Department of Mathematics and Natural Sciences
MAT216: Linear Algebra & Fourier Analysis
Summer 2023

ASSIGNMENT 3

Faculty Name: **Mark:** 50

---

*Make a **Front Page** by yourself, mentioning your #name, #ID, and #section. (Compulsory)*

---

1. Calculate the basis and dimension of the four fundamental subspaces of the matrix A, $(5\times 3 = 15)$

(a) $A = \begin{bmatrix} 1 & 0 & 3 & -3 \\ 2 & 3 & 2 & -6 \\ 2 & 2 & 3 & -6 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 0 & 3 \\ 3 & -4 & 2 \\ 7 & 9 & -2 \end{bmatrix}$ (c) $A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 1 & 6 \\ 2 & 1 & 9 \end{bmatrix}$

2. Which of the following sets of vectors are bases for the subspace S, $(2.5 \times 4 = 10)$

(a) $v_1 = (4, 1), v_2 = (-7, -8)$ $S = \mathbb{R}^2$

(b) $v_1 = (4, 8), v_2 = (-12, -24)$ $S = \mathbb{R}^2$

(c) $v_1 = (3, 1, -4), v_2 = (2, 5, 6), v_3 = (1, 4, 8)$ $S = \mathbb{R}^3$

(d) $v_1 = (1, 6, 4), v_2 = (2, 4, -1), v_3 = (-1, 2, 5)$ $S = \mathbb{R}^3$

3. Determine whether A is diagonalizable. If so, find $P^{-1}AP$. $(5 \times 2 = 10)$

(a) $A = \begin{bmatrix} 19 & -9 & -6 \\ 25 & -11 & -9 \\ 17 & -9 & -4 \end{bmatrix}$ (b) $A = \begin{bmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{bmatrix}$

4. Find the orthogonal complement($W^\perp$) and basis of $W^\perp$ of the subspace W, where, $(5 \times 2 = 10)$

(a) $W = span\left\{\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 4 \end{bmatrix}\right\}$ (b) $W = span\left\{\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 22 \\ -4 \\ 4 \end{bmatrix}, \begin{bmatrix} 23 \\ -4 \\ 6 \end{bmatrix}\right\}$

5. Use the Gram–Schmidt process to transform the basis (v_1, v_2, v_3) into an orthonormal basis. (5)

$$v_1 = (1, 0, 0), v_2 = (3, 7, -2), v_3 = (0, 4, 1)$$