Sept 28, 2006

Economics 730

Economic Development

PROBLEM SET 1 ANSWERS

1.

$$\max_{x,l,A} U(x,l)$$
$$\text{s.t. } px + rA_H \leq F(L,A) + rA_r$$

where

$$A = A_H + A_E - A_r$$
$$l = L_E - L$$

rearrange to get

$$\max U(x, L_E - L)$$
$$\text{s.t. } px \leq F(L,A) - rA + rA_E$$

Notice here that we can't just notice that $F - rA$ is profits, and claim that increasing the rhs of the bc is good, so we must be maximizing profits. This is because L enters both preferences and the production function. In fact, if the production function is NOT crts, we won't generically choose to maximize profits. Marginal products might vary across farms depending upon preferences. With crts, we're in better shape. The budget constraint becomes

$$px \leq L\{f(\frac{A}{L}) - r\frac{A}{L}\} + rA_E$$

and we are done. You can choose L and $\frac{A}{L}$ independently, and for any L you want to maximize $f(\frac{A}{L}) - r\frac{A}{L}$. So you are maximizing per-labor hour profits – with constant returns to scale that means marginal products are equalized across all plots. The scale of production is influenced by preferences, but input intensities are not, and profits are maximized at $f'((\frac{A}{L})^*) = r$.

2. This would have been easier if I had said "output" instead of "yield" in the problem statement, sorry. From 1, we know $\frac{A}{L} = (\frac{A}{L})^*$. So $T(1+\theta)$ is fixed at some constant k. Increases in θ have to be balanced by proportional declines in T. The regression

$$\ln y_i = \alpha + \beta \ln T_i + X_i \gamma + \varepsilon_i$$

is misspecified, because yield actually depends on quality adjusted land. To make the point most clearly, suppose that L^* is constant across households, and we have $CRTS$. Everyone, then, holds the same amount of quality adjusted land, and all the observed variation in T_i is exactly countered by a propotionate inverse variation in θ_i. Then $\varepsilon_i = 1 + \theta_i + \tilde{\varepsilon}_i$, where $\tilde{\varepsilon}_i$ is, say, measurement error in y_i. Our estimated coefficient β will be 0! More generally, $\hat{\beta}$ is biased down because of the negative correlation between unobserved land quality and observed T.

3.

A. No problem here. Preferences are now $U(x, l, h(x, l, \eta))$, and the budget constraint (after re-arranging) is

$$px + wl \leq F(L, A) - rA - wL + rA_E + wL_E.$$

The separation argument from class goes straight through.

B. Now, of course, separation doesn't go through. Preferences are as above, but the budget constraint is now

$$px + wl \leq F(L, A, h) - rA - wL + rA_E + wL_E$$

Now a household's production choices might be influenced by its preferences or its health endowment. For example, we'll have

$$\begin{aligned} \frac{\partial F}{\partial L} - w &= 0 \\ \frac{\partial F}{\partial A} - r &= 0 \end{aligned}$$

and all looks well. However, we also have

$$\frac{\partial U}{\partial h} = \lambda \frac{\partial F}{\partial h}$$

so health is not chosen to maximize profits, and as long as $\frac{\partial^2 F}{\partial L \partial h} \neq 0$ (and similarly for A), factor inputs will change with η.

4. **Empirical project.** See ps1ans.do