Simulation of Iodine-Xenon Transients After Reactor Shutdown

Comparison of RK4 and Matrix Exponential Methods

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Introduction

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Why Important?

- Xe-135 is a strong neutron absorber.
- Accurate simulation ensures reactor safety.

Nuclear Physics Background

Isotopes Involved

- **Iodine-135 (I-135)**: Decays to Xe-135.
- **Xenon-135 (Xe-135)**: Produced directly and from I-135.
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Key Challenge

Xe-135 has a high neutron absorption cross-section.

Bateman equations and Parameters

Bateman equations

The equations for the I-135 and Xe-135 populations in a reactor, where λ_T is the decay constant for tellurium-135:

$$\frac{d}{dt}I(t) = \lambda_T T(t) - \lambda_I I(t)$$

$$\frac{d}{dt}X(t) = \lambda_I I(t) - \lambda_X X(t)$$

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Fixed Nuclear Constants

Symbol	Description	Value
λ_I	Decay constant of I-135	2.874×10^{-5} s^{-1}
λ_X	Decay constant of Xe-135	2.027×10^{-5} s^{-1}
Y_I, Y_X	Fission yields	0.061, 0.003
σ_{aX}	Absorption cross-section of Xe-135	2.75×10^{-18} cm^2

Equilibrium Before Shutdown

The equilibrium concentrations I_0 and X_0 represent the steady-state levels of Iodine-135 and Xenon-135 during full-power operation of the reactor.

Initial Concentrations

$$I_0 = \frac{Y_I \cdot \Sigma_f \cdot \phi}{\lambda_I}$$

$$X_0 = \frac{Y_X \cdot \Sigma_f \cdot \phi + \lambda_I I_0}{\lambda_X + \sigma_{aX} \cdot \phi}$$

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Significance

- As Initial Conditions
- Incorrection of values leading to inaccurate poisioning forecast

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- Step-by-step numerical integration
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Matrix Exponential

- The matrix exponential is a matrix function on square matrices analogous to the ordinary exponential function.
- It is used to solve systems of linear differential equations.
- Efficient and compact via $\exp(At)$

Analytical Reference Solution

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Xe-135 Concentration Over Time

$$X(t) = X_0 e^{-\lambda_X t} + \frac{\lambda_I I_0}{\lambda_X - \lambda_I}(e^{-\lambda_I t} - e^{-\lambda_X t})$$

- Used as benchmark for simulation accuracy.

Simulation Setup

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The simulation makes several simplifying assumptions for tractability and clarity:

- All neutrons are treated as if they belong to a single energy group.
- This neglects the energy dependence of cross-sections and reactions.
- The reactor core is assumed homogeneous — spatial variations in flux and nuclide concentrations are ignored.
- Only I-135 and Xe-135 are modeled.
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Parameters

- Time range: up to 70 hours
- Flux levels: 20% to 100% of base flux
- Step size: 3600 seconds (1 hour)

Simulation Workflow

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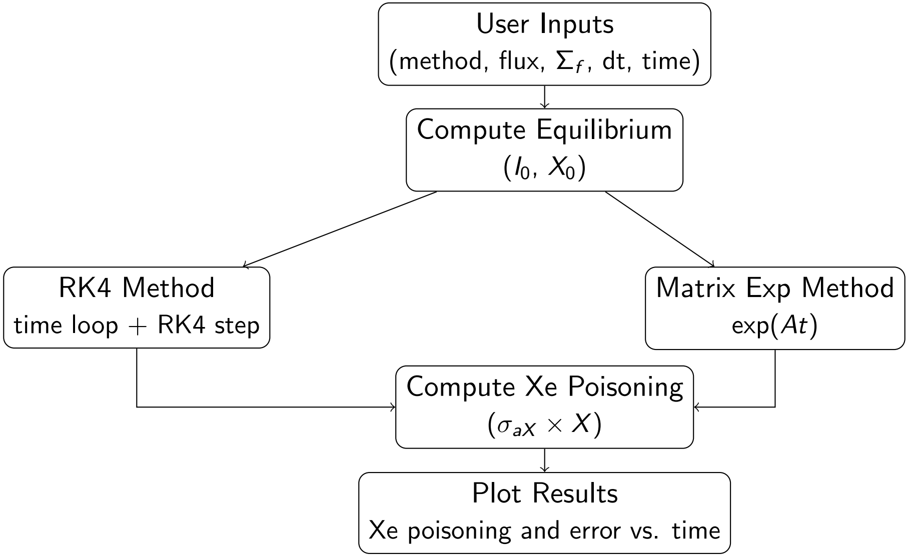

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Average Runtime Over 300 Runs

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RK4	0.001493 s
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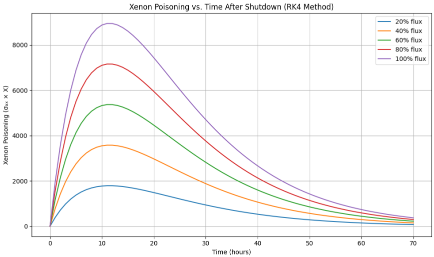

Fig: RK4 Method

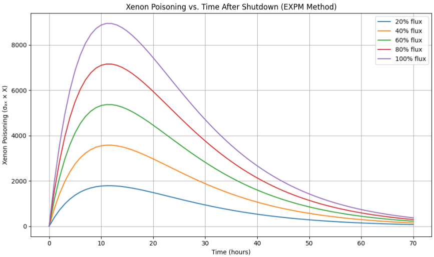

Fig: EXPM Method

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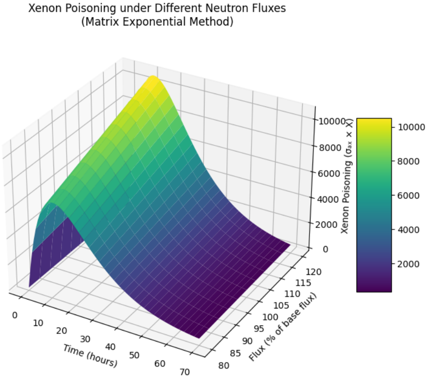

Figure: 3D plot of expm Method

Xenon Poisoning vs. Time

Behavior After Shutdown

- Xe-135 builds up then decays.

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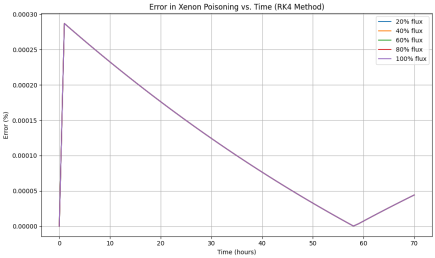

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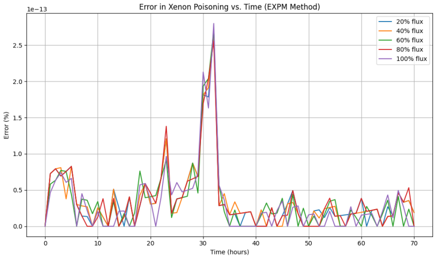

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- In expm, there's a small spike in error around 30–35 hours, which might be due to: floating-point precision issues or slight rounding artifacts in matrix operations.
- The RK4 error behavior is largely independent of the flux.
- The smoothness of the curve shows RK4 is stable and doesn't exhibit erratic behavior.

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