

Science-2 (Mid Sem Exam)
(2024)

Q1a. Consider the Compartmental models (Susceptible-Recovery-Infection) of epidemiology. If at any time t , the density of the susceptible, infected, and recovered population is captured by s , i , and r respectively, construct the associated differential equation of each compartment. Explain each term in one or two sentences. Here the rate of infection is β , rate of recovery is γ , and take $s(t) + i(t) + r(t) = 1$. 3

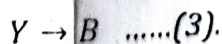
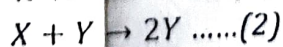
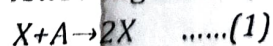
Q1b. At initial phase of the disease spread, what should be the relation between β and γ such that the disease starts to propagate (exponentially)? 3

Q2. For a certain class of distributions, it is possible to create pseudo-random numbers from uniformly distributed random numbers by finding a mathematical transformation (inverse transform method). If z (drawn from uniform distribution) is the random element chosen from 0 to 1, and the target distribution is $f = k e^{-yk}$, then find the relation between y and z . k is constant. 3

Q3a. In 19th century, Robert Brown, a Scottish botanist, observed that pollen grains suspended in water, instead of remaining stationary or falling downwards, would trace out a random zig-zagging pattern. Using Langevin's approach, write down (with explanation) the equation of motion of one single pollen grain. 4

Q3b. Specify which principles/axioms (of statistical thermodynamics) are required for solving the equation of motion of a large number of pollen grains. Using these axioms, calculate the average square displacement ($\langle x^2 \rangle$) of a large number pollen grains. 2+3=5

Q4a. The Predator-prey system consists of two kinds of animals. One of which preys on the other. If X symbolize prey, Y the predator, and A the food of the prey, we can write (this is also called as Lotka-Volterra model) following rate equations:



Construct the associated master equation for the model. k_1, k_2, k_3 are the rate constants of equations 1, 2, and 3 respectively. 3

Q4b. Write down the related ODE models for the abovementioned rate reactions (assuming A is constant). Explain each term in one to two sentences. 3

$$\begin{aligned} k_1 A x \Delta t, (x, x') \\ k_2 x y \Delta t; (x-1, y+1) \\ k_3 y \Delta t; (x, y-1) \end{aligned}$$

Q5: If $f(x, y) = x^3 + x^2y^3 - 2y^2$, find $f_x(1, 2)$ and $f_y(2, 1)$. Show that $u(x, y) = e^x \sin y$ is a solution of Laplace Equation $\frac{\partial^2 u(x, y)}{\partial x^2} + \frac{\partial^2 u(x, y)}{\partial y^2} = 0$. 1+1+2=4

Q6: If $f''(x_i) = \frac{f_{i+1} + f_{i-1} - 2f_i}{\Delta x^2}$, and if the heat conduction equation is $\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$, then show that $T_i^{j+1} = T_i^j + (T_{i+1}^j + T_{i-1}^j - 2T_i^j) \alpha \frac{\Delta t}{\Delta x^2}$. 3

— END —

T. Toli

Choose your own dataset — *user?* *purpose?*

All tasks & project in terms of 3.

Intro - submit a 2page report on your dataset - *does?* *purpose?*

Science 2 Mid Sem

Total Marks: 40

Objective type Questions (4x2 = 8 marks)

✓ 1. Find the probability of decay of a nuclei in the next unit time, if the decay constant = $\log_e(3)$.

- a. 0.67
- b. 0.33
- c. 1.67
- d. 1.33

✓ 2. Let $S = \{s_1, s_2, s_3, \dots, s_\infty\}$ be an infinitely long sequence generated by a Random Number Generator. Also assume that each s_i is between $[0,1]$. What is the arithmetic mean of the sequence?

- a. 0.25
- b. 1
- c. 0
- d. 0.5

✓ 3. Find a,b,c by balancing the following chemical equation: $aAl + bO_2 \rightarrow cAl_2O_3$. Which of the following is correct?

- a. (4, 2, 3)
- b. (4, 3, 2)
- c. (2, 3, 1)
- d. (2, 3, 2)

Romeo Juliet
Rabbit
logistics eqn
RC circuit

✓ 4. Consider the harmonic oscillator, where,

$$\dot{x} = v, \text{ and}$$

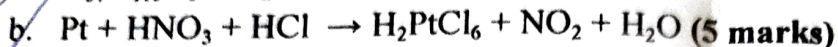
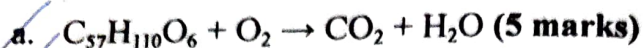
$$\dot{v} = -\omega^2 x$$

The orbits in the (x-v) plane can be captured by which of the following?

- a. Parabola
- b. Hyperbola
- c. Ellipse
- d. None of the above

Long Answer Type Questions (10+10+12 = 32 marks)

✓ 1. Balance the following chemical equations using a system of linear equations:



✓ 2. Consider the classic 'Rabbit vs Sheep' model. The equations are:

$$\dot{x} = x(3 - x - 2y)$$

$$\dot{y} = y(2 - x - y)$$

✓ a) Find all the Fixed Points of the above system. (2 marks)

✓ b) Construct the Jacobian around each of the fixed points. (2 marks)

✓ c) Find the eigenvalues for the different fixed points and use these eigenvalues to classify the fixed points as Stable/Unstable/Saddle points. (4 marks)

✓ d) State the Principle of Competitive Exclusion and verify it using the example of this system. (2 marks)

✓ 3. The simple random walk problem in one dimension. Let n_1 denote the number of steps to the right, n_2 the number of steps to the left, and N ($n_1 + n_2$) the total number of steps. Assuming successive steps are independent of each other:

p = probability that the step is to the right

$q = 1 - p$ = probability that the step is to the left

✓ a) Calculate the probability $W(n_1)$ of taking (in a total of N steps) n_1 steps to the right and $n_2 = N - n_1$ steps to the left, in any order. (3 marks)

✓ b) Verify the normalization of this probability: (3 marks)

$$\sum_{n_1=0}^N W(n_1) = 1$$

✓ c) What is the mean number of steps to the right? (6 marks)