-1, +1\}^n$) are inverse of themselves.
  - Can represent variable assignment $\{x = a, y = b, z = c\}$ using bipolar vectors X, Y, and Z added into a vector $(X * A) + (Y * B) + (Z * C)$. To recover the value of x, multiply by X:
    $X * (X * A) + X * (Y * B) + X * (Z * C)) = A + \textsf{noise} + \textsf{noise} \sim A$
- Vector elements permutation,
  - Also invertible; dissimilar; enormous number of permutations.
  - Useful to represent structures, e.g. lists: Π_1 for CAR Π_2 for CDR: (a, b) represented with $\Pi_1(a) + \Pi_2(b)$

(In highly-dimensional spaces, most vectors are dissimilar; cosine or Pearson correlation of 0.25 indicate close similarity.)