1.3.9

Max: $x_1 + x_2 + x_3$

Subject to,

$$x_1 + x_2 \geq 3$$

$$x_1 + 2x_2 + x_3 \geq 4$$

$$2x_1 + x_2 + x_3 \leq 2$$

$$x_1, x_2, x_3 \geq 0$$

Solution:

To solve this problem, we use some step:

Step-1: If any constrains have negative constant on the right side, multiply by -1

Step-2: Introduce slack variable in each $\leq$ constraint.

Step-3: Introduce surplus variable and an artificial variable in each $\geq$ constraint.

Step-4: Introduce artificial variable in each $=$ constraint.

Step-5: For each artificial variable, a_i, add $-Ma_i$ to the objective function

So,

form the modified problem:

Let, Max = P.

So, $P = x_1 + x_2 + x_3 - Ma_1 - Ma_2$

$x_1 + x_2 - S_1 + a_1 = 3$

$x_1 + 2x_2 + x_3 - S_2 + a_2 = 4$

$2x_1 + x_2 + x_3 + S_3 = 2$

$x_1, x_2, x_3, S_1, S_2, S_3, a_1, a_2 \geq 0$

from objective function,

$-x_1 - x_2 - x_3 + Ma_1 + Ma_2 + P = 0$

Now,

$$\begin{array}{c} \\ R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}
\begin{array}{cccccccccc} x_1 & x_2 & x_3 & S_1 & S_2 & S_3 & a_1 & a_2 & P & \\ \end{array}$$

$$\begin{array}{c|cccccccccc}
 & x_1 & x_2 & x_3 & S_1 & S_2 & S_3 & a_1 & a_2 & P & \\
\hline
R_1 & 1 & 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 3 \\
R_2 & 1 & 2 & 1 & 0 & -1 & 0 & 0 & 1 & 0 & 4 \\
R_3 & 2 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 2 \\
R_4 & -1 & -1 & -1 & 0 & 0 & 0 & M & M & 1 & 0
\end{array}$$

$\uparrow \quad \uparrow$

$-mR_1 + R_4 \to R_4$

$$\begin{array}{c}
\left[\begin{array}{ccccccccc|c}
1 & 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 3 \\
1 & 2 & 1 & 0 & -1 & 0 & 0 & 1 & 0 & 4 \\
2 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 2 \\
-m-1 & -m-1 & -1 & -m & 0 & 0 & 0 & m & 1 & -3m
\end{array}\right]
\begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}
\end{array}$$

$-mR_2 + R_4 \to R_4$

$$\begin{array}{cccccccccc}
x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & a_1 & a_2 & P & \\
\end{array}$$

$$\left[\begin{array}{ccccccccc|c}
1 & 1 & 0 & -1 & 0 & 0 & \boxed{1} & 0 & 0 & 3 \\
1 & 2 & 1 & 0 & -1 & 0 & 0 & \boxed{1} & 0 & 4 \\
2 & 1 & 1 & 0 & 0 & \boxed{1} & 0 & 0 & 0 & 2 \\
-2m-1 & -3m-1 & -m-1 & -m & m & 0 & 0 & 0 & \boxed{1} & -7m
\end{array}\right]$$

Basic variable : s_3, a_1, a_2, P

Nonbasic variable : $x_1, x_2, x_3, s_1, s_2,$

we consider:

$x_1 = 0,\ x_2 = 0,\ x_3 = 0,\ s_1 = 0,\ s_2 = 0$

$s_2 = 3,\ a_1 = 3,\ a_2 = 4$: these basic variable

must be non-negative for a solution to exist.

So, yes, solution exist.

$$
\begin{array}{c|c}
 & \begin{array}{ccccccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & a_1 & a_2 & P & \\ \end{array} \\
\begin{array}{c} s_3 \\ a_1 \\ a_2 \\ P \end{array} &
\left[\begin{array}{ccccccccc|c}
1 & 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 3 \\
1 & 2 & 1 & 0 & -1 & 0 & 0 & 1 & 0 & 4 \\
2 & 1 \text{ Pivot} & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 2 \\
-2m-1 & -3m-1 & -m-1 & -m & m & 0 & 0 & 0 & 1 & -7m
\end{array}\right]
\end{array}
$$

$-m$ is a very large negative number. We pick most negative number

$3/1 = 3$, $4/2 = 2$, $2/1 = 2$. We pick most smallest element.

$-1R_3 + R_1 \rightarrow R_1$

$-2R_3 + R_2 \rightarrow R_2$

$(3m+1)R_3 + R_4 \rightarrow R_4$

$$
\begin{array}{c|c}
 & \begin{array}{ccccccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & a_1 & a_2 & P & \\ \end{array} \\
\begin{array}{c} s_3 \\ a_1 \\ x_2 \\ P \end{array} &
\left[\begin{array}{ccccccccc|c}
-1 & 0 & -1 & -1 \text{ Pivot} & 0 & -1 & 1 & 0 & 0 & 1 \\
-3 & 0 & -1 & 0 & -1 & -2 & 0 & 1 & 0 & 0 \\
2 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 2 \\
4m+1 & 0 & 2m & -m & m & 3m+1 & 0 & 0 & 1 & -\cancel{7m}\,2
\end{array}\right]
\end{array}
$$

we get, $s_3 = 1$, $a_1 = 0$, $x_2 = 2$, $P = 2$, $x_1 = 0$, $x_3 = 0$, $a_2 = 0$, $s_2 = 0$, $s_1 = 0$.

max, $P = 2$.

when, $x_1 = 0$, $x_2 = 2$, $x_3 = 0$.