Problem Set 3: Single Agent Dynamic Discrete Choice Model: Rust (1987)
ECON 532 (Point: 15%)
Due Dec 9 11:59pm
No late submission allowed

This problem set is designed to help you to understand the nested fixed point algorithm and conditional choice probability algorithm for estimation of single agent dynamic discrete choice model. In particular, we will first simulate fake data a la Rust (1997). Then, we will use both algorithms to estimate the model parameters. All notations in the problem set are from Rust (1987).

1 Model

Payoffs: As in Rust (1987), Harold Zurcher's flow utility function is given by

$$u(x_t, i_t, \theta_1) + \varepsilon_t(i_t) = \begin{cases} -RC - c(0, \theta_1) + \varepsilon_t(1) & \text{if } i_t{=}1 \\ -c(x_t, \theta_1) + \varepsilon_t(0) & \text{if } i_t{=}0, \end{cases}$$

where RC is the replacement cost; $x_t \in \{0, 1, 2\}$ is the mileage-bin for the bus and $c(\cdot)$ is the cost function to be estimated; ε_t is a random variable, distributed as extreme value type I, independently of the other parameters; $i_t = 1$ indicates that Zurcher decided to replace the bus.

State Transition: If $i_t = 0$, then the Bus's mileage might increase. The state transition probabilities follow a multinomial distribution on the set $\{0, 1, 2\}$. The transition probability is given by

$$\theta_{3j} = \Pr\{x_{t+1} = x_t + j | x_t, i_t = 0\},$$

and $j \in \{0, 1, 2\}$, and zero otherwise.

If $i_t = 1$, then $x_{t+1} = 0$ with probability 1.

Objective Function: HZ makes engine replacement decision i_t in every period t to maximize

$$\sum_{t=1}^{\infty} \beta^t (u(x_t, i_t, \theta_1) + \varepsilon_t(i_t))$$

Parameters: Let the true parameters and functions be given by following

- $c(x, \theta_1) = \theta_1 x$ where $\theta_1 = 0.05$
- $(\theta_{30}, \theta_{31}, \theta_{32}) = (0.3, 0.5, 0.2)$
- $RC = 10$
- $\beta = 0.99$
- $\varepsilon_t(0)$ and $\varepsilon_t(1)$ are drawn from i.i.d. bivariate extreme value process, with mean (0,0) and variance $(\pi^2/6, \pi^2/6)$
- Initial mileages of all buses are zero.
- The number of buses: $N = 100$

2 Questions and Simulation

1. Write the choice specific value functions, and describe the problem as an optimal stopping problem.
2. Write the optimal stopping rule in terms of the flow utilities and expected continuation values. Use this expression to derive the choice probabilities in terms of the continuation values.
3. Compute $EV(x, i, \theta_1)$. Explain the procedure.
4. Graph $EV(x, i, \theta_1)$ for each $x \in \{0, 1, 2, \ldots, 10\}$ and $i \in \{0, 1\}$. Comment on the graph.
5. Simulate a fake dataset with x_t and optimal decisions i_t for $T = 1000$ periods. Present summary statistics and describe HZ's investment decision in the data.

3 Estimation and Counterfactual analysis

1. Estimate the parameters using nested fixed point algorithm.
2. Estimate the parameters using conditional choice probability algorithm a la Hotz and Miller (1993). Compare the two sets of estimates. Comment.
3. Now, let's assume that HZ's engine is more costly, but has lower operating cost per period. That is, new parameter values are $\theta_1 = 0.02$ and $RC = 20$. Would HZ prefer this engine to the original one? Use your preferred estimates.

4. Compute HZ's demand function for the two bus engines, assuming that he has access to only one of the engines. i.e. How many engines does HZ demand per period (in expectation) as a function of RC?

5. Compute the total values of the two engines, assuming that the marginal cost of production is RC. What is the maximum R&D cost of the more efficient engine, the one with $\theta_1 = 0.02$, that justifies developing it?