

From the given scenario, we extract the eqn.

$$\text{Max, } Z = 5x_1 - 4x_2 + 3x_3$$

subject to,

$$2x_1 + x_2 - 6x_3 = 20$$

$$6x_1 + 5x_2 + 10x_3 \leq 76$$

$$8x_1 - 3x_2 + 6x_3 \leq 50$$

$$x_1, x_2, x_3 \geq 0$$

Now, we convert the LPP into standard formula.

$$\text{Max, } Z = 5x_1 - 4x_2 + 3x_3 + 0x_5 + 0x_6$$

subject to,

$$2x_1 + x_2 - 6x_3 + x_4 = 20$$

$$6x_1 + 5x_2 + 10x_3 + x_5 = 76$$

$$8x_1 - 3x_2 + 6x_3 + x_6 = 50$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

So,



## Phase-I

Auxiliary LPP

$$\text{Max, } z^* = 0x_1 + 0x_2 + 0x_3 + 0x_5 + 0x_6 - x_4$$

subject to,

$$2x_1 + x_2 - 6x_3 + x_4 = 20$$

$$6x_1 + 5x_2 + 10x_3 + x_5 = 76$$

$$8x_1 - 3x_2 + 6x_3 + x_6 = 50$$

So, the matrix formula  $AX=B$ , we get

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ 2 & 1 & -6 & 1 & 0 & 0 \\ 6 & 5 & 10 & 0 & 1 & 0 \\ 8 & -3 & 6 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 20 \\ 76 \\ 50 \end{pmatrix}$$

Identity matrix



| CB <sub>i</sub> | C <sub>j</sub>                  | 0              | 0              | 0              | -1             | 0              | 0              | Solution<br>X <sub>B</sub> | Ratio |
|-----------------|---------------------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------------------|-------|
|                 | Basic variable                  | x <sub>1</sub> | x <sub>2</sub> | x <sub>3</sub> | x <sub>4</sub> | x <sub>5</sub> | x <sub>6</sub> |                            |       |
| -1              | x <sub>4</sub>                  | 2              | 1              | -6             | 1              | 0              | 0              | 20                         | 10    |
| 0               | x <sub>5</sub>                  | 6              | 5              | 10             | 0              | 1              | 0              | 76                         | 12.6  |
| 0               | x <sub>6</sub>                  | 8              | -3             | 6              | 0              | 0              | 1              | 50                         | 6.25  |
|                 | z <sub>j</sub>                  | -2             | -1             | 6              | -1             | 0              | 0              | -20                        |       |
|                 | C <sub>j</sub> - z <sub>j</sub> | 2↑             | 1              | -6             | 1              | 0              | 0              |                            |       |

| CB <sub>i</sub> | C <sub>j</sub>                  | 0              | 0              | 0              | -1             | 0              | 0              | Solution<br>X <sub>B</sub> | Ratio                  |
|-----------------|---------------------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------------------|------------------------|
|                 | Basic variable                  | x <sub>1</sub> | x <sub>2</sub> | x <sub>3</sub> | x <sub>4</sub> | x <sub>5</sub> | x <sub>6</sub> |                            |                        |
| -1              | x <sub>4</sub>                  | 0              | 7/4            | -15/4          | 1              | 0              | -1/4           | 15/2                       | 4.29                   |
| 0               | x <sub>5</sub>                  | 0              | 29/4           | 11/2           | 0              | 1              | -3/4           | 77/2                       | 5.31                   |
| 0               | x <sub>1</sub>                  | 1              | -3/8           | 3/4            | 0              | 0              | 1/8            | 25/4                       | The value is negative. |
|                 | z <sub>j</sub>                  | 0              | -7/4           | 15/4           | -1             | 0              | 1/4            | -15/2                      |                        |
|                 | C <sub>j</sub> - z <sub>j</sub> | 0              | 7/4↑           | -15/4          | 0              | 0              | -1/4           |                            |                        |

Formula:

$$\text{new value} = \text{old value} - \frac{\text{corr. row} \times \text{corr. column}}{\text{pivot}}$$



| CB <sub>i</sub> | C <sub>j</sub>                  | 0              | 0              | 0               | -1              | 0              | 0              | 0 | Solution<br>X <sub>B</sub> | Ratio |
|-----------------|---------------------------------|----------------|----------------|-----------------|-----------------|----------------|----------------|---|----------------------------|-------|
|                 | Basic Variable                  | x <sub>1</sub> | x <sub>2</sub> | x <sub>3</sub>  | x <sub>4</sub>  | x <sub>5</sub> | x <sub>6</sub> |   |                            |       |
| 0               | x <sub>2</sub>                  | 0              | 1              | $-\frac{15}{7}$ | $\frac{4}{7}$   | 0              | $-\frac{1}{7}$ |   | $\frac{30}{7}$             |       |
| 0               | x <sub>5</sub>                  | 0              | 0              | $\frac{256}{7}$ | $-\frac{29}{7}$ | 1              | $\frac{2}{7}$  |   | $\frac{52}{7}$             |       |
| 0               | x <sub>1</sub>                  | 1              | 0              | $-\frac{6}{7}$  | $\frac{3}{14}$  | 0              | $\frac{1}{14}$ |   | $\frac{55}{7}$             |       |
|                 | Z <sub>j</sub>                  | 0              | 0              | 0               | 0               | 0              | 0              |   | Z* = 0                     |       |
|                 | C <sub>j</sub> - Z <sub>j</sub> | 0              | 0              | 0               | -1              | 0              | 0              |   |                            |       |

As,  $Z^* = 0$  and no artificial variable appear in basis and  $C_j - Z_j \leq 0$ . So, it has a feasible solution and we go to phase II.

### Phase-II

Consider the final simplex table of phase-I. Consider the actual cost associated with the original variables. Delete the artificial variable  $x_4$  column from the table.



| $C_B$ | $C_j$          | 5     | -4    | 3                | 0     | 0                | 0               | Solution $x_B$        | Ratio |
|-------|----------------|-------|-------|------------------|-------|------------------|-----------------|-----------------------|-------|
|       | Basic Variable | $x_1$ | $x_2$ | $x_3$            | $x_5$ | $x_6$            |                 |                       |       |
| -4    | $x_2$          | 0     | 1     | $-\frac{30}{7}$  | 0     | $-\frac{1}{7}$   |                 | $\frac{30}{7}$        |       |
| 0     | $x_5$          | 0     | 0     | $\frac{256}{7}$  | 1     | $\frac{2}{7}$    |                 | $\frac{54}{7}$        |       |
| 5     | $x_1$          | 1     | 0     | $-\frac{6}{7}$   | 0     | $\frac{1}{14}$   |                 | $\frac{55}{7}$        |       |
|       | $Z_j$          | 5     | -4    | $\frac{180}{7}$  | 0     | 0                | $\frac{13}{14}$ |                       |       |
|       | $C_j - Z_j$    | 0     | 0     | $-\frac{159}{7}$ | 0     | $-\frac{13}{14}$ |                 | $Z^* = \frac{155}{7}$ |       |

As, all  $C_j - Z_j \leq 0$ .

$$\text{So, } x_1 = \frac{55}{7}, x_2 = \frac{30}{7}, x_3 = 0$$

$$\therefore \max, Z = \left( 5 \times \frac{55}{7} - 4 \times \frac{30}{7} + 3 \times 0 \right)$$

$$= \frac{155}{7}$$

(Ans)