Computational Economics: Problem Set 2

S M Sajid Al Sanai

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1 Dynamic Entry and Exit Game

1.1 Choice Probabilities

There are a total of (6) distinct choice probabilities which are faced by firms. These represent the probability a particular firm i chooses to take a specific action a_{it} as a function of the current demand state $M_t \in s_t = \big(a_{1t-1}, a_{2t-1}, M_t\big)$ faced.

Our profit function is additive in our standard normal shock ϵ_{it} which is identically and independently distributed, and the former is determined as a function of our opponent's decision a_{it}.

Equilibrium conditions hold when being active is the best response for expected private information regarding $\epsilon_{it}(a_{it})$, and where $\sigma_i(s)$ denotes ex ante beliefs of firm i that firm $j \neq i$ will choose to be active in the state s.

$$(1 - \sigma_i(s)) \cdot (\lambda M_t - \delta a_{-i,t} - (1 - a_{i,t-1})\psi + \epsilon_{it}(1)) + \sigma_i(s) \cdot \epsilon_{it}(0) \geq 0 \tag{1}$$

$$p_i(s) = 1 - \Phi[(1 - \sigma_i(s)) \cdot (\lambda M_t - \delta a_{-i,t} - (1 - a_{i,t-1}) \cdot \psi)] \tag{2}$$

$$\mathbf{p} - \mathbf{\Psi}(\sigma, \theta) = \mathbf{0} \tag{3}$$

Given consistency in beliefs,

$$p_i(s) = 1 - \Phi[(1 - p_i(s)) \cdot (\lambda M_t - \delta a_{-i,t} - (1 - a_{i,t-1}) \cdot \psi)] \tag{4}$$

$$\mathbf{p} - \mathbf{\Psi}(\mathbf{p}, \theta) = \mathbf{0} \tag{5}$$

Where $\mathbf{p}$ is the matrix of conditional probabilities,

$$\mathbf{p} = \mathbf{\Psi}(\mathbf{p}, \theta) = (p_i(s)_{i=1}^2)_{s=1}^3 \tag{6}$$

$$\mathbf{p} = \begin{pmatrix} p_1(s=1) & p_2(s=1) \\ p_1(s=2) & p_2(s=2) \\ p_1(s=3) & p_2(s=3) \end{pmatrix} \tag{7}$$

1.2 Value Function

A Markov Perfect Equilibrium is characterised by $(\mathbf{a}, \sigma)$ representing a best response a_i for firm i to the opposite firm's action a_{-i} given beliefs σ_i across all states s in the state space. Additionally, all firms will employ Markovian strategies and hold beliefs consistent with their strategies.

Given that the profit function is additive in shocks,

$$\Pi_{it}(a_t, s_t, \epsilon_{it}) = \Pi_{it}(a_t, s_t) + \epsilon_{it}(a_{it}) \tag{8}$$

$$\Pi_{it}(a_t, s_t, \epsilon_{it}) = \mathbf{1}(a_{it} = 1) \cdot [\lambda M_t - \delta a_{-i,t} - (1 - a_{i,t-1}) \cdot \psi] + \epsilon_{it}(a_{it}) \tag{9}$$

$$\begin{aligned} \bar{\Pi}_{it}(a_t, s_t, \epsilon_{it}) = \mathbf{1}(a_{it} = 1) \cdot [\lambda M_t - \delta a_{-i,t} - (1 - a_{i,t-1}) \cdot \psi] \\ + \sum_{k=1}^{K} \epsilon_{it}^{k}(a_{it}) \cdot \mathbf{1}(a_i^t = k) \end{aligned} \tag{10}$$

Through simplification we then arrive at our ex ante value function,

$$\begin{aligned} V_i(s, \sigma_i) = \sum_{a \in A} \sigma_i(a|s) \cdot [\mathbf{1}(a_{it} = 1) \cdot [\lambda M_t - \delta a_{-i,t} - (1 - a_{i,t-1}) \cdot \psi] \\ + \beta \sum_{s' \in S} g(a, s, s') \cdot V_i(s', \sigma_i)] + \sum_{k=1}^{K} E_{\epsilon}[\epsilon_i^k | a_i = k] \cdot \sigma_i(a_i = k|s) \end{aligned} \tag{11}$$

$$\begin{aligned} V_i(s, \sigma_i) = \sum_{a \in A} \sigma_i(a|s) \cdot [\Pi_i(a, s) + \beta \sum_{s' \in S} g(a, s, s') \cdot V_i(s', \sigma_i)] \\ + \sum_{k=1}^{K} E_{\epsilon}[\epsilon_i^k | a_i = k] \cdot \sigma_i(a_i = k|s) \end{aligned} \tag{12}$$

Subsequent conversion to matrix form yields the closed form solution of the value function,

$$\mathbf{V}_i(\sigma_i) = \sigma_i \mathbf{\Pi}_i + \beta \sigma_i \mathbf{G} \mathbf{V}_i(\sigma_i) + \mathbf{D}_i(\sigma_i) \tag{13}$$

$$\mathbf{V}_i(\sigma_i) - \beta \sigma_i \mathbf{G} \mathbf{V}_i(\sigma_i) = \sigma_i \mathbf{\Pi}_i + \mathbf{D}_i(\sigma_i) \tag{14}$$

$$(\mathbf{I} - \beta \sigma_i \mathbf{G}) \mathbf{V}_i(\sigma_i) = \sigma_i \mathbf{\Pi}_i + \mathbf{D}_i(\sigma_i) \tag{15}$$

$$\mathbf{V}_i(\sigma_i) = (\mathbf{I} - \beta \sigma_i \mathbf{G})^{-1} [\sigma_i \mathbf{\Pi}_i + \mathbf{D}_i(\sigma_i)] \tag{16}$$

Parameters that have been specified include $\lambda = 2, \delta = 2, \psi = 1.5$, and $\beta = 0.95$.
The matrix designated as a component of the state variable vector is as follows,

$$M = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1.5 \end{pmatrix} \tag{17}$$

The matrix designated as the Markovian state transition probabilities for M is as follows,

$$\mathbf{G} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} 0.5 & 0.5 \\ 0.4 & 0.6 \end{pmatrix} \tag{18}$$