chapter-4

General Equation of second degree

conic
(i) central
(ii) non-central

1. $y^2 = 4ax$; Parabola } non-central

2. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; Ellipse

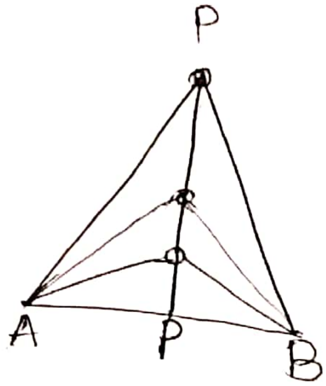
3. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, Hyperbola

central

4. $xy = c$, Rectangular hyperbola

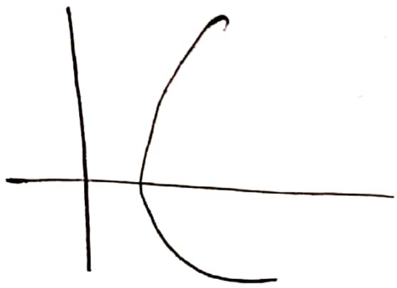
[# $x^2+y^2=a^2$], circle

conic: If a point P moves in a plane such a way that the ratio of its distance PS from a fixed point S in the plane to its perpendicular distance PM from a fixed straight line XM in it, is always a constant, the locus of the point P is called conic.



PA = PB

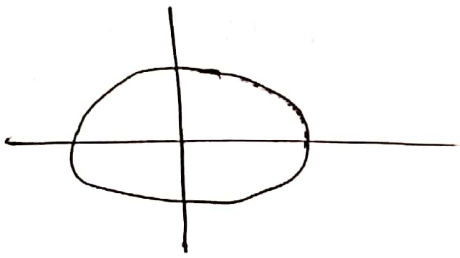
1.



Parabola
পরাবৃত্ত

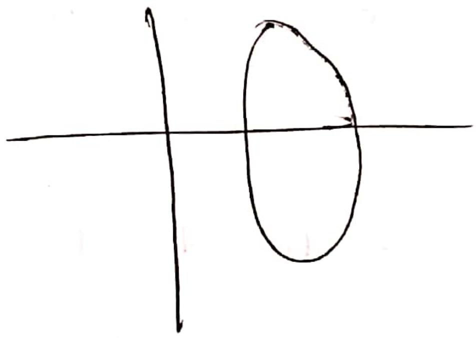
e = 1

2.



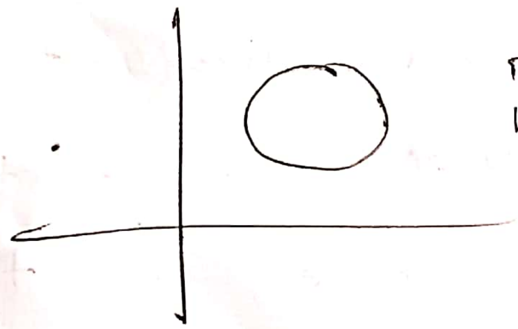
Ellipse (উপবৃত্ত)

3.



hyperbola
(অধিবৃত্ত)

4.



rectangular
hyperbola

(1000 / 500)

\#

Given that,

$$2x^2 - 7xy + 3y^2 + x + 7y - 6 = 0 \quad \text{---(i)}$$

compairing (i) with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

where,

$$a = 2, \quad b = 3, \quad c = -6$$
$$f = \frac{7}{2}, \quad g = \frac{1}{2}, \quad h = -\frac{7}{2}$$

since the equation (i) may represents a pair of straight lines, we have

Δ

$$\Rightarrow abc + 2fgh - af^2 - bg^2 - ch^2$$

$$\Rightarrow 2 \cdot 3 \cdot (-6) + 2 \cdot \frac{7}{2} \cdot \frac{1}{2} \cdot \left(-\frac{7}{2}\right) - 2 \cdot \left(\frac{7}{2}\right)^2 - 3 \cdot \left(\frac{1}{2}\right)^2 - (-6) \cdot \left(-\frac{7}{2}\right)^2$$

$$= -36 - \frac{49}{4} - \frac{98}{4} - \frac{3}{4} + 147$$

$$= \frac{-144 - 49 - 98 - 3}{4}$$

08-11-2022

Chapter-4

General Equation of second Degree

Formulae:

The general equation of second degree

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

will represent

(i) a Parabolla if $\Delta \neq 0$ and $ab=h^2$

(ii) an ellipse if $\Delta \neq 0$ and $ab-h^2>0$

(iii) a hyperbola if $\Delta \neq 0$ and $ab-h^2<0$

Q. Test the nature of the conic given by the following equation

$$x^2+2xy+y^2-6x+10y+25=0$$

Soln:

Given,

$$x^2+2xy+y^2-6x+10y+25=0 \quad \text{---} \quad (i)$$

compairing (i) with,

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

we have,

$$a=1, \quad b=1, \quad c=25$$

$$f=5, \quad g=-3, \quad h=1$$

Now,

$$\Delta = abc+2fgh-af^2-bg^2-ch^2$$

$$=1.1.25+2.5.(-3).1-1.(5)^2-1.(-3)^2-25.(1)^2$$

$$=-64$$

$$\neq 0$$

Again,

$$ab-h^2$$

$$=1.1.-(1)^2$$

$$=1-1$$

$$=0$$

since, $\Delta \neq 0$ and $ab-h^2=0$, so the given equation represents an Parabola.

④ Given,

$4x^2+9y^2-8x+36y-31=0$ —(i)

compairing (i) with

$ax^2+2hxy+by^2+2gx+2fy+c=0$,

we have,

$a=4$, $b=9$, $c=-31$

$f=18$, $g=-4$, $h=0$

Now,

$\Delta = abc+2fgh-af^2-bg^2-ch^2$

$= 4.9.(-31)+2.18.(-4).0-4.(18)^2$

$-9.(-4)^2-(-31)(0)^2$

$= -1116+0-1296-144$

$= -2556$

$\neq 0$

Again,

$ab-h^2$

$= 4.9-0^2$

$= 36$

since $\Delta \neq 0$ and $ab - h^2 > 0$, so the given equation represents an ellipse.

Q. Reduce the following equation to standard form
$$8x^2 + 4xy + 5y^2 - 16x - 14y + 13 = 0$$

Soln.:

Given,
$$8x^2 + 4xy + 5y^2 - 16x - 14y + 13 = 0 \quad \text{--- (i)}$$

Compairing (i) with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we have

$a = 8$, $b = 5$, $c = 13$
$f = -7$, $g = -8$, $h = 2$

Now, $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$
$= 8 \cdot 5 \cdot 13 + 2 \cdot (-7) \cdot (-8) \cdot 2 - 8 \cdot (-7)^2 - 5 \cdot (-8)^2 - 13 \cdot (2)^2$

$= \cancel{232}$ $= -20$
$\neq 0$

Again,

$$ab - h^2 = 8 \cdot 5 - 2^2$$

$$= 36 > 0$$

since, $a \neq 0$ and $ab - h^2 > 0$,

so the given equation represents an ellipse.

13-11-2022

let (α, β) be the centre of the ellipse, where

$$\alpha = \frac{hf - bg}{ab - h^2}, \qquad \beta = \frac{gh - af}{ab - h^2}$$

$$= \frac{2(-7) - 5(-8)}{8 \cdot 5 - 2^2} \qquad = \frac{(-8 \cdot 2) - 8 \cdot (-7)}{36}$$

$$= \frac{-14 + 40}{36} \qquad = \frac{-16 + 56}{36}$$

$$= \frac{26}{36} \qquad = \frac{40}{36}$$

$$= \frac{13}{18} \qquad = \frac{10}{9}$$

$\therefore \left(\frac{13}{18}, \frac{10}{9}\right)$ is the centre of the ellipse

New constant,

$$c' = g\alpha + f\beta + c$$

$$= -8 \cdot \frac{13}{18} + (-7) \cdot \frac{10}{9} + 13$$

$$= \frac{-104}{18} - \frac{70}{9} + 13$$

$$= -\frac{52}{9} - \frac{70}{9} + 13$$

$$= \frac{-52 - 70 + 117}{9}$$

$$= -5/9$$

$\therefore$ The equation of conic referred as origin is

$$8x^2 + 4xy + 5y^2 - \frac{5}{9} = 0 \longrightarrow (2)$$

When the xy term is removed by Rotation of axes, let the reduced equation be

$$a_1x^2+b_1y^2-\frac{5}{9}=0 \quad —— (3)$$

where,

$$a_1+b_1=a+b \quad \text{and} \quad a_1b_1=ab-h^2$$

$$\therefore \; a_1+b_1=8+5=13 \quad —— (4)$$

$$a_1b_1=36 \quad —— (5)$$

solving (4) & (5), we have

$$a_1=9, \quad b_1=4$$

$$(3) \Rightarrow \quad 9x^2+4y^2=\frac{5}{9}$$

$$\Rightarrow \frac{x^2}{\frac{5}{81}}+\frac{y^2}{\frac{5}{36}}=1$$

$$\Rightarrow \frac{x^2}{\left(\frac{\sqrt{5}}{9}\right)^2}+\frac{y^2}{\left(\frac{\sqrt{5}}{6}\right)^2}=1. \quad \text{Ans.}$$

15-11-2022

chapter-4

Q.1 Reduce the following equation is standard form

$5x^2 - 24xy - 5y^2 + 4x + 58y - 59 = 0$

Given,

$5x^2 - 24xy - 5y^2 + 4x + 58y - 59 = 0$ —— (i)

compairing (i) with $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, we have,

$a = 5$, $b = -5$, $c = -59$

$f = 29$, $g = 2$, $h = -12$

Now,

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

$= 5 \cdot (-5) \cdot (-59) + 2 \cdot 29 \cdot 2 \cdot (-12) - 5(29)^2 - (-5)(2)^2 - (-59)(-12)^2$

$= 147 - 1392 - 4205 + 20 + 8496$

$= 4394$

$\neq 0$

Again,

$\Delta \neq 0$ and $ab-h^2 < 0$, so

the given equation represents

a hyperbola

Let (α, β) be the centre of the hyperbolla, where,

$$\alpha = \frac{hf-bg}{ab-h^2}, \quad \beta = \frac{gh-af}{ab-h^2}$$

$$= \frac{-12.29-(-5).2}{-169} \qquad = \frac{2.(-12)-5.29}{-169}$$

$$= \frac{-348+10}{-169} \qquad = \frac{-24-145}{-169}$$

$$= \frac{-338}{-169} \qquad = \frac{-169}{-169}$$

$$= 2 \qquad = 1$$

$\therefore$ $(2,1)$ is the centre of the hyperbola.

New constant

$$c' = g\alpha + f\beta + c$$
$$= 2.2 + 29.1 - 59$$
$$= 32 - 29$$
$$= -26$$

The equation of conic referred as origin is

$$5x^2 - 24xy - 5y^2 - 26 = 0 \quad \text{——} \quad (2)$$

When xy term is removed by rotation of axes,

let the reduce equation be,

$$a_1x^2 + b_1y^2 - 26 = 0 \quad \text{——} \quad (3)$$

Where, $a_1 + b_1 = a + b$ and $a_1b_1 = ab - h^2$

Now, $a_1 + b_1 = 5 - 5 = 0$ — (4)

and $a_1b_1 = -169$

$$\therefore a_1 - b_1 = \sqrt{(a_1+b_1)^2 - 4a_1b_1}$$
$$= \sqrt{0^2 - 4(-169)} = \sqrt{676}$$
$$= 26 \quad \text{——} \quad (5)$$

solving (4) & (5), we have

$a_1 = 13$, $b_1 = -13$

(3) $\Rightarrow$

$13x^2 - 13y^2 = 26$

$\Rightarrow \frac{x^2}{2} - \frac{y^2}{2} = 1$