$f(t) \in C[0,1]$
n

$$B_n(f(t);x) = \sum_{k=0}^{n} nkx^k(1-x)^{n-k} f\left(\frac{k}{n}\right)$$

$b_{n,k}(x)$

0. $\sum_{k=0}^{n} b_{n,k}(x) = 1$

$\sum_{k=0}^{n} kb_{n,k}(x) = nx$

$\sum_{k=0}^{n} k^2 b_{n,k}(x) = n(n-1)x^2 + nx$

$\sum_{k=0}^{n} k^3 b_{n,k}(x) = n(n-1)(n-2)x^3 + 3n(n-1)x^2 + nx$

$\sum_{k=0}^{n} b_{n,k}(x) = \sum_{k=0}^{n} x^k(1-x)^{n-k} = (x+1-x)^n = 1$

$\sum_{k=0}^{n} kb_{n,k}(x) = \sum_{k=0}^{n} k\frac{n!}{k!(n-k)!}x^k(1-x)^{n-k} = 0 + \sum_{k=1}^{n} k\frac{n!}{k!(n-k)!}x^k(1-x)^{n-k} = \sum_{k=1}^{n} \frac{n!}{(k-1)!(n-k)!}x^k(1-x)^{n-k} = \sum_{k=0}^{n-1} \frac{n(n-1)!}{k!(n-1-k)!}x^{k+1}(1-x)^{n-1-k} = nx\sum_{k=0}^{n-1} b_{n-1,k}(x) = nx$

$\sum_{k=0}^{n} k^2 b_{n,k}(x) = \sum_{k=0}^{n} k^2\frac{n!}{k!(n-k)!}x^k(1-x)^{n-k} = 0 + \sum_{k=1}^{n} k^2\frac{n!}{k!(n-k)!}x^k(1-x)^{n-k} = \sum_{k=1}^{n} k\frac{n!}{(k-1)!(n-k)!}x^k(1-x)^{n-k} = \sum_{k=0}^{n-1}(k+1)\frac{n(n-1)!}{k!(n-1-k)!}x^{k+1}(1-x)^{n-1-k} = nx\sum_{k=0}^{n-1}(k+1)b_{n-1,k}(x) = nx\left\{\sum_{k=0}^{n-1} kb_{n-1,k}(x) + \sum_{k=0}^{n-1} b_{n-1,k}(x)\right\} = nx\Big\{(n-1)x+1\Big\} = n(n-1)x^2 + nx$

$\sum_{k=0}^{n} k^3 b_{n,k}(x) = \sum_{k=0}^{n} k^3\frac{n!}{k!(n-k)!}x^k(1-x)^{n-k} = 0 + \sum_{k=1}^{n} k^3\frac{n!}{k!(n-k)!}x^k(1-$

$1]$
$=$
$n(n-1)(n-2)x^3+(1+2)n(n-1)x^2+nx$
$=$
$n(n-1)(n-2)x^3+3n(n-1)x^2+nx$
$f(t)$
$[a,b]$
$[0,\infty)$
$B_n(f;x)$
$B_n(1;x)\to 1$
$n\to\infty$
$B_n(t;x)\to x$
$n\to\infty$
$B_n(t^2;x)\to x^2$
$n\to\infty$
$B_n(f;x)\to f(x)$
$n\to\infty$
$B_n(1;x)=\sum_{k=0}^{n} b_{n,k}(x)\cdot 1 = 1 \to 1$
$n\to\infty$
$B_n(t;x)=\sum_{k=0}^{n} b_{n,k}(x)\cdot (kn) = 1n\cdot nx = x \to x$
$n\to\infty$
$B_n(t^2;x)=\sum_{k=0}^{n} b_{n,k}(x)\cdot (k^2n^2) = 1n^2\cdot [n(n-1)x^2+nx] \to x^2$
$n\to\infty$
$B_n(f;x)\to f(x)$
$n\to\infty$