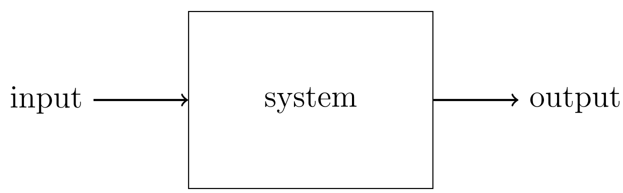
# 1 Signals & Systems - Introduction

Signal : Any time varying physical phenomenon that is intended to convey information is called as signal .

Signal is a function of time .
e.g : human voice , voltage on telephone wires , electric signal , . . .

System : system is a device which operates on signals according to its characteristics .




e.g : Communication System

# 2 Topics

- Introduction
- L.T.I (Linear Time Invariants)
- F.S (Fourier Series)
- F.T (Fourier Transform)
- L.T (Laplace Transform)
- Z.T

# 3 Point

A Signal $f_1(t)$ can be represented in terms of another signal $f_2(t)$ as

$$f_1(t) = c_{12} f_2(t)$$

where

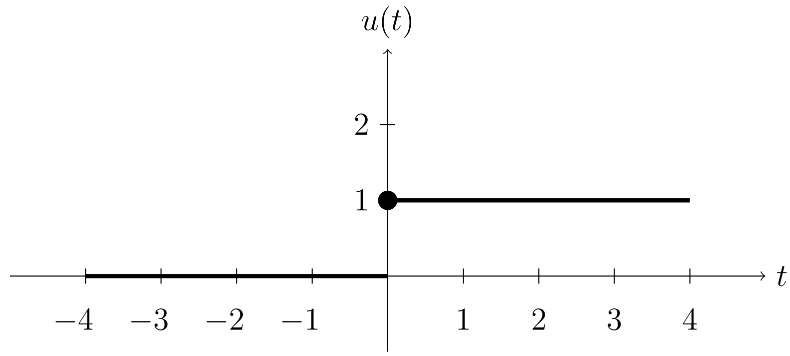
$$c_{12} = \frac{\int_{t_1}^{t_2} f_1(t) f_2(t) dt}{\int_{t_1}^{t_2} |f_2|^2(t)}$$

# 4 Basic Signals

- Unit Step Signal
- Impulse function
- Signum function
- Exponential Signal
- Unit Ramp Signal
- Parabolic Signal
- Rectangular Signal
- Triangular Pulse
- Sinusoidal Signal
- Sinc function
- sampling function

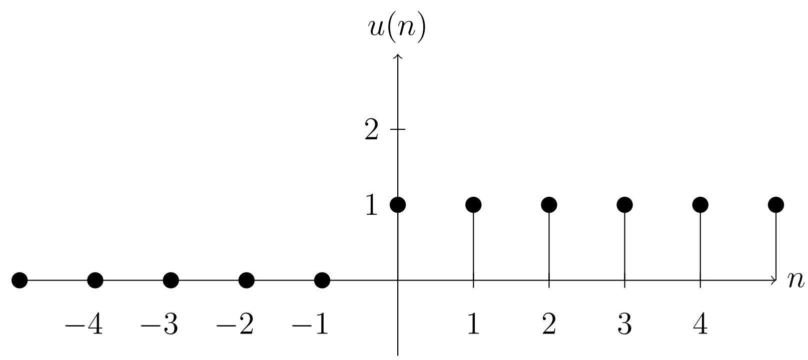
# 5 Unit Step function

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$




### 5.1 Discrete

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



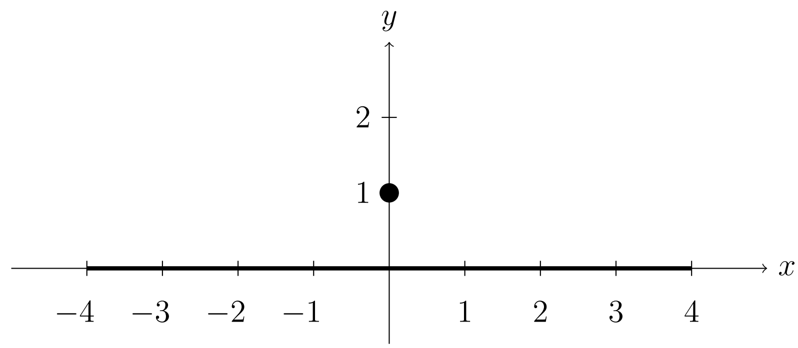

### 5.2 Properties

$$(u(n))^n = u(t)$$
$$[u(t - t_0)]^k = u(t - t_0)$$
$$u(at) = u(t)$$

## 6 Impulse Function

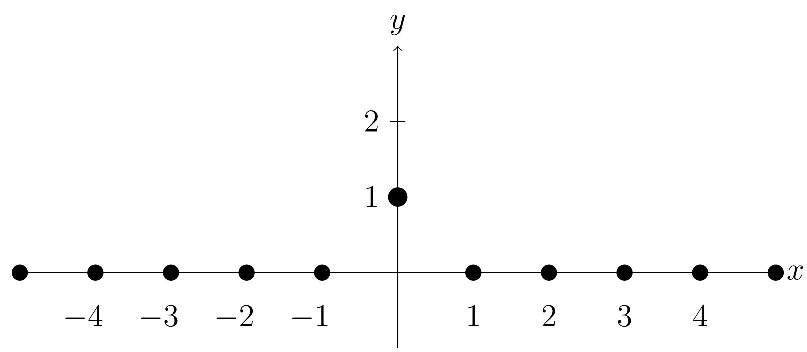
It is denoted by $\delta(t)$

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$




## 6.1 Discrete

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$




## 6.2 Properties

$$\int_{-\infty}^{+\infty} \delta(t) dt = 1$$

$$\delta(n-k) = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$$

$$\delta(n) = u(n) - u(n-1)$$

$$f(t)\delta(t) = f(0)$$

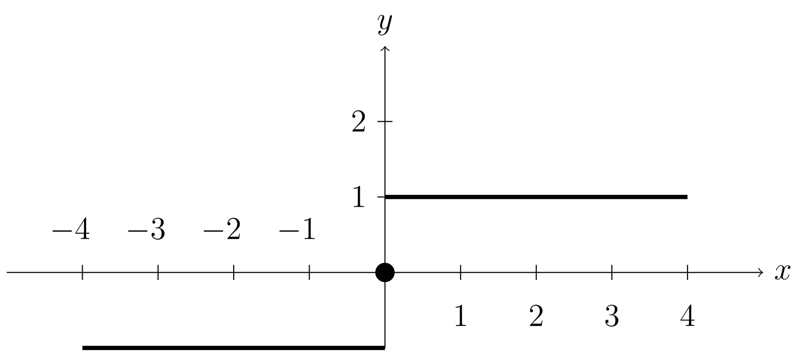
$$\delta(t-t_0)f(t) = f(t_0)$$

$$\delta(kt) = \frac{1}{|k|}\delta(t)$$

# 7 Signum function

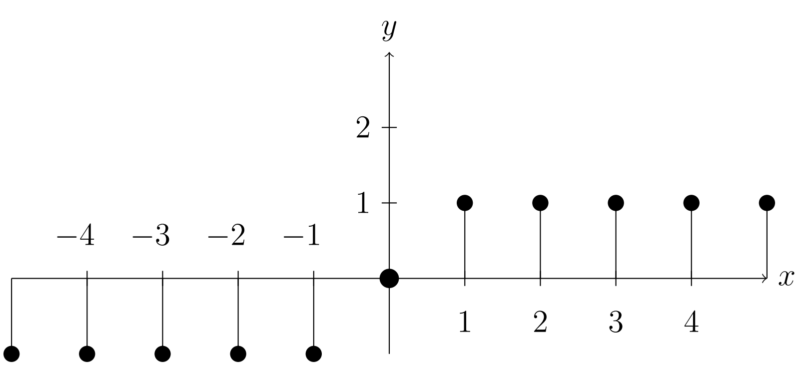
It is denoted with $sgn(t)$ , $sgn(n)$

$$sgn(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$




## 7.1 Discrete

$$sgn(n) = \begin{cases} 1 & n > 0 \\ 0 & n = 0 \\ -1 & n < 0 \end{cases}$$



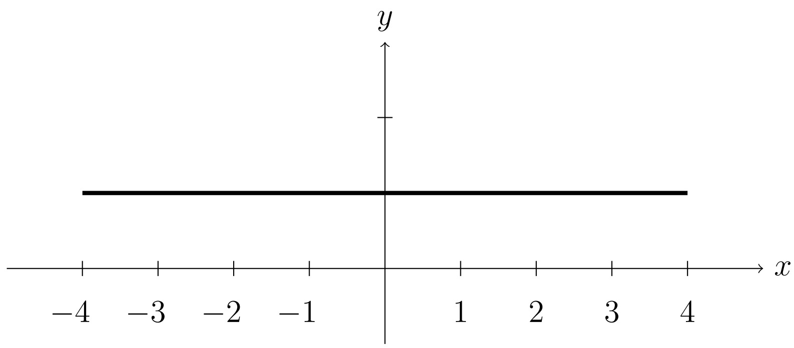

# 8 Exponential Signal

This signal is in the form

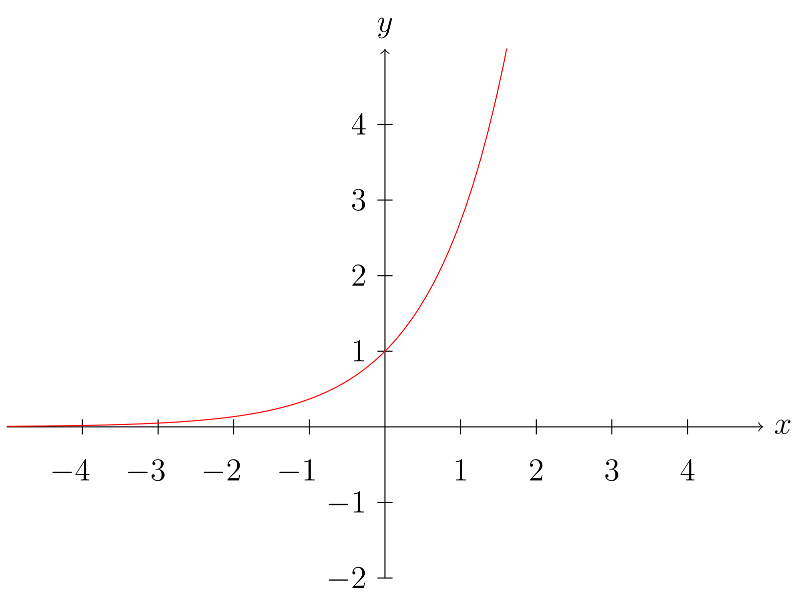
$$x(t) = e^{\alpha t}$$

The shape of exponential depends on $\alpha$

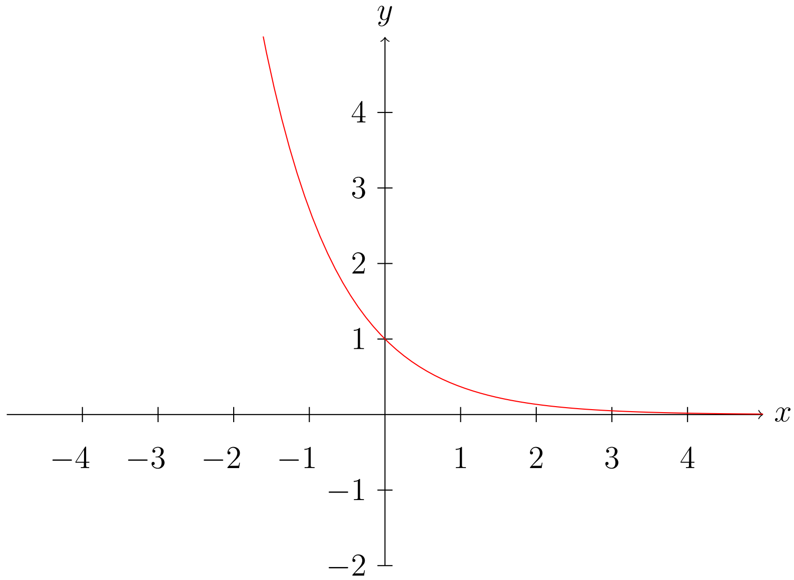
$$\alpha = 0 \Rightarrow x(t) = e^{0.t} = e^0 = 1$$




$$\alpha > 0 \xRightarrow{\alpha=3} x(t) = e^{3t}$$



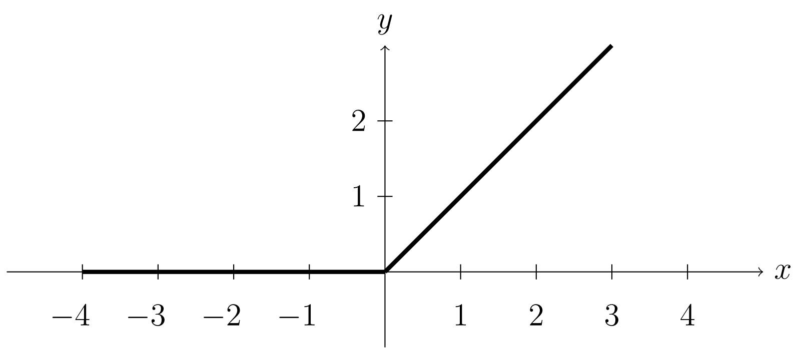

$$\alpha < 0 \xRightarrow{\alpha=-3} x(t) = e^{-3t}$$




# 9 Unit Ramp Signal

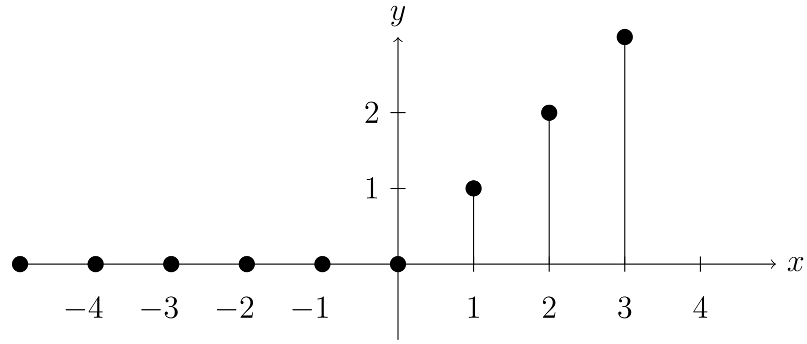
It is denoted by $x(t)$

$$R(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$




## 9.1 Discrete

$$R(n) = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$




## 9.2 Properties

$$R(t) = \int u(t)dt$$

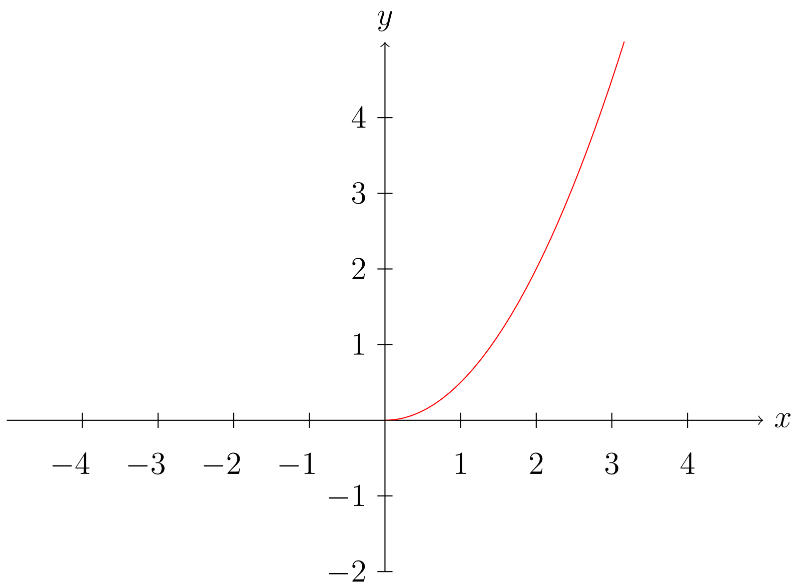
$$u(t) = \frac{dR(t)}{dt}$$

$$\left.\begin{array}{l} \int \delta(t)dt = u(t) \\ \int u(t)dt = R(t) \end{array}\right\} \Rightarrow \iint \delta(t)dt = R(t)$$

$$\left.\begin{array}{l} \delta(t) = \dfrac{du(t)}{dt} \\ u(t) = \dfrac{dR(t)}{dt} \end{array}\right\} \Rightarrow \delta(t) = \frac{d^2R(t)}{dt^2}$$

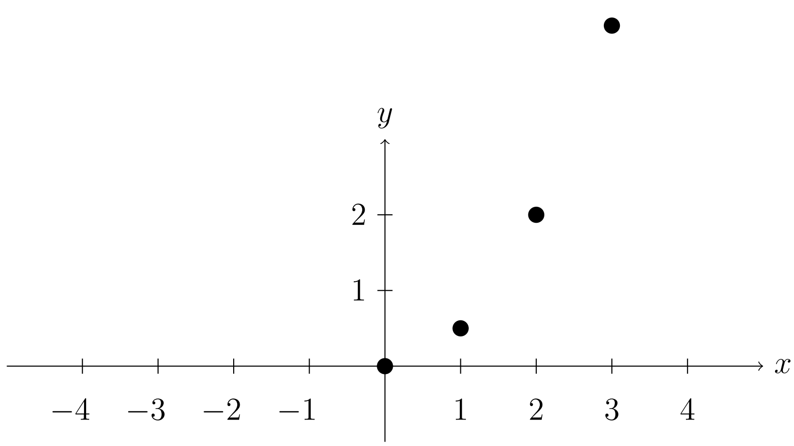
# 10 Unit Parabolic Signal

$$x(t) = \begin{cases} \dfrac{t^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$




### 10.1 Discrete

$$x(n) = \begin{cases} \frac{n^2}{2} & n \geq 0 \\ 0 & n < 0 \end{cases}$$



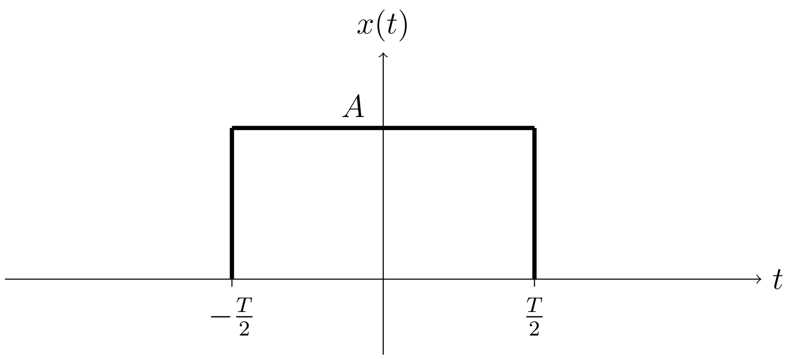

## 11 Rectangular Pulse

let it be denoted as $x(t)$ :

$$x(t) = A \times Rect(\frac{t}{T})$$

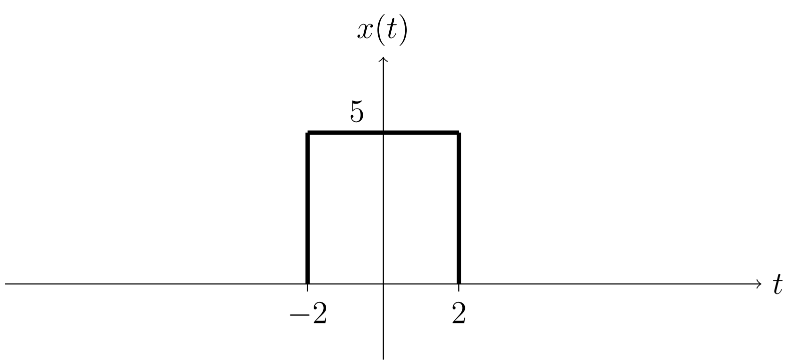
where

$$\begin{cases} A = \text{amplitue of rectangle} \\ \\ T = \text{period of rectangle} \end{cases}$$




e.g :

$$x(t) = 5 \times Rect(\frac{t}{4})$$

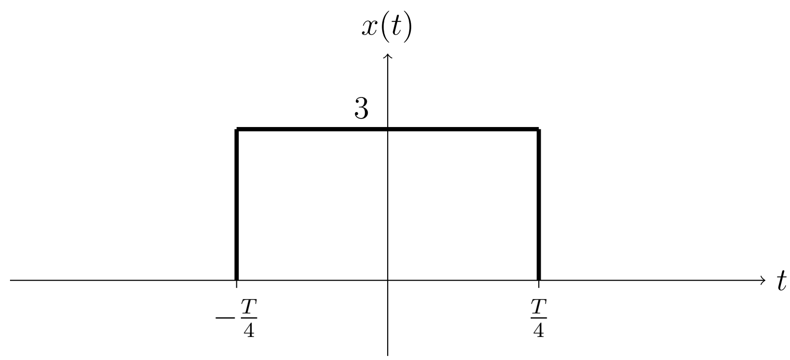



## 11.1 example

$$x(t) = A \times Rect(\frac{t}{T})$$

e.g :

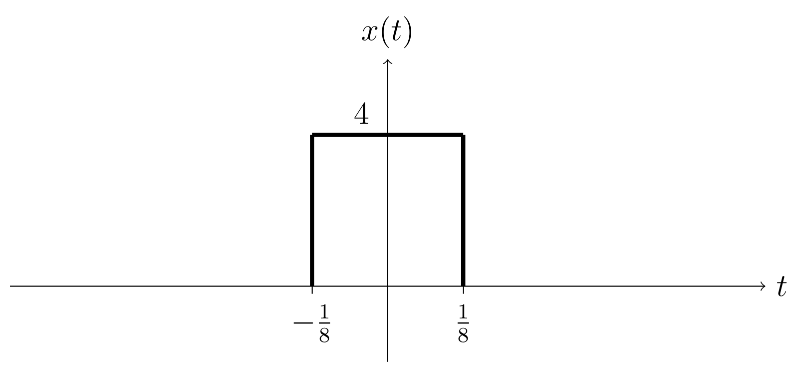
$$
\begin{aligned}
x(t) &= 3 \times Rect(\frac{2t}{T}) \\
&= 3 \times Rect(\frac{t}{\frac{T}{2}}) \\
&\Rightarrow \begin{cases} A = 3 \\ T = \dfrac{T}{2} \end{cases}
\end{aligned}
$$




e.g :

$$x(t) = 4 \times Rect(4t)$$

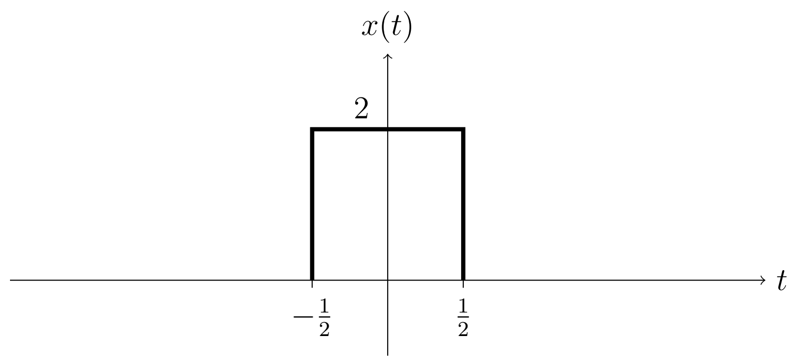
$$\Rightarrow \begin{cases} A = 4 \\ T = \frac{1}{4} \end{cases}$$




e.g :

$$x(t) = 2 \times Rect(t)$$

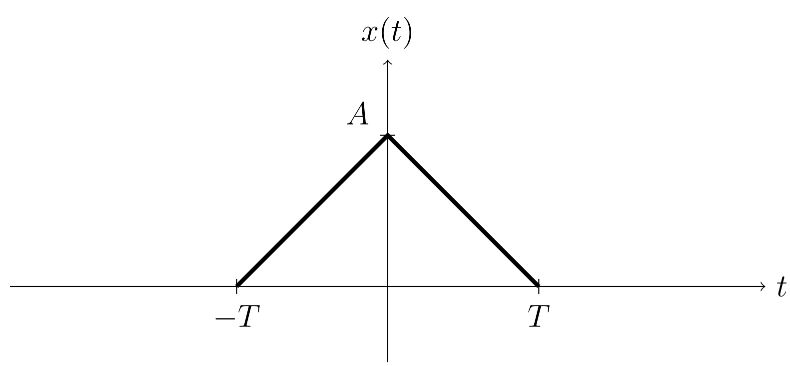
$$\Rightarrow \begin{cases} A = 2 \\ T = 1 \end{cases}$$




# 12 Triangular Signal

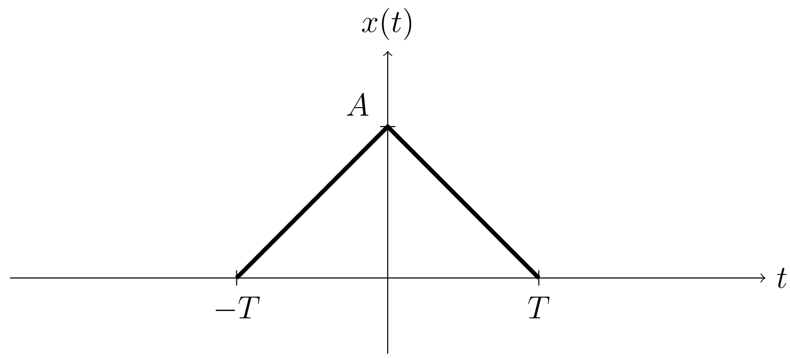
let it be denoted as x(t) :

$$x(t) = A \times \left(1 - \frac{|t|}{T}\right) \Rightarrow \begin{cases} A = \text{amplitude} \\ \\ T = \text{period} \end{cases}$$



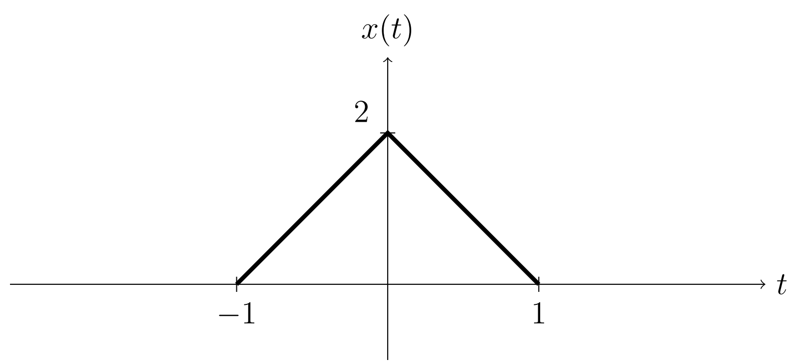
## 12.1 examples

$$x(t) = A \times \left(1 - \frac{|t|}{T}\right)$$



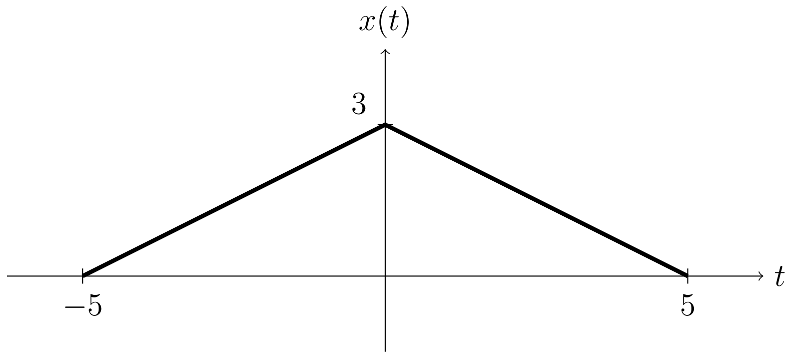

e.g :

$$x(t) = 2 \times \left(1 - \frac{|t|}{1}\right)$$



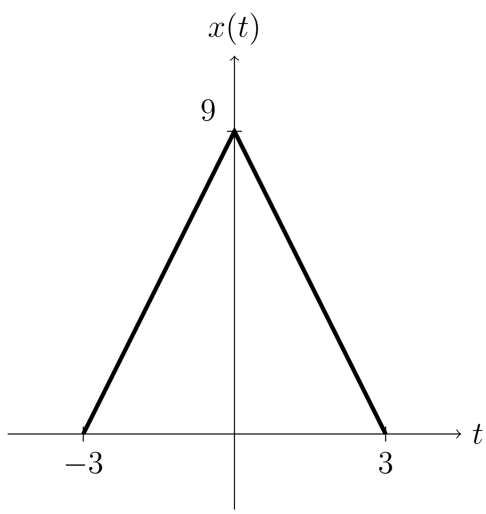

e.g :

$$x(t) = 3 \times \left(1 - \frac{|t|}{5}\right)$$




e.g :

$$x(t) = 9 \times \left(1 - \frac{|t|}{3}\right)$$



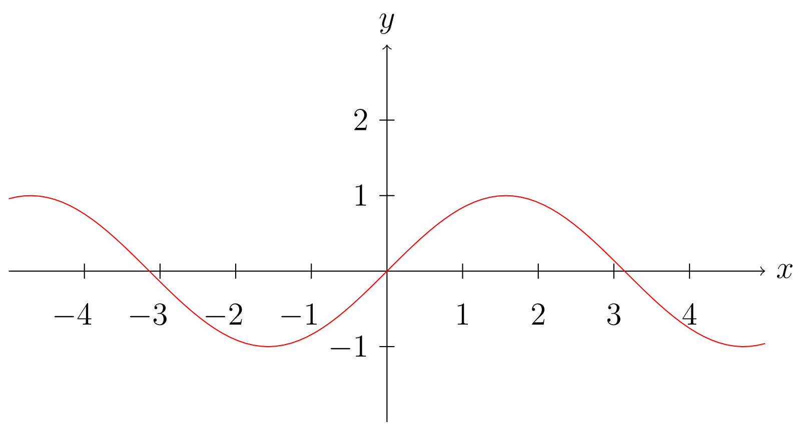

# 13 Sinusoidal Signal

let it be denoted with :

$$x(t) = A\cos(\omega_0 \pm \phi)$$
$$x(t) = A\sin(\omega_0 \pm \phi)$$

where :

$$\phi \rightarrow \text{phase shift}$$




# 14 Operations On Signals

In general we can vary two parameters :

1. amplitude
2. time

Operations can be performed on amplitude are :

- Scaling
- Addition
- Subtraction
- Multiplication

Operations can be performed on time are :

- Shifting
- Scaling
- Reversed

# 15 Amplitude Related Operations

## 15.1 Scaling

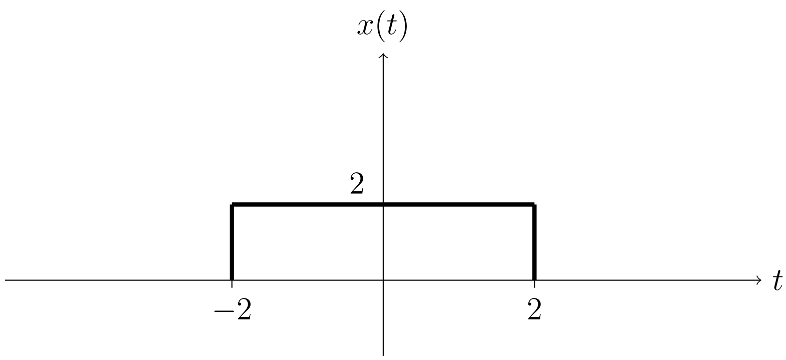
$$y(t) = c \times x(t)$$

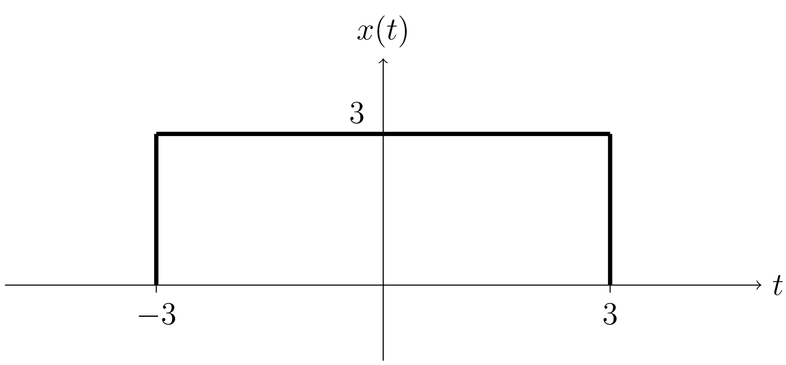
e.g :

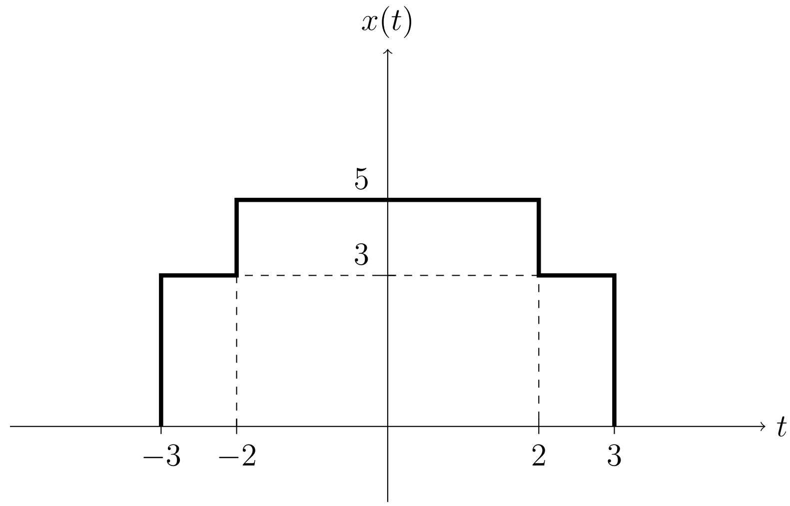
$$x(t) = 4 \times \cos(t)$$

$$y(t) = 2 \times x(t) = 2 \times 4 \times \cos(t) = 8 \times \cos(t)$$

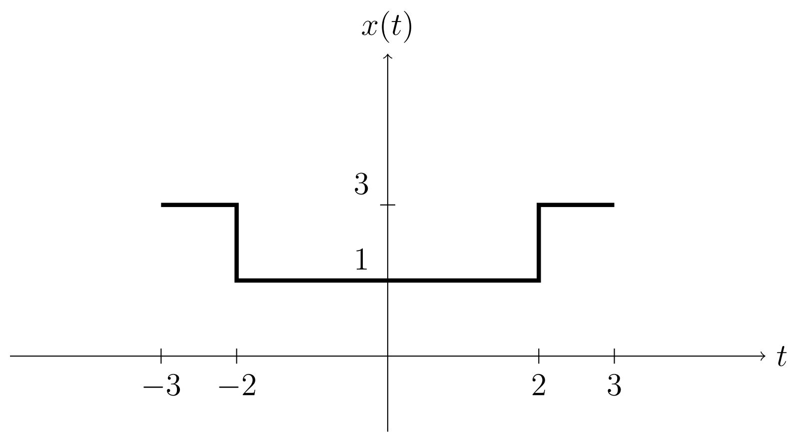
## 15.2 Addition






### 15.3 Substraction



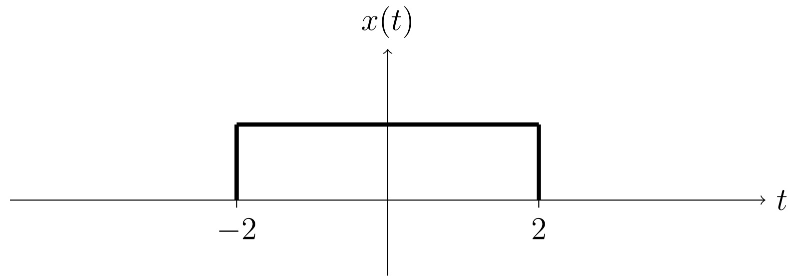

### 15.4 Multiplication

## 16 Time Related Operations

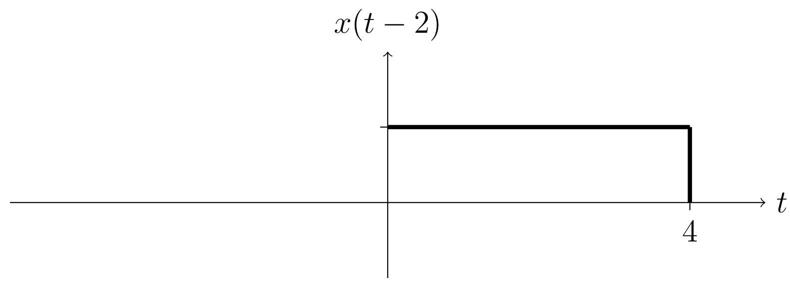
### 16.1 Time Shifting

$$x(t) \to x(t - t_0)$$
$$x(t) \to x(t + t_0)$$

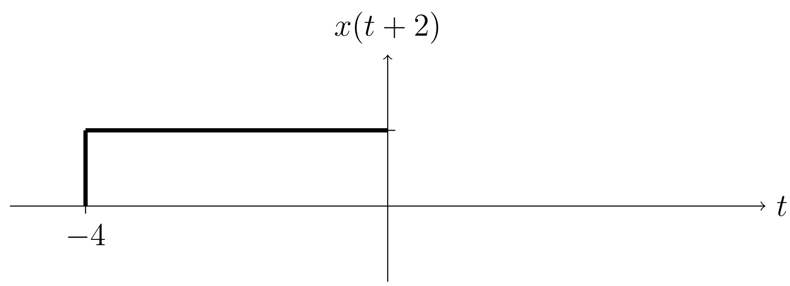
e.g : $x(t)$




e.g : $x(t-2)$




e.g : $x(t+2)$

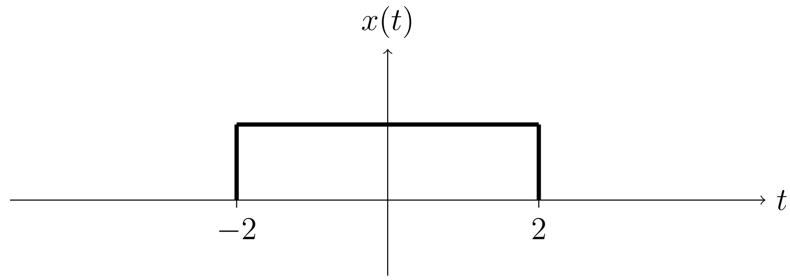



## 16.2 Time Scaling

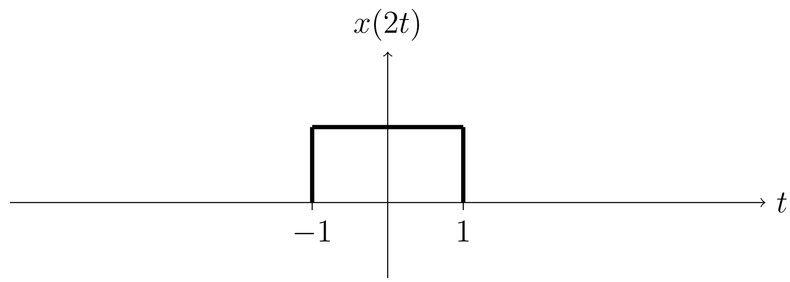
$$x(t) \to x(at)$$

$$|a| > 1 \to \text{companion of signal}$$
$$|a| < 1 \to \text{expansion of signal}$$

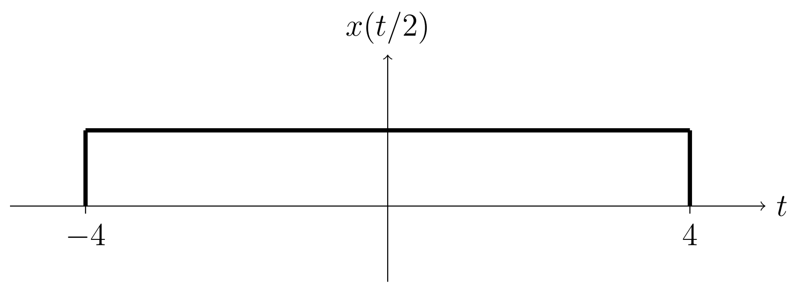
e.g : $x(t)$




e.g : $x(2t)$



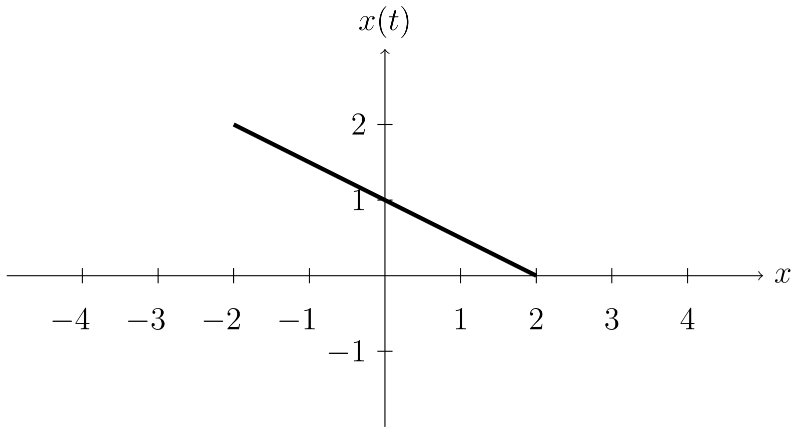

e.g : $x(t/2)$



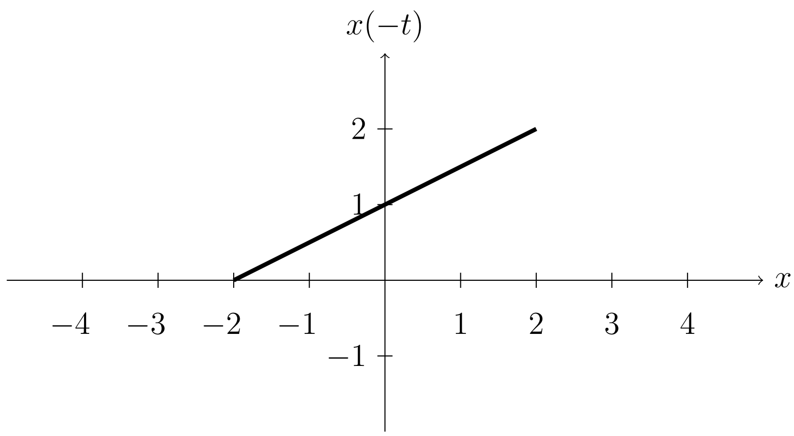

## 16.3 Time Reversal

$$x(t) \to x(-t)$$

e.g : $x(t)$




$\Rightarrow x(-t)$




# 17 Point

Time-Scaling Won't Work for Unit Step function because :

$$\frac{0}{2} = 0$$
$$\frac{\infty}{2} = \infty$$

# 18 Classification of Signals

1. Continuous time & Discrete time Signals

2. Deterministic & non-Deterministic (Random) Signals

3. Even & Odd Signals

4. Periodic & Aperiodic Signals

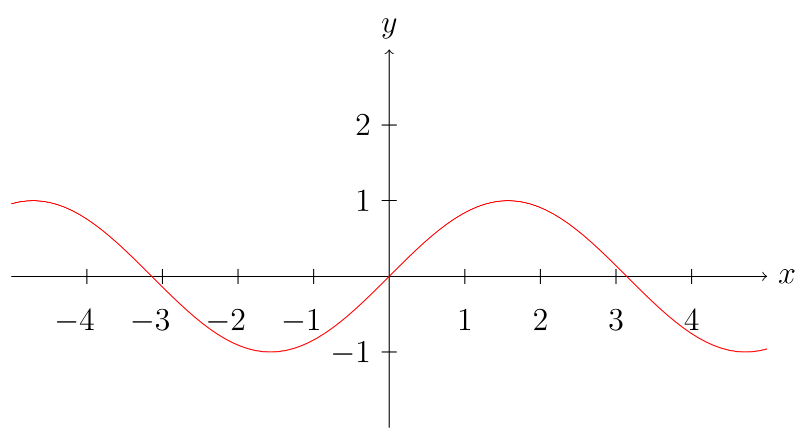
5. Energy Signals & Power Signals

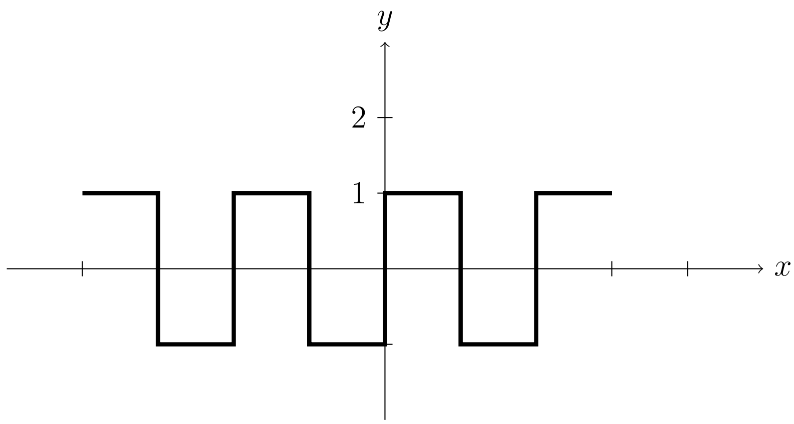
6. Real & Imaginary Signals

## 19 Continuous time & Discrete time Signals

A Signal which is defined for all values of $t$ is called continuous time signals .
A Signal which is defined only at discrete interval of time is called discrete signal

- for discrete signals, time is discrete, amplitude is continuous .
- for digital signal both amplitude & time are discrete .






# 20 Deterministic & non-Deterministic Signals

A signal is said to be deterministic , if There is no uncertainty with respect to its value at any instant of time .
A non-Deterministic Signal is the one which There is uncertainty at any particular instant of time .

# 21 Even & Odd Signals

A Signal is said to be even when it satisfies the condition below :

$$x(-t) = x(t)$$

e.g : $\cos(t), t^2, t^4, \ldots$
A Signal is said to be odd when it satisfies the condition below :

$$x(-t) = -x(t)$$

e.g : $\sin(t), t, t^3, t^5, \ldots$

## 21.1 the even & odd components of the signal

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

# 22 Periodic & Aperiodic Signals

A Signal $x(t)$ is said to be periodic if it satisfies the consition :

$$x(t) = x(t + T)$$

the smallest value of T which satisfies the above condition is called "fundamental time period"

when two signals of same frequency are added the result signal is also sinusoidal signal & periodic .

when two signals of different frequency are added the result may be periodic or non-periodic .

# 23 Energy Signal & Power Signal

## 23.1 finite duration

$$E = \int_{-T}^{T} x^2(t)dt$$

$$P = \frac{1}{T}\int_{\frac{T}{2}}^{-\frac{T}{2}} x^2(t)dt$$

### 23.2 infinite duration

$$E = \lim_{T \to \infty} \int_{-T}^{T} x^2(t)dt$$

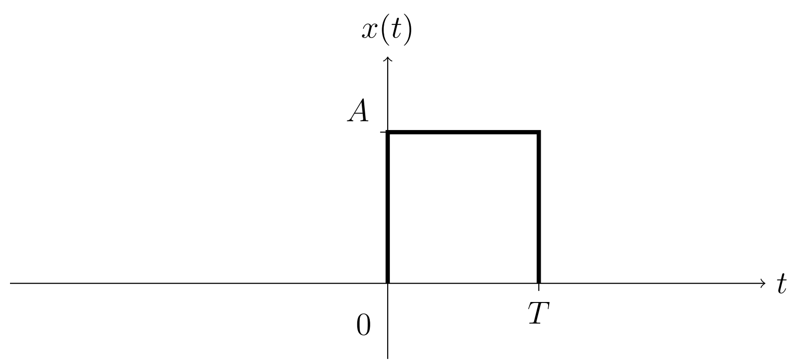
$$P = \lim_{T \to \infty} \frac{1}{T} \int_{\frac{T}{2}}^{-\frac{T}{2}} x^2(t)dt$$

### 23.3 Note

- Power of Energy Signal = 0
- Energy of Power Signal = $\infty$

## 24 Energy & Power Signal Example

$$x(t) = \begin{cases} A & 0 < t < T \\ 0 & other.wise \end{cases}$$



$$E = \int_{-T}^{T} x^2(t)dt$$

$$= \int_{0}^{T} A^2 dt$$

$$= A^2 \int_{0}^{T} dt$$

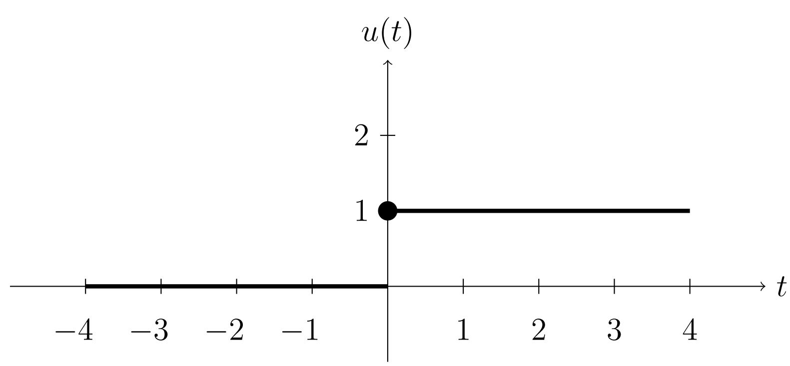
$$= A^2 \left[t\right]_0^T$$

$$= A^2 \left[T - 0\right]$$

$$= A^2 T$$

# 25 Energy & Power of Unit Step function

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



$$E = \lim_{T\to\infty} \int_{-T}^{T} x^2(t)dt$$

$$= \lim_{T\to\infty} \int_{0}^{\infty} 1^2 dt$$

$$= \lim_{T\to\infty} (t)_0^\infty$$

$$= \infty$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 1dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} (t)_0^T$$

$$= \lim_{T\to\infty} \frac{1}{2T}(T-0)$$

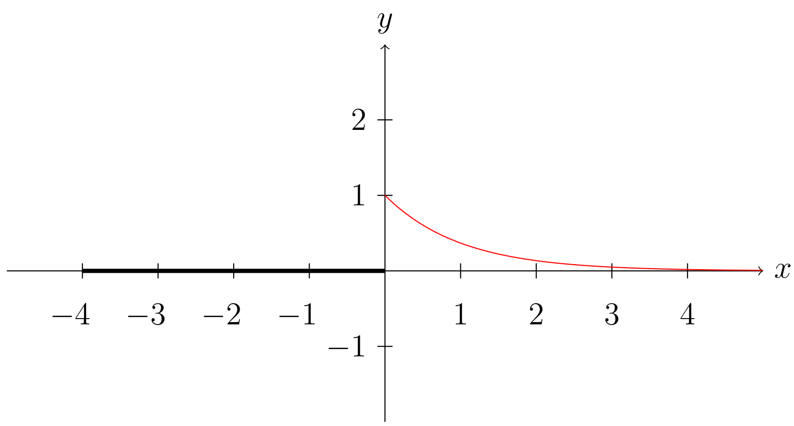
$$= \lim_{T\to\infty} \frac{1}{2T} \times T$$

$$= \frac{1}{2}$$

# 26 Energy & Power Signal of Exponential with Negative Power

$$e^{-at}u(t)$$

$$t < 0 \rightarrow u(t) = 0 \quad \Rightarrow$$



$$E = \lim_{T\to\infty} \int_{-T}^{T} x^2(t)dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} (e^{-at}u(t))^2 dt$$

$$u(t) = 1$$

$$= \lim_{T\to\infty} \int_{0}^{T} (e^{-at})^2 dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} e^{-2at} dt$$

$$= \lim_{T\to\infty} \left[\frac{e^{-2at}}{-2a}\right]_0^T$$

$$= \lim_{T\to\infty} \left[\frac{e^{-2at}}{-2a} - \frac{e^0}{-2a}\right]$$

$$\lim_{u\to\infty} e^{-u} = 0$$

$$= 0 - \frac{1}{-2a}$$

$$= \frac{1}{2a}$$

# 27 Note

- Energy of $x(t) = E$
- Energy of $x(at) = \dfrac{E}{a}$

e.g :

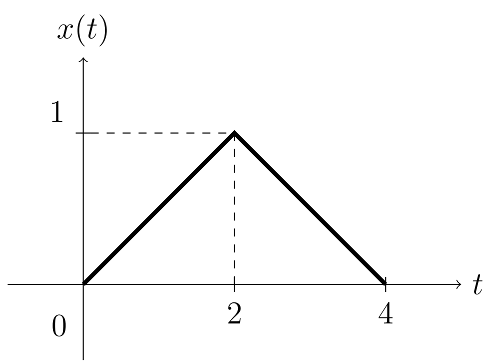
the energy of the signal

$$x(t) = e^{-5t}u(t)$$

is $\dfrac{1}{10}$ then the energy of the time scaled version of signal $x(2t)$ is :

$$E(x(2t)) = \frac{\frac{1}{10}}{2} = \frac{1}{20}$$

# 28 Calculating Energy of Triangular Signal



$$E = \int_{-T}^{T} x^2(t)dt$$

$$= \int_{0}^{1} x^2(t)dt + \int_{1}^{2} x^2(t)dt$$

$$= \int_{0}^{1} t^2(t)dt + \int_{1}^{2} (2-t)^2(t)dt$$

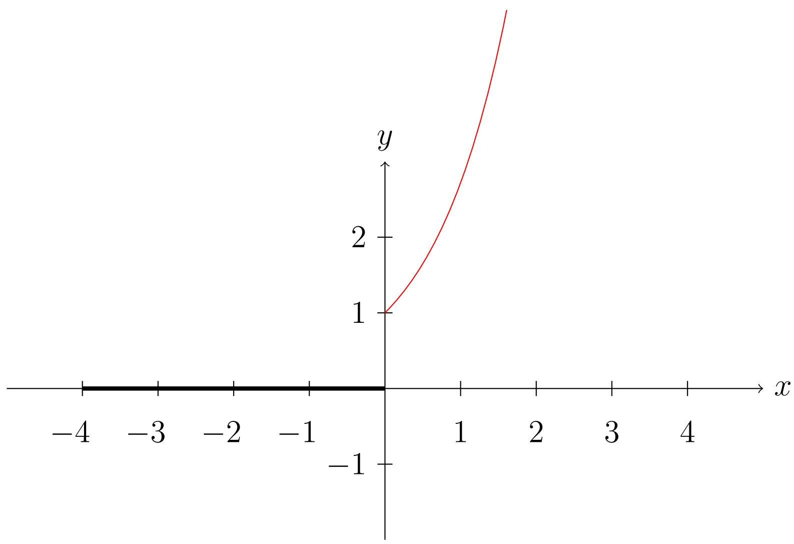
$$= \left[\frac{t^3}{3}\right]_0^1 + \int_{1}^{2} (t^2 - 4t + 4)dt$$

$$= \left[\frac{t^3}{3}\right]_0^1 + \left[\frac{t^3}{3}\right]_1^2 - \left[\frac{4t^2}{2}\right]_1^2 + [4t]_1^2$$

# 29 Energy & Power of Exponential Signal With Positive Power

$$x(t) = e^{at}u(t)$$

$$t < 0 \to u(t) = 0 \quad \Rightarrow$$


y
2
1
x
−4
−3
−2
−1
1
2
3
4
−1

$$E = \lim_{T \to \infty} \int_{-T}^{T} x^2(t) dt$$

$$= \lim_{T \to \infty} \int_{-T}^{T} (e^{-at} u(t))^2 dt$$

$u(t) = 1$

$$= \lim_{T \to \infty} \int_{0}^{T} e^{2at} dt$$

$$= \lim_{T \to \infty} \left[ \frac{e^{2at}}{2a} \right]_0^T$$

$$= \lim_{T \to \infty} \left[ \frac{e^{2aT}}{2a} - \frac{e^0}{2a} \right]$$

$$= \infty$$

$$\Rightarrow E = \infty$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} (e^{-at}u(t))^2 dt$$

$u(t) = 1$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} e^{2at} dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{e^{2at}}{2a}\right]_0^T$$

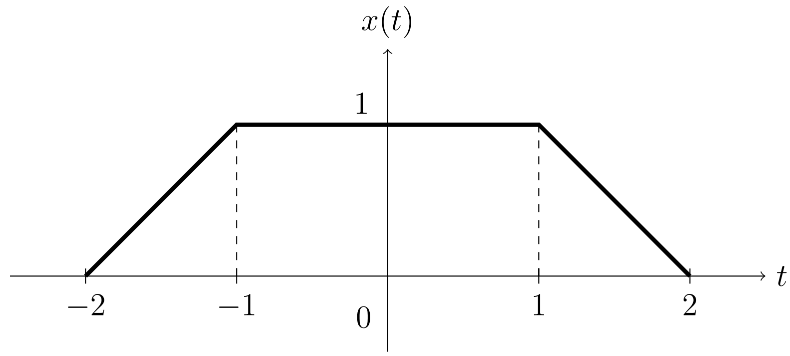
$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{e^{2aT}}{2a} - \frac{e^0}{2a}\right]$$

$$= 0$$

$$\Rightarrow \quad P = 0$$

# 30 Energy & Power Signal Example

Determine The Energy of The Signal given below :



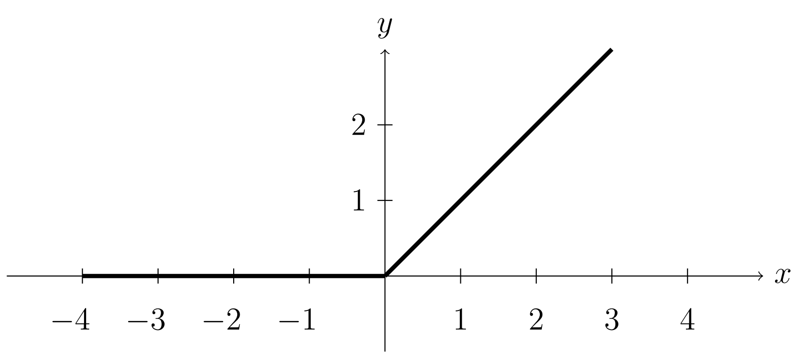

$$E = \int_{-T}^{T} x^2(t)dt$$

$$= \int_{-2}^{2} x^2(t)dt$$

$$= \int_{-2}^{-1} (t+2)^2(t)dt + \int_{-1}^{1} (1)^2(t)dt + \int_{1}^{2} (2-t)^2(t)dt$$

# 31 Energy & Power of Ramp Signal

$$R(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$



$$E = \lim_{T\to\infty} \int_{-T}^{T} x^2(t)dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} x^2(t)dt$$

$$= \lim_{T\to\infty} \int_{0}^{T} t^2(t)dt$$

$$= \lim_{T\to\infty} \left[\frac{t^3}{3}\right]_0^T$$

$$= \lim_{T\to\infty} \frac{T^3}{3}$$

$$\Rightarrow \quad E = \infty$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} x^2(t)dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} t^2(t)dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \left[\frac{t^3}{3}\right]_0^T$$

$$= \lim_{T\to\infty} \frac{1}{2T} \times \frac{T^3}{3}$$

$$= \lim_{T\to\infty} \frac{T^2}{6}$$

$$\Rightarrow \quad P = \infty$$

# 32 Classification of Systems

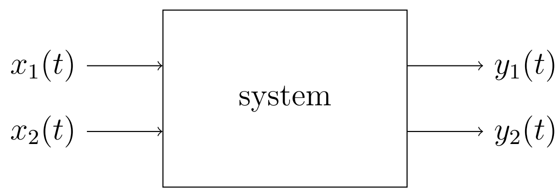
1. Linear & non-Linear Systems
2. Time-Variant & Time-Invariant Systems
3. Linear Time Variant (LTV) & Linear Time Invariant Systems (LTI)
4. Static & Dynamic Systems
5. Casual & non-Casual Systems
6. Invertible & non-Invertible Systems

7. Stable & unStable Systems

# 33 Linear & non-Linear Systems

A System is said to be Linear if it satisfies the superposition principle, consider a system with input $x_1(t)$ , $x_2(t)$ and output $y_1(t)$ , $y_2(t)$ for be Linear :

$$T[a_1x_1(t) + a_2x_2(t)] = a_1T[x_1(t)] + a_2T[x_2(t)]$$




#### 33.0.1 example

$$y(t) = x^2(t)$$

$$T[x_1(t)] = x_1^2(t) \Rightarrow a_1T[x_1(t)] = a_1x_1^2(t)$$
$$T[x_2(t)] = x_2^2(t) \Rightarrow a_2T[x_2(t)] = a_2x_2^2(t)$$

$$T[a_1x_1(t) + a_2x_2(t)] = [a_1x_1(t) + a_2x_2(t)]^2$$

$$\Rightarrow T[a_1x_1(t) + a_2x_2(t)] \neq a_1T[x_1(t)] + a_2T[x_2(t)]$$

$\Rightarrow$ The System is non-Linear

### 33.0.2 example

$$y(t) = x(t)$$

$$T[x_1(t)] = x_1(t) \Rightarrow a_1 T[x_1(t)] = a_1 x_1(t)$$
$$T[x_2(t)] = x_2(t) \Rightarrow a_2 T[x_2(t)] = a_2 x_2(t)$$

$$T[a_1 x_1(t) + a_2 x_2(t)] = [a_1 x_1(t) + a_2 x_2(t)]$$

$$\Rightarrow T[a_1 x_1(t) + a_2 x_2(t)] = a_1 T[x_1(t)] + a_2 T[x_2(t)]$$

$\Rightarrow$ The System is Linear

# 34 Time-Variant & Time-Invariant Systems

A System is said to be time variant if its input, output characteristics change with time , otherwise said to be time invariant .

The condition for time invariant is

$$y(n,k) = y(n-k)$$

where

$$y(n,k) = T[x(n-k)]$$

### 34.0.1 example

$$y(n) = x(n) + x(n-2)$$

$$\begin{aligned} y(n,k) &= T[x(n-k)] \\ &= x(n-k) + x(n-2-k) \end{aligned}$$

$$\begin{aligned}y(n-k) &= x(n-k) + x(n-k-2)\\ &\Rightarrow y(n,k) = y(n-k)\\ &\Rightarrow \text{System is Time-Invariant}\end{aligned}$$

#### 34.0.2 example

$$y(n) = x(n) + nx(n-3)$$

$$\begin{aligned}y(n,k) &= T[x(n-k)]\\ &= x(n-k) + nx(n-k-3)\end{aligned}$$

$$\begin{aligned}y(n-k) &= x(n-k) + (n-k)x(n-k-3)\\ &\Rightarrow y(n,k) \neq y(n-k)\\ &\Rightarrow \text{System is Time Variant}\end{aligned}$$

# 35 Linear Time Variant (LTV) & Linear Time Invariant Systems (LTI)

A System is said to be LTV when it satisfies both Linearity & Time Variant .

A System is said to be LTI when it satisfies both Linearity & Time InVariant .

#### 35.0.1 example

check for linearity

$$y(n) = nx^2(n)$$

$$\begin{aligned}y_1(t) = T[x_1(t)] = nx_1^2(t) \Rightarrow a_1T[x_1(t)] = a_1nx_1^2(t)\\ y_2(t) = T[x_2(t)] = nx_2^2(t) \Rightarrow a_2T[x_2(t)] = a_2nx_2^2(t)\end{aligned}$$

$$T[a_1x_1(t) + a_2x_2(t)] = n[a_1x_1(t) + a_2x_2(t)]^2$$

$$\Rightarrow T[a_1x_1(t) + a_2x_2(t)] \neq a_1T[x_1(t)] + a_2T[x_2(t)]$$

$$\Rightarrow \text{The System is non-Linear}$$

check for time variant or invariant .

$$y(n) = nx^2(n)$$

$$\begin{aligned} y(n,k) &= T[x(n-k)] \\ &= nx^2(n-k) \end{aligned}$$

$$\begin{aligned} y(n-k) &= (n-k)x^2(n-k) \\ &\Rightarrow y(n,k) \neq y(n-k) \\ &\Rightarrow \text{System is Time Variant} \end{aligned}$$

### 35.0.2 example

$$y(n) = x(n-2)$$

check for linearity

$$\begin{aligned} y_1(t) = T[x_1(t)] = x_1(t-2) \Rightarrow a_1T[x_1(t)] = a_1x_1(t-2) \\ y_2(t) = T[x_2(t)] = x_2(t-2) \Rightarrow a_2T[x_2(t)] = a_2x_2(t-2) \end{aligned}$$

$$T[a_1x_1(t) + a_2x_2(t)] = a_1x_1(t-2) + a_2x_2(t-2)$$

$$\Rightarrow T[a_1x_1(t) + a_2x_2(t)] = a_1T[x_1(t)] + a_2T[x_2(t)]$$

$$\Rightarrow \text{The System is Linear}$$

check for time variant or invariant .

$$y(n) = x(n-2)$$

$$\begin{aligned} y(n,k) &= T[x(n-k)] \\ &= x(n-k-2) \end{aligned}$$

$$\begin{aligned} y(n-k) &= x(n-k-2) \\ &\Rightarrow y(n,k) = y(n-k) \\ &\Rightarrow \text{System is Time Invariant} \end{aligned}$$

## 36 Static & Dynamic Systems

Static System is memoryless system where as Dynamic System is with memory System .

e.g :

$$\begin{aligned} y(n) &= x(n) \\ y(0) &= x(0) \rightarrow \text{Static} \end{aligned}$$

e.g :

$$\begin{aligned} y(t) &= 2x^2(t) \\ y(-2) &= 2x^2(-2) \rightarrow \text{Static} \end{aligned}$$

e.g :

$$\begin{aligned} y(n) &= x(n) + x(n-1) \\ y(1) &= x(1) + x(1-1) \\ &= x(1) + x(0) \rightarrow \text{Dynamic} \end{aligned}$$

e.g :

$$\begin{aligned} y(t) &= x(t) + x(t+3) \\ y(-1) &= x(-1) + x(-1+3) \\ &= x(-1) + x(2) \rightarrow \text{Dynamic} \end{aligned}$$

# 37 Casual & non-Casual Systems

A System is said to be casual if its response is dependent upon present & past inputs & doesn't depends upon future input .

for a non-casual System The output depends upon future input too .

e.g :

$$\begin{aligned} y(n) &= x(n) + \frac{1}{x(n-1)} \\ y(1) &= x(1) + \frac{1}{x(1-1)} \\ y(1) &= x(1) + \frac{1}{x(0)} \Rightarrow \text{Casual} \end{aligned}$$

e.g :

$$\begin{aligned} y(t) &= 2x(t) + \frac{1}{x^2(t)} \\ y(0) &= 2x(0) + \frac{1}{x^2(0)} \\ &= x(1) + \frac{1}{x(0)} \Rightarrow \text{Casual} \end{aligned}$$

e.g :

$$
\begin{aligned}
y(n) &= x(n) + \frac{1}{2x(n+1)} \\
y(0) &= x(0) + \frac{1}{2x(1)} \\
&= x(1) + \frac{1}{x(0)} \Rightarrow \text{non-Casual}
\end{aligned}
$$

e.g :

$$
\begin{aligned}
y(t) &= x(t) + x(t-1) + \frac{1}{x(t+1)} \\
y(0) &= x(0) + x(0-1) + \frac{1}{x(0+1)} \\
y(0) &= x(0) + x(-1) + \frac{1}{x(1)} \Rightarrow \text{non-Casual}
\end{aligned}
$$

All Static Systems Are Casual but not Vice Versa

All non-Casual Systems Are Dynamic but not Vice Versa

# 38 Invertible & non-Invertible Systems

A System is said to be Invertible if the input of the system appears at the output

$$x(t) \longrightarrow \boxed{h_1(t)} \longrightarrow \boxed{h_2(t)} \longrightarrow y(t) = x(t)$$

if $y(t) \neq x(t)$ then the system is non-invertible

# 39 Stable & unStable Systems

A System is said to be stable when it produces bounded output for a bounded input .

e.g :

$$y(n) = x^2(n) \rightarrow \text{stable}$$

e.g :

$$y(t) = \int x(t)dt \rightarrow \text{unstable}$$