Solution #2

1. For the circuit in Fig. 1, obtain v_1 and v_2.

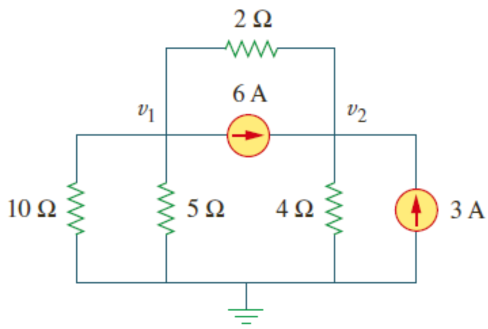


Fig. 1. For Prob. 1.

Solution:

At node 1,

$$\frac{-v_1}{10} - \frac{v_1}{5} = 6 + \frac{v_1 - v_2}{2} \rightarrow 60 = -8v_1 + 5v_2 \qquad (1)$$

At node 2,

$$\frac{v_2}{4} = 3 + 6 + \frac{v_1 - v_2}{2} \rightarrow 36 = -2v_1 + 3v_2 \qquad (2)$$

Solving (1) and (2),

$$v_1 = 0 \text{ V}, v_2 = 12 \text{ V}$$

2. Given the circuit in Fig. 2, calculate the currents i_1 through i_4. (4%)

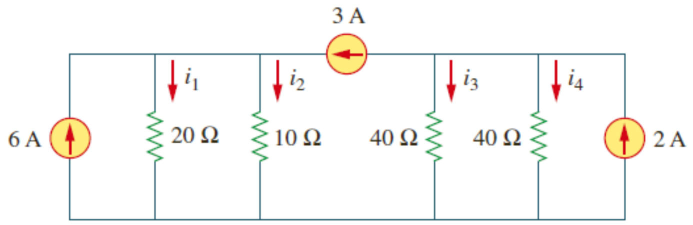
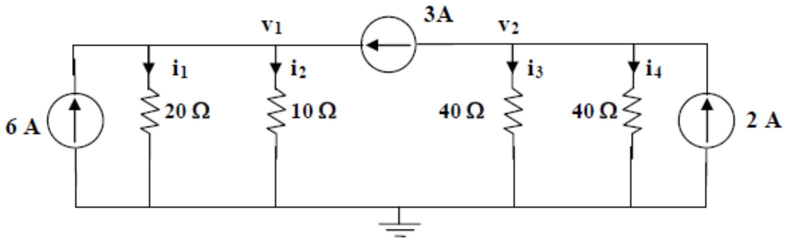


Fig. 2. For Prob. 2.

Solution:



At node 1,

$$-6 - 3 + \frac{v_1}{20} + \frac{v_1}{10} = 0 \rightarrow v_1 = 9 * \left(\frac{20}{3}\right) = 60 \text{ V} \qquad (1)$$

At node 2,

$$3 - 2 + \frac{v_2}{40} + \frac{v_2}{40} = 0 \rightarrow v_2 = -1 * \left(\frac{1600}{80}\right) = -20 \text{ V} \qquad (2)$$

$$i_1 = \frac{v_1}{20} = 3 \text{ A}, \; i_2 = \frac{v_1}{10} = 6 \text{ A},$$

$$i_3 = \frac{v_2}{40} = -500 \text{ mA}, \; i_4 = \frac{v_2}{40} = -500 \text{ mA}.$$

3. Solve for V_1 in the circuit of Fig. 3 using nodal analysis.

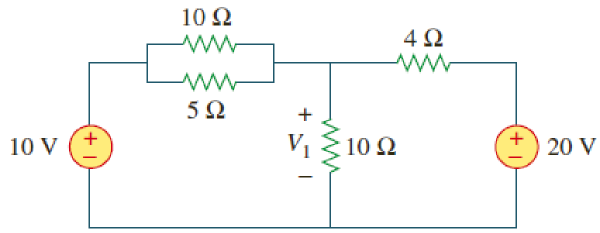


Fig. 3. For Prob. 3.

Solution:

Set the bottom of the circuit as the reference node.

At node 1:

$$\frac{(V_1-10)}{5} + \frac{(V_1-10)}{10} + \frac{(V_1-0)}{10} + \frac{(V_1-20)}{4} = 0,$$

$$(0.2 + 0.1 + 0.1 + 0.25)V_1 = 2 + 1 + 5,$$

$V_1 = 8/0.65 = 160/13 = 12.308$ V.

4. Using nodal analysis, find v_o in the circuit of Fig. 4.

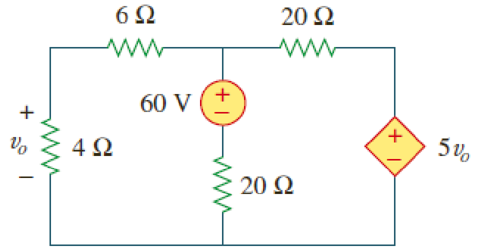
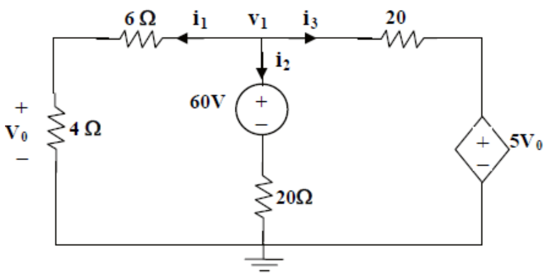


Fig. 4. For Prob. 4.

Solution:



$$i_1 + i_2 + i_3 = 0 \rightarrow \frac{V_1}{10} + \frac{(V_1 - 60) - 0}{20} + \frac{V_1 - 5V_0}{20} = 0$$

But $V_0 = \frac{2}{5}V_1$, so that $2V_1 + V_1 - 60 + V_1 - 2V_1 = 0$

or $v_1 = 60/2 = 30$ V, therefore $v_0 = 2v_1/5 = 12$ V.

5. Apply nodal analysis to find i_0 and the power dissipated in each resistor in the circuit of Fig. 5.

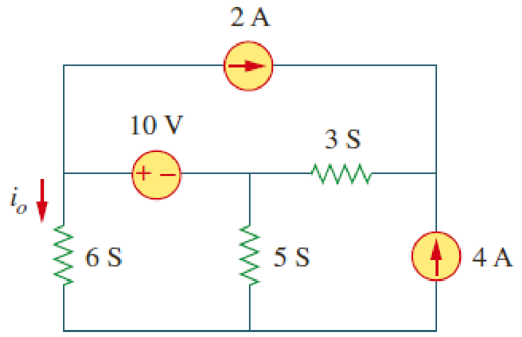
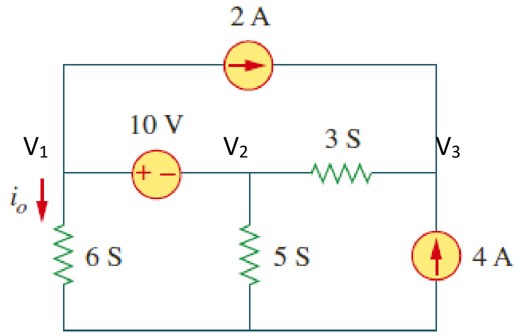


Fig. 5. For Prob. 5.

Solution:



Nodes 1 and 2 form a supernode so that $v_1 = v_2 + 10$ (1)

At the supernode, $2 + 6v_1 + 5v_2 = 3(v_3 - v_2) \rightarrow 2 + 6v_1 + 8v_2 = 3v_3$ (2)

At node 3, $2 + 4 = 3(v_3 - v_2) \rightarrow v_3 = v_2 + 2$ (3)

Substituting (1) and (3) into (2),

$$2 + 6v_2 + 60 + 8v_2 = 3v_2 + 6 \rightarrow v_2 = \frac{-56}{11} = 5.1 \text{ V}$$

$$v_1 = v_2 + 10 = \frac{54}{11} = 4.9 \text{ V}$$

$$i_0 = 6v_1 = 29.45\text{A}$$

$$P_{65} = v_1^2 G = (\frac{54}{11})^2 6 = 144.6\text{W}$$

$$P_{55} = v_2^2 G = (\frac{-56}{11})^2 5 = 129.6\text{W}$$

$$P_{35} = (v_2 - v_3)^2 G = (2)^2 3 = 12\text{W}$$

6. Determine voltages v_1 through v_3 in the circuit of Fig. 6 using nodal analysis.

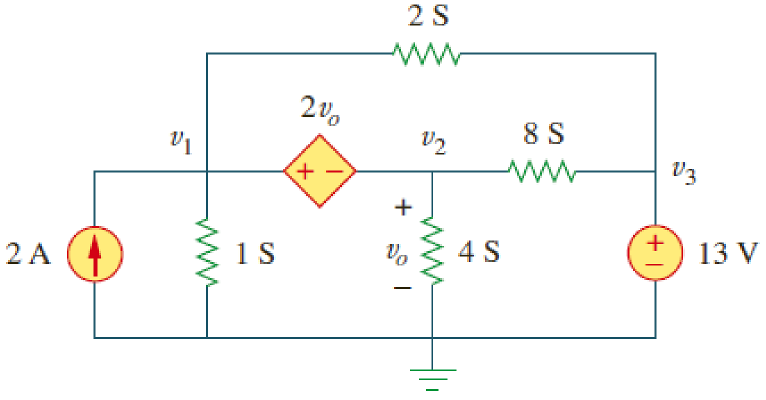
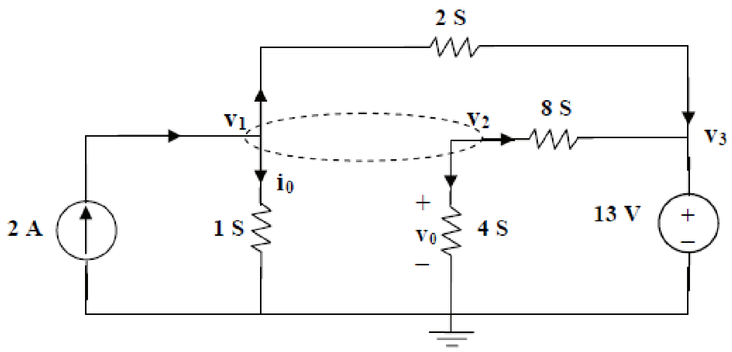


Fig.6. For Prob. 6.

Solution:



At the supernode,

$2 = v_1 + 2\,(v_1 - v_3) + 8(v_2 – v_3) + 4v_2$, which leads to $2 = 3v_1 + 12v_2 - 10v_3$ (1)

But $v_1 = v_2 + 2v_0$ and $v_0 = v_2$.

Hence $v_1 = 3v_2$ (2)

$v_3 = 13$ V (3)

Substituting (2) and (3) with (1) gives, $2 = 9\,v_2 + 12v_2 - 130$

v_1 =132/7 V= **18.858 V** , v_2 =44/7 V= **6.286 V** v_3 = **13 V**

7. Use nodal analysis to find v_1, v_2, v_3 and in the circuit of Fig. 7.

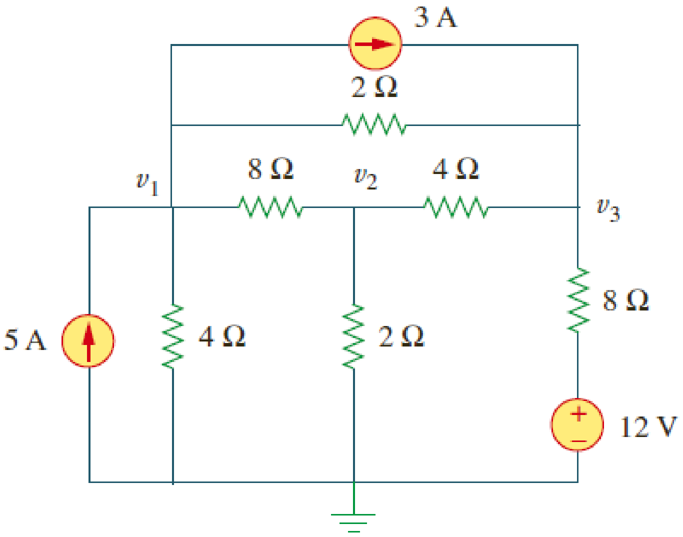


Fig. 7. For Prob. 7.

Solution:

At node 1,

$$5 = 3 + \frac{v_1 - v_3}{2} + \frac{v_1 - v_2}{8} + \frac{v_1}{4} \rightarrow 16 = 7v_1 - v_2 - 4v_3 \quad (1)$$

At node 2,

$$\frac{v_1 - v_2}{8} = \frac{v_2 - v_3}{4} + \frac{v_2}{2} \rightarrow 0 = -v_1 + 7v_2 - 2v_3 \quad (2)$$

At node 3,

$$3 + \frac{12 - v_3}{8} + \frac{v_1 - v_3}{2} + \frac{v_2 - v_3}{4} = 0 \rightarrow -36 = 4v_1 + 2v_2 - 7v_3 \quad (3)$$

Form (1) to (3),

$$\begin{pmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ 4 & 2 & -7 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 16 \\ 0 \\ -36 \end{pmatrix} \longrightarrow AV = B$$

Using MATLAB,

$$V = A^{-1}B = \begin{bmatrix} 10 \\ 4.933 \\ 12.267 \end{bmatrix} \longrightarrow \underline{V_1 = 10\text{ V},\ V_2 = 4.933\text{ V},\ V_3 = 12.267\text{ V}}$$

8. For the circuit in Fig. 8, find v_1, v_2 and v_3 using nodal analysis.

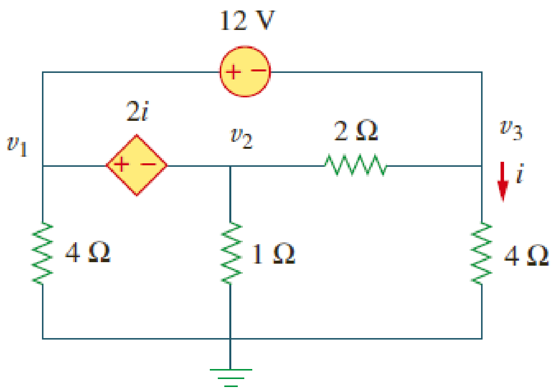
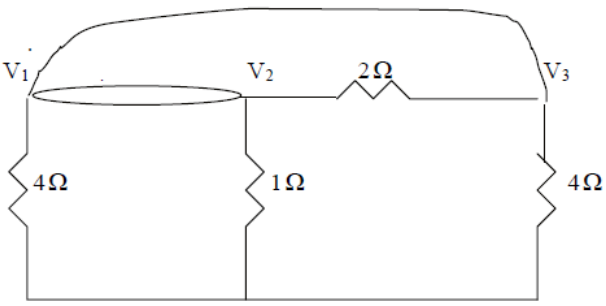


Fig. 8. For Prob. 8.

Solution:

Nodes 1 and 2 form a supernode; so do nodes 1 and 3. Hence

$$\frac{V_1}{4} + \frac{V_2}{1} + \frac{V_3}{4} = 0 \qquad \longrightarrow \qquad V_1 + 4V_2 + V_3 = 0 \qquad (1)$$



Between nodes 1 and 3,

$$-V_1 + 12 + V_3 = 0 \qquad \longrightarrow \qquad V_3 = V_1 - 12 \qquad (2)$$

Similarly, between nodes 1 and 2,

$$V_1 = V_2 + 2i \qquad (3)$$

But $i = V_3 / 4$. Combining this with (2) and (3) gives

$$V_2 = 6 + V_1 / 2 \qquad (4)$$

Solving (1), (2) , and (4) leads to

$$\underline{\mathbf{V_1 = -3V, \quad V_2 = 4.5V, V_3 = -15V}}$$

9. Use nodal analysis and *MATLAB* to find V_0 in the circuit of Fig. 9.(*ML*)

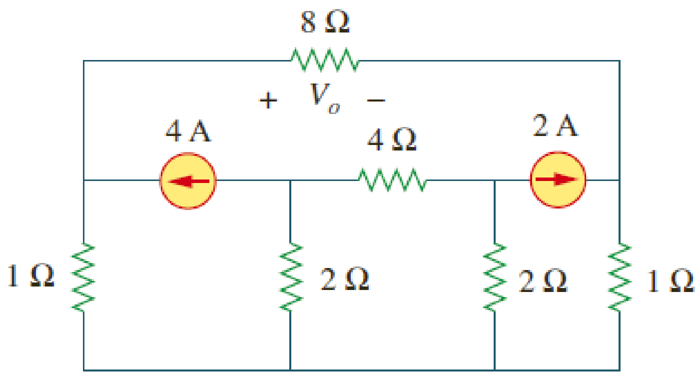
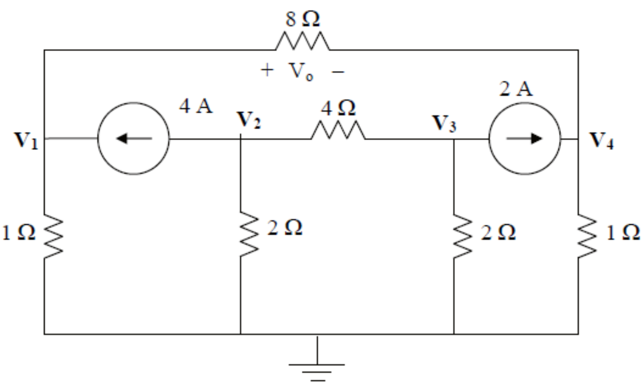


Fig. 9. For Prob. 9.

Solution:

Consider the circuit below .



Use nodal analysis at nodes 1~4,

$$\frac{V_1-0}{1}-4+\frac{V_1-V_4}{8}=0 \rightarrow 1.125V_1-0.125V_4=4 \qquad (1)$$

$$+4+\frac{V_2-0}{2}+\frac{V_2-V_3}{4}=0 \rightarrow 0.75V_2-0.25V_3=-4 \qquad (2)$$

$$\frac{V_3-V_2}{4}+\frac{V_3-0}{2}+2=0 \rightarrow -0.25V_2+0.75V_3=-2 \qquad (3)$$

$$-2+\frac{V_4-V_1}{8}+\frac{V_4-0}{1}=0 \rightarrow -0.125V_1+1.125V_4=2 \qquad (4)$$

$$\begin{bmatrix} 1.125 & 0 & 0 & -0.125 \\ 0 & 0.75 & -0.25 & 0 \\ 0 & -0.25 & 0.75 & 0 \\ -0.125 & 0 & 0 & 1.125 \end{bmatrix} V = \begin{bmatrix} 4 \\ -4 \\ -2 \\ 2 \end{bmatrix}$$

Now we can use MATLAB to solve for the unknown node voltage.

```
>> Y=[1.125,0,0,-0.125;0,0.75,-0.25,0;0,-0.25,0.75,0;-0.125,0,0,1.125]

Y =
```

```
    1.1250         0         0   -0.1250
         0    0.7500   -0.2500         0
         0   -0.2500    0.7500         0
   -0.1250         0         0    1.1250

>> I=[4,-4,-2,2]'

I =

     4
    -4
    -2
     2

>> V=inv(Y)*I

V =

    3.8000
   -7.0000
   -5.0000
    2.2000
```

$V_o = V_1 - V_4 = 3.8 - 2.2 = \mathbf{1.6\ V}$.

10. Calculate the node voltages v_1, v_2, and v_3 in the circuit of Fig. 10.(*ML*)

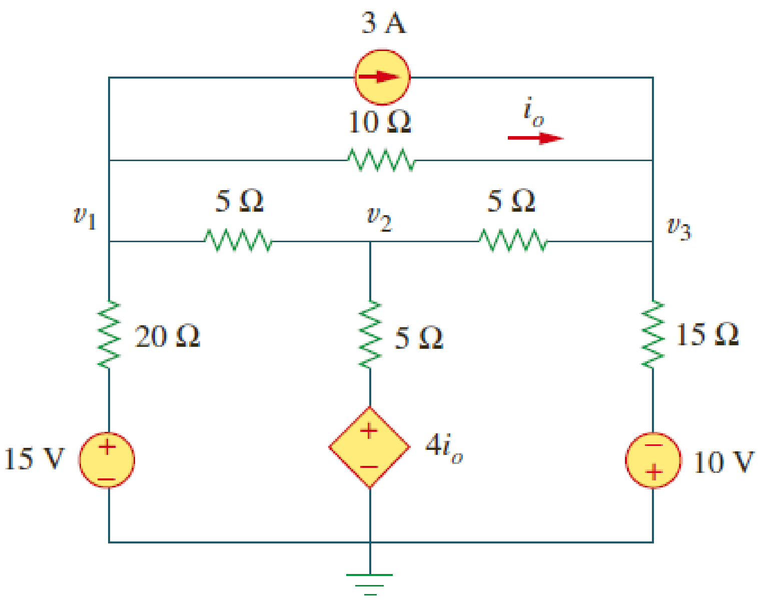


Fig.10. For Prob.10.

Solution:

At node 1,

$$\frac{15-V_1}{20} = 3 + \frac{V_1 - V_3}{10} + \frac{V_1 - V_2}{5} \longrightarrow -45 = 7V_1 - 4V_2 - 2V_3 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{5} + \frac{4I_o - V_2}{5} = \frac{V_2 - V_3}{5} \qquad (2)$$

But $I_o = \frac{V_1 - V_3}{10}$. Hence, (2) becomes

$$0 = 7V_1 - 15V_2 + 3V_3 \qquad (3)$$

At node 3,

$$3 + \frac{V_1 - V_3}{10} + \frac{-10 - V_3}{15} + \frac{V_2 - V_3}{5} = 0 \quad \longrightarrow \quad 70 = -3V_1 - 6V_2 + 11V_3 \qquad (4)$$

Putting (1), (3), and (4) in matrix form produces

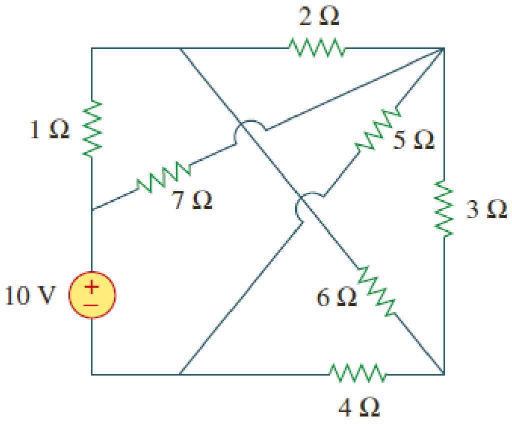
$$\begin{pmatrix} 7 & -4 & -2 \\ 7 & -15 & 3 \\ -3 & -6 & 11 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} -45 \\ 0 \\ 70 \end{pmatrix} \quad \longrightarrow \quad AV = B$$

Using MATLAB leads to

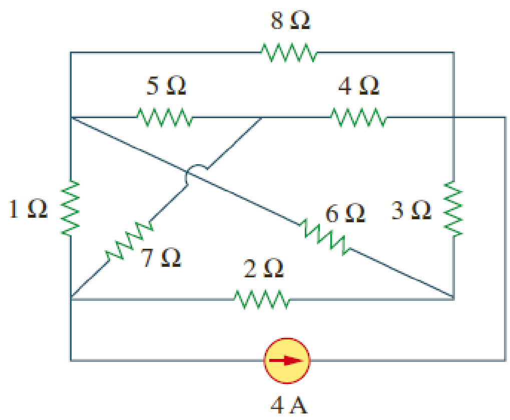
$$V = A^{-1}B = \begin{pmatrix} -7.19 \\ -2.78 \\ 2.89 \end{pmatrix}$$

Thus, $V_1 = \mathbf{-7.19V}$; $V_2 = \mathbf{-2.78V}$; $V_3 = \mathbf{2.89V}$.

11. Determine which of the circuits in Fig. 11 is planar and redraw it with no crossing branches.



(a)

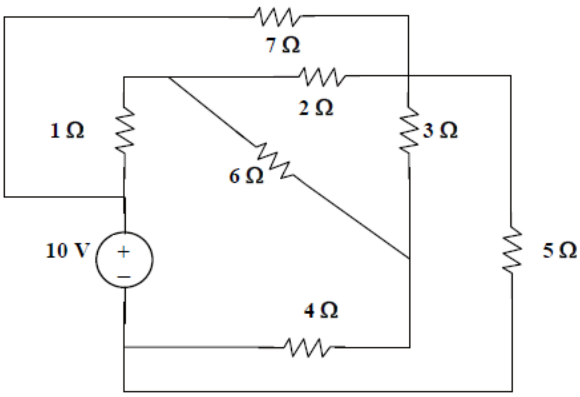


(b)

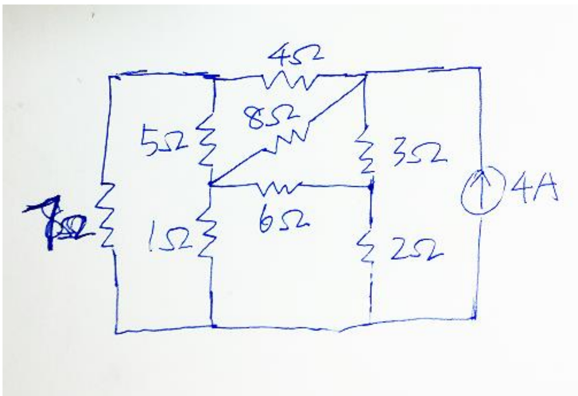
Fig. 11. For Prob. 11.

Solution:

(a) This is a planar circuit because it can be redraw as shown below,



(b) This is a planar circuit because it can be redraw as shown below,



12. Apply mesh analysis to the circuit in Fig. 12 and obtain *Io*. (*ML*)

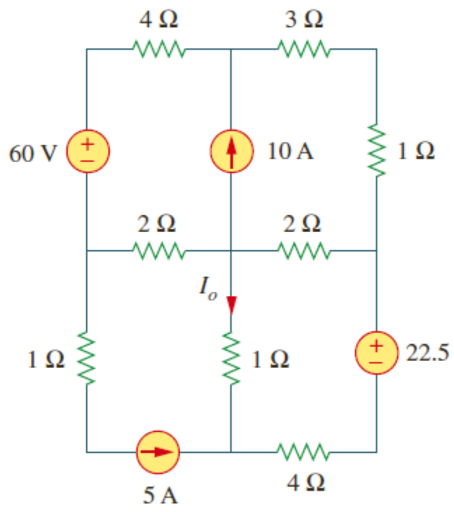
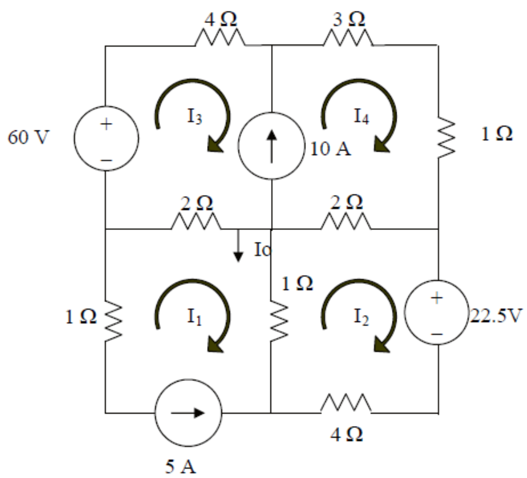


Fig. 12. For Prob.12.

Solution:

Consider the circuit below with the mesh currents.



$$I_1 = -5A \qquad (1)$$

$$1(I_2 - I_1) + 2(I_2 - I_4) + 22.5 + 4I_2 = 0$$

$$7I_2 - 2I_4 = -27.5 \qquad (2)$$

$$-60 + 4I_3 + 3I_4 + 1I_4 + 2(I_4 - I_2) + 2(I_3 - I_1) = 0 \;\; super\ mesh$$

$$-2I_2 + 6I_3 + 6I_4 = 60 - 10 = 50 \qquad (3)$$

But, we need one more equation, so we use the constraint equation $-I_3+I_4 = 10$. This now gives us three equations with three unknows.

$$\begin{bmatrix} 7 & 0 & -1 \\ -2 & 6 & 6 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -27.5 \\ 50 \\ 10 \end{bmatrix}$$

We can now use MATLAB to solve the problem.

```
>> Z=[7,0,-1;-2,6,6;0,-1,0]
```

Z =

```
     7     0    -1
    -2     6     6
     0    -1     0
```
>> V=[–27.5,50,10]'

V =
```
  –27.5
   50
   10
```
>> I=inv(Z)*V

I =
```
  –1.3750
  –10.0000
   17.8750
```

$$I_0 = I_1 - I_2 = -5 - (-1.375)\,\text{A} = -3.625\,\text{A}$$

13. For the bridge network in Fig. 13, find i_0 using mesh analysis. (*ML*)

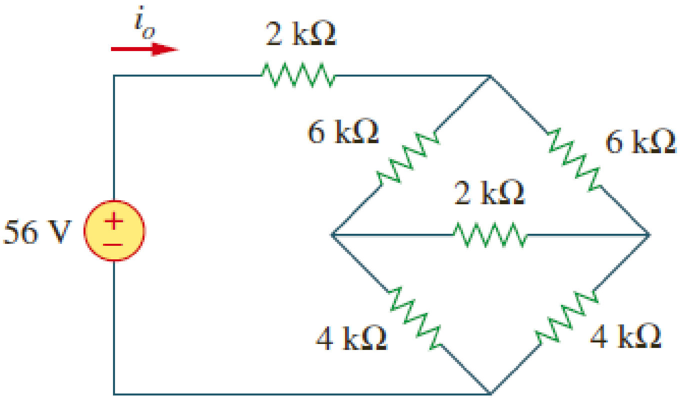
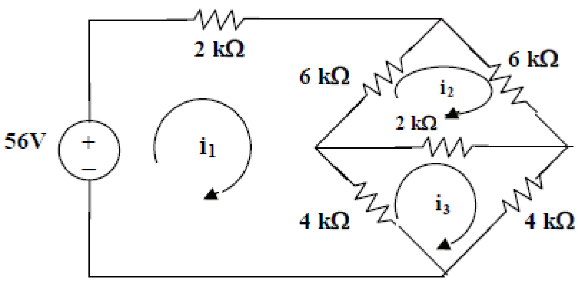


Fig. 13. For Prob.13.

Solution:



Assume all currents are in mA and apply mesh analysis for mesh 1.

$$-56 + 12i_1 - 6i_2 - 4i_3 = 0 \text{ or } 6i_1 - 3i_2 - 2i_3 = 28 \qquad (1)$$

For mesh 2,

$-6i_1 + 14i_2 - 2i_3 = 0$ or $-3i_1 + 7i_2 - i_3 \qquad = 0$ (2)

For mesh 3,
$-4i_1 - 2i_2 + 10i_3 = 0$ or $-2i_1 - i_2 + 5i_3 \qquad = 0$ (3)
Solving (1), (2), and (3) using MATLAB, we obtain,

$i_o = i_1 = \mathbf{8\ mA}$.

14. Use mesh analysis to obtain i_0 in the circuit of Fig.14.

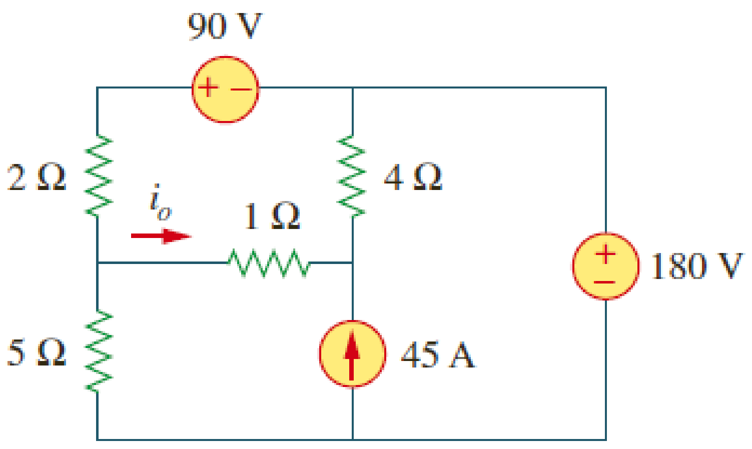
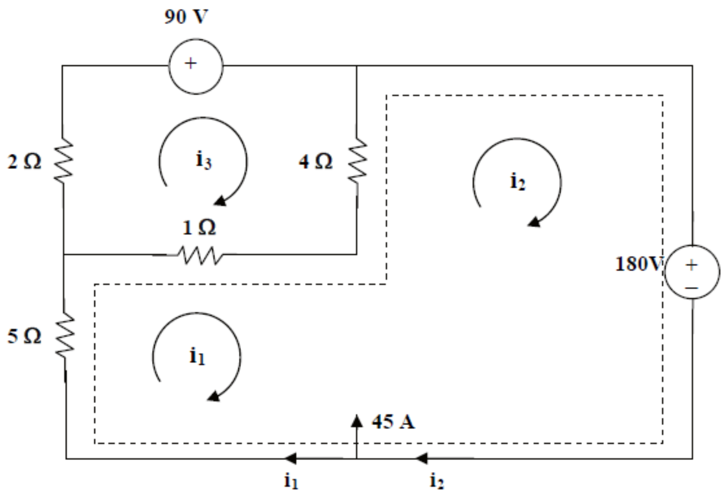


Fig.14. For Prob.14.

Solution:



Loop 1 and 2 form a supermesh. For the supermesh,

$6i_1 + 4i_2 - 5i_3 + 180 = 0$ (1)

For loop 3,

$-i_1 - 4i_2 + 7i_3 + 90 = 0$ (2)

Also,
$i_2 = 45 + i_1$ (3)

Solving (1) to (3),

$i_1 = -46,\ i_3 = -20;\qquad i_o = i_1 - i_3 = \mathbf{-26\ A}$

15. Use mesh analysis to find the current i_0 in the circuit of Fig. 15. (*ML*)

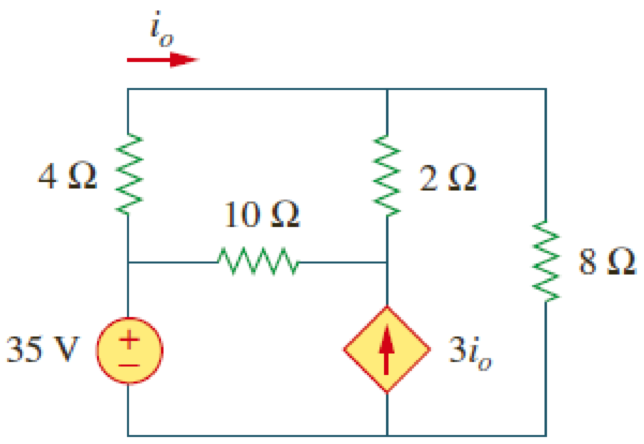
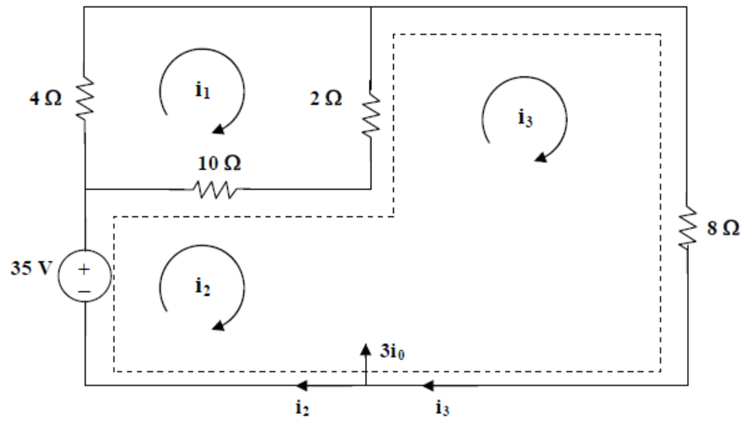


Fig.15. For Prob. 15.

Solution:



For loop 1, $16i_1 - 10i_2 - 2i_3 = 0$ which leads to $8i_1 - 5i_2 - i_3 = 0$ (1)

For the supermesh, $-35 + 10i_2 - 10i_1 + 10i_3 - 2i_1 = 0$

Or $\quad -6i_1 + 5i_2 + 5i_3 = 17.5$ (2)

Also, $3i_0 = i_3 - i_2$ and $i_0 = i_1$ which leads to $3i_1 = i_3 - i_2$ (3)

Solving (1), (2), and (3), we obtain $i_1 = 1.0098$ and

$i_0 = i_1 = \mathbf{1.0098\ A}$

16. Find the mesh currents in the circuit of Fig. 16. (*ML*)

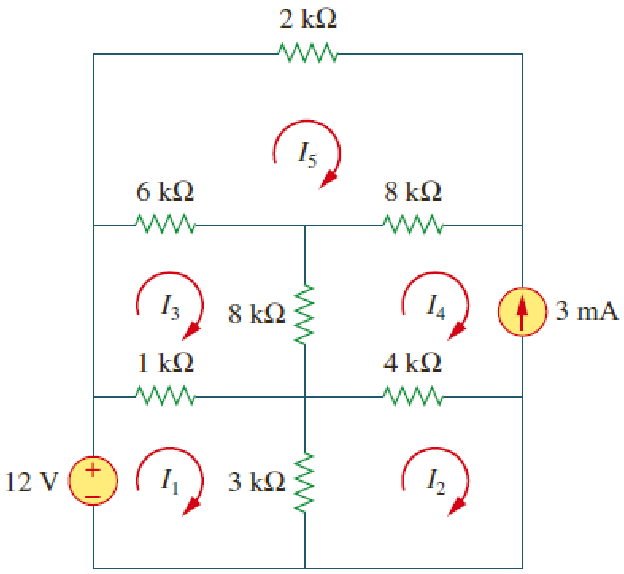


Fig. 16. For Prob. 16.

Solution:

Applying mesh analysis leads to;

$$-12 + 4kI_1 - 3kI_2 - 1kI_3 = 0 \quad (1)$$

$$-3kI_1 + 7kI_2 - 4kI_4 = 0 \text{ or } -3kI_1 + 7kI_2 = -12 \quad (2)$$

$$-1kI_1 + 15kI_3 - 8kI_4 - 6kI_5 = 0 \text{ or } -1kI_1 + 15kI_3 - 6k = -24 \quad (3)$$

$$I_4 = -3mA \quad (4)$$

$$-6kI_3 - 8kI_4 + 16kI_5 = 0 \text{ or } -6kI_3 + 16kI_5 = -24 \quad (5)$$

Putting these in matrix form (having substituted I_4 = 3mA in the above),

$$\begin{bmatrix} 4 & -3 & -1 & 0 \\ -3 & 7 & 0 & 0 \\ -1 & 0 & 15 & -6 \\ 0 & 0 & -6 & 16 \end{bmatrix} k \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_5 \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \\ -24 \\ -24 \end{bmatrix}$$

ZI = V

Using MATLAB,

```
>> Z = [4,-3,-1,0;-3,7,0,0;-1,0,15,-6;0,0,-6,16]
Z =
4 -3 -1 0
-3 7 0 0
-1 0 15 -6
0 0 -6 16
>> V = [12,-12,-24,-24]'
```

V =
12
-12
-24
-24
We obtain,
>> I = inv(Z)*V

I= 1.6196 mA
-1.0202 mA
-2.461 mA
-3 mA
-2.423 mA

17. Find *vo* and *io* in the circuit of Fig. 17. (*ML*)

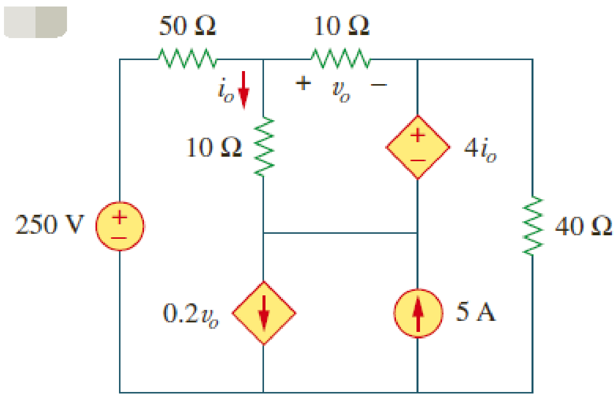
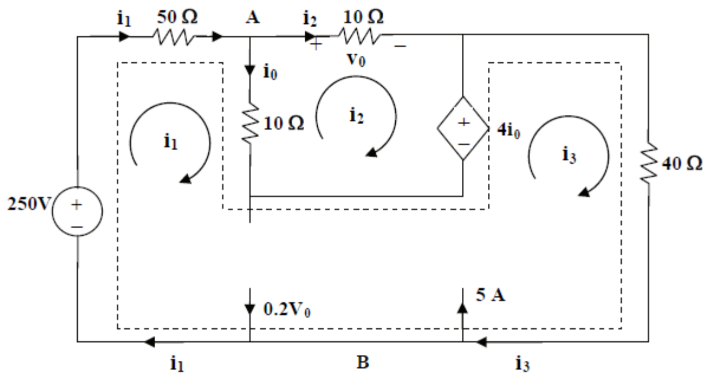


Fig. 17. For Prob. 17.

Solution:



For mesh 2,

$20i_2 - 10i_1 + 4i_0 = 0$ (1)

But at node A, $i_0 = i_1 - i_2$ so that (1) becomes $i_1 = (16/6)i_2$ (2)

For the supermesh,

$-250 + 50i_1 + 10(i_1 - i_2) - 4i_0 + 40i_3 = 0$ or $28i_1 - 3i_2 + 20i_3 = 125$ (3)

At node B, $i_3 + 0.2v_0 = 2 + i_1$ (4)

But, $v_0 = 10i_2$,so that (4) becomes $i_3 = 5 + (2/3)i_2$ (5)

Solving (1) to (5) , $i_2 = 0.2941$ A,

$v_0 = 10i_2 =$ **2.941 volts**, $i_0 = i_1 - i_2 = (5/3)i_2 =$ **490.2mA**.

18. Write the node-voltage equations by inspection and then determine values of V_1 and V_2 in the circuit of Fig. 18.

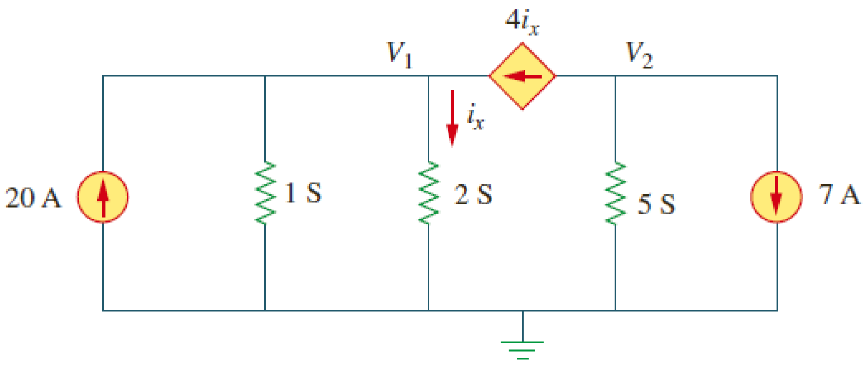


Fig. 18. For Prob. 18.

Solution:

$$\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix} V = \begin{bmatrix} 4I_x + 20 \\ -4I_x - 7 \end{bmatrix}$$

With two equations and three unknowns, we need a constraint equation,

$I_x = 2V_1$, thus the matrix equation becomes,

$$\begin{bmatrix} -5 & 0 \\ 8 & 5 \end{bmatrix} V = \begin{bmatrix} 20 \\ -7 \end{bmatrix}$$

This results in $V_1 = 20/(-5) =$ **–4 V** and $V_2 = [-8(-4) - 7]/5 = [32 - 7]/5 =$ **5 V**.

19. By inspection, obtain the mesh-current equations for the circuit in Fig. 19.

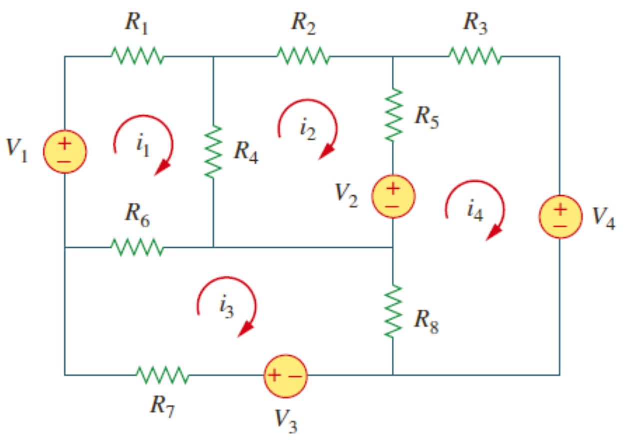


Fig. 19. For Prob. 19.

Solution:

$R_{11} = R_1 + R_4 + R_6$, $R_{22} = R_2 + R_4 + R_5$, $R_{33} = R_6 + R_7 + R_8$

$R_{44} = R_3 + R_5 + R_8$, $R_{12} = -R_4$, $R_{13} = -R_6$, $R_{14} = 0$, $R_{23} = 0$

$R_{24} = -R_5$, $R_{34} = -R_8$, again, we note that $R_{ij} = R_{ji}$ for all i not equal to j.

The input voltage vector is

$$= \begin{bmatrix} V_1 \\ -V_2 \\ V_3 \\ V_2 - V_4 \end{bmatrix}$$

$$\begin{bmatrix} R_1 + R_4 + R_6 & -R_4 & -R_6 & 0 \\ -R_4 & R_2 + R_4 + R_5 & 0 & -R_5 \\ -R_6 & 0 & R_6 + R_7 + R_8 & -R_8 \\ 0 & -R_5 & -R_8 & R_3 + R_5 + R_8 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \\ V_3 \\ V_2 - V_4 \end{bmatrix}$$

20. For the transistor circuit of Fig. 20, find *vo*. Take β= 200, V_{BE} = 0.7 V.

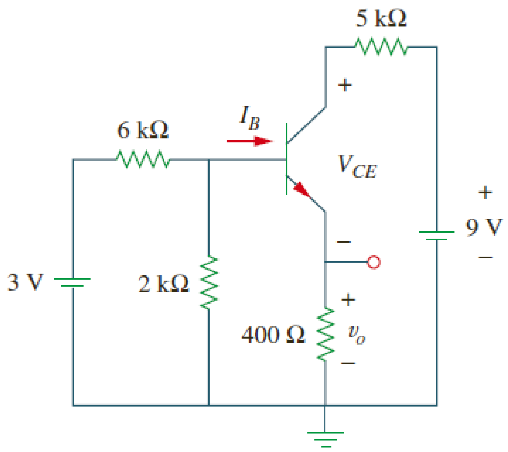
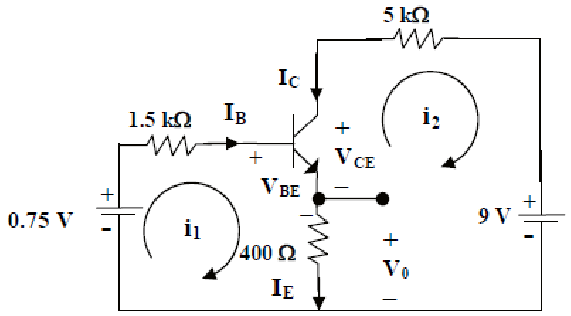


Fig. 20. For Prob. 20.

Solution:

We first determine the Thevenin equivalent for the input circuit.

R_{Th} = 6||2 = 6x2/8 = 1.5 k   and V_{Th} = 2(3)/(2+6) = 0.75 volts



For loop 1, $-0.75 + 1.5kI_B + V_{BE} + 400I_E = 0 = -0.75 + 0.7 + 1500I_B + 400(1 +\beta)I_B$

$I_B = 0.05/81{,}900 =$ **0.61 uA**

$v_0 = 400I_E = 400(1 + \beta)I_B =$ **49 mV**