



Mathematical Formulation

Key Equations

1. Pairwise Likelihood Computation:

$$P(X|W) = \prod_{i,j} P(x_i, x_j|W)$$

2. Posterior Update:

$$P(W|X) = \frac{P(X|W)P(W)}{\sum_W P(X|W)P(W)}$$

3. Weight Update Rule:

$$W_{new} = W_{current} \cdot (1 + \alpha(1 - \frac{P_1}{P_2}))$$

where:

- P_1 : posterior probability of true class
- P_2 : maximum posterior probability of other classes
- α : learning rate

4. Multivariate Normal PDF for Feature Pairs:

$$P(x_i, x_j|W) = \frac{1}{2\pi|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x - \mu)^T \Sigma^{-1}(x - \mu)\right)$$

5. Final Prediction:

$$y_{pred} = \operatorname{argmax}_c \sum_{i,j} \log P(x_i, x_j|W_c) + \log P(W_c)$$

Implementation Details

1. Feature Processing:

$$\text{scaled}_x = \frac{x - \mu}{\sigma + \epsilon}$$

2. Numerical Stability:

$$\log_likelihood = \log(P(X|W) + \epsilon)$$

3. Weight Bounds:

$$W_{final} = \text{clamp}(W_{new}, \epsilon, 10.0)$$