On Solving Partial Differential Equations with Neural Networks (The Deep Ritz Method)

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Outline

Introduction

"The key is in not spending time, but in investing it"

- *Stephen R. Covey.*

Finite Element Method

Poisson Equation on Unit Square - Strong Form

$$\begin{cases} -\Delta u = f(x) & , on\Omega = [0 \times 1] \times [0 \times 1] - [0, 1) \\ u = 0 & , on\partial\Omega \end{cases}$$

Finite Element Method

Poisson Equation on Unit Square - Weak Form

$\sum_{i=1}^{M} c_i < \phi_i, \phi_j >=< f, \phi_j > \qquad i = 1, ..., M,$

where

$u_h = \sum_{i=1}^{M} c_i \phi_i \;\; and \in V_h = span\{\phi_i\}_{i=1}^{M}$

Finite Element Method

Quite Complex, eh?

Lets cheat!

Deep Ritz Method

Universal Approximation Theorem

Let $\phi : \mathbb{R} \rightarrow \mathbb{R}$ be a nonconstant, bounded, and continuous function. Let I_m denote the m-dimensional unit hypercube $[0 \times 1]^m$. The space of real-valued continuous functions on I_m is denoted by $C(I_m)$. Then, given any $\epsilon > 0$ and any function $f \in C(I_m)$, there exist an integer N, real constants $v_i, b_i \in \mathbb{R}$ and real vectors $w_i \in \mathbb{R}^m$ for i=1,...,N, such that we may have define : $F(x) = \sum_{i=1}^{N} v_i \phi(w_i^T x + b_i)$ as an approximate realization of the function f; that is, $|F(x) - f(x)| < \epsilon$ for all $x \in I_m$.

Deep Ritz Method

Poisson Equation - Neural Networks Form

$u_\theta(x) = a.f_n(x) \circ \ldots \circ f_1(x) + b$

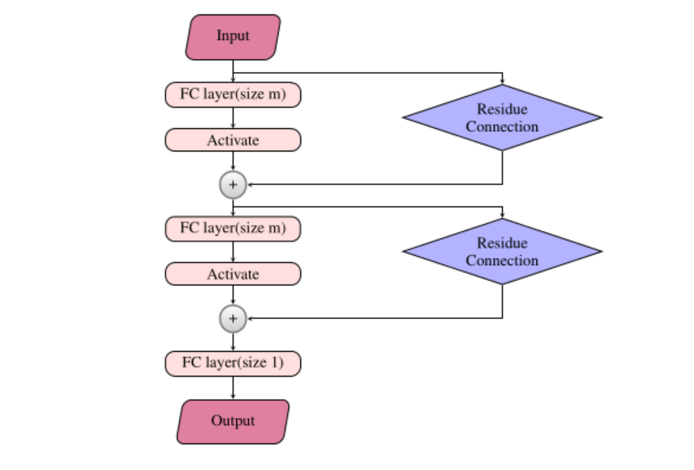
where

$f_i(x) = \phi(W_i 2.\phi(W_i 1.x + b_{i1}) + b_{i2}) + x$ and $a \in \mathbb{R}^m$, $b \in \mathbb{R}$

and $\phi(x) = \max\{x^3, 0\}$

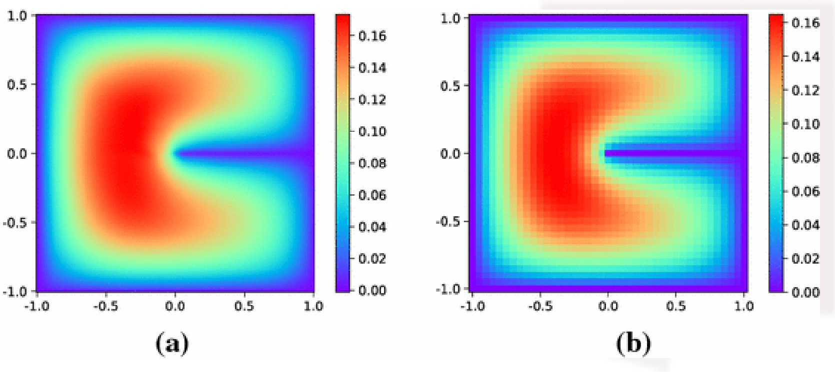
Deep Ritz Method

Poisson Equation - Neural Networks Form



Deep Ritz Method

Figure: (a) DRM - 811 parameter, (b) FDM - 1681 parameter



Conclusion

- Bypass the calculation of the basis

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- Better accuracy in less cost

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- Bypass the calculation of the basis
- Better accuracy in less cost
- Domain and Problem Agnostic

Q&A

"Ask, and it shall be given you!"

Matthew 7:7

- The Deep Ritz Method
- Universal Approximation Theorem
- Quotes about the time