Project #5 : Parametric Earth Model

When a tuning fork is stuck, the tines vibrate back and forth producing sound. A good tuning fork produces a pure tone – a single frequency. This is why it can be used to simultaneously tune instruments. When a bell is struck, it also vibrates and produces sound. Because the bell is not designed as a tuning fork, it produces a variety of tones which combine into a rich sound that makes a bell much more pleasant to hear. The bell has an infinite variety of resonant vibrational modes, the audible ones falling within the range of human hearing. These resonance frequencies are determined by the bell geometry and the properties of the material that make up the bell (density and stiffness).

The Earth itself is a type of bell which vibrates when struck. When an earthquake occurs the Earth reacts by vibrating in some combination of its natural resonance frequencies. Because the Earth is so large and earthquakes have very low energy compared to the gravitational and stiffness energy of the Earth, the vibrations are extremely low amplitude and of very low frequency. The vibration of lowest frequency has a period of about 43 minutes which corresponds to a frequency of about 0.00039 Hz, far below the typical range of human hearing (20 Hz to 20 kHz).

Sensitive seismographic equipment can detect many such resonance frequencies of the Earth. Because the structure of the Earth is essentially only radially dependent (consists of homogeneous and isotropic layers), the vibrations can be classed into three types and computed using a combination of gravitational theory and elasticity theory. Toroidal modes consist of purely non-radial shear motion and do not result in shape changes. Radial modes consist of purely radial expansion and compression. Spheroidal modes are a combination of shear and compressional motion and are characterized by nontrivial (though very small) shape changes. The details of these types of modes are not important for our work here, but are well documented.

The expected resonance frequencies of the Earth can be numerically computed given an Earth model consisting of radial functions of the density, $\rho(r)$, shear modulus, $\mu(r)$ and the second Lamé parameter, $\lambda(r)$. Earth scientists have a standard model which divides the Earth into several layers defining parts of the core, mantle and crust, and within each layer the model functions are given by parameterized polynomial functions of order three or less. Altogether, an full Earth model consists of about 55 free parameters defining ρ, μ and λ, which should be chosen so that the calculated resonance frequencies agree with the measured resonance frequencies. The calculated resonance frequencies are determined by numerical integration of up to six simultaneous ordinary differential equations with appropriate boundary conditions. These computations are **not** the main subject of this project and you will be supplied working code.

In this project, you will use 108 experimentally-determined toroidal mode oscillation periods to produce an optimal Earth model in 36 free parameters. The supplied code will compute a list of user-requested resonance periods given user-supplied model parameters. You will create an objective function which uses the supplied code and solve this optimization problem with derivative-free code of your composition.

Task 1. Create your own Nelder-Mead optimization function that can be used to solve black-box optimization problems.

Task 2. Upload the Toroidal Mode calculation function (ToroidalPeriods.m or TF.py) and the sample objective function calculator (matlabtest.m or pythontest.py) and understand the basic strategy. This code is somewhat optimized for speed though it does declare data on each call. The user can change the measured data if desired, but there is no anticipated need to do so. The user can also modify how the measured and computed data are compared – that is, change the norm in the test functions. Note: R and Fortran users should make an appropriate call to the python or matlab function.

Task 3. Solve the Earth Model problem using your Nelder-Mead code, three different comparison norms and a few different initial parameter vectors. One suggested initial decision variable vector is given in the test codes.

Task 4. Provide a short report on your optimization results and the appropriateness and efficiency of the Nelder-Mead algorithm. Compare the efficiency of Nelder-Mead when applied to one or more of the previous Projects.