DPCN (Quiz 2)

Set1

1a. Assume the degree distribution of a network (symmetric and binary) follows a power law $p_k = Ck^{-\gamma}$, Calculate the normalization constant C. Find out the condition such that you can obtain the finite C. **(Marks 3)**

1b. Assume the degree distribution of network (symmetric and binary) follows a power law $p_k = Ck^{-\gamma}$. Find out the first moment ($\langle k \rangle$), and the second moment ($\langle k^2 \rangle$). Explain in which cases these moments will diverge. **(Marks 4)**

2. Prove that, in Erdos-Renyi Graph, GCC will pop-up if mean degree ($\langle k \rangle$) is greater than 1.

(Marks 5)

3. If $\boldsymbol{x_{n+1} = f(x_n)}$; Find the condition (show the derivation) where the system will show a stable fixed point

$$(\, \boldsymbol{x_{n+1} = f(x_n) = x^*} \,).$$

$(i)\boldsymbol{f(x_n) = x_n^2}$; $(\mathbf{ii})\boldsymbol{f(x_n) = \cos x_n}$; Find the fixed points and find the stability conditions.

(Marks 2+3=5)

QUIZ 1 (Set IV)

Dynamical processes in complex networks
(Full marks: 20)

January 29, 2024

Question 1 [2+1=3 Marks]

For a star graph with $N = 5$ nodes, what is the average path length of the graph? What is the diameter of the graph?

Question 2 [2+1=3 Marks]

For the graph given in Figure 1, calculate the clustering coefficients of the nodes labelled X and Y.

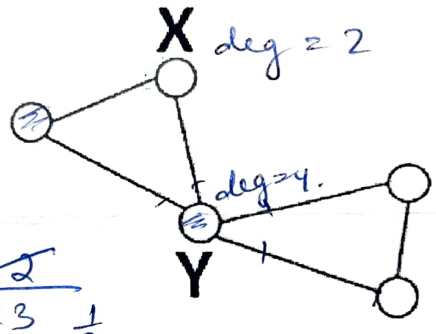


Figure 1

In reference to this graph, which statements are correct?

(a) Degree of Y is greater than X.

(b) Node betweenness of X is greater than Y.

(c) Closeness centrality of X is smaller than Y.

Question 3 [2+2=4 Marks]

Calculate the (a) node betweenness and (b) closeness of nodes labelled X and Y in the graph in Figure 2.

Figure 2

Question 4 [1+1+2=4 Marks]

What is the main difference between a Eulerian path and a Hamiltonian path of a graph?

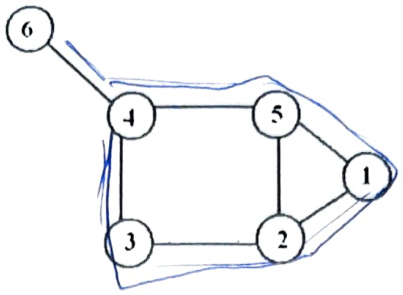


Figure 3

In the graph given in Figure 3, is there any Hamiltonian path/ Eulerian path/ Eulerian cycle? justify your answer. If not, how can you make an Eulerian path?

Question 5 [2 Marks]

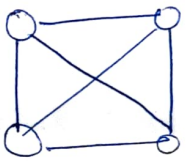
For an all-to-all connected network (binary, undirected, without self-loops) with number of nodes N, which of the following is/are true?

(a) Network is symmetric.

(b) Diameter of the network is N.

(c) Average path length of the network is 1.

(d) Average degree of the network $N - 1$.



Select the right option(s) and provide a one-line reasoning for why EACH of the options were correct or incorrect. Only attempting the options will NOT fetch any marks.

Question 6 [2 Marks]

Given a probability (denoted by p) representing the likelihood of a connection between two nodes or the ratio of existing links to the total potential edges, what is the relationship between p, the average degree $\langle K \rangle$, and the number of nodes N?

Question 7 [2 Marks]

Adjacency matrix of a directed graph can be written as:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Write the in-degree and out-degree sequence of this graph.