Introduction to Calculus

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Functions

x	0	1	2	3
y	3	4	−1	6

- $f(x) = x^2$
- Vertical line test
- Domain
- Range

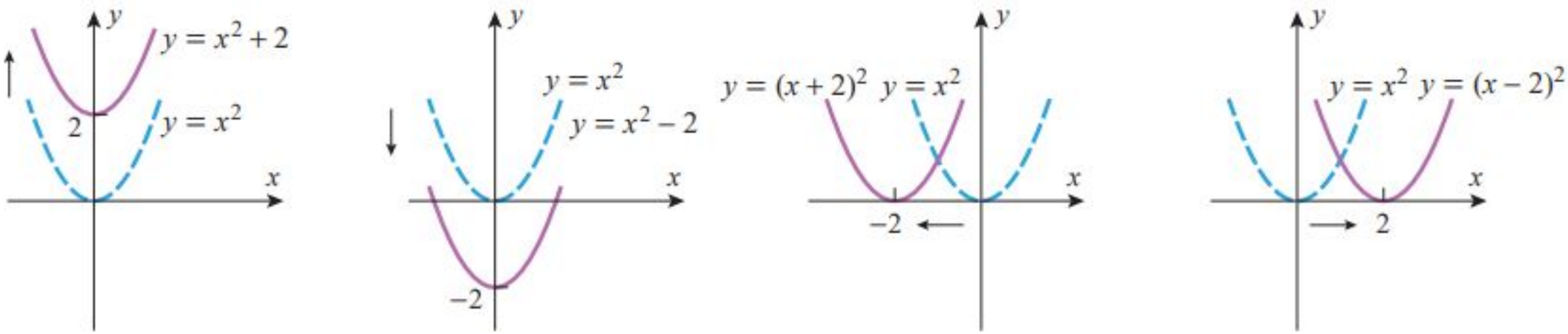
Piece-wise function

$$f(x) = \begin{cases} 0, & x \leq -1 \\ \sqrt{1 - x^2}, & -1 < x < 1 \\ x, & x \geq 1 \end{cases}$$

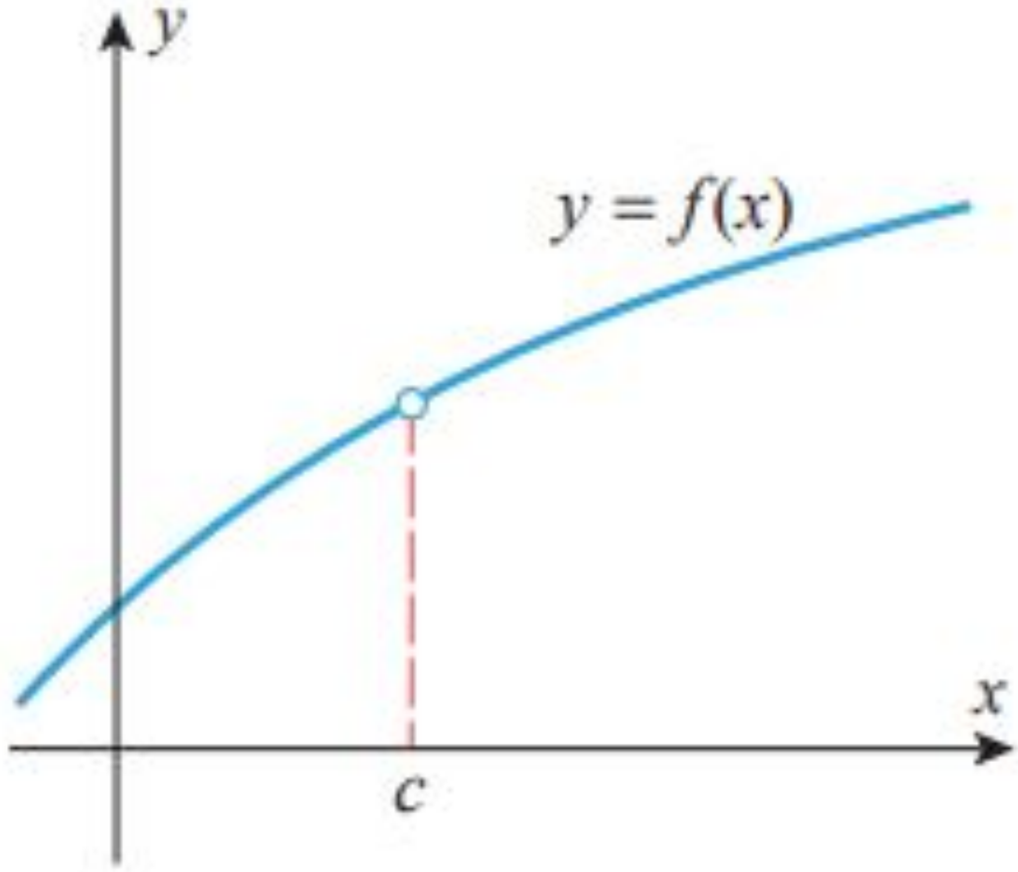
Composition

- Composition of sin x and x^3

Translations

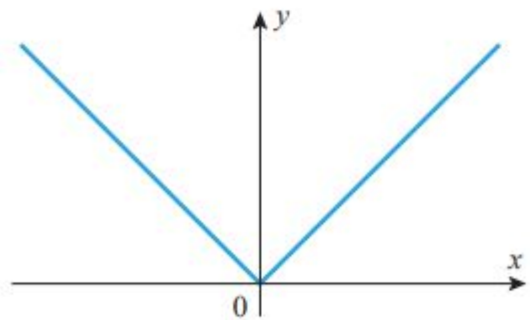


Continuous function



Derivative

- Rate of change
- Tangent to the curve
- Slope
- Velocity
- Differentiability



$y = |x|$

Some common derivatives

- Constant
- Power function
- Constant multiply by function
- Sums and differences
- Product and quotient rule
- Trigonometric function

Chain rule

▶ **Example** Find dw/dt if $w = \tan x$ and $x = 4t^3 + t$.

Solution. In this case the chain rule computations take the form

$$\begin{aligned}
\frac{dw}{dt} &= \frac{dw}{dx} \cdot \frac{dx}{dt} \\
&= \frac{d}{dx}[\tan x] \cdot \frac{d}{dt}[4t^3 + t] \\
&= (\sec^2 x) \cdot (12t^2 + 1) \\
&= [\sec^2(4t^3 + t)] \cdot (12t^2 + 1) = (12t^2 + 1)\sec^2(4t^3 + t) \quad \blacktriangleleft
\end{aligned}$$

► **Example** Find dy/dx if $y = \cos(x^3)$.

Solution. Let $u = x^3$ and express y as $y = \cos u$. Applying Formula (1) yields

$$\begin{aligned}
\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\
&= \frac{d}{du}[\cos u] \cdot \frac{d}{dx}[x^3] \\
&= (-\sin u) \cdot (3x^2) \\
&= (-\sin(x^3)) \cdot (3x^2) = -3x^2 \sin(x^3)
\end{aligned}$$ ◄

Integration

Extreme values

- Maxima / Minima

THEOREM (*Second Derivative Test*) *Suppose that f is twice differentiable at the point x_0.*

(a) *If $f'(x_0) = 0$ and $f''(x_0) > 0$, then f has a relative minimum at x_0.*

(b) *If $f'(x_0) = 0$ and $f''(x_0) < 0$, then f has a relative maximum at x_0.*

(c) *If $f'(x_0) = 0$ and $f''(x_0) = 0$, then the test is inconclusive; that is, f may have a relative maximum, a relative minimum, or neither at x_0.*