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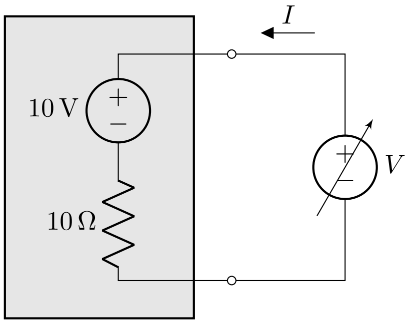
Brac University
Semester: Spring 2023
Course Code: CSE250
Circuits And Electronics

Assessment: *Midterm*
Duration: 1 hour 30 minutes
Date: March 5, 2023
Full Marks (incl. bonus 6): 56

- ✓ No washroom breaks. Phones must be turned off. Using/carrying any notes during the exam is not allowed.
- ✓ At the end of the exam, both the **answer script** and the **question paper** must be returned to invigilator.
- ✓ All **3 questions** are compulsory. Marks allotted for each question are mentioned beside each question.
- ✓ Write your answers inside the indicated boxes where applicable.
- ✓ Symbols have their usual meanings.

■ Question 1 of 3 *[CO1, CO3] [20 marks]*

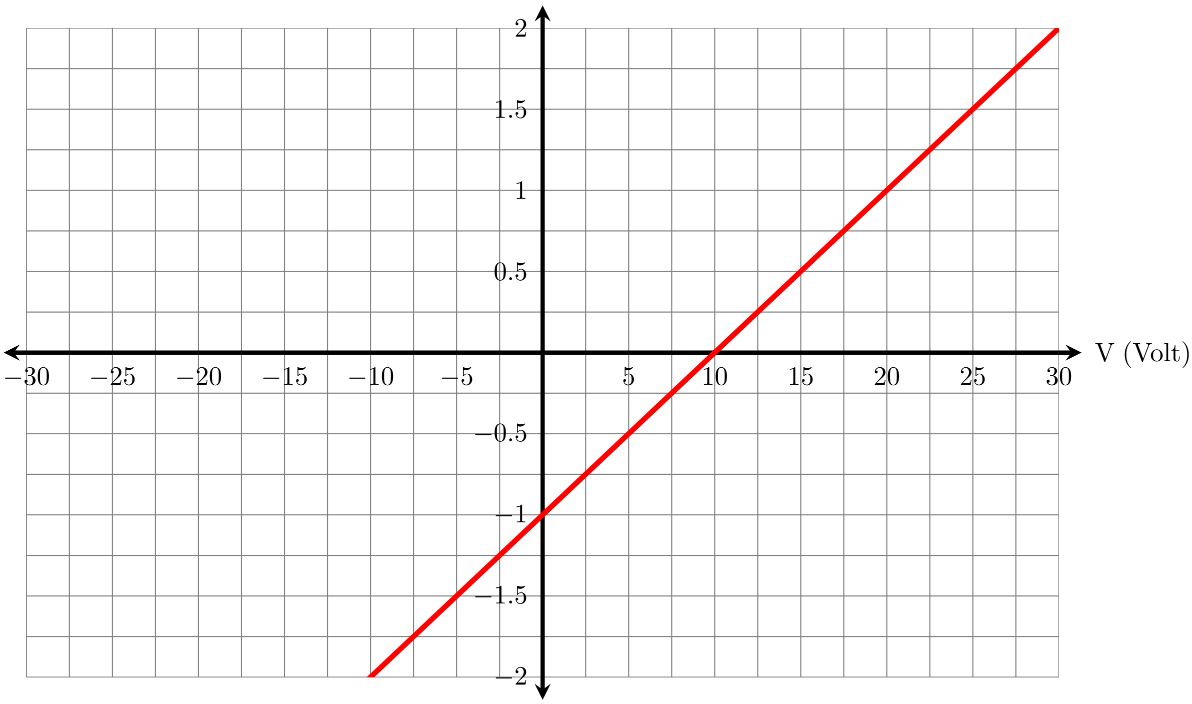
(a) In order to test the $I - V$ characteristics of a two-terminal linear circuit (inside the gray box), the following circuit was constructed.



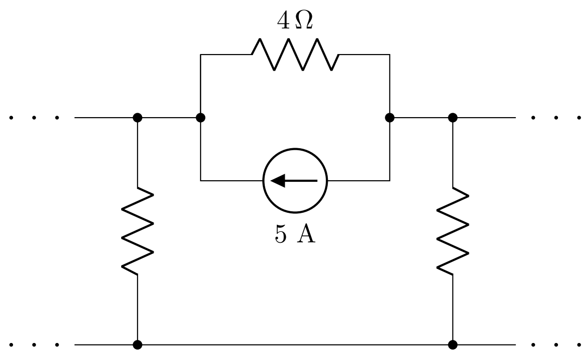
(i) *[1 mark]* **Determine** the relationship between I and V, where V is the applied voltage difference across the test circuit that is varied and I is the current through it. In the following box write I in terms of V.

$$I = \frac{1}{10}V - 1$$

(ii) *[2 marks]* Based on your answer in (i), plot the $I - V$ characteristics of the test circuit in the following grid.

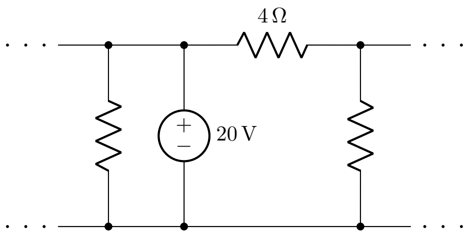


(b) *[2 marks]* Which one is the correct **Source Transformation** of the following circuitry?



Cross-out or fill-in the checkbox (☐) at the top-left corner of the correct answer.

☐



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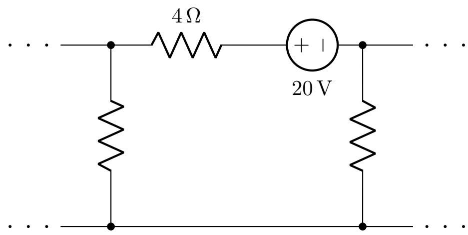

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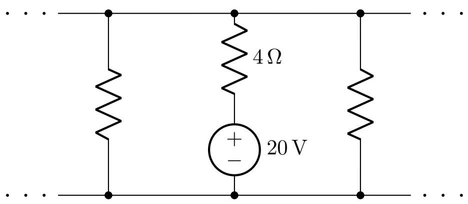

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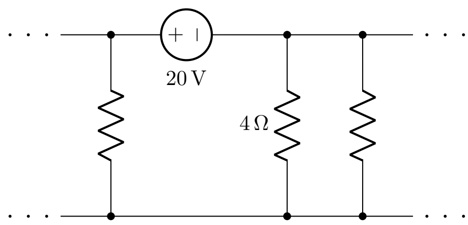
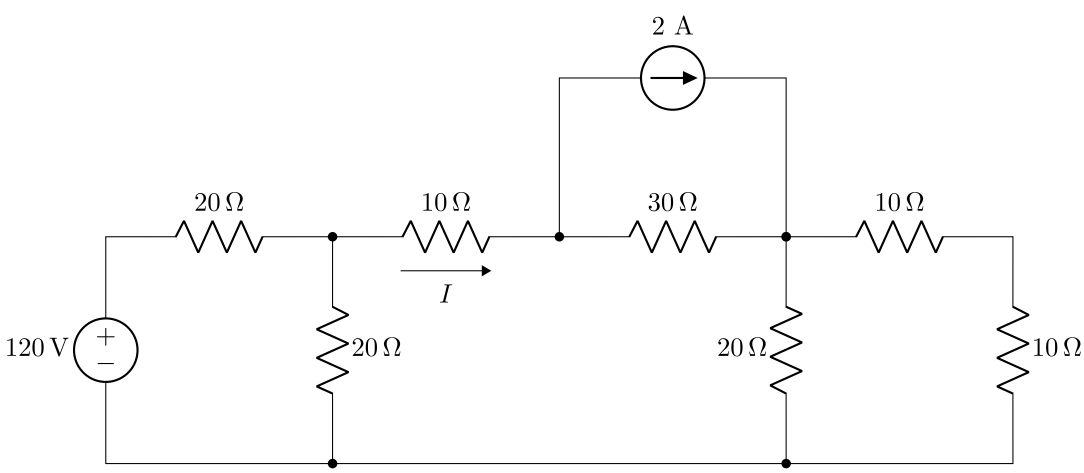
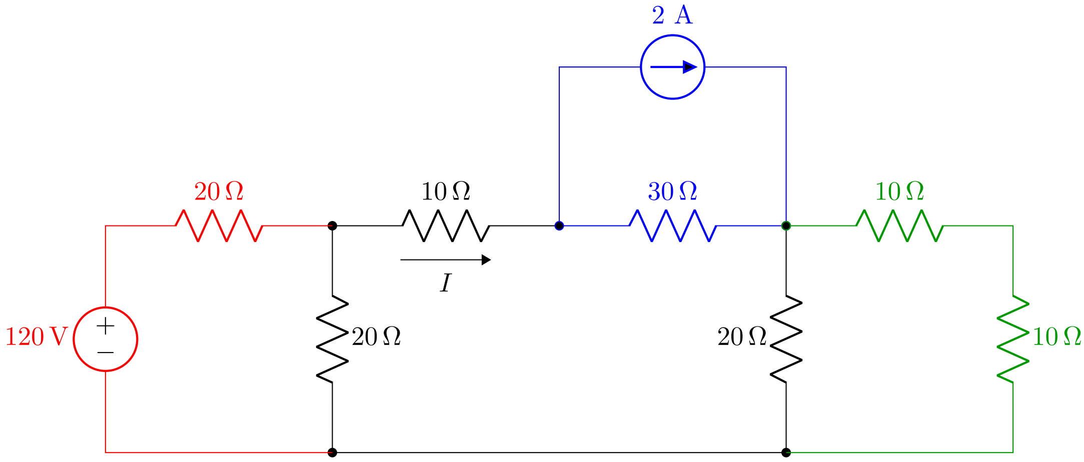


(c) *[15 marks]* **Determine** the current I as shown in the circuit below using **Superposition Principle** and/or **Source Transformation**.



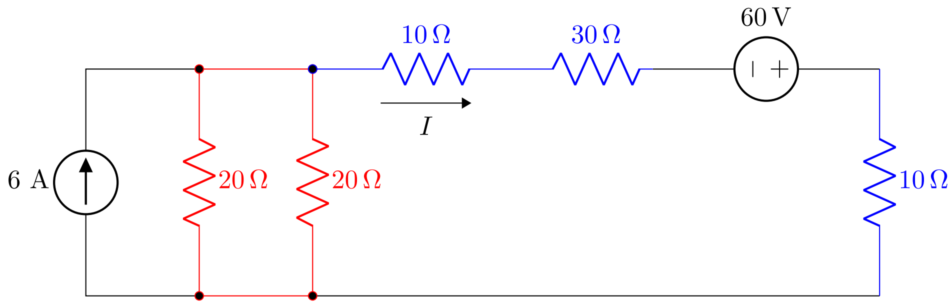
Solution:

Source Transformation Method:



- Transforming the 120 V voltage source in series with the 20 Ω resistor into a current source in parallel with a resistor.
- Transforming the 2 A current source in parallel with the 30 Ω resistor into a voltage source in series with a resistor.
- The two 10 Ω resistors in the rightmost part of the circuit are in series ($10 + 10 = 20\ \Omega$) and the series combination is parallel with the 20 Ω resistor.

  $\Rightarrow\ 20 \mid\mid (10 + 10) = 20\ \Omega$

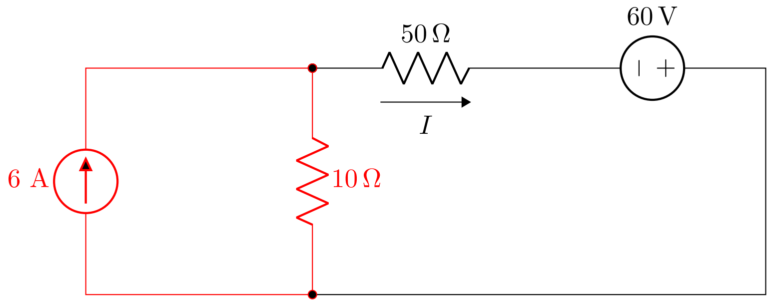


- The two 20 Ω resistors are parallel.

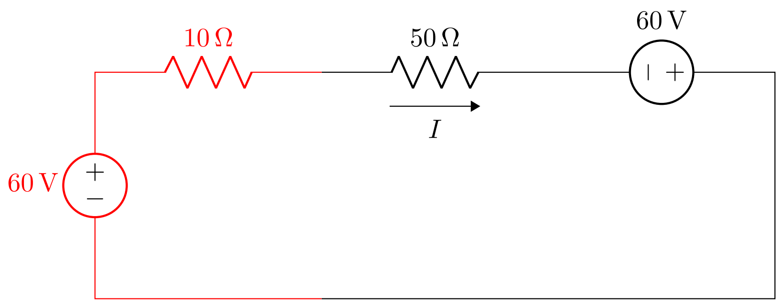
  $\Rightarrow\ 20 \mid\mid 20 = 10\ \Omega$ and

- The 10 Ω, 30 Ω, and 10 Ω resistors are parallel.

  $\Rightarrow\ 10 + 30 + 10 = 50\ \Omega$



- Transforming the 6 A current source in parallel with the 10 Ω resistor into a voltage source in series with a resistor.



- Replacing the two voltage sources by one: the value of the resultant source is,

  $\Rightarrow\ 60 + 60 = 120\ V$

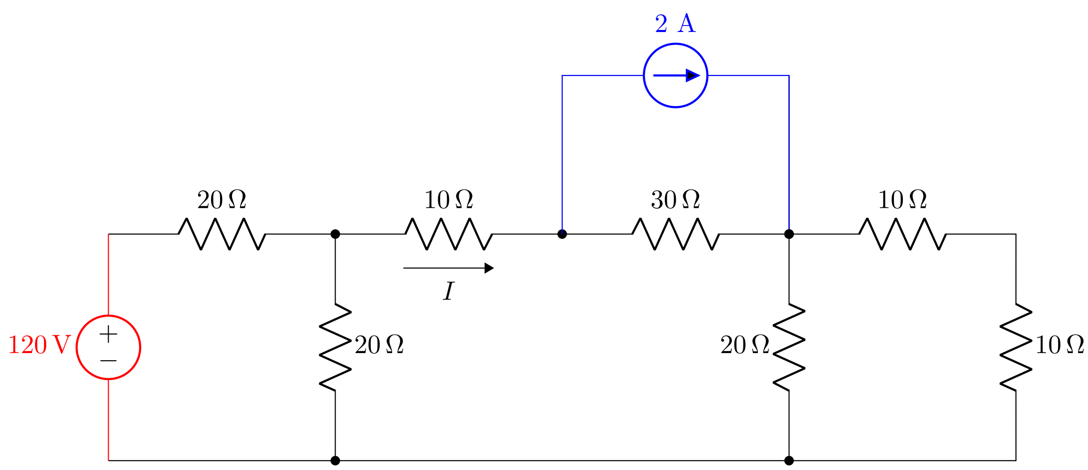
  So, the current I can be calculated as,

  $I = \frac{120}{10+50}\ (A)$

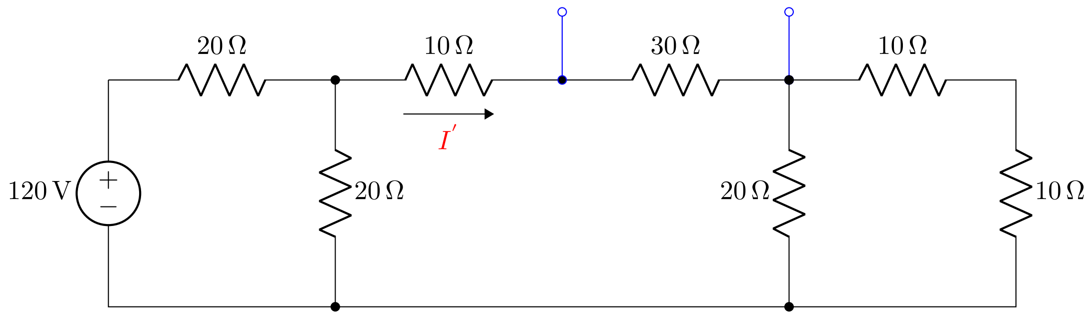
  $\Rightarrow \boxed{I = 2\ A}$

Superpostion Principle:

- There are two independent sources in the given circuit: the 120 V voltage source and the 2 A current source.



- Let's first calculate the contribution from the 120 V source only ($I^{'}$). Turning off the 2 A source (open circuit), the circuit looks like the one shown below.



- The circuit can be solved in several ways. Let's do some series-parallel combination of resistors to reduce the circuit.

- $10\ \Omega$ and $10\ \Omega$ are in series.

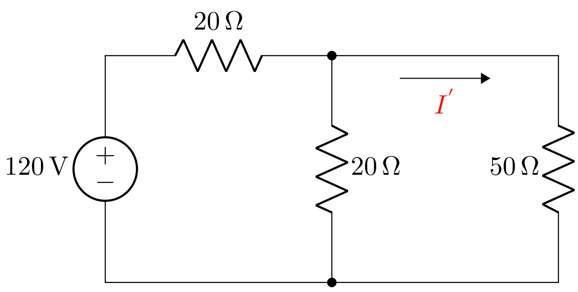
  $\Rightarrow\ 10\ \Omega + 10\ \Omega = 20\ \Omega$

- Then their combination ($20\ \Omega$) is in parallel with the other $20\ \Omega$.

  $\Rightarrow\ 20\ \Omega \ ||\ 20\ \Omega = 10\ \Omega$

- Then $10\ \Omega$, $30\ \Omega$, and $30\ \Omega$ are in series.

  $\Rightarrow\ 10\ \Omega + 30\ \Omega + 10\ \Omega = 50\ \Omega$

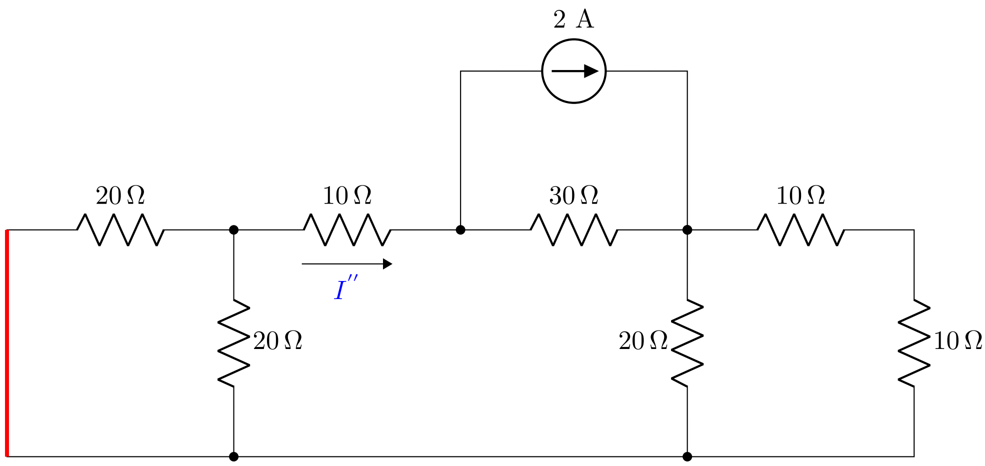


- The voltage across the parallel combination of the $20\ \Omega$ and $50\ \Omega$ can be found using the voltage divider rule as,

  $\frac{(20\ ||\ 50)}{20+(20\ ||\ 50)} \times 120\ V = 50\ V$

  So, $\boxed{I^{'} = \frac{50}{50} = 1\ A}$

- Now, for the 2 A current source, turning off the 120 V source (short circuit), the circuit looks like the one shown below.



- The circuit can be solved in several ways. Let's do some series-parallel combination of resistors to reduce the circuit.

- $10\ \Omega$ and $10\ \Omega$ are in series.

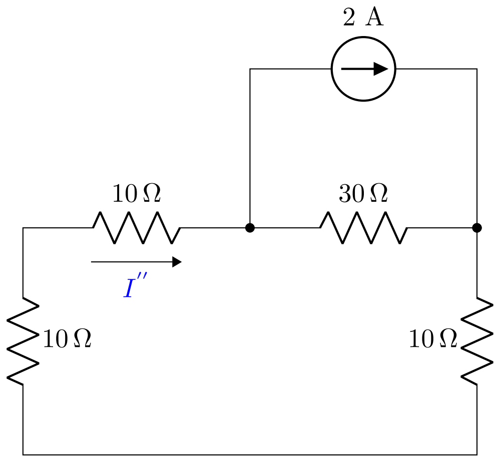
  $\Rightarrow\ 10\ \Omega + 10\ \Omega = 20\ \Omega$

- Then their combination (20 Ω) is in parallel with the other 20 Ω.

  $\Rightarrow\ 20\ \Omega\ ||\ 20\ \Omega = 10\ \Omega$

- In the left side, 20 Ω and 20 Ω are in parallel.

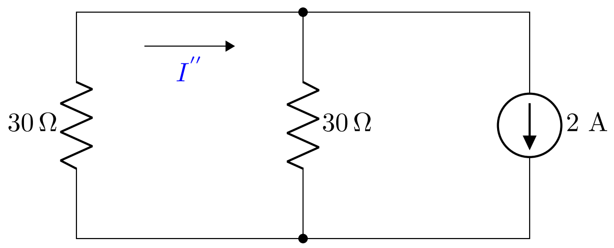
  $\Rightarrow\ 20\ \Omega\ ||\ 20\ \Omega = 10\ \Omega$



- We may reduce further. The three 10 Ω resistors are in series where the current I'' flows.

  $\Rightarrow\ 10\ \Omega + 10\ \Omega + 10\ \Omega = 30\ \Omega$

  Now the circuit becomes,



- The current I'' will be halved through each of the 30 Ω resistors,

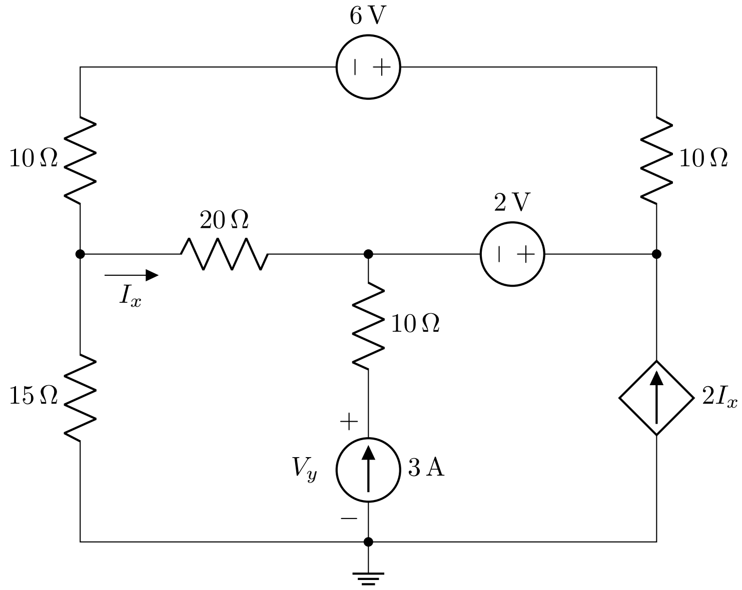
  So, $\boxed{I'' = 1\ A}$

  According to the Superposition Principle, the total current will be the algebraic summation of the two contributions of the two independent sources. That is,

  $I = I' + I'' = 1 + 1\ (A)$

  $\boxed{\Rightarrow\ I = 2\ A}$

■ Question 2 of 3 *[CO2, CO4] [20 marks]*



Apply Nodal/Mesh analysis to answer the following questions:

(a) *[1 mark]* Which analysis method should be more advantageous in solving the above circuit?

> **Solution:** Mesh analysis

(b) *[15 marks]* Find all the node voltages/mesh currents in the circuit.

> **Solution:** Mesh currents: $\pm\ 1.4\ A,\ \mp\ 1.6\ A,\ \pm\ 0.6\ A.$
> Node voltages: $21\ V,\ 37\ V,\ 39\ V.$

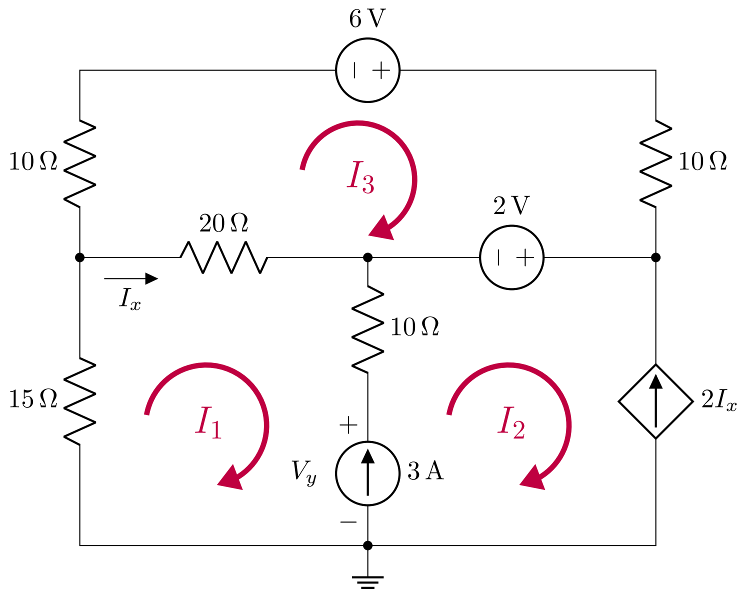
(c) *[2 marks]* Find V_y, the voltage across the 5 A current source.

> **Solution:** $67\ V$

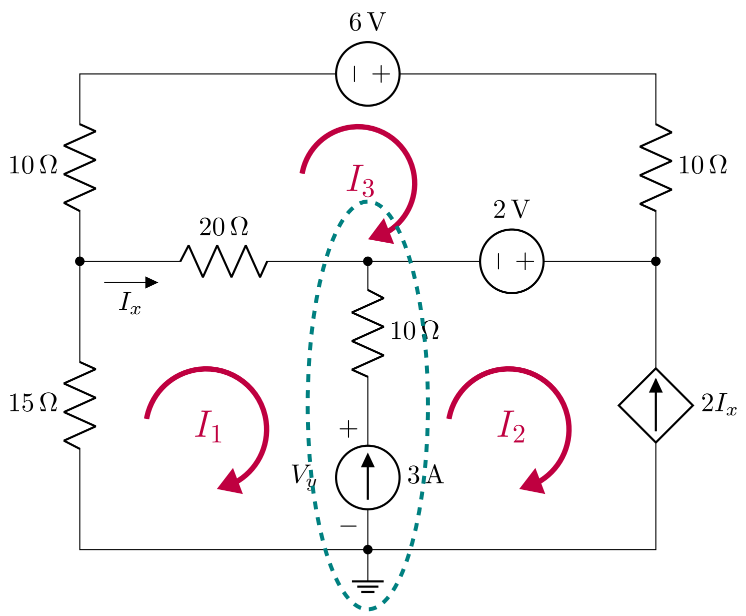
(d) *[2 marks]* How much **power** is the 5 A current source consuming/supplying to the circuit?
Also mention whether the source is supplying or consuming power.

> **Solution:** −201 W, supplying

> **Solution:**
>
> (b) There are 3 meshes in the given circuit. Let's assign I_1, I_2, and I_3, in ampere units, as the mesh currents, all taken in clockwise direction.



The 3 A current source forms a Supermesh between loops 1 and 2 as shown below.



- From loop 2, we can directly write,

  $I_2 = -2I_X$

  where, $I_x = I_1 - I_3$

  $\Rightarrow \; I_2 = -2(I_1 - I_3)$

  $\Rightarrow \; 2I_1 + I_2 - 2I_3 = 0 - - - - - - - - -(eqn.\ 1)$

  From the Supermesh, we can write using KCL,

  $I_2 - I_1 = 3$

  $\Rightarrow \; I_1 - I_2 = -3 - - - - - - - - -(eqn.\ 2)$

- Applying KVL at loop 3,

$-6 + 10I_3 + 2 + 20(I_3 - I_1) + 10I_3 = 0$

$\Rightarrow \ -20I_1 + 40I_3 = 4 - - - - - - - - -(eqn.\ 3)$

Solving equations 1, 2, and 3, we get,

$\boxed{I_1 = -1.4\ A}$

$\boxed{I_2 = 1.6\ A}$

$\boxed{I_3 = -0.6\ A}$

(c) To find V_y, the voltage across the 3 A source, using KVL at loop 1,

$15I_1 + 20(I_1 - I_3) + 10(I_1 - I_2) + V_y = 0$

Substituting for I_1, I_2, and I_3,

$\boxed{V_y = 67\ V}$

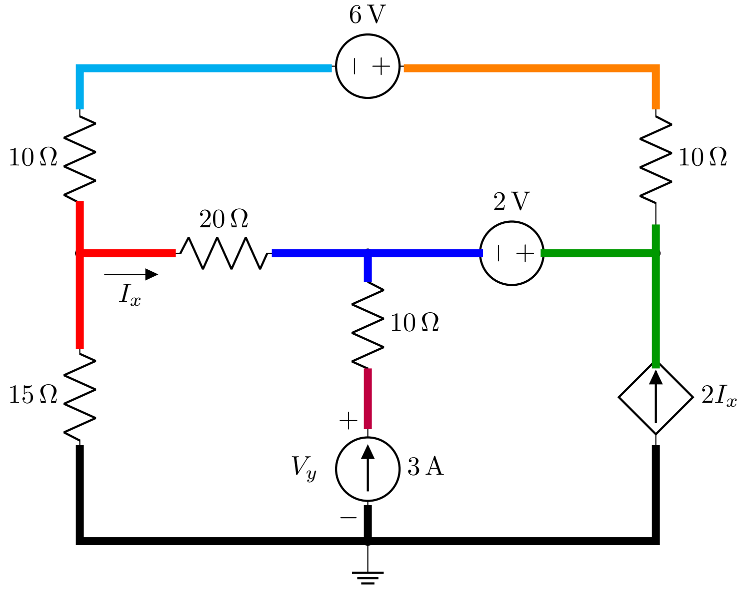
(d) The power of the 3 A current source according to passive sign convention is,

$P_{3\ A} = -V_y \times 5\ (Watt)$

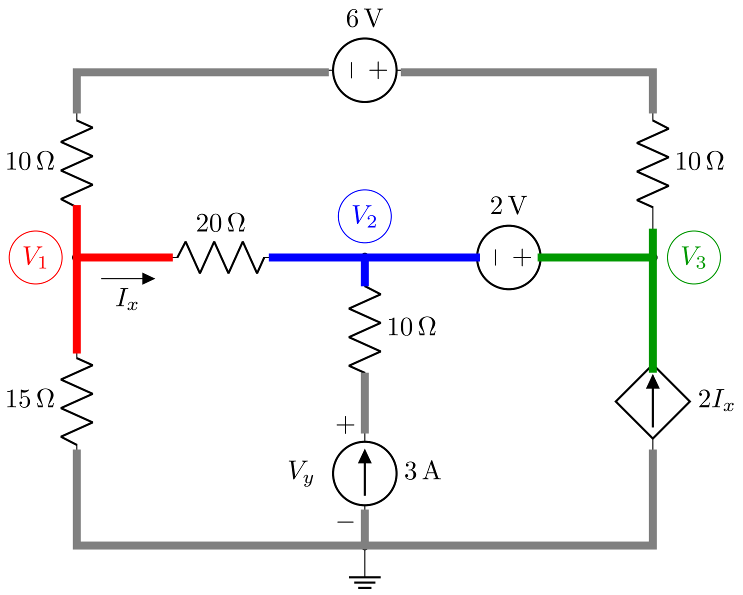
$\Rightarrow \ \boxed{P_{3\ A} = -67 \times 3 = -201\ (W)}$

Nodal Analysis Method: general approach

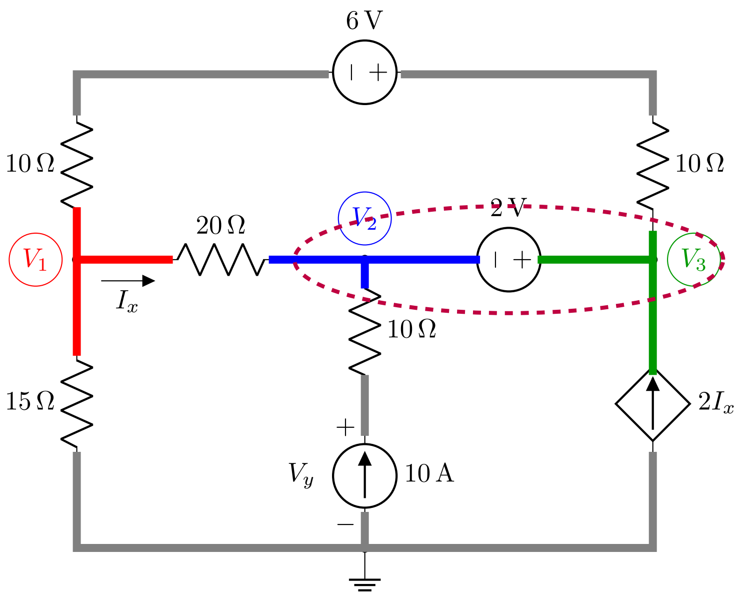
(b) There are 7 nodes in the given circuit as marked in the following diagram.



- But in the general approach of nodal analysis, we don't have to consider all the nodes. We have to consider only 3 nodes (red, blue, and green).



- The 2 V source forms a Supermesh between nodes 2 (V_2) and 3 (V_3).



- From the Supernode we can write,

  $V_3 - V_2 = 2\ V$

  $\Rightarrow\ V_2 - V_3 = -2\ V - - - - - - - - -(eqn.\ 1)$

- Applying KCL at nodes 2 (V_2) and (V_3),

  $3 + 2I_x = \frac{V_2 - V_1}{20} + \frac{V_3 - V_1 - 6}{10+10}$

  The current I_x through the 20 Ω resistor can be written as,

  $I_x = \frac{V_1 - V_2}{20}$

  Substituting for I_x,

$3 + 2\left(\frac{V_1 - V_2}{20}\right) = \frac{V_2 - V_1}{20} + \frac{V_3 - V_1 - 6}{10+10}$

$\Rightarrow\ 4V_1 - 3V_2 - V_3 = -66 - - - - - - - - -(eqn.\ 2)$

- Finally, applying KCL at node 1 (V_1),

$\frac{V_1 - 0}{15} + \frac{V_1 - V_2}{20} + \frac{V_1 - V_3 + 6}{10+10} = 0$

$\Rightarrow\ 10V_1 - 3V_2 - 3V_3 = -18 - - - - - - - - -(eqn.\ 3)$

Solving the 3 equations we get the node voltages.

$\boxed{V_1 = 21\ V}$

$\boxed{V_2 = 37\ V}$

$\boxed{V_3 = 39\ V}$

(c) The voltage V_y can be found by applying KVL through the loop consisting of the 3 A source. That is, one way is,

$V_2 - V_y + (3 \times 5) = 0$

Substituting for $V_2 = 37\ V$,

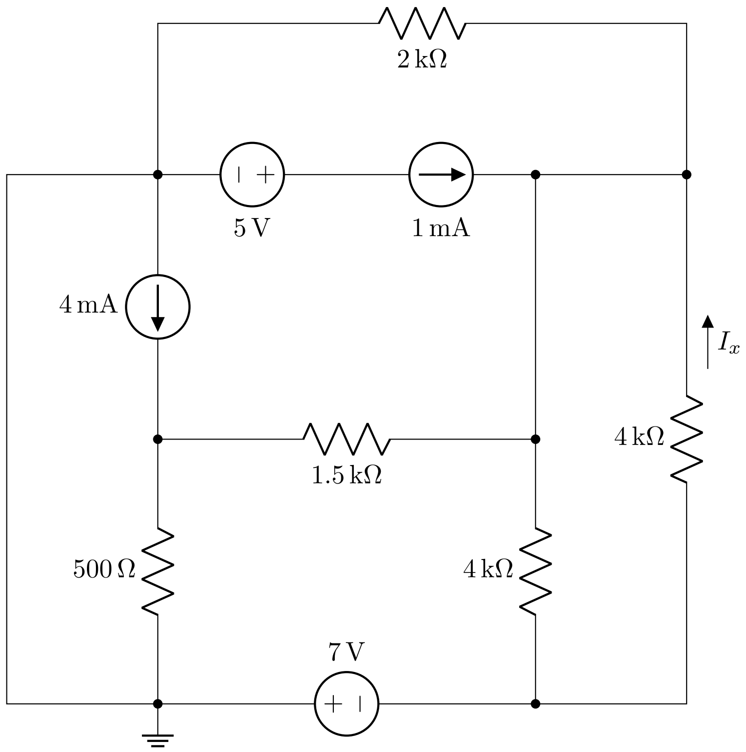
$\Rightarrow\ \boxed{V_y = 67\ V}$

(d) The power of the 3 A current source according to passive sign convention is,

$P_{3\ A} = -V_y \times 3\ (Watt)$

$\Rightarrow\ \boxed{P_{3\ A} = -67 \times 3 = -201\ (W)}$

■ Question 3 of 3 *[CO2, CO4] [16 marks]*



Apply Nodal/Mesh analysis to answer the following questions:

(a) *[1 mark]* Which analysis method should be more advantageous in solving the above circuit?

> **Solution:** Nodal analysis

(b) *[14 marks]* Find all the node voltages/mesh currents in the circuit.

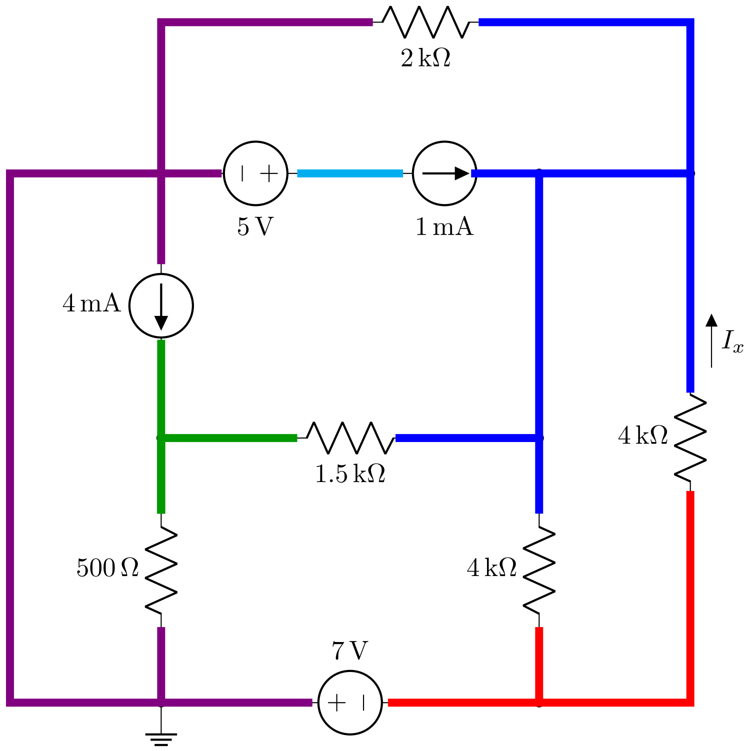
> **Solution:** Node voltages: $-1\ V,\ 1.25\ V.$
> Mesh currents: $\pm\ 5.5\ mA,\ \mp\ 3.5\ mA,\ \pm\ 1.5\ mA,\ \pm 1.5\ mA,\ \pm 0.5\ mA$

(c) *[1 mark]* Find I_x, the amount of current through the $6\,\mathrm{k\Omega}$ resistor.

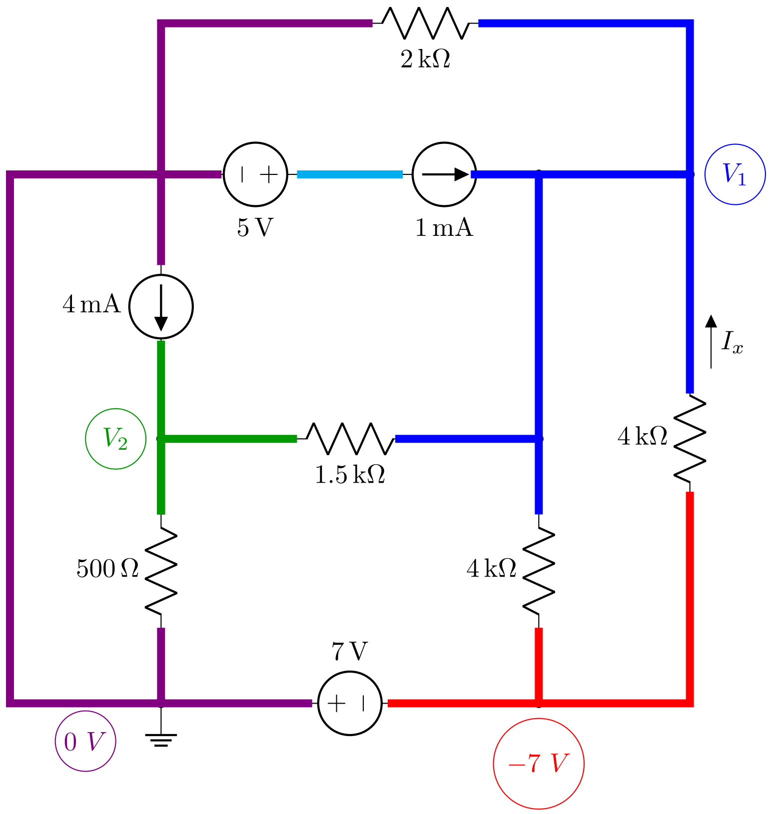
> **Solution:** $-1.5\,\mathrm{mA}$

Solution: Nodal Analysis Method (General approach)

(b) There are 4 nodes in the given circuit apart from the ground as shown in the figure below.



- The red marked node voltage is $-7\ V$, as can be seen from the figure.
- In the general approach of nodal analysis, we don't have to consider the node colored as cyan.
- Let's assign V_1, V_2 as the remaining node variables (see the figure below).



- Applying KCL at node 1 (V_1),

  $1 = \frac{V_1-0}{2} + \frac{V_1-(-7)}{4} + \frac{V_1-(-7)}{4} + \frac{V_1-V_2}{1.5}$

  $\Rightarrow\ 10V_1 - 4V_2 = -15 - - - - - - - - -(eqn.\ 1)$

- Applying KCL at node 2 (V_2),

  $4 = \frac{V_2-V_1}{1.5} + \frac{V_2-0}{0.5}$

  $\Rightarrow\ V_1 - 4V_2 = -6 - - - - - - - - -(eqn.\ 2)$

  Solving equations 1 and 2 we get,

  $\boxed{V_1 = -1\ V}$

  $\boxed{V_2 = 1.25\ V}$

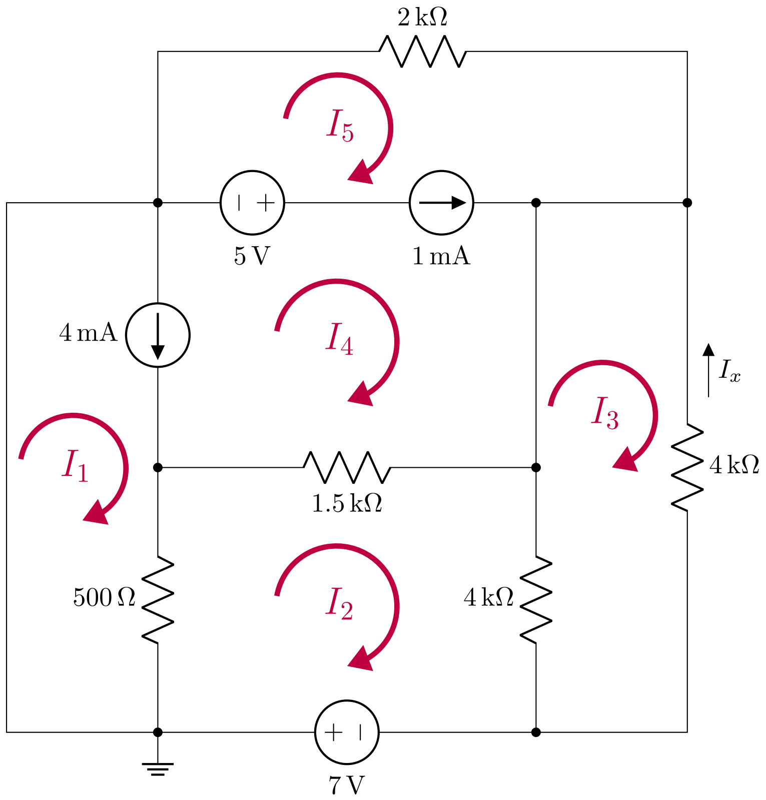
(c) The current I_x through the 4 kΩ resistor is thus,

$I_x = \frac{-7-V_1}{4}$ (mA)

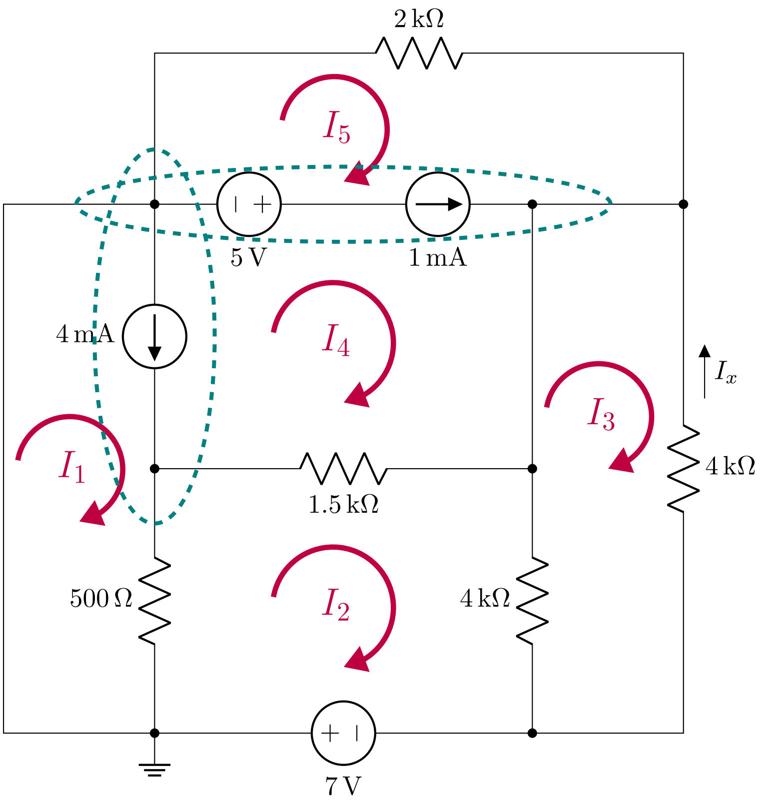
$\Rightarrow \boxed{I_x = -1.5\ mA}$

Mesh Analysis Method:

(b) There are 5 meshes in the given circuit. Let's assign I_1, I_2, I_3, I_4, and I_5, in milliampere units, as the mesh currents.



- The 1 mA current source and the 4 mA current source form two Supermeshes between meshes 4 & 5 and 1 & 4 respectively.



From the two Supermeshes, we can write for the current sources,

$I_4 - I_5 = 1 - - - - - - - - -(eqn.\ 1)$ and

$I_1 - I_4 = 4 - - - - - - - - -(eqn.\ 2)$

- Now, applying KVL at loop 3,

$4I_3 + 4(I_3 - I_2) = 0$

$\Rightarrow\ 8I_3 - 4I_2 = 0 - - - - - - - - -(eqn.\ 3)$

- Applying KVL at loop 2,

$-7 + 0.5(I_2 - I_1) + 1.5(I_2 - I_4) + 4(I_2 - I_3) = 0$

$\Rightarrow\ I_1 - 12I_2 + 8I_3 + 3I_4 = -14 - - - - - - - - -(eqn.\ 4)$

- Now, applying KVL along loops 5, 4, and 1,

$2I_5 + 1.5(I_4 - I_2) + 0.5(I_1 - I_2) = 0$

$\Rightarrow\ I_1 - 4I_2 + 3I_4 + 4I_5 = 0 - - - - - - - - -(eqn.\ 5)$

Solving equations 1 to 5,

$\boxed{I_1 = 5.5\ mA}$

$\boxed{I_2 = 3.5\ mA}$

$\boxed{I_3 = 1.5\ mA}$

$\boxed{I_4 = 1.5\ mA}$

$\boxed{I_5 = 0.5\ mA}$

(c) It can be seen that, the current through the 4 $k\Omega$ resistor is $-I_3$.

So, $\boxed{I_x = -I_3 = -1.5\ mA}$