Assignment - 1

(Q2) A and B are TRUE
X and Y are FALSE

A	¬A	B	¬B	X	¬X	Y	¬Y
T	F	T	F	F	T	F	T

(a) $\neg (A \vee X)$
$\Rightarrow \neg A \wedge \neg X$
$\Rightarrow$ (False) $\wedge$ (True)
$\Rightarrow$ False

(b) $A \vee (X \wedge Y)$
$\Rightarrow$ (True) $\vee$ (False)
$\Rightarrow$ True

X	Y	A	X∧Y	A∨(X∧Y)
F	F	T	F	T

(c) $A \wedge (X \vee (B \wedge Y))$
$\Rightarrow A \wedge (B \wedge Y)$
$\Rightarrow$ (True) $\wedge$ (False)
$\Rightarrow$ False

A	B	X	Y	B∧Y	X∨(B∧Y)	A∧(X∨(B∧
T	T	F	F	F	F	F

(d) $[(A \wedge X) \vee \neg B] \wedge \neg [(A \wedge X) \vee \neg B]$

A	B	X	¬B	A∧X	(A∧X)∨¬B	¬[(A∧X)∨¬B]
T	T	F	F	F	F	T

$\Rightarrow$ [(False) $\vee \neg$(True)] $\wedge \neg$ [(False) $\vee \neg$(True)]
$\Rightarrow$ [(False) $\vee$ (False)] $\wedge \neg$ [(False) $\vee$ (False)]

$\Rightarrow$ False $\wedge \neg$ (False)
$\Rightarrow$ False $\wedge$ True = False

(e) ~~$(R \wedge Q) \wedge (\neg A \vee X)$~~

(f) $[(X \wedge Y) \rightarrow A] \rightarrow [X \rightarrow (Y \rightarrow A)]$

First lets evaluate: $[(X \wedge Y) \rightarrow A]$

$\Rightarrow$ ((False) $\wedge$ (False)) $\rightarrow$ True
$\Rightarrow$ False $\rightarrow$ True
$\Rightarrow$ True

Now: $X \rightarrow (Y \rightarrow A)$
$\Rightarrow$ False $\rightarrow$ (False $\rightarrow$ True)
$\Rightarrow$ False $\rightarrow$ (True)
$\Rightarrow$ True

So $[(X \wedge Y) \rightarrow A] \rightarrow [X \rightarrow (Y \rightarrow A)]$ = True $\rightarrow$ True
= True

(Q1) (a) " If it rains, Raju carries an umbrella. Raju is carrying an umbrella, therefore it will rain."

Here we assume; P = " it will rain"
Q = " Raju is carrying an umbrella".
$P \rightarrow Q$ = " If it rains, Raju carries an umbrella".

To determine the validity of an argument using truth table, we will evaluate all possible combinations of truth values of P & Q.

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	T	T

From the truth table, we see that the above argument is not valid because the conclusion (P) doesn't follow $P \rightarrow Q$ & Q logically. There is a row where Q is ~~False~~ TRUE & P is FALSE which contradicts the conclusion. Thus, the argument is logically invalid.

(b) Let P = "weather is warm"
Q = "sky is clear"
R = "we go swimming"
S = "we go boating"

So, the statement can be presented logically as -:

$$(P \wedge Q) \rightarrow (R \vee S),\ \neg(\neg R \rightarrow \neg Q) \therefore (P \vee S)$$

P	Q	R	S	P∧Q	R∨S	¬R	¬Q	¬R→¬Q	P∨S
T	T	T	T	T	T	F	F	T	T
T	T	T	F	T	T	F	F	T	T
T	T	F	T	T	T	T	F	F	T
T	T	F	F	T	F	T	F	F	T
T	F	T	T	F	T	F	T	T	T
T	F	T	F	F	T	F	T	T	T
T	F	F	T	F	T	T	T	T	T
F	F	F	F	F	F	T	T	T	F

From the truth table we can see that there is a row where the conclusion (PVS) is FALSE but the premises (P∧Q)→(RVS) and ~(~R→~Q) are TRUE. So the conclusion doesn't follow the premises logically. Hence the statement is logically invalid.

(Q3) (a) $P \rightarrow \sim Q$, $\sim Q \rightarrow R$, $\Rightarrow P \rightarrow R$

- FORMAL PROOF

~~let us assume P is True Then P → ~Q ⇒ ~Q is true ⇒ Q is false.~~ Assume P as premise. From P→Q, we can ~~Now ~Q → R ⇒ R is true (∵ Q is true)~~ deduce ~Q. From ~Q→R, we can deduce R. ∴ From premises P→R ~~∴ if P is true then R is also true~~. can be derived.

- RESOLUTION METHOD

We know $A \rightarrow B \equiv A \vee B$

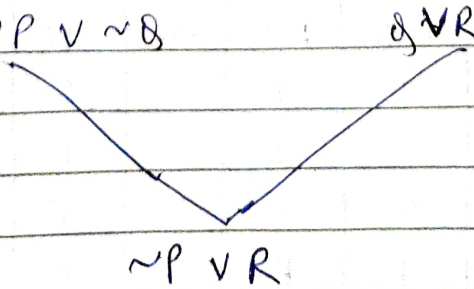
Then, we get clauses:

$P \rightarrow \sim Q \equiv \sim P \vee \sim Q$ — (1)

$\sim Q \rightarrow R \equiv Q \vee R$ — (2)

{CNF form}

So;



∴ we get a new clause (~PVR) as there are

no contradictions ; so this must be valid.

$\therefore \sim P \vee R \equiv P \rightarrow R$ is a logically valid statement.

(b) The statement tells us that with no premise, the proposition is true.

$((P \vee Q) \wedge \neg P) \rightarrow Q$ is true.

For this to be true, the proposition should be a tautology.

- FORMAL METHOD :

$[(P \vee Q) \wedge \neg P] \rightarrow Q.$

$\Rightarrow \neg [(P \vee Q) \wedge \neg P] \vee Q.$

$\Rightarrow \neg (P \vee Q) \vee P \vee Q$

$\Rightarrow \neg (P \vee Q) \vee (P \vee Q) \equiv T \quad \{\neg P \vee P = T\}$

- METHOD OF RESOLUTION

Given, ① $P \vee Q$ & ② $\neg P$

$\Rightarrow \neg P \vee F$ ③ $\{A \vee F = A\}$

$\Rightarrow Q \vee F$ ④ $\{① \& ③ : \text{Resolution}\}$

$\Rightarrow Q$ ⑤ $\{A \vee F = A\}$

$\therefore \dfrac{P \vee Q \quad \neg P}{\therefore Q}$ is a valid argument.

$\therefore [(P \vee Q) \wedge \neg P] \rightarrow Q$ is a tautology.