S.L.C Model Question 2
Optional Maths

F.M: 100

P.M: 32

Attempts the questions.

Group-A [8x(2+2)=32]

1.a) If f and g be two functions defined by

f={(1,2),(3,4),(5,6)} and g={(2,7),(4,11),(6,15)}. Find g and f.

b) State remainder theorem and find the remainder when $3x^3$-$5x^2$+2x-3 is divided by x-2 with the help of remainder theorem.

2.a) Shubham joins a job with starting salary Rs 8000. If he gets an annual increament of Rs 750 every year, find his salary in 7^{th} year of his service.

b) Under what condition product of two matrices P and Q is comformable? Is P Q exist ? If

P = $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ and Q = $\begin{pmatrix} 4 & -5 & 6 \\ 3 & 1 & 0 \end{pmatrix}$

3.a) Find the inverse of given matrix: $\begin{pmatrix} 6 & -15 \\ -3 & 8 \end{pmatrix}$

b. Show that the straight line 2x+y+2=0 is perpendicular to the line passing through the points (3,4) (4,3) and (5,5).

4.a) Show that two straight lines represented by $qx^2+2xy^2-9y^2=0$ are perpendicular to each other.

b) Find the equation of the circle in which the end points of diameter are (3,4) and (4,-7).

5.a) Prove that

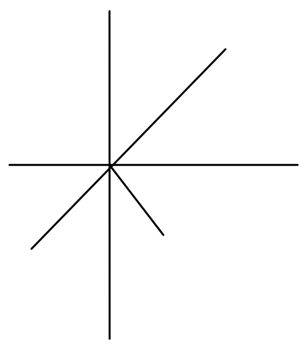
$$\frac{1+\cos \square}{\sin \square} = \cot\theta$$

b) Prove that

1-2 $\sin^2 (45^o-\theta)= \sin 2\theta$

6.a) If $\sin\frac{\theta}{3} = \frac{3}{5}$, prove that $\sin\theta = \frac{117}{125}$

b)Solve : sin x + Tan x= 0
[$0^\circ \leq$ □ $\leq 180^\circ$]
7.a) State the conditions on what two vectors $\underset{□}{\leftarrow}$ □□□ $\underset{□}{\rightarrow}$ are parallel and perpendicular.
b). If A(-1,-1), B(5,-1) and c(2,5) are the vertices of Δ□□□ then find the position vector of centroid G of Δ□□□.
8.a) From the adjoining figure write down the co-ordinates of B' and B''.



b). To what transformation is the matrix $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ associated ? Use the matrix to transform the point (-2, -3).

Group – B [17x4=68]
9) If f(x) = 3x+5, q(x) =x-2 and $f^{-1}g(x)=6$, find the value of x.
10) Solve: $2x^3+3x^2-11x-6=0$
11) If 6th term of an A.P. is 56 and the sum of its first four term is 70, find 11th term.
12) Solve graphically the quadratic equation $x^2=3-2x$
13) Solve by matrix method
$X+y=3,\ 2^{2x+y}=16.$
14) Find the eq^n of line parallel to x+y=0 and passing through centroid of Δ□□□ with vertices A(0,0), B(2,4) and C(4,0).
15). Prove that the homogeneous equation of second degree $ax^2+2hxy+by^2=0$ always represents a pair of straight lines passing through the origin.
16). A circle passes through the origin making the intercepts 5 units and 4 units from x and y-axis respectively. Find the equation of the circle.
17) Prove that: $\cos 10^\circ - \sqrt{3}\sin 10^\circ = 2\cos 70^\circ$.
18) If A+B+C= Π , Prove that

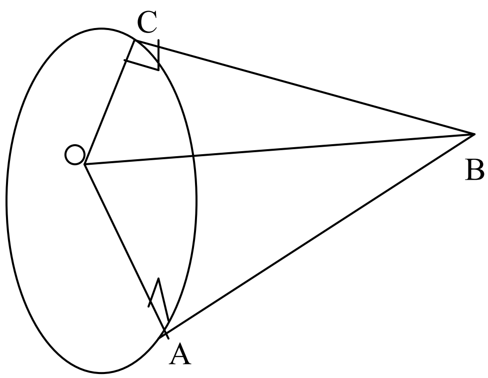
sinA cosB cosC+sinB cosC cosA+sinC cosA cosB=sinA.sinB.sinC

19) Solve:

Secx.Tanx=$\sqrt{2}$ $[0^\circ \leq \square \leq 360^\circ]$

20) An aeroplane flying horizontally 750 m. Above the ground is observed at an elevation of 60 $^\circ$.If after 5 second the elevation is observed to be 30 $^\circ$,find the speed of aeroplane in km/hr.

21) From the given information in the figure , prove vertically that AB=BC.



22) Image of A(2,3) is A’(6,9) and image of B(2,1) is B’(6,3). Find the scale factor and centre of enlargement. Using same centre and scale factor enlarge the line segments joining the points P(4,5) and Q(8,7).

23) A square ABCD having vertices A(1,0), B(4,0),c(1,3) and D(4,3) is maped to a square A’B’C’D by 2x2 matrix so that the vertices of image are A’(2,0), B’(8,0), C’(2,6) and D’(8,6).Find the transformation matrix.

24) Find mean deviation from median and its coefficient:

Marks	0-10	0-20	0-30	0-40	0-50
No.of Students	2	5	15	20	30

25) The sum of squares of 10 terms in 38500. If their variance is 825. Find the coefficient of variance.

Set-2
Solution,

1)a. f= {(1,2), (3,4),(5,6)}
g={(2,7), (4,11), (6,15)}
gof={(1,7), (3,11), (5,15)} Ans.

b. If a function f(x) is divided by(x-a) then f(a) is remainder.
Let, $f(x) = 3x^3-5x^2+2x-3$
x-a= x-2
$\therefore$ a= 2
R= f(a)
= f(2)
$=3(2)^3-5(2)^2+2(2)-3$
=24-20+4-3
=4+4-3= 5 Ans.

2)a.
a=8000
d=750
t7=?

tn= a+(n-1)d
t7=8000+(7-1)x750
=8000+6x750
=8000+4500
= Rs 12,500 Ans.

b. Product of two matrices P and Q conformable if number of column of P is equal to number of row of Q.

Now,

Order of P = 2x2

Order of Q = 2x3

Since, No. of column of P is equal to no. of row of Q, PQ exist.

3)a.

Let, a= $\begin{pmatrix} 6 & -15 \\ -3 & 8 \end{pmatrix}$

$\begin{vmatrix} 6 & -15 \\ -3 & 8 \end{vmatrix}$

= 48-45
= 3

$A^1 = \frac{Adj.A}{|A|} = \frac{1}{3}\begin{pmatrix} 8 & 15 \\ 3 & 6 \end{pmatrix} = \begin{pmatrix} 8/3 & 5 \\ 1 & 2 \end{pmatrix}$ Ans

b. slope of line 2x+y+2 = 0 is

$m_1 = -2$

slope of line joining points (3,4) and (5,5) is

$m_2 = \frac{5-4}{5-3} = \frac{1}{2}$

They are perpendicular to each other,

$m_1.m_2 = -1$

$-2x\frac{1}{2} = -1$

$-1 = -1$

Hence proved.

4)a. Compairing $9x^2+2xy-9y^2 = 0$ with
$ax^2+2hxy+3y^2 = 0$, we get

a= 9

b= -9

h= 1

$a+b = 9-9 = 0$
Hence, two st. lines represented by $9x^2+2xy-9y^2=0$ are perpendicular to each other.

b.Find the equation of the circle in which the end points of diameter are (3,4) and (2,-7).
Solution,
$(x_1,y_1) = (3,4)$
$(x_2,y_2) = (2,-7)$
We know that, eq^n of circle in diameter form is
$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$
Or, $(x-3)(x-2) + (y-4)(y+7) = 0$
Or, $x^2-5x+6+y^2+3y-28 = 0$
$\therefore\ x^2+y^2-5x+3y-22 = 0$ Ans.

5)a. L.H.S $= \frac{1+\cos\theta}{\sin\theta}$
$=$

b. L.H.S $= 1-2\sin^2(45°-\theta)$
$= \cos2(45°-\theta)$
$= \cos2(90°-2\theta)$
$= \sin2\theta$= R.H.S proved.

6)a. $\sin\theta/3 = \frac{3}{5}$
We know
$\sin3\theta = 3\sin\theta.4\sin^3\theta$
$\therefore\ \sin\theta = 3\sin\theta/3-4\sin^3\theta/3$
$3\left(\frac{3}{5}\right)-4\left(\frac{3}{5}\right)^3$

$= \frac{9}{25} - \frac{4x27}{125}$

$= \frac{9x25-108}{125}$

$=\frac{225-108}{125}$

$= \frac{117}{125}$ proved.

b. Sinx + tanx = 0

or, $\sin x + \frac{\sin x}{\cos x} = 0$

or, $\frac{\sin x.\cos x+\sin x}{\cos x} = 0$

or, sinx(cosx+1)= 0

∴ sinx= 0 --------eqn 1

Cosx+1 =0 --------eqn 2

From 1

Sinx = 0

Sinx= sin0°, sin180°

∴ x= 0°, 180°

From 2

Cos+1=0

Cosx=-1 (not needed)

∴ x=0°, 180° Ans.

7(a) Two vectors $\vec{a}$ & $\vec{b}$ are parallel if $\vec{a}$ =k$\vec{b}$ & perpendicular if $\vec{a}.\vec{b}$=0

(b) let, O be the origin of reference.

i.e. position vector of A(i.e.O.$\vec{A}$)= (-1,-1)

" " " B(i.e.O$\vec{B}$)= (5,-1)

" " " C(i.e.O$\vec{C}$)=(2,5) ∴ Position vector G(i.e.O$\vec{G}$) = 1/3(O$\vec{A}$+O$\vec{B}$+O$\vec{C}$)

=1/3(-1+5+2,-1-1+5)

=1/3(6,3)

=(2,1) answer.

(8) (a)

From B to B', there is rotation about origin in 180° & From B' to B'' there reflection along y-axis.

i.e. B(2,4) $\xrightarrow{R(O,180^\circ}$ B'(-2,-4) $\xrightarrow{y-axis}$ B''(-2,4)

Hence the co-ordinate of B' is (-2,-4)

& the co-ordinate of B is (-2,4)

(b) If reflection in y=-x

Now, $\begin{pmatrix}-2\\-3\end{pmatrix}\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}= \begin{pmatrix}0+3\\2+0\end{pmatrix}=\begin{pmatrix}3\\2\end{pmatrix}$

∴Image of given point = (3,2)

(9) Let y=f(x0

Y=3x+5

Interchanging y & x

$\therefore x=3y+5$

$\therefore y=\frac{x-5}{3}$ $\therefore f^{-1}(x)=\frac{x-5}{3}$

Now, f^{-1} g(x)= 6

Or, f^{-1} (x-2) = 6

Or, $\frac{x-2-5}{3}$ = 6

Or, x-7=18

$\therefore$X=25 answer.

(10) $2x^3+3x^2-11x-6=0$

Let, f(x)= $2x^3+3x^2-11x-6$

The factor of 6 are 1,2,3,6

$f(2)=2(2)^3+3(2)^2-11(2)-6=0$

$f(x)= 2.2^3+3x^2-11x-6=0$

=16+12-22-6=0

i.e.x-2 is a factor of f(x)

Now, $2x^3+3x^2-11x-6=0$

Or, $2x^3-3x^2+7x^2-11x=0$

Or, $2x^2(x-2)+7(x-2)+3(x-2)=0$

or, $(x-2)(2x^2+7x+3)=0$

or, $\{(x-2)2x^2+6x+x+3\}$=0

or, (x-2) $\{2x(x+3)+6x+x+3\}$=0

or, (x-2) $\{2x(x+3)+1(x+3\}=0$

or,(x-2)(x+3)(2x+1)=0

so,x=2,-3,-1/2answer.

11.16=56

a+(6-1)=56

a+5d=56…………………(i)

$s_4=70$

n/2 $[\![2a+(n-1)d]\!]$=70

of,4/2 $[\![2a+(4-1)d]\!]$

or,2 $[\![2a+3d]\!]=70$

or,2a+3d=35…………………(ii)

a=17/2

now,

2a+10d=112

2a+3d =35

\- - -

7d=77

d=11

Put the value of d in (i)

A+5x11=56

a=1

Now

tn=1=(11-1)x11

=1+10x11

=111answer.
12.x^2=3-2x
x^2+2x-3=0
let, y= x^2+2x-3
y=x^2=-2x+3
y=x^2
y=-2x+3……………(ii)
From(1)
Y=x2, a=1,b=0, c=0
Vertex= $\left(-\frac{b}{2a}, \middle| 4ac-\frac{b^2}{4a}\right)$=(0,0)
Figure…..

X=1,-3answer.
13
.x+y=3………(1)
2^{2x+y}=2^4
2x+y=4…………(2)
Let, A=$\begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$, X=$\begin{pmatrix} x \\ y \end{pmatrix}$
$|A|$=$\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}$=1-2=-1
Adj:A=$\begin{vmatrix} 1 & -1 \\ -2 & 1 \end{vmatrix}$
A-1=$\frac{Adj:A}{|A|}$=-1$\begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix}$$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$=$\begin{pmatrix} -3+4 \\ 6-4 \end{pmatrix}$=$\begin{pmatrix} 1 \\ 2 \end{pmatrix}$
x=1answer
y=2answer.
14.Here, centroid of $\Delta ABC = \left(-\frac{0+2+4}{3}, \middle| 4ac-\frac{b^2}{4a} 0+4+0/3\right)$
=(2,4/3)
Now, Equation of line passing through (2,4/3) is
y-4/3=m(x-2)……….1
slope of x+y=0 is m_1=-1
by condition of parallel of two lies,
m=m_1
m=-1
therefore the required equation is
y-4/3=-1(x-2)
3y-4=-3x+6
Therefore 3x+3y=10 answer.

15

We have,

$Ax^2+2hxy+by^2=0$……………..(i)

Or,$by^2+2hxy+ax^2=0$

If $b\neq0$, the equation can be written as

$y^2+\frac{2h}{b}xy+\frac{a}{b}x^2=0$

$(\frac{y}{x})^2+2h(\frac{y}{x})+(\frac{a}{b})=0$

This is quadratic equation in $(\frac{y}{x})$.So, it has two m1 and m2.

Then $\frac{y}{x}=m^1$ and $\frac{y}{x}=m^2$

$Y=m^1x$ &$y=m^2x$

These two equations are the equations of straight lines passing through origin.

If b=0, then eq^n (1) becomes

$ax^2+2hxy=0$

Or $x(ax+2hy)=0$ which represents two st. lines $x+0$ and $ax+2hy=0$.

These two lines pass through origin.

16. Let P(h,k) be the center at circle.

OA=5 Figure……

OB=4

A=(5,0)

B=(0,4)

Since ∢ ABC=90°, AB is diameter.

Equation of circle is

$(x-s^1)(x-x^2)+(y-y^1)(y-y^2)=0$

Or,$(x-5)(x-0)+(y-0)(y-4)=0$

Or, $x(x-5)+y(y-4)=0$

Of,$x^2=5x+y^2-4y=0$

$X^2+y^2-5x-4y=0$ answer.

17. L.H.S.= $\cos10°-\sqrt{3}\sin10°$

$=2(\frac{1}{2}\cos10°-\frac{1}{\sqrt{3}}3\sin10°)$

$=2(\cos60°.\cos10°-\sin60°\sin10°)$

$=2\cos(60°+10°)=2\cos70°$=RHS proved.

18.$A+B+C=\pi$

$A+B=\pi-C$

Or $Tan(A+B)=Tan(\pi-C)$

Or,$\frac{TanA+TanB}{1-TanA.TanB}=-TanC$

Or,TanA+TanB=TanA.TanB.TanC-TanC.

Or,TanA+TanB+TanC=TanA.TanB.TanC

or$\frac{SinA}{CosA}+\frac{SinB}{CosB}+\frac{SinC}{CosC}=\frac{SinA.SinB.SinC}{CosA.CosB.CosC}$

or, $\frac{SinA.CosB.CosC+SinBCosACosC+SinCCosACosB}{CosA.CosB.CosC}=\frac{SinA.SinB.SinC}{CosA.CosB.CosC}$

SinACosBCosC+SinBCosACosC+SinCCosACosB=SinASinBSinC Proved.

19.$Secx.Tanx=\sqrt{2}$

Or,$\frac{1}{Cosx}.\frac{Sinx}{Cosx}=\sqrt{2}$

Or, $Sinx=\sqrt{2}\cos^2x$

or Sinx= $\sqrt{2}-\sqrt{2}\cos^2x$

or, $\sqrt{2}Sin^2x+Sinx-\sqrt{2}=0$

or, $\sqrt{2}Sin^2x+2Sinx-Sinx-\sqrt{2}=0$

Or, $\sqrt{2}Sinx(Sinx+\sqrt{2})-1(Sinx-\sqrt{2})=0$

Or,$(Sinx+\sqrt{2})\ (Sinx-1)=0$

Or,$Sinx+\sqrt{2}=0$……(i)

or, $\sqrt{2}Sinx-1=0$……(ii)

From (i)

Or,$Sinx+\sqrt{2}=0$

Or,$Sinx=-\sqrt{2}$ doesn't defined

From(ii)

$\sqrt{2}Sinx-1=0$

$Sinx=1/\sqrt{2}$

$Sinx=Sin45°,Sin(90°X2-45°)$

$X=45°,135°$ Answer.

(20) Let,AB be the night of an aeroplane.AD be distance covered by the aeroplane in 5 second.

Type equation here.

Figure……………………………

$\sphericalangle AEB=60°$

i$\sphericalangle DEC = 30°$

AB=750m

From right ΔABE,

$Tan60°=\frac{AB}{BE}$

$\sqrt{3}=\frac{750}{BE}$

$BE=\frac{750}{\sqrt{3}}=\frac{750}{\sqrt{3}}x\frac{\sqrt{3}}{\sqrt{3}}=\frac{750\sqrt{3}}{3}=250\sqrt{3}m.$

From right ΔDCE,

$Tan30°=\frac{DC}{CE}$

or$\frac{1}{\sqrt{3}}=\frac{750}{CB+BE}$

or$\frac{1}{\sqrt{3}}=\frac{750}{Bc+250\sqrt{3}}$

or,$BC+250\sqrt{3}=750\sqrt{3}$

or $BC=750\sqrt{3}-250\sqrt{3}$

$BC=500\sqrt{3}$

$Speed=\frac{500\sqrt{3}}{5}=100\sqrt{3}=\frac{100\sqrt{3}X6060}{1000}=360\sqrt{3}+625.5km/hr.$

21.To prove:AB=BC

Figure….

Proof:

(1)$\overrightarrow{OB}=\overrightarrow{Oc}+\overrightarrow{CB}$ (by triangle low of vector addition)
(2) $\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}$(by triangle low of vector addition)
(3) $\overrightarrow{OC}+\overrightarrow{CB}=\overrightarrow{OA}+\overrightarrow{AB}$ (from 1 & 2)
Or, $\overline{(\overrightarrow{OC}+\overrightarrow{CB})}^2=(\overrightarrow{OA}+\overrightarrow{AB})^2$
or, $\overrightarrow{OC}^2+2\overrightarrow{OC}.\overrightarrow{CB}+\overrightarrow{CB}^2=\overrightarrow{OA}^2+2.\overline{OA.\overrightarrow{AB}}+\overrightarrow{AB}^2$
or, $OC^2+0+CB^2=OA^2+0+AB^2$
or,$CB^2=AB^2$
AB=BC answer.
22. Let(a,b) be ceter & k be scale factor
We know,
$(x,y)\xrightarrow{E(a,b)K)}(k(x-a)+k(y-b)+b)$
$A(2,3)\rightarrow A'(6,9)$
$B(2,1)\rightarrow B'(6,30$
i.e.$A(2,3)\xrightarrow{E(a,b)K)}(k(2-a)+k(3-b)+b)$
i.e.k(2-a)+a=6
or2k-ka+a=6….(i)
or,3k-kb+b=9…(ii)
$B(2,1)\rightarrow B'(k(2-a)+a,k(1-b)+b)$
2k-ka+a=6…….(iii)
k-kb+b=3……..(iv)
Solving (iii)&(iv)
2k-ka+a=6
<u>k-kb+b=3</u>
2k=6
K=3

Putting the value of k in (3)
2X3-3a+a=6
-2a=0
A=o
Putting the value of k in (iv)
3-3b+b=3
b=0
Center=(4,5)=(0,0)
Scale factor=k=3.
Now,
$P(4,5)\xrightarrow{E(o,o)3)}P'(12,15)$
$Q(8,7)\xrightarrow{E(0,0)3)}Q'(24,21)$
23. Let, $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be the transformation matrix.
$=\begin{pmatrix} 1 & 4 & 1 & 4 \\ 0 & 0 & 0 & 3 \end{pmatrix}$
$=\begin{pmatrix} 2 & 8 & 2 & 8 \\ 0 & 0 & 6 & 6 \end{pmatrix}$

$=\begin{pmatrix} a+0 & 4a+0 & a+3b & 4a+3b \\ c+0 & 4c+0 & c+3d & 4c+4d \end{pmatrix} = \begin{pmatrix} 2 & 8 & 2 & 8 \\ 0 & 0 & 6 & 6 \end{pmatrix}$

$=\begin{pmatrix} a & 4a & a+3 & 4a+3b \\ c & 4c & c+3d & 4c+4d \end{pmatrix} \begin{pmatrix} 2 & 8 & 2 & 8 \\ 0 & 0 & 6 & 6 \end{pmatrix}$

a=2

C=2

a+3b=2…………………..(i)

4a+3b=8…………………(ii)

C+3d=6…………………..(iii)

4c+cd=6……………………(iv)

Put a=2 in (i)

2+3b=2

b=0

Put c=0 in (iii)

0+3d=6

d=2

therefore the required transformation matrix= $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$answer.

24.

Marks	No.of Students(c.f.)	f.	M.V.	IdI=Ix-medI	fIdI
0-10	2	0-10	2	5	50
0-20	5	10-20	3	15	45
0-30	15	20-30	10	25	100
0-40	20	30-40	5	35	25
0-50	30	40-50	10	45	150
		N=30			EfIdI=370

N/2=30/2=15

Median class=20-30

L=20

h=10

c.f.=5

f.=10

Med.=$L+\frac{N/2-c.f.}{f} \times h$

$=20+\frac{15-5}{10} \times 10$

=20+10=30

M.D.=EfIdI/N=370/30=12.3

25.Here,

Ex^2=38500

N=10

σ^2=825

We Know,

$\sigma^2 = \frac{\Sigma x2}{N} - \left(\frac{\Sigma x}{N}\right)^2$

Or, $825 = \frac{38500}{10} - \left(\frac{\Sigma x}{10}\right)^2$

Or, $\left(\frac{\Sigma x}{10}\right)^2 = 3025$

or $\left(\frac{\Sigma x}{10}\right) = \sqrt{3025}$

$\vec{X} = 55$

$\sigma = \sqrt{variance}$

$= \sqrt{825}$

$= 28.72$

C.V. $= \frac{\sigma}{\vec{X}}$ X100%=28.72x100%/55=52.2%