= :LNLSHGLH RWHY HQ« HQF\NORSHGLH

LNR

7HQWR ÏO£QHN QHQ¯ R]GURMRYDQ¿ GRVWDWHÏQ@H WHG\ REVDKRYDW LQI NWHU« MH RY ÝHLWD

-VWH OL V SRSLVRYDQ¿P S HGPÝWHP VH]Q£PHQL SRPR]WH WYU]HQ¯ GR SROHÝ QQ\FK]GURM¯ FÝURKRGQ«]GURMH

,OXVWUDFH OLQH£UQ¯ UHJUHVH

/LQH£UQ¯ UHJUHVH MH PDWHPDWLFN£ PHWRGD SRX@¯YDQ£ SUR S ERG1 Y JUDIX NRX ERGHFK UHSUH]HQWXM¯F¯FK PÝ HQ£ GDW S HGSRNO£G£ @H MHMLFK [RY« VRX DGQLFH MVRX S HVQ VRX DGQLFH PRKRX E¿W]DW¯@HQ\ Q£KRGQRX FK\ERX S L @H]£YLVORVW \ QD [O]H JUD²FN\ Y\M£G LW S ¯PNRX 3RN SURO@¯PH S ¯PNRX WDN S L RGHÏ¯W£Q¯]JUDIX EXGH PÝ HQ«KR ERGX D \SVLORQRYRX KRGQRWRX OH@¯F¯ QD S OLQH£UQ¯ UHJUHVH MH QDOH]HQ¯ WDNRY« S ¯PN\ DE\ VRX RGFK\OHN E\O FR QHMPHQ#¯ /LQH£UQ¯ UHJUHVL O]H]REHF IXQNF¯ QH@ S ¯PNRX 7HUPLQ¯ QHSRXÏ¯YDQ« 1@H R]QDÏRYDW GY ÏVWHÏQÝ RGOL#Q« YÝFL

/LQH£UQ¯ UHJUHVH S HGVWDYXMH DSUR[LPDFL GDQ¿FK KRGQRW S ¯PNRX PHWRGRX QHMPHQ#¯FK ÏWYHUF¯ 3RNXG WXWR S ¯PNX Y\M£G ¯PH URYQLF¯ $y = b_1 + b_2 x$ MHGQ£ VH R QDOH]HQ¯ RSWLP£OQ¯FK KRGQRW b_1 D b_2

9 REHFQÝM#¯P S ¯SDGÝ MH1@H UHJUHVQ¯ IXQNFH UHJUHVQ¯ PRGHO S¯ GDQ¿FK KRGQRW $[x_i, y_i]$ D WDNRY« IXQNFH $y = f(x, b_1, \ldots, b_k)$ NWHUR O]H Y\M£G LW MDNR OLQH£UQ¯ NRPELQDFL IXQNF¯ f_k

$y = b_1 f_1(x) + \ldots + b_k f_k(x)$ .RH²FLHQW\ $b_1, \ldots, b_k$ VH RSÝW XUÏXM¯ PHWRGRX QHMPHQ#¯FK ÏWYHUF1

+RPRVNHGDVWLFLWD KRPRJHQLWD YH YDULDQFL GDW MH EÝ@Q¿P S HGSRNODGHP P1@H Y«VW N S HNUR ÏHQ¯ KRGQRW Y\VYÝWOHQR@HN NRH²FLHQWX MLVW¿FK S ¯SDGHFK MH WHG\ ]DYHGHQD YH£@HQRX UHJUHVL

$SUR[LPDFH S¯PNRX

8YD@XMPH IXQNÏQ¯ $f(x) = ax + b$ Y LaORVW

6RXÏHW ÏWYHUF1 SDN EXGH Y\SDGDW WDNWR

$$S(a,b) = \sum_{i=1}^{n} [f(x_i) - y_i]^2 = \sum_{i=1}^{n} (ax_i + b - y_i)^2$$

NG[$[x_i, y_i]$ MVRX VRX DGQLFH DSUR[LPRYDQ¿FK ERG1

$EKFKRP QD#OL PLQLPXP VRXÏWX a b DW OL NDE\ FLHQW]HQ£]£YLVORVW YKRGQÝ DSUR[LPRYDOD GDQ£ GDWD SDUFLDOQ¯ GHULYDF VRXÏWX ÏWYHUF1 URYQ\ QXOH

$$0 = \frac{\partial S}{\partial a} = 2\sum_{i=1}^{n}(ax_i + b - y_i)x_i$$

$$0 = \frac{\partial S}{\partial b} = 2\sum_{i=1}^{n}(ax_i + b - y_i)$$

œSUDYDPL REGU@¯PH VRXVWDYX

$$a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i$$

$$a\sum_{i=1}^{n} x_i + bn = \sum_{i=1}^{n} y_i$$

/]H XN£]DW PD@LFH WR VRXVWDY\ UHJXODUQ¯ Y#HF $n \geq 2$ D PÝ WHG\ SU£YÝ MHGQR H#HQ¯ 2EHFQÝ O]H WDN« XN£]DW @H ÏWYHUF1 PLQLPXP

-HM¯P H#HQ¯P SUR NRQNUHWQ¯ x_i, y_i GRVWDQHPH NRQHÏQÝ KOHGDQp KRGQRW\ SDUDPHWU1 a b

^?GLVSOD\VW\OH D ^?IUDF ^Q?VXP ^[B^L`\E

^\B^L```^Q?VXP ^[B^L`A^ `` ?OHIW ?VXP ^[B^

$$b = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{n \sum x_i^2 - (\sum x_i)^2}$$

3RGREQ¿ SRVWXS O]H DSOLNRYDW QD MDN¿NROLY GUXK]£

3RNXG MH ND@G£ KRGQRWD]DW σ_i ¯@HQD PLÝQ¯RX H FKÝERX OLND
U1]Q¿PL S ¯VWURML MH Y¿KRGQ«]DKUQRXW L WRWR GR

$$\langle x\rangle = \sum_{i=1}^{n} \frac{x_i}{\sigma_i^2}$$ SRWRP GRVW£Y£PH

$$a = \frac{\langle 1\rangle\langle xy\rangle - \langle x\rangle\langle y\rangle}{\langle 1\rangle\langle x^2\rangle - \langle x\rangle^2}$$

$$b = \frac{\langle y\rangle\langle x^2\rangle - \langle xy\rangle\langle x\rangle}{\langle 1\rangle\langle x^2\rangle - \langle x\rangle^2}$$

3 ¯PND SURFK£]HM¯F¯ SRÏ£WNHP

3RNXG MH SR@DGRY£QR DE\ S ¯PND SURFK£]HOD SRÏ£WNH
$y = ax$ 3UR NRQVWDQWX a]QHD WR GYRGLW Q£VOHGXM¯F¯ Y]WDK

$$a = \frac{\sum x_i y_i}{\sum x_i^2}$$

0£PH OL]£YLVORVW $y = ax$ DWKRGQRW\ MVRX]DW¯@HQ\ FKUERX σ_i SRGKDG
SDUDPHWUX $a = \frac{\langle xy\rangle}{\langle x^2\rangle}$ MH X@LWR $\langle x\rangle = \sum_{i=1}^{n} \frac{x}{\sigma_i^2}$ D σ_i]QDÏ¯
FK\EX VPÝURGDWQRX RGFKÝONX HQ¯

'£OH SUR SDUDPHWU a SODWt $V[a] = \frac{1}{\langle x^2\rangle}$

9¿SRÏHW QD SRÏ¯WDÏL

0DWOD XPR@ XMH SRX@ ¯W 3RO\QRPLDO ; < NGH SRVOHGQ¯
SDUDPHWU G£Y£ @H KOHG£PH NRH²FLHQW\ SRO\QRPX SUYQ¯
>QHGRVWXSQ@¿]GURM

9 ([FHOX &DOFXOXV /LEUH2´ IILFH 2´SHQ2´IILFH RUJ]H NRH²FLHQWLW IXQNF¯
6/23(< ; >@ @D NRQVWDQWX 7(5&(37 < >@ @3 ¯SDGQÝ
O]H RED NRH²FLHQW\ ]MLVWLW PDWLFRYÝ IXQNF¯
ÏHVN«P ([FHOX VH WDWR IXQNFH (Q]Y£

2EHFQ£ OLQH£UQ¯ UHJUHVH

,OXVWUDFH KOHG£Q¯ RSWLP£OQ¯ OLQH£UQ¯ NRPELQDFH S HGVWDYXMH SURVWRU YH NWHU«P VH QDFK£]¯ Y#HFK NRPELQDFH G; G; 9HNWRU HGVWDYXMH YHNWRU KRG NH NWHU«P VH DSUR[LPDFH LEO¯@LW V QHMPHQ#¯ PR FK\ERX\¥G; UHVSHNWLYH GUXKRX PRFQLQRX W«WR FK\ SRSLV YL] NDSLWROD .ROP« Y HNWRUX PHWRGD QHMPHQ#¯FK ÏWYHUF1

9 REHFQÝM#¯P S ¯SDGÝ MH PR@Q« $[x_i, y_i]$ $i = 1, \ldots, n$ QRWDPL SURO@LW $y = f(x, b_1, \ldots, b_k)$ VHVWDYHQRX MDNR OLQH£UQ¯ NRPELQDFL k IXQNF¯ $y = b_1 f_1(x) + \ldots + b_k f_k(x)$ NGH $f_1(x), \ldots, f_k(x)$ MVRX OLERYROQ«]SUDYLGOD OLQH£UQÝ QH]£YLVO« IXQNFH 5HJUHVH VH QD]¿Y£ OLQH£UQ¯ IXQNÏQ¯ S $y = f(x, b_1, \ldots, b_k)$ MH OLQH£UQ¯ Y $b_1, \ldots, b_k$ SURPÏQQ¿FK WHG\ Y NRH²FLHQWHFK NWHU« SRGUREXMHPH UHJUHVL -L IRUPXORYDW DOJHEUDLFN\ PHWRGD QHMPHQ#¯FK ÏWYHUF1

/LQH£UQ¯ UHJUHV¯ MH WHG\ L Y¿#H SRSVDQ£ SURO@HQ¯ $f_1(x) = x$ $f_2(x) = 1$ $f(x) = b_1 x + b_2$ DOH WDNÄ SDUDERO $f_1(x) = x^2$ $f_2(x) = x$ $f_3(x) = 1$ $f(x) = b_1 x^2 + b_2 x + b_3$ QHER REHFQÏ SRO\QRPHP 3R]QDPHQHMPH @H V SURO@HQ¯P PQR@LQ\ ERG1 SDUDE SRO\QRPHP VH P1@HPH Y OLWHUDWX H VHWNDW SRG SRMP SRO\QRPLFN£ ÏL SRO\QRPLFN£ OLQH£UQ¯ UHJUHVH >@

.RH²FLHQW\ $b_1, \ldots, b_k$ MVRX Y\SRÏWHQ\ PHWRGRX QHMPHQ#¯FK ÏW DE\ VRXÏHW GUXK¿FK PRFQLQ RGFK\OHN PRGHOX RG GDQ¿

$$S = \sum_{i=1}^{n} (y_i - f(x_i, b_1, \ldots, b_k))^2 = \sum_{i=1}^{n} (b_1 f_1(x_i) + \ldots + b_k f_k(x_i) - y_i)^2,$$

E\O PLQLP£OQ¯

]S1VRE Y¿SRÏWX SDUFL£OQ¯ GHULYDFH

3UR NRH²FLHQW\ NWHU« PLQLPDOL]XM S YR¿#VH XSRHGWDQW N@ Y#HFKQ\ SUYQ¯ SDUFL£OQ¯ GHULYDFH NULW«ULD SRGOH W URYQ\ QXOH

$$\frac{\partial S}{\partial b_1} = \ldots = \frac{\partial S}{\partial b_k} = 0$$

'DO#¯PL ¼SUDYDPL VH VRXVWDYX OLQH£UQ¯FK URYQLF

$$\begin{array}{ccccccc} a_{11}b_1 & + & \ldots & + & a_{1k}b_k & = & a_1 \\ \vdots & & \ddots & & \vdots & = & \vdots \\ a_{k1}b_1 & + & \ldots & + & a_{kk}b_k & = & a_k \end{array}$$

.GH MHGQRWOLY« a_{jk} D a_j]QDPHQDM¯

$$a_{jk} = \sum_{i=1}^{n} f_j(x_i) f_k(x_i)$$

$$a_j = \sum_{i=1}^{n} f_j(x_i) y_i$$

9¿#H XYHG HQRX VRXVWDYX OLQH£UQ¯FK URYQLF OH#LW QÝNWHURX] PHWRG XYHG ÏO£QN VRXVWDYD OLQH£UQ¯FK URYQLF

]S1VRE Y¿SRÏWX S HXUÏHQ£ VRXVWDYD URYQLF

-LQ¿P]S1VREHP MDN Y\SRÏ¯WDW KOHGDQ« NRHILFLHQW\ M VRXVWDY\ URYQLF Y\ H#HQ¯ RSÝW PHWRGRX QHMPHQ#¯FK SRQÝNXG RGOL#Q¿P SRVWXSHP 3 HXUÏHQ£ VRXVWDYD URY Q£VOHGRYQÝ

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

$$\mathbf{A} = \begin{bmatrix} f_1(x_1) & \ldots & f_k(x_1) \\ \vdots & & \vdots \\ f_1(x_n) & \ldots & f_k(x_n) \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} b_1 \\ \vdots \\ b_k \end{bmatrix}$$

[redacted]

+OHGDQ« NRH²FLHQW\ XP¯VWÝQ]«H YHD YSHNWRUX OLQH
QH]£YLVORVWL VORXSF\1 PDWLFH]WDKHP

$$\mathbf{x} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{b}$$

9¿SRÏHW QD SRÏ¯WDÏL

0DWOOD XPR@ XMH VRXVWDY[U#YQLF HOPL VQDGQR SRPRF¯
?]SÝWQ« ORP¯WND $? E (NYLYDOHQWQ¯ /, 'L QHF\
PODGLYLGH $> E@

9 ([FHO & DO[(/EXUH 2´FH 2SHQ2´FH RI]H Y¿#H VHVWDYHQRX
S HXUÏHQRX VRXVWDYX URYQLF H#LW SRX@LW¯P PDWLFRY
^ /,1(67 NQRZQB\ V NQRZQB[V> F@QVW@Y ÏHVN«P ([FHOX
/,15(*5(6(SROHB\ SROHB[@ENGH SUYQ¯ SDUDPHWU
ÏHVN SROHBMH VYLVO£ REODVW EXQÝN REVDKXM¯F¿VOR@N
SDUDPHWU NQRZQB[ÏHVN SROHB[MH REODVW REVDKXM¯F SUYN\ P
9¿VOHGQ¿ YHNWRU FK£]¯ YH YRGRURYQ« REODVWL S LÏHP@
XP¯VWÝQ\ Y EX N£FK Y RSDÏQ« EP N MR DGEX FHOQ MYEFH YOHY
MH QHMY¯FH YSUDYR 7RWR VWDVQ UHWÏH¿W Y WRPWR S ¯N
URYHQ QXOH VSU£YQ« SRX@LW E WHG\`MH

2Y#HP QHMMHGQRGX##¯P]S1VREHP RGKDGX SDUDPHWU1 P
ÏWYHUF1 MH SRX@LW¯ HNRQRPHWULFN«KR VRIWZDUX MDN
QHER 5 NGH H[LVWXM¯ REHFQ« S ¯ND]\ SUR MHMLFK Y¿SR
SURJUDP\ XPR@ XM¯ MHGQRGX#H WHVWRYDW S HGSRNODG

3 HYRG PRFQLQQ« D H[SRQHQFL£OQ¯ UHJUHVH QD

1D OLQH£UQ¯ SUREO«P O]H WUDQVIRUPRYDW L XQRU[LPDFL
$f(x) = a \cdot x^b$ QHER DSUR[LPDFL H[SRQHQFL£OQ $f(x) = a \cdot b^x$

0RFQLQQ£ IXQNFH

3UREO«P MDN DSUR[LPRY[redacted]1MRLYQ¯ GDWD O]H S HY«VW
QD SRGREQ¿ SUREO«P UHYQ¯FH N LYN\

S LÏHP@ P¯VWR]H SV£W