Bahria University
Discovering Knowledge

ASSIGNMENT 1: CLO 2

TOPICS:

- Propositional Logic
- Laws of Logic
- Rules of inference

INSTRUCTIONS:

- Max Number of Students in a Group: 2
- Deadline: 6th April, 2022
- Marks will be deducted for late submission.

LAWS OF LOGIC

1. Law of double negation:

$$\neg\neg p \iff p.$$

2. Absorption Laws:

$$p \wedge (p \vee q) \iff p,$$
$$p \vee (p \wedge q) \iff p.$$

3. Idempotent Laws:

$$p \wedge p \iff p,$$
$$p \vee p \iff p.$$

4. Inverse Laws:

$$p \wedge \neg p \iff F,$$
$$p \vee \neg p \iff T.$$

5. Identity Laws:

$$p \wedge T \iff p,$$
$$p \vee F \iff p.$$

6. Domination Laws:

$$p \wedge F \iff F,$$
$$p \vee T \iff T.$$

7. Commutative Laws:

$$(p \wedge q) \Longleftrightarrow (q \wedge p),$$
$$(p \vee q) \Longleftrightarrow (q \vee p).$$

8. Associative Laws:

$$(p \wedge (q \wedge r)) \Longleftrightarrow ((p \wedge q) \wedge r),$$
$$(p \vee (q \vee r)) \Longleftrightarrow ((p \vee q) \wedge r).$$

9. Distributive Laws:

$$(p \wedge (q \vee r)) \Longleftrightarrow ((p \wedge q) \vee (p \wedge r)),$$
$$(p \vee (q \wedge r)) \Longleftrightarrow ((p \vee q) \wedge (p \vee r)).$$

10. Contrapositive Law:

$$(p \rightarrow q) \quad \Longleftrightarrow \quad (\neg q \rightarrow \neg p)$$

11. De Morgan's Laws:

$$\neg(p \vee q) \Longleftrightarrow (\neg p \wedge \neg q),$$
$$\neg(p \wedge q) \Longleftrightarrow (\neg p \vee \neg q).$$

12. No specific name is given, but this law is one of the most frequently used laws in logical proof.

$$(p \rightarrow q) \quad \Longleftrightarrow \quad (\neg p \vee q).$$

Figure 1: Laws of Logic

RULES OF INFERENCE

Rule of Inference	Tautology	Name
p $p \to q$ $\therefore q$	$(p \wedge (p \to q)) \to q$	Modus Ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \wedge (p \to q)) \to \neg p$	Modus Tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \wedge (q \to r)) \to (p \to r)$	Hypothetical syllogism
$\neg p$ $p \vee q$ $\therefore q$	$(\neg p \wedge (p \vee q)) \to q$	Disjunctive Syllogism
p $\therefore (p \vee q)$	$p \to (p \vee q)$	Addition
$(p \wedge q) \to r$ $\therefore p \to (q \to r)$	$((p \wedge q) \to r) \to (p \to (q \to r))$	Exportation
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \to q \vee r$	Resolution

Figure 2: Rules of Inference

IMPORTANT!

Primitive Statements for the given below questions are same as mentioned in Problem 4.

PROBLEM 1:

If $p \rightarrow q$ is false, what is the truth value of

$$((\neg p) \wedge q) \longleftrightarrow (p \vee q)?$$

PROBLEM 2:

Construct the truth tables for the following:

1. $(a \longrightarrow T) \wedge (F \longrightarrow b)$
2. $(F \vee a) \longrightarrow (b \wedge F)$
3. $(a \vee b) \wedge (a \vee \neg b)$

T refers to tautology

F refers to contradiction

PROBLEM 3:

Which of the following is a tautology?

1. $(a \longleftrightarrow b) \rightarrow (a \wedge b)$,
2. $(a \longleftrightarrow b) \longleftrightarrow (a \wedge b) \vee (\neg a \wedge \neg b)$

PROBLEM 4:

Let p, q, r denote the following statements about a triangle ABC.

$p:$ Triangle ABC is isosceles;
$q:$ Triangle ABC is equilateral;
$r:$ Triangle ABC is equiangular.

Translate each of the following into an English sentence.

1. $q \longrightarrow p$
2. $\neg p \longrightarrow \neg q$
3. $q \longleftrightarrow r$
4. $p \wedge \neg q$
5. $r \longrightarrow p$

PROBLEM 5:

Let p, q, r denote primitive statements. Use truth tables to prove the following logical equivalences.

1. $p \rightarrow (q \wedge r) \iff (p \rightarrow q) \wedge (p \rightarrow r)$
2. $[(p \vee q) \rightarrow r] \iff [(p \rightarrow r) \wedge (q \rightarrow r)]$

PROBLEM 6:

Let p, q, r denote primitive statements. Use the laws of logic to show that

$$[p \longrightarrow (q \vee r)] \iff [(p \wedge \neg q) \longrightarrow r].$$

PROBLEM 7:

Let $p, q,$ and r be primitive statements. Write the dual for the following statements.

1. $q \longrightarrow p$
2. $p \longrightarrow (q \wedge r)$
3. $p \longleftrightarrow q$

PROBLEM 8:

Show that

$$((a \wedge b) \longrightarrow c) \Longleftrightarrow ((a \longrightarrow c) \vee (b \longrightarrow c)).$$

PROBLEM 9:

Simplify

$$(p \wedge (\neg r \vee q \vee \neg q)) \vee ((r \vee t \vee \neg r) \wedge \neg q).$$

PROBLEM 10:

Simplify

$$(p \vee (p \wedge q) \vee (p \wedge q \wedge \neg r)) \wedge ((p \wedge r \wedge t) \vee t).$$

PROBLEM 11:

The following is a logical proof for

$$(p \wedge (p \rightarrow q) \wedge (s \vee r) \wedge (r \rightarrow \neg q)) \rightarrow (s \vee t).$$

Refer to the laws of logic and inference rule, and give reasons to justify each step of the proof.

steps		reasons
1.	p	
2.	$p \rightarrow q$	
3.	q	
4.	$r \rightarrow \neg q$	
5.	$q \rightarrow \neg r$	
6.	$\neg r$	
7.	$s \vee r$	
8.	s	
9.	$s \vee t$	

PROBLEM 12:

State which rule of inference is used in the argument:

If it rains today, then we will not have a barbecue today. If we do not have a barbecue today, then we will not have a barbecue tomorrow. Therefore, if it rains today, then we will not have a barbecue tomorrow.

PROBLEM 13:

It is known that:

1. It is not sunny this afternoon, and it is colder than yesterday.
2. We will go swimming only if it is sunny.
3. If we do not go swimming, we will play basketball.
4. If we play basketball, we will go home early.

• Can you conclude “we will go home early”?

PROBLEM 14:

It is known that:

1. If you send me an email, then I will finish my program.
2. If you do not send me an email, then I will go to sleep early.
3. If I go to sleep early, I will wake up refreshed.

• Can you conclude “If I do not finish my program, then I will wake up refreshed”?