Computational Economics

Homework Assignment 2

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Solve the following questions. Students are encouraged to work as a group, but each student should write her/his own answer. Please indicate the name(s) of classmates you worked with. Please also submit your computer code.

1 Computing an MPE for an Entry/Exit Game

Consider a dynamic game of entry/exit with two players. Time is discrete; $t = 1, 2, \ldots$. At the beginning of each time period, players who stayed in the market in the previous period choose whether to stay in or exit, and players who stayed out of the market in the previous period decide whether to enter or stay out of the market. That is, their choice is binary, i.e., $\alpha_{it} \in \{0, 1\}$ where 0 is for "out of market" and 1 is for "in the market". Let $a_t \equiv (a_{1t}, a_{2t})$. When it enters, a player incurs the cost of entry φ which is assumed to be common across players and across periods. Furthermore, player i receives player- and alternative-specific shocks of $\varepsilon_{it} \equiv (\varepsilon_{it}(0), \varepsilon_{it}(1))$, which is private information and is assumed to be independent across players, alternatives, and periods. A market-level variable, M_t, also affects players' profit, and we assume M_t follows an first-order Markov process, characterized by $f(M_{t+1}|M_t)$. Specifically, we assume that M_t can take two different values, $M_t \in \{m_1, m_2\}$. The transition probability matrix for M is given by

$$\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix},$$

where $b_{ij} = \Pr(M_{t+1} = m_j | M_t = m_i)$. Let $s_t \equiv (M_t, a_{1t-1}, a_{2t-1})$ and $\varepsilon_t \equiv (\varepsilon_{1t}, \varepsilon_{2t})$ be the vector of common knowledge and private information variables, respectively. When making a decision at time t, player i observes s_t and ε_{it}. A player's current profits depend on the realization of M_t, on its own private information ε_{it}, and on the vector of players' current decisions $a_{t-1} \equiv (a_{1t-1}, a_{2t-1})$.

Let $\Pi_{it}(a_t, s_t, \varepsilon_{it})$ be player i's payoff at t if a set of decisions is given by a_t, the state variable is s_t, and the privately-observed payoff shocks are given by ε_{it}. We assume the payoff is additive

$$\Pi_{it}\left(a_t, s_t, \varepsilon_{it}\right) = \Pi_{it}\left(a_t, s_t\right) + \varepsilon_{it}\left(a_{it}\right),$$

and is written as

$$\Pi_{it}\left(a_t, s_t, \varepsilon_{it}\right) = \begin{cases} \lambda M_t - \delta a_{-i,t} - \left(1 - a_{i,t-1}\right)\varphi + \varepsilon_{it}\left(1\right) & \text{if } a_{it} = 1 \\ \varepsilon_{it}\left(0\right) & \text{if } a_{it} = 0 \end{cases}.$$

The timing of the game is as follows. At the beginning of period t, ε_t and the market-level variable M_t realize. After observing their private shocks and M_t, players simultaneously make an entry/exit decision. The current period payoff (it depends on entry decisions that have just been made and on the realization of M_t) realizes.

Assume $\lambda = 2$, $\delta = 2$, $m_1 = 1$, $m_2 = 1.5$, $b_{11} = 0.5$, $b_{21} = 0.4$, and $\varphi = 1.5$. Set $\beta = 0.95$. Assume that ε follows the iid extreme value distribution. Now you are going to compute a symmetric MPE.

1. How many distinct choice probabilities are there?

2. Write down the Bellman equation. Then, using the ex-ante (integrated) value functions, write down the problem as a system of linear equations. That is, express the ex-ante value functions as a solution to a linear system.

3. Compute a symmetric MPE in the following way. First construct a transition function for any given choice probabilities. Second, for any initial guess of conditional choice probabilities, calculate the ex-ante value functions using the transition function and the system of linear equations that you obtained above. Third, update choice probabilities (i.e., calculate Ψ in our notation we discussed in class). Finally, go back to step 2 and keep iterating. Comment on your results.