Science-2 (Mid Sem Exam)
(2024)

Q1a. Consider the Compartmental models (Susceptible-Recovery-Infection) of epidemiology. If at any time t, the density of the susceptible, infected, and recovered population is captured by $s, i,$ and r respectively, construct the associated differential equation of each compartment. Explain each term in one or two sentences. Here the rate of infection is β, rate of recovery is γ, and take $s(t) + i(t) + r(t) = 1$. **3**

Q1b. At initial phase of the disease spread, what should be the relation between β and γ such that the disease starts to propagate (exponentially)? **3**

Q2. For a certain class of distributions, it is possible to create pseudo-random numbers from uniformly distributed random numbers by finding a mathematical transformation (inverse transform method). If z (drawn from uniform distribution) is the random element chosen from 0 to 1, and the target distribution is $f = k\, e^{-yk}$, then find the relation between y and z. k is constant. **3**

Q3a. In 19th century, Robert Brown, a Scottish botanist, observed that pollen grains suspended in water, instead of remaining stationary or falling downwards, would trace out a random zig-zagging pattern. Using Langevin's approach, write down (with explanation) the equation of motion of one single pollen grain. **4**

Q3b. Specify which principles/axioms (of statistical thermodynamics) are required for solving the equation of motion of a large number of pollen grains.
Using these axioms, calculate the average square displacement ($\langle x^2 \rangle$) of a large number pollen grains. **2+3=5**

Q4a. The Predator-prey system consists of two kinds of animals. One of which preys on the other. If X symbolize prey, Y the predator, and A the food of the prey, we can write (this is also called as Lotka-Volterra model) following rate equations:

$$X + A \rightarrow 2X \quad \text{......(1)}$$
$$X + Y \rightarrow 2Y \quad \text{......(2)}$$
$$Y \rightarrow B \quad \text{......(3).}$$

Construct the associated master equation for the model. k_1, k_2, k_3 are the rate constants of equations 1,2, and 3 respectively. **3**

Q4b. Write down the related ODE models for the abovementioned rate reactions (assuming A is constant). Explain each term in one to two sentences. **3**

$k_1 \alpha x \Delta t, (x \alpha' x')$
$k_2 x y \Delta t; (x-1, y+1)$
$k_3 y \Delta t; (x, y-1)$

Q5: If $f(x,y) = x^3 + x^2y^3 - 2y^2$, find $f_x(1,2)$ and $f_y(2,1)$. Show that $u(x,y) = e^x \sin y$ is a solution of Laplace Equation $\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = 0$. **1+1+2=4**

Q6: If $f''(x_i) = \frac{f_{i+1}+f_{i-1}-2f_i}{\Delta x^2}$, and if the heat conduction equation is $\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$, then show that $T_i^{j+1} = T_i^j + (T_{i+1}^j + T_{i-1}^j - 2\,T_i^j\ )\alpha \frac{\Delta t}{\Delta x^2}$. **3**

1

— END —

L Hali

Choose your own dataset → user? → purpose?

All labs & project in teams of 3.

Intro - submit a 2page report on your dataset - users? purpose?

Science 2 Mid Sem

Total Marks: 40

Objective type Questions (4x2 = 8 marks)

1. Find the probability of decay of a nuclei in the next unit time, if the decay constant = $log_e(3)$.
   a. 0.67
   b. 0.33
   c. 1.67
   d. 1.33

2. Let S = {s_1, s_2, s_3, …… s_∞} be an infinitely long sequence generated by a Random Number Generator. Also assume that each s_i is between [0,1]. What is the arithmetic mean of the sequence?
   a. 0.25
   b. 1
   c. 0
   d. 0.5

3. Find a,b,c by balancing the following chemical equation: $aAl + bO_2 \rightarrow cAl_2O_3$. Which of the following is correct?
   a. (4, 2, 3)
   b. (4, 3, 2)
   c. (2, 3, 1)
   d. (2, 3, 2)

Romeo Juliet
Rabbit
Logistics eqn
RC circuit

4. Consider the harmonic oscillator, where,

$$\dot{x} = v, \text{ and}$$
$$\dot{v} = -\omega^2 x$$

The orbits in the (x-v) plane can be captured by which of the following?
   a. Parabola
   b. Hyperbola
   c. Ellipse
   d. None of the above

Long Answer Type Questions (10+10+12 = 32 marks)

1. Balance the following chemical equations using a system of linear equations:

   a. $C_{57}H_{110}O_6 + O_2 \rightarrow CO_2 + H_2O$ **(5 marks)**
   b. $Pt + HNO_3 + HCl \rightarrow H_2PtCl_6 + NO_2 + H_2O$ **(5 marks)**

2. Consider the classic 'Rabbit vs Sheep' model. The equations are:

$$\dot{x} = x(3 - x - 2y)$$
$$\dot{y} = y(2 - x - y)$$

   a) Find all the Fixed Points of the above system. **(2 marks)**
   b) Construct the Jacobian around each of the fixed points. **(2 marks)**
   c) Find the eigenvalues for the different fixed points and use these eigenvalues to classify the fixed points as Stable/Unstable/Saddle points. **(4 marks)**
   d) State the Principle of Competitive Exclusion and verify it using the example of this system. **(2 marks)**

3. The simple random walk problem in one dimension. Let n_1 denote the number of steps to the right, n_2 the number of steps to the left, and N ($n_1 + n_2$) the total number of steps. Assuming successive steps are independent of each other:

$$p = \text{probability that the step is to the right}$$
$$q = 1 - p = \text{probability that the step is to the left}$$

   a) Calculate the probability $W(n_1)$ of taking (in a total of N steps) n_1 steps to the right and $n_2 = N - n_1$ steps to the left, in any order. **(3 marks)**
   b) Verify the normalization of this probability: **(3 marks)**

$$\sum_{n_1=0}^{N} W(n_1) = 1$$

   c) What is the mean number of steps to the right? **(6 marks)**