Computational Economics

Homework Assignment 1

Yuya Takahashi

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Solve the following questions. Students are encouraged to work as a group, but each student should write her/his own answer. Please indicate the name(s) of classmates you worked with. Please also submit your computer code.

1. Consider an infinite-horizon Ramsey model given by

$$\max_{\{K_t\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t u\left(C_t\right)$$

subject to

$$\begin{aligned} F\left(K_t\right) &\geq C_t + I_t \\ K_0 &= K \end{aligned}$$

where

$$I_t = K_{t+1} - (1-\delta) K_t$$

and non-negativity constraints on C_t and K_{t+1}.

(a) Assume $u\left(C_t\right) = \ln\left(C_t\right)$ and $F\left(K_t\right) = AK_t^a$ with $\beta = 0.6$, $A = 20$, $\alpha = 0.3$, and $\delta = 0.5$. Write the Bellman function and compute the value function using value function iterations. Plot $V(K)$ over the range $K \in [0, 12]$.

(b) Assume $u\left(C_t\right) = \ln\left(C_t\right)$ and $F\left(K_t\right) = K_t^a$ with $\delta = 1$.

i. Use value function iterations to approximate the value function for your choice of (α, β).

ii. Use guess and verify to obtain the value function analytically.

iii. Compare (plot) the value functions obtained in (i) and (ii).

2. Consider a Rust-type model. The data generating process is as follows:

- x_t can take on 11 values, i.e. $x_t \in \{0, 1, 2, \dots, 10\}$.
- At any time period,

$$x_{t+1} = \begin{cases} \min\{x_t + 1,\ 10\}, & \text{with probability } \lambda, \\ x_t, & \text{with probability } 1 - \lambda \end{cases}$$

  Note that λ does not depend on the replacement decision i_t.

- The agent's replacement decision is made at the beginning of each period and is effective immediately (i.e. if $i_t = 1$, the agent uses machine with mileage 0 and the next period state x_{t+1} is 1 with probability λ and 0 with probability $1 - \lambda$).
- The per-period maintenance cost for a bus with mileage x is $C(x, \theta) = \theta_1 x + \theta_2 x^2$.
- The cost of replacement is θ_3.
- That is, the per-period utility function is given by

$$u(x_t, i_t, \epsilon_{1t}, \epsilon_{2t}; \boldsymbol{\theta}) = \begin{cases} -\theta_1 x_t - \theta_2 x_t^2 + \epsilon_{0t}, & \text{if } i_t = 0, \\ -\theta_3 + \epsilon_{1t}, & \text{if } i_t = 1 \end{cases}$$

  where $(\epsilon_{0t}, \epsilon_{1t})$ follows the iid Type 1 Extreme Value distribution.

- The decision maker discounts future by $\beta = 0.95$.

(a) Write down the ML estimator for λ and solve for it (note that this does not require any nonlinear optimization).

(b) Set $\theta_1 = 0.3$, $\theta_2 = 0.0$, $\theta_3 = 4.0$ and solve the DP problem using $\lambda = 0.8$. Report the probability of replacement for every state. To do so, try both (1) the choice-specific value function formulation and (2) the integrated value function formulation. Compare the results. Note: assume the Euler constant is 0.5772.

(c) Simulate the model over $T = 5{,}000$ time periods, using random draws (download "draw.out" from the course website) and the policy function obtained in part (b). Set $x_1 = 0$.

(d) Estimate parameters of the model using MLE with a nested fixed point algorithm:

   i. Guess initial parameter values (you can use $\theta_1 = 0.3$, $\theta_2 = 0.0$, $\theta_3 = 4.0$ as your initial guess);

   ii. Solve DP problem;

iii. Calculate the probability of replacement at each state;

iv. Use model predictions (the probability calculated in the previous step) and data to derive a likelihood;

v. Search over parameter values by repeating steps $(i) - (iv)$.

(e) Using the estimated parameters, conduct the following policy experiments:

i. Calculate the expected long-run replacement probability for bus engines by forward simulation;

ii. Calculate the expected long-run replacement probability for bus engines by simulating the steady-state distribution and compare the result with the previous one;

iii. Suppose that the government reduces replacement cost θ_3 by 10 percent with an investment subsidy. Predict the effect of this subsidy on the long-run replacement probability.