10. DYNAMICS OF A HEATED TANK WITH PI TEMPERATURE CONTROL

10.1 Numerical Methods

Solution of ordinary differential equations, generation of step functions, simulation of a proportional integral controller.

10.2 Concepts Utilized

Closed loop dynamics of a process including first order lag and dead time. Padé approximation of time delay.

10.3 Course Useage

Process Dynamics and Control

10.4 Problem Statement

A continuous process system consisting of a well-stirred tank, heater and PI temperature controller is depicted in Figure (4). The feed stream of liquid with density of ρ in kg/m^3 and heat capacity of C in kJ / kg·°C flows into the heated tank at a constant rate of W in kg/min and temperature T_i in °C. The volume of the tank is V in m^3. It is desired to heat this stream to a higher set point temperature T_r in °C. The outlet temperature is measured by a thermocouple as T_m in °C, and the required heater input q in kJ/min is adjusted by a PI temperature controller. The control objective is to maintain $T_0 = T_r$ in the presence of a change in inlet temperature T_i which differs from the steady state design temperature of T_{is}.

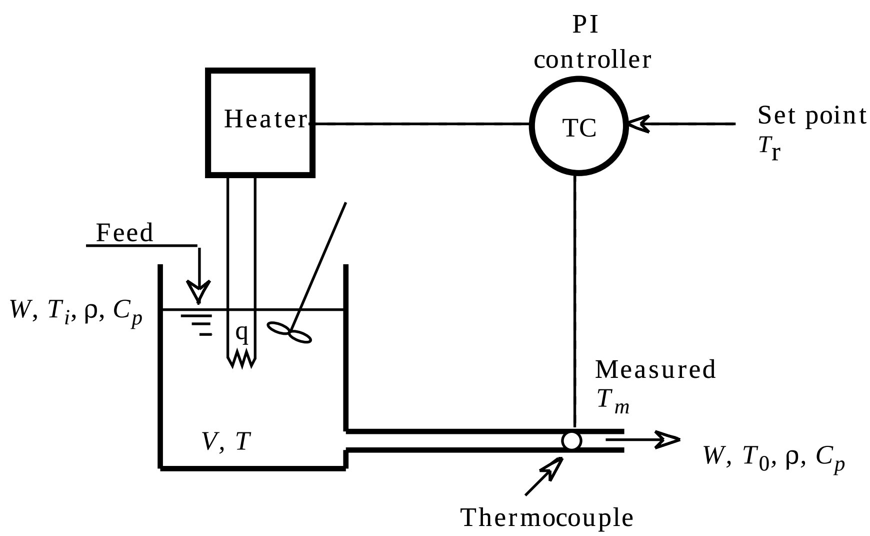

Figure 4 Well Mixed Tank with Heater and Temperature Controller

Modeling and Control Equations

An energy balance on the stirred tank yields

$$\frac{dT}{dt} = \frac{WC_p(T_i - T) + q}{\rho V C_p} \tag{40}$$

with initial condition $T = T_r$ at $t = 0$ which corresponds to steady state operation at the set point temperature T_r.

The thermocouple for temperature sensing in the outlet stream is described by a first order system plus the dead time τ_d which is the time for the output flow to reach the measurement point. The dead time expression is given by

$$T_0(t) = T(t - \tau_d) \tag{41}$$

The effect of dead time may be calculated for this situation by the Padé approximation which is a first order differential equation for the measured temperature.

$$\frac{dT_0}{dt} = \left[T - T_0 - \left(\frac{\tau_d}{2}\right)\left(\frac{dT}{dt}\right)\right]\frac{2}{\tau_d} \qquad \text{I. C. } T_0 = T_r \text{ at } t = 0 \text{ (steady state)} \tag{42}$$

The above equation is used to generated the temperature input to the thermocouple, T_0.

The thermocouple shielding and electronics are modeled by a first order system for the input temperature T_0 given by

$$\frac{dT_m}{dt} = \frac{T_0 - T_m}{\tau_m} \qquad \text{I. C. } T_m = T_r \text{ at } t = 0 \text{ (steady state)} \tag{43}$$

where the thermocouple time constant τ_m is known.

The energy input to the tank, q, as manipulated by the proportional/integral (PI) controller can be described by

$$q = q_s + K_c(T_r - T_m) + \frac{K_c}{\tau_I}\int_0^t (T_r - T_m)dt \tag{44}$$

where K_c is the proportional gain of the controller, τ_I is the integral time constant or reset time. The q_s in the above equation is the energy input required at steady state for the design conditions as calculated by

$$q_s = WC_p(T_r - T_{is}) \tag{45}$$

The integral in Equation (44) can be conveniently be calculated by defining a new variable as

$$\frac{d}{dt}(errsum) = T_r - T_m \qquad \text{I. C. } errsum = 0 \text{ at } t = 0 \text{ (steady state)} \tag{46}$$

Thus Equation (44) becomes

$$q = q_s + K_c(T_r - T_m) + \frac{K_c}{\tau_I}(errsum) \tag{47}$$

Let us consider some of the interesting aspects of this system as it responds to a variety of parameter and operational changes.The numerical values of the system and control parameters in Table (4) will be considered as leading to baseline steady state operation.

Table 4 Baseline System and Control Parameters for Problem 10

$\rho V C_p$ = 4000 kJ/°C	$W C_p$ = 500 kJ/min·°C
T_{is} = 60 °C	T_r = 80 °C
τ_d = 1 min	τ_m = 5 min
K_c = 50 kJ/min·°C	τ_I = 2 min

(a) Demonstrate the open loop performance (set $K_c = 0$) of this system when the system is initially operating at design steady state at a temperature of 80°C, and inlet temperature T_i is suddenly changed to 40°C at time t = 10 min. Plot the temperatures T, T_0, and T_m to steady state, and verify that Padé approximation for 1 min of dead time given in Equation (42) is working properly.

(b) Demonstrate the closed loop performance of the system for the conditions of part (a) and the baseline parameters from Table (4). Plot temperatures T, T_0, and T_m to steady state.

(c) Repeat part (b) with K_c = 500 kJ/min·°C.

(d) Repeat part (c) for proportional only control action by setting the term $K_c/\tau_I = 0$.

(e) Implement limits on q (as per Equation (47)) so that the maximum is 2.6 times the baseline steady state value and the minimum is zero. Demonstrate the system response from baseline steady state for a proportional only controller when the set point is changed from 80°C to 90°C at t = 10 min. K_c = 5000 kJ/min·°C. Plot q and q_{lim} versus time to steady state to demonstrate the limits. Also plot the temperatures T, T_0, and T_m to steady state to indicate controller performance