



On Solving Partial Differential Equations with Neural Networks (The Deep Ritz Method)

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Outline

Introduction

Problem Definition

Results

Conclusion





Introduction

"The key is in not spending time, but in investing it"
- *Stephen R. Covey.*



Finite Element Method

Poisson Equation on Unit Square - Strong Form

$$\begin{cases} -\Delta u = f(x) \\ u = 0 \end{cases} \quad , \text{ on } \Omega = [0 \times 1] \times [0 \times 1] - [0, 1) \\ \quad , \text{ on } \partial\Omega$$



Finite Element Method

Poisson Equation on Unit Square - Weak Form

$$\sum_{i=1}^M c_i \langle \phi_i, \phi_j \rangle = \langle f, \phi_j \rangle \quad i = 1, \dots, M,$$

where

$$u_h = \sum_{i=1}^M c_i \phi_i \text{ and } \in V_h = \text{span}\{\phi_i\}_{i=1}^M$$



Finite Element Method

Quite Complex, eh?

Let's cheat!





Deep Ritz Method

Universal Approximation Theorem

Let $\phi : \mathbb{R} \rightarrow \mathbb{R}$ be a nonconstant, bounded, and continuous function. Let I_m denote the m -dimensional unit hypercube $[0 \times 1]^m$. The space of real-valued continuous functions on I_m is denoted by $C(I_m)$. Then, given any $\epsilon > 0$ and any function $f \in C(I_m)$, there exist an integer N , real constants $v_i, b_i \in \mathbb{R}$ and real vectors $w_i \in \mathbb{R}^m$ for $i=1, \dots, N$, such that we may have define : $F(x) = \sum_{i=1}^N v_i \phi(w_i^T x + b_i)$ as an approximate realization of the function f ; that is, $|F(x) - f(x)| < \epsilon$ for all $x \in I_m$.



Deep Ritz Method

Poisson Equation - Neural Networks Form

$$u_{\theta}(x) = a.f_n(x) \circ \dots \circ f_1(x) + b$$

where

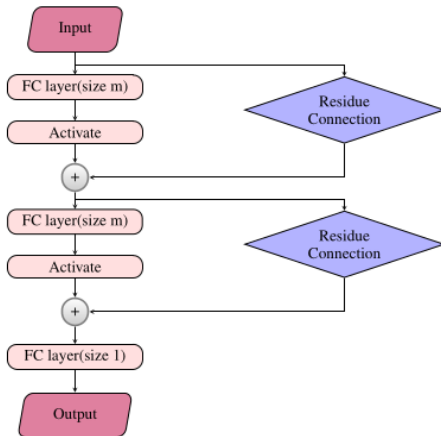
$$f_i(x) = \phi(W_i2.\phi(W_i1.x + b_{i1}) + b_{i2}) + x \text{ and } a \in \mathbb{R}^m, b \in \mathbb{R}$$

and $\phi(x) = \max\{x^3, 0\}$



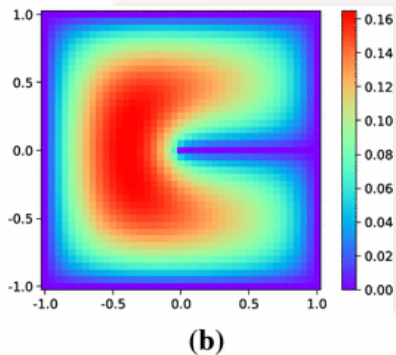
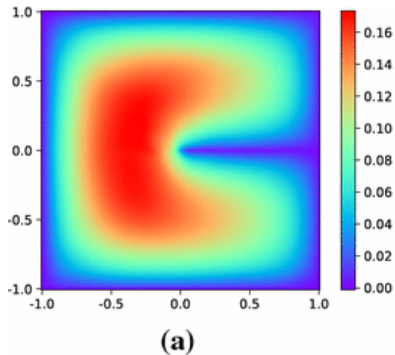
Deep Ritz Method

Poisson Equation - Neural Networks Form



Deep Ritz Method

Figure: (a) DRM - 811 parameter, (b) FDM - 1681 parameter





Conclusion

- Bypassing the calculation of the basis





Conclusion

- Bypassing the calculation of the basis
- Better accuracy in less cost





Conclusion

- Bypassing the calculation of the basis
- Better accuracy in less cost
- Domain and Problem Agnostic





Q&A

"Ask, and it shall be given you!"

Matthew 7:7



References

- The Deep Ritz Method
- Universal Approximation Theorem
- Quotes about the time

