6. HEAT EXCHANGE IN A SERIES OF TANKS

6.1 Numerical Methods

Solution of simultaneous first order ordinary differential equations.

6.2 Concepts Utilized

Unsteady state energy balances, dynamic response of well mixed heated tanks in series.

6.3 Course Useage

Heat Transfer.

6.4 Problem Statement

Three tanks in series are used to preheat a multicomponent oil solution before it is fed to a distillation column for separation as shown in Figure (2). Each tank is initially filled with 1000 kg of oil at 20°*C*. Saturated steam at a temperature of 250°*C* condenses within coils immersed in each tank. The oil is fed into the first tank at the rate of 100 kg/min and overflows into the second and the third tanks at the same flow rate. The temperature of the oil fed to the first tank is 20°*C*. The tanks are well mixed so that the temperature inside the tanks is uniform, and the outlet stream temperature is the temperature within the tank. The heat capacity, C_p, of the oil is 2.0 KJ/kg. For a particular tank, the rate at which heat is transferred to the oil from the steam coil is given by the expression

$$Q = UA(T_{steam} - T) \tag{18}$$

where UA = 10 kJ/min·°*C* is the product of the heat transfer coefficient and the area of the coil for each tank, T = temperature of the oil in the tank in °*C*, and Q = rate of heat transferred in kJ/min.

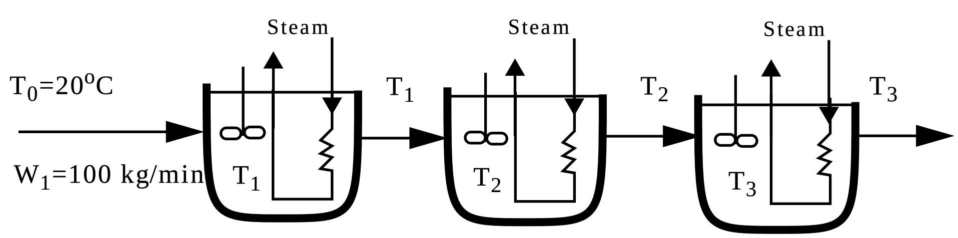

Figure 2 Series of Tanks for Oil Heating

Energy balances can be made on each of the individual tanks. In these balances, the mass flow rate to each tank will remain at the same fixed value. Thus $W = W_1 = W_2 = W_3$. The mass in each tank will be assumed constant as the tank volume and oil density are assumed to be constant. Thus $M = M_1 = M_2 = M_3$. For the first tank, the energy balance can be expressed by

Accumulation = Input - Output

$$MC_p\frac{dT_1}{dt} = WC_pT_0 + UA(T_{steam} - T_1) - WC_pT_1 \tag{19}$$

Note that the unsteady state mass balance is not needed for tank 1 or any other tanks since the mass in each tank does not change with time. The above differential equation can be rearranged and explicitly solved for the derivative which is the usual format for numerical solution.

$$\frac{dT_1}{dt} = [WC_p(T_0 - T_1) + UA(T_{steam} - T_1)]/(MC_p) \tag{20}$$

Similarly for the second tank

$$\frac{dT_2}{dt} = [WC_p(T_1 - T_2) + UA(T_{steam} - T_2)]/(MC_p) \tag{21}$$

For the third tank

$$\frac{dT_3}{dt} = [WC_p(T_2 - T_3) + UA(T_{steam} - T_3)]/(MC_p) \tag{22}$$

Determine the steady state temperatures in all three tanks. What time interval will be required for T_3 to reach 99% of this steady state value during startup?