The Two-Stage Assembly Flow-Shop Scheduling Problem with Setup Time for Minimize Tardiness and Earliness

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ABSTRACT
Global competition and the requirement to control production costs, make factories design production and assembly lines that can be produced by combining small, diverse, and multiple components. The assembly flow-shop is a combined production system in which different production operations are carried out independently and simultaneously. In this problem, some machines process different jobs. Then, these jobs are transferred into the assembly stage. In this article, the assembly flow-shop scheduling model is being investigated and solved for minimization of setup times, tardiness, and earliness. The value of the objective function is analyzed by varying the parameters for different problems.

KEYWORDS: Scheduling, two-stage flow-shop, tardiness, earliness, delivery time, setup time.

1. Introduction

Assembly flow-shop is a combined system in which different production operations are carried out independently and simultaneously. All jobs in this work environment require accurate scheduling for work progress. Scheduling is a type of decision-making process to optimize one or several objectives and has turned into an essential need for factories in the present market.

This research aims to make scheduling problems close to reality. The paper also aims to involve earliness times in scheduling problems to show that earliness time significantly impacts customers' satisfaction. Furthermore, the necessity and importance of the current research are that the maximum customers' satisfaction is obtained by optimizing goods delivery time.

The problem consists of n different jobs that each is processed on $m + 1$ different machines. There are m machines in the manufacturing stage and only one machine is incorporated in the assembly stage. Each machine needs to be set up before processing. Material and primary components are always available without any failure or problem. In the manufacturing stage, a given machine can only be operating at a specific time and each job can only be processed on one machine. In the assembly stage, there is only one job station consisting of one machine to assembly the jobs. The jobs associated with a product are all packed in this stage and finally, the product is delivered to the customer.

Two-stage assembly flow-shop scheduling problems have been analyzed in different ways. Kazemi et al. [1], for instance, consider the two-stage assembly flow-shop scheduling problem with delivery and tardiness costs.

One item that can make the solutions more accurate is considering the setup time of machinery. This problem has been solved using the genetic algorithm (GA) to minimize tardiness [2]. In the cases the setup time of machinery is assumed time-independent, the completion time will be more accurate [3].

In another study, the scheduling problem was addressed using a metaheuristic algorithm aiming for maximizing the production capacity, where setup times are flexible specifically [4].

In recent years, various pieces of research have been adopted to deal with scheduling problems. Other studies focusing on this problem are described in Table 1.

Table 1. Studies on the scheduling problem

Reference	Publication year	Parameter			Objective function		Method		
		Setup	Tardiness	Earliness	Single-objective	Multi-objective	heuristic	metaheuristic	Exact
[1]	2017		*		*			*	
[2]	2013	*	*		*			*	
[3]	2006	*	*		*			*	
[4]	2018	*	*		*			*	
[5]	2011				*		*		
[6]	2018		*		*		*		
[7]	2018		*		*			*	
[8]	2016		*		*			*	
[9]	2011	*	*		*			*	
[10]	2012		*		*			*	
[11]	2010		*		*			*	
[12]	2015	*	*		*			*	
[13]	2017		*		*			*	
[14]	2016	*	*		*			*	
[15]	2007	*	*		*			*	
[16]	2018		*		*				*
[17]	2017		*			*	*		
[18]	2018		*	*	*			*	
[19]	2015		*		*			*	

1. Two-stage assembly flow-shop scheduling problem

1.1. Problem assumptions

1) There are *n* available jobs with *m* + 1 operations, of which *m* operations are in the processing stage and one is in the assembly stage.
2) All jobs are available at time zero.
3) Machines are always available and their failure rate is negligible.
4) It is essential to do the jobs of a given customer consequently, then go to the assembly stage. In other words, jobs i1, i2, i3, … all need to be terminated in the processing stage and then they go to the assembly stage to assemble produce j belonging to the corresponding customer.
5) Setup times are independent of the processing time and are definite.
6) A job is not allowed to be terminated, i.e. there is no return to back or emergency repairs.
7) Delivery times and costs are allowed and definite.
8) Each machine can do one single job at a time, and each job can be performed by only one machine.

1.2. Indices, parameters, and variables

Indices:

i: jobs $i=1,2,3, \ldots, n_j$

j: customer number $j=1,2,3, \ldots, H$

m: Machinery number (first stage) $m=1,2, \ldots, M$

k: job ranking number (first stage) $k=1,2, \ldots, N$

b: category number $b=1,2,3, \ldots, N$

Parameters:

SM_{ji}: Assembly time of job i related to customer j

dd_{ij}: Delivery time of job i related to customer j

D_j: Delivery cost of jobs to customer j

α: Per unit cost of earliness

β: Per unit cost of tardiness

n_j: Order number of customer j

Variables:

P'_{mk} : Processing time of jobs in step k that are processed on machine m in the first stage.

F_{km}: Completion time of jobs in step k that are processed on machine m in the first stage.

FF_k: Time when the *k*th jobs are ready for assembly operations in the second stage.

SM'_k: Assembly time of jobs in step k that are processed in the second stage.

C_{bj}: Completion time of category b that belongs to customer j

C_{ij}: Completion time of job i that belongs to customer j

CT_k: Completion time of the (k-1)th job entered to the assembly stage

R_k: Time when assembly operations on the kth job starts on the assembly machine in the second stage.

T_{ij}: Tardiness time of job i related to customer j

E_{ij}: Delivery time of job i related to customer j

S_{km}: Setup time of the job in position k that is processed on machine m in the first stage

X_{ijk}: In the first stage, if job i related to customer j is in position k, this variable will be 1, and is 0 otherwise.

Y_k: When the kth job enters the assembly stage, this variable will be 1, and is zero otherwise.

Z_{ijb}: This variable is 1 when job i belonging to customer j is in category b, and is 0 otherwise.

V_{jb}: This variable is 1 when a job belonging to customer j is in category b, and is 0 otherwise.

The following model is extracted from [1] and extended for the scheduling problem.

$$\min \sum_{j=1}^{H}\sum_{i=1}^{nj}\beta \times T_{ij} + \sum_{j=1}^{H}\sum_{b=1}^{N} D_j \times V_{jb} + \sum_{j=1}^{H}\sum_{i=1}^{nj}\alpha \times E_{ij} \,. \tag{1}$$

$$T_{ij} \geq 0 \quad \forall_{i,j} \tag{2}$$

$$T_{ij} \geq R_{ij} - d_{ij} \quad \forall_{i,j} \tag{3}$$

$$E_{ij} \geq 0 \quad \forall_{i,j} \tag{4}$$

$$E_{ij} \geq d_{ij} - R_{ij} \quad \forall_{i,j} \tag{5}$$

$$\sum_{b=1}^{N} Z_{ijb} = 1 \quad \forall_{i,j} \tag{6}$$

$$\sum_{j=1}^{H} Z_{ijb} \leq 1 \quad \forall_{i,b} \tag{7}$$

$$R_{ij} + M(1 - Z_{ijb}) \geq C_{bj} \quad \forall_{j,b,i} \tag{8}$$

$$C_{bj} + M(1 - Z_{ijb}) \geq C_{ij} \quad \forall_{j,b,i} \tag{9}$$

$$C_{ij} + M(1 - X_{ijk}) \geq C_k \quad \forall_{j,b,i} \tag{10}$$

$$\sum_{k=1}^{N} X_{ijk} = 1 \quad \forall_{i,j} \tag{11}$$

$$\sum_{j=1}^{H}\sum_{i=1}^{mj} X_{ijk} = 1 \quad \forall_{k} \tag{12}$$

$$C_1 = Y_1 \times (FF_1 + SM_1') \tag{13}$$

$$C_k = Y_k \times (R_k + SM_k') \quad \forall_{k,k\neq 1} \tag{14}$$

$$R_k \geq FF_k \quad \forall_{k,k\neq 1} \tag{15}$$

$$R_k \geq CT_k \quad \forall_{k,k\neq 1} \tag{16}$$

$$CT_k \geq C_{(e-1)} \quad \forall_{k,k\neq 1,e=2,3,\ldots,k} \tag{17}$$

$$FF_k \geq F_{km} \quad \forall_{k,m} \tag{18}$$

$$F_{km} = \sum_{a=1}^{k} P_{km}' \quad \forall_{k,m} \tag{19}$$

$$P_{hm}' = \sum_{j=1}^{H}\sum_{i=1}^{mj}\left(X_{ijk} \times \left(P_{ijm} + S_{kn} \right)\right) \quad \forall_{k,m} \tag{20}$$

$$SM_k' = \sum_{j=1}^{H}\sum_{i=1}^{nj}\left(X_{ijk} \times SM_{ij} \right) \quad \forall_{k} \tag{21}$$

$$\sum_{i=1}^{nj} Z_{ijb} + M(1 - V_{jb}) > 0 \quad \forall_{j,b} \tag{22}$$

$$\sum_{i=1}^{nj} Z_{ijb} + M \times V_{jb} \leq 0 \quad \forall_{j,b} \tag{23}$$

Equation (1) is the objective function, which includes minimization of transport, earliness, and tardiness. Constraints (2)-(3) calculate tardiness. Equations (4)-(5) calculate earliness for each job. Constraint (6) ensures the existence of a job in a category. Constraint (7) prevents the jobs of different customers from being in the same category. Constraint (8) calculates the transport time. Constraint (9) calculates the competition time of categories, which is equal to the longest competition time of jobs for that category. Constraint (10) calculates job I related to customer j. In constraints (11) and (12), each machine can perform a job at a given time and each job can be processed on a specific machine. Constraints (13)-(14) find the competition time of a job for the first and *k*th ranks. Constraints (15), (16), (17), and (18) are dependent on variables definition. Constraint (19) finds the competition time of jobs that depend on processing time. Constraint (20) calculates the processing time of the *k*th job according to the setup time of machines. Constraint (21) finds the assembly time of the *k*th rank. Constraints (22)-(23), ensure that a category appears when it contains at least one job.

2. Solution and results

The presented model was solved in GAMS software, and the output of its objective function was 4.051. This result shows that if job *i* in position *k* and category *b* is processed on machine *m*, then after the processing time it goes to the assembly stage that has one machine, the whole process time should be completed in a time of at least 4.051. Comparing this problem with other studies can be realized in three ways:

1. Comparison with references in which the earliness time and the setup time of machinery are not considered.

Table 2. Results of a comparison between the considered problem (1) and other references

Reference	No. of jobs	No. of machines	No. of customers	No. of job places	Initial results	Secondary results
[8]	4	4	2	2	5.010	4.734
[17]	2	3	7	2	5.387	5.012
[11]	6	1	2	4	5.453	5.067
[18]	5	3	4	3	5.403	5.056

Assume that a product has been produced under the conditions given in [17]. The customer needs this product at time a, and if the product is not delivered to the customer at the proper time, the industrial unit will incur a cost named tardiness cost. This problem aims to minimize this cost subject to all mentioned conditions. Considering the parameters of the problem [17], the initial result of the problem will be 5.378, meaning that the product was produced in such a condition that it has been delivered to the customer with delay. Also, the obtained result is not realistic as it neglects the setup time of machinery and earliness. It is assumed that a product is produced considering the earliness and setup parameters. The customer returns the product and believes that the good has been delivered sooner and he does not need it. In such a situation, the organization incurs a cost named earliness cost. Considering the setup time of machinery for producing a new product, the secondary result is 5.012. This indicates that better and more realistic solutions will be obtained by adding more parameters to the problems.

2. Comparison with references that consider the earliness time but neglect setup time of machinery.

Table 3. Results of a comparison between the considered problem (2) and other references

Reference	No. of jobs	No. of machines	No. of customers	No. of job places	Initial results	Secondary results
[18]	3	4	2	3	5.013	4.312

3. Comparison with references that neglect the earliness time but consider setup time of machinery

Table 4. Results of a comparison between the considered problem (3) and other references

Reference	No. of jobs	No. of machines	No. of customers	No. of job places	Initial results	Secondary results
[9]	4	4	2	3	5.288	4.563
[4]	2	3	2	7	4.912	4.30012

The goal of this type of comparison is to analyze each case separately to prove that adding a new parameter to the problem leads the problem to a more realistic situation and provides more accurate solutions.

4. Analysis of numerical results

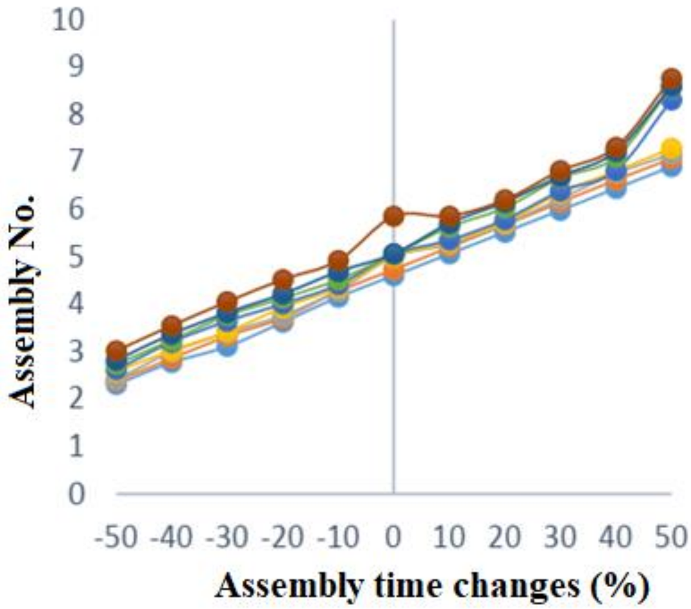

Fig. 1. Sensitivity analysis of assembly time

According to Fig. 1, as the assembly time of the job increases, even under conditions such as considering the earliness and setup time of machinery, the objective function becomes worse again but its growth reduces.

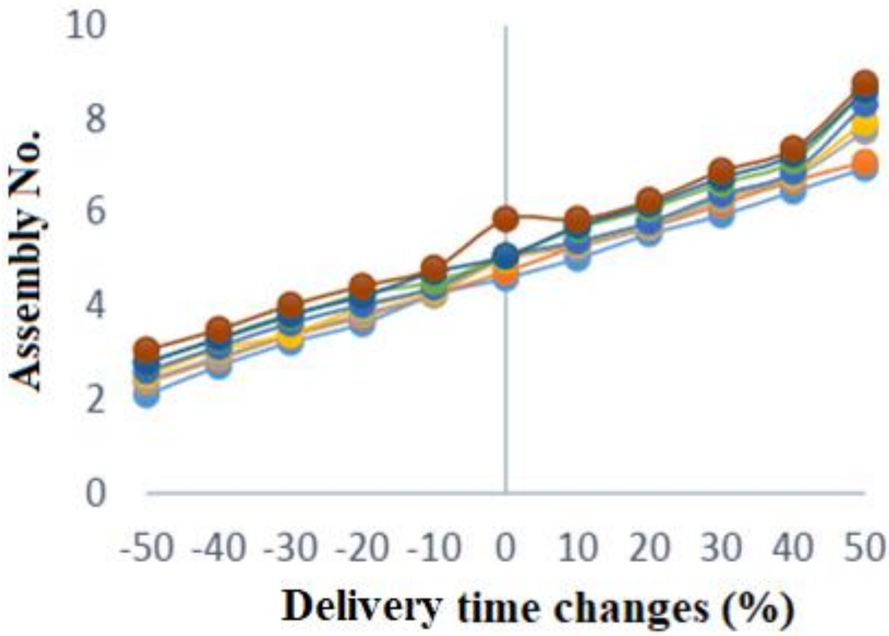

Fig. 2. Sensitivity analysis of delivery time

Fig. 2 shows that as the delivery time of jobs in a production line reduces, considering conditions such as earliness and setup time of machinery, the objective function of the problem becomes better but its growth reduces.

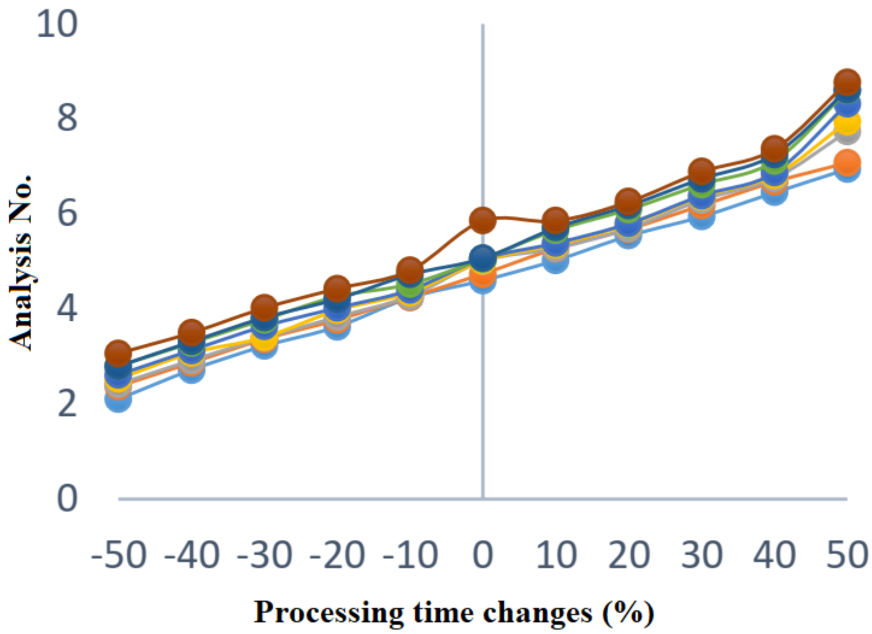

Fig. 3. Sensitivity analysis of processing time

As is seen in Fig. 3, by reducing the assembly time of the job in the form of assumed oscillations, the production time of the product reduces accordingly. However, note that this curve is descending in some problems, and reducing the assembly time as 10% and 20% oscillations gives a worse solution. This states that, in these problems, reducing assembly time of jobs to some extent has no positive impact on minimizing the delivery time of a product to the customer.

5. Conclusion

As mentioned, assembly flow-shop is a combined production system in which different production operations are carried out independently and simultaneously. In this research, the assembly flow-shop scheduling problem was modeled and solved in GAMS considering the setup time to minimize the earliness and tardiness. The results were then compared with those of other papers, showing that with increasing the number of parameters in the problem more acceptable and realistic solutions are found. On the other hand, sensitivity analyses of assembly time, delivery time, and processing time were conducted. These changes and their related results highlight the importance of earliness and tardiness to which the manufacturing units should pay more attention to reduce their cost. This problem can also be applied by assuming uncertainty in the parameters and/or the existence of two or several machines in the assembly stage where failure is allowed.

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