Department of Computer Science and Engineering (CSE)
BRAC University

Summer 2023

CSE250 - Circuits and Electronics

CIRCUIT LAWS

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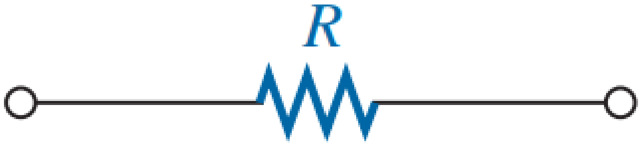
Resistance



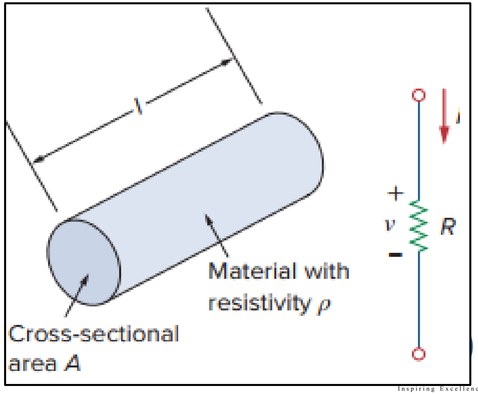
- What determines the level of current that results when a particular voltage is applied across a wire? *The answers lie in the fact that there is an opposition to the flow of charge in the system that depends on the components of the circuit. This opposition to the flow of charge through an electrical circuit, called resistance*.

- This opposition, due primarily to collisions and friction between the free electrons and other electrons, ions, and atoms in the path of motion, converts the supplied electrical energy into heat that raises the temperature of the electrical component and surrounding medium.

- So, *resistance* is a physical property of materials that refers to the ability to resist current.

- The resistance of any material with a uniform cross-sectional area A depends on A and its length ℓ. Mathematically, $R = \rho \frac{L}{A}$, where ρ is known as the resistivity of the material in ohm-meters ($\Omega - m$).

- The measuring unit for resistance is ohm (Ω)



Conductance

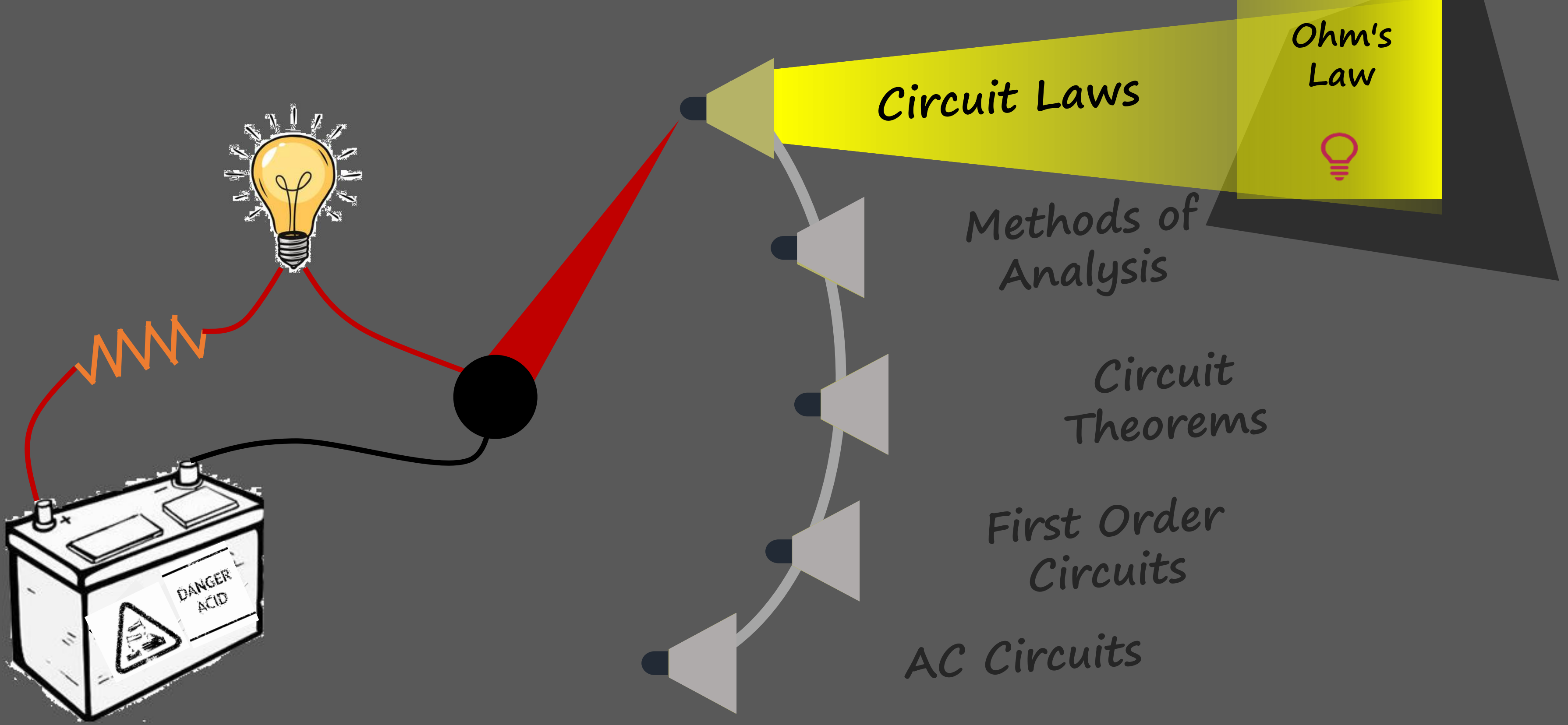
- By finding the reciprocal of the resistance of a material, we have a measure of how well the material conducts electricity. The quantity is called conductance, has the symbol G, and is measured in $siemens$ (S) or $mhos$ $(\mho)$

- So, the *conductance* is a measure of how well an element will conduct electric current.

- $G = \frac{1}{R}$ $\quad [1\,\mho = 1\,A/V = 1\,Siemen\,(S)]$

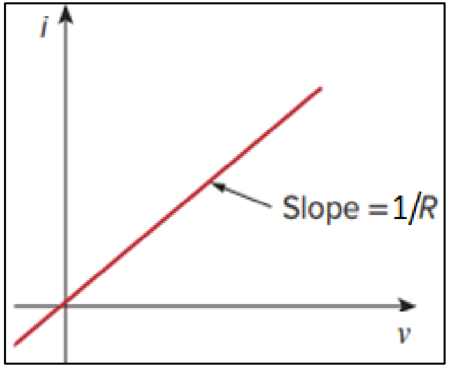
- $G = \frac{1}{R} = \frac{A}{\rho L} = \frac{\sigma A}{L}$, where $\sigma = \frac{1}{\rho}$ is a material-specific parameter called *conductivity*, measured in *siemens per meter* (Sm^{-1})

Course Outlines: broad themes



Ohm's Law

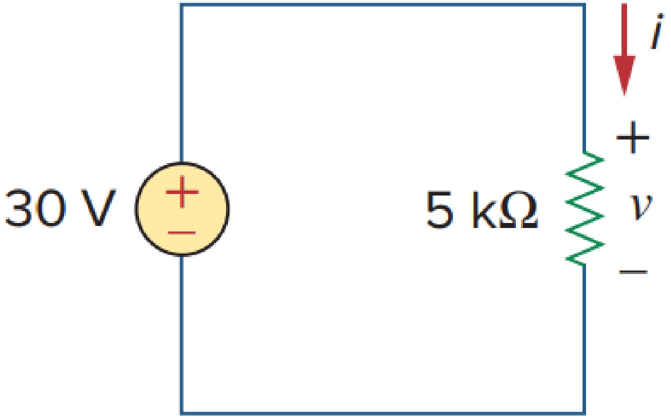
- One of the basic equations for any physical system is, $Effect = \frac{Cause}{Opposition}$. Every conversion of energy from one form to another can be related to this equation.
- In electric circuits, the effect we are trying to establish is the flow of charge, or current. The potential difference, or voltage, between two points is the cause ("pressure"), and the opposition is the resistance encountered. Substituting the terms,
- $Current = \frac{Voltage}{Resistanee} \quad \Rightarrow \quad I = \frac{V}{R}$



- *Ohm's law* states that the voltage across a resistor is directly proportional to the current flowing through the resistor.
- That is, $v \propto i$ or $v = Ri$. Ohm defined the constant of proportionality for a resistor to be the resistance, R, measured in *ohm* (Ω). [1 Ω = 1 V/A]
- $p = vi = i^2R = \frac{v^2}{R}$ *(always +ve for passive elements like resistor)*

Example 1

- In the circuit shown below, calculate the current i, the conductance G, and the power p.



Solution

The voltage across the resistor is the same as the source voltage $(30\ V)$ because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is,

$$i = \frac{v}{R} = \frac{30}{5 \times 10^3} = 0.006\ A = 6\ mA$$

The conductance is,

$$G = \frac{1}{R} = \frac{1}{5 \times 10^3} = 0.0002\ S = 0.2\ mS$$

The power can be calculated in various ways

$$p = vi = 30 \times (6 \times 10^{-3}) = 0.18\ W = 180\ mW$$

Or,

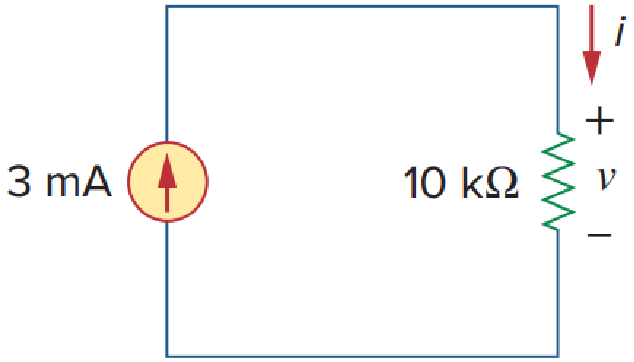
$$p = i^2 R = (6 \times 10^{-3})^2 \times (5 \times 10^3) = 180\ mW$$

Or,

$$p = \frac{v^2}{R} = \frac{30^2}{5 \times 10^3} = 180\ mW$$

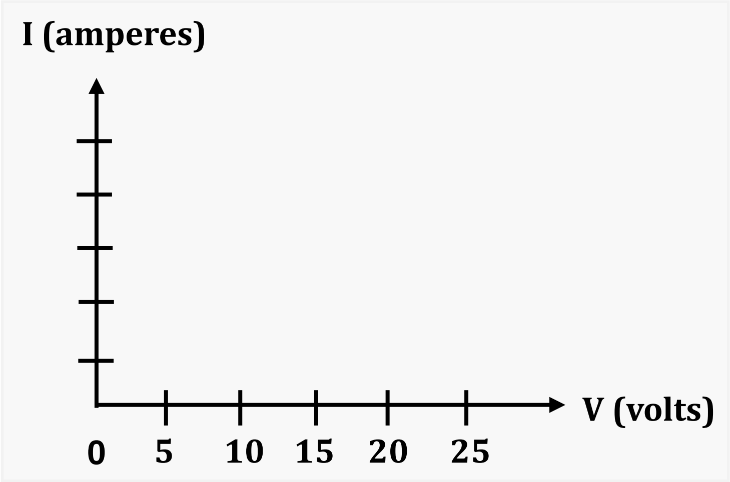
Problem 1

i. For the circuit shown below, calculate the voltage v, the conductance G, and the power p.



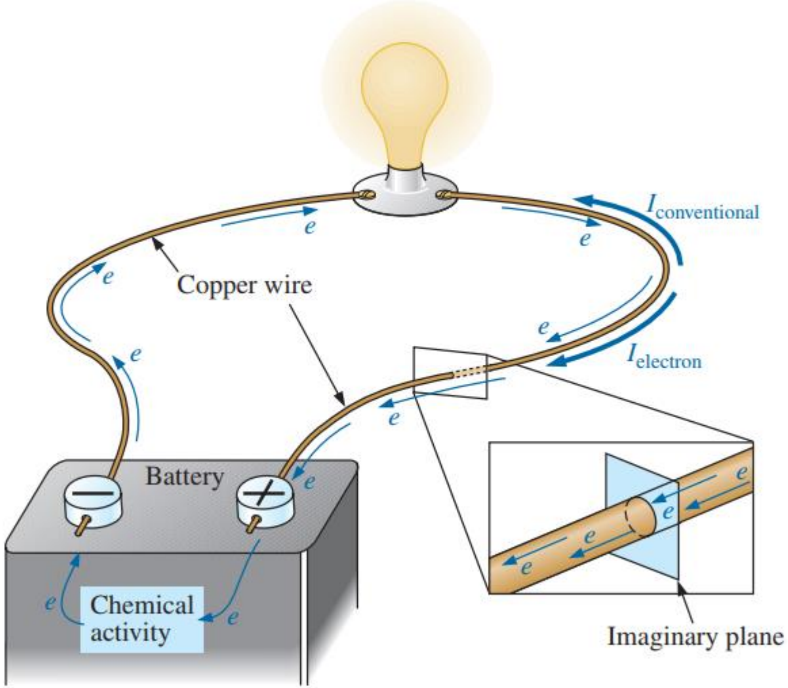
Ans: (i) $v = 30\ V$; $G = 100\ \mu S$; $p = 90\ mW$

ii. Draw the $I - V$ characteristics of a $10\ k\Omega$ resistor using the following template. Label the axes appropriately.



Problem 2

- Find the hot resistance of a light bulb rated $60\ W, 120\ V$.



Ans: $\mathbf{R} = \mathbf{240\ \Omega}$

Problem 3

(a) Calculate current i when the switch is in position 1.

(b) Find the current when the switch is in position 2.

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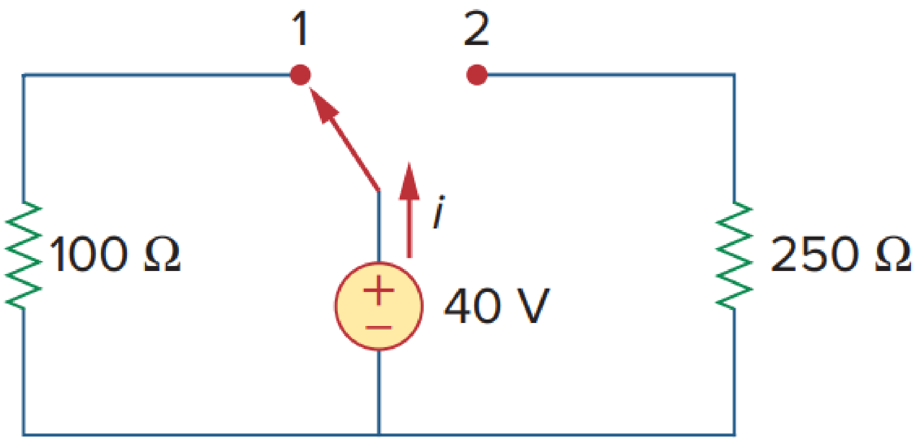
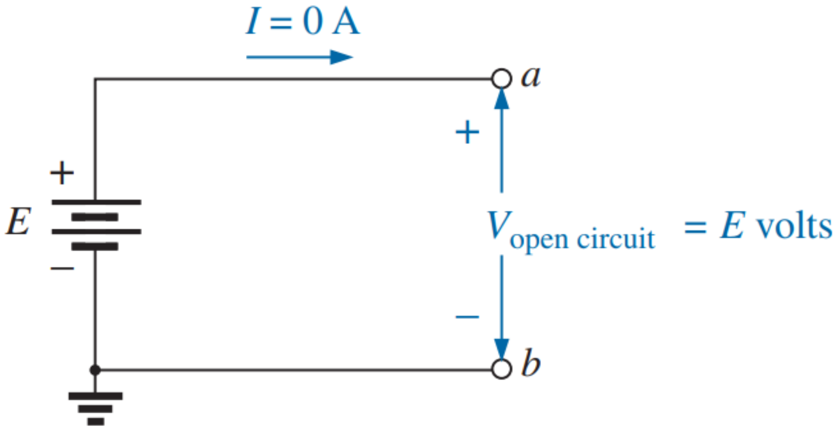
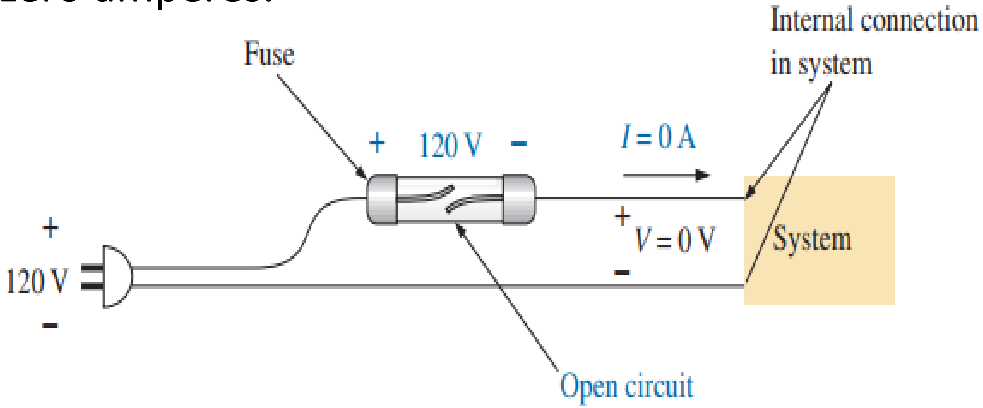


Ans: $(a)\ \boldsymbol{i = 0.4\ A}$; $(b)\ \boldsymbol{i = 0.16\ A}$.

Open circuit

- An *open circuit* is two isolated terminals not connected by an element of any kind.
- Any element with $R \to \infty$ is an open circuit. $i = 0 = \lim_{R \to \infty} \frac{v}{R}$
- Indicating that, an open circuit can have a potential difference (voltage) across its terminals, but the current is always zero amperes.

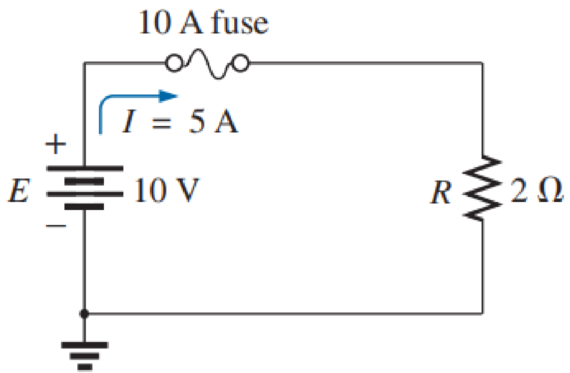


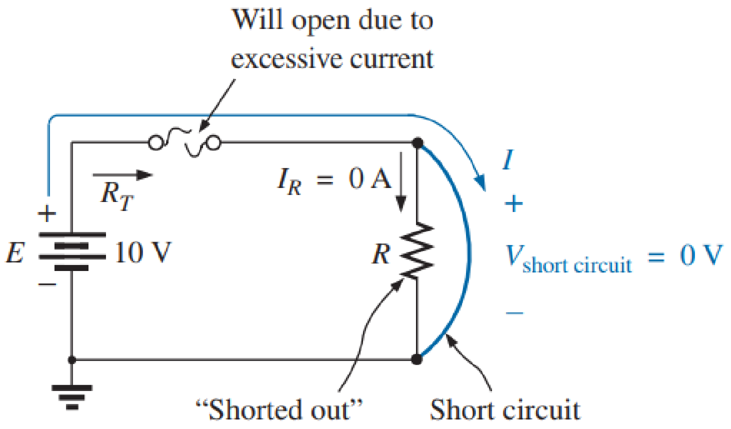


In the event of an excessive current flow, a fuse opens to protect appliances.

Short circuit

- A *short circuit* is a very low resistance, direct connection between two terminals of a network
- Any element with $R = 0$ is a short circuit. $v = 0 = \lim_{R \to 0} iR$
- Indicating that, a short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero volts.



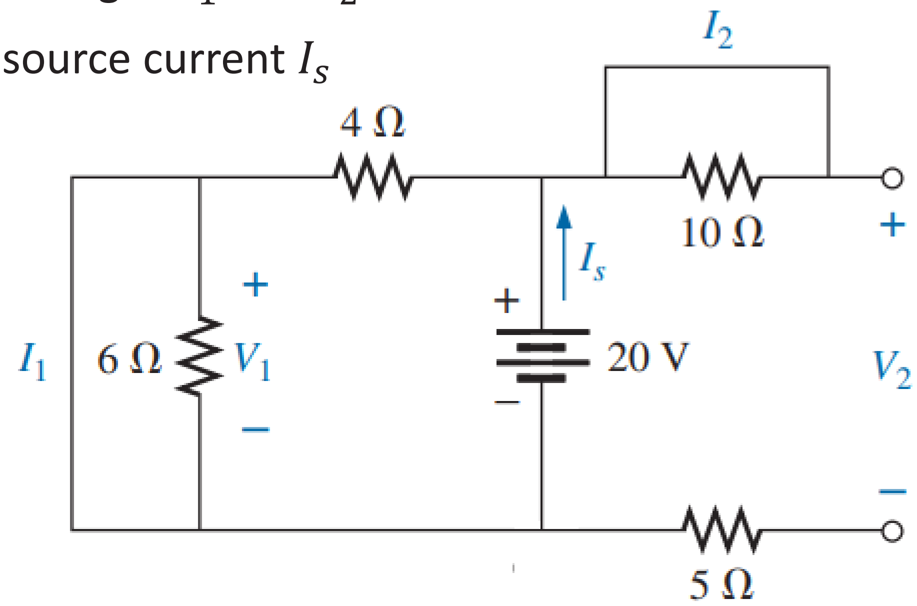


Problem 4

a. Determine the short circuit currents I_1 and I_2.

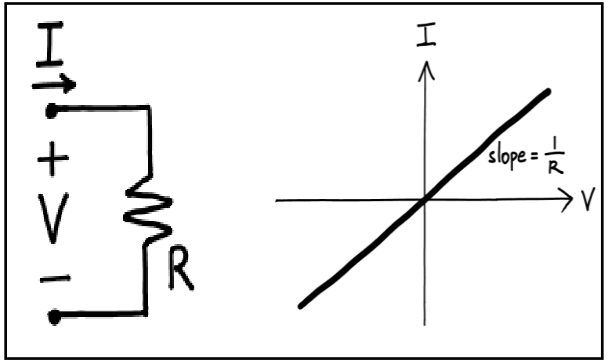
b. The voltages V_1 and V_2.

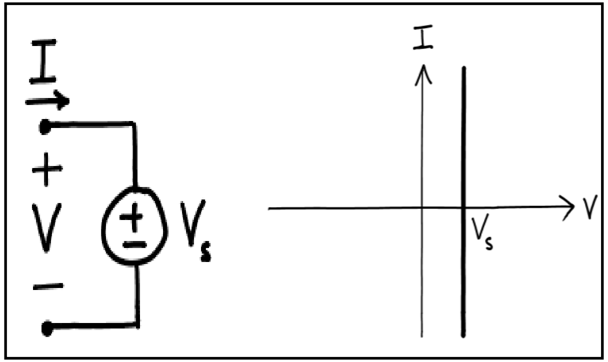
c. The source current I_s

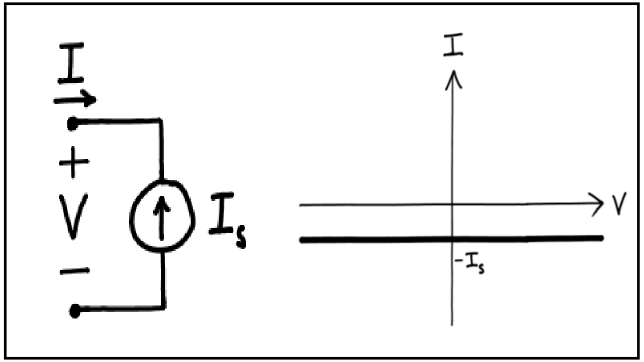


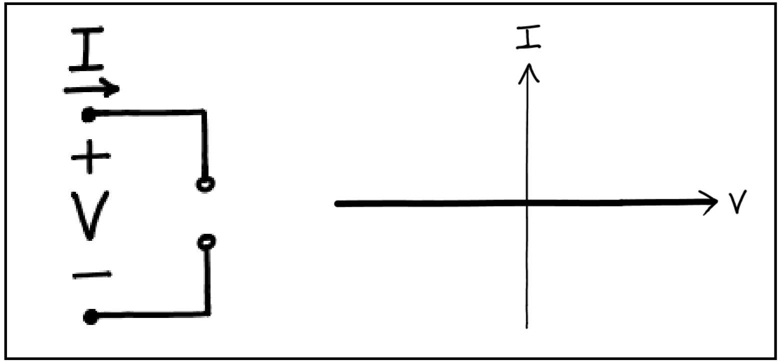
Ans:
a. 5 A, 0 A
b. 0 V, 20 V
c. 5 A

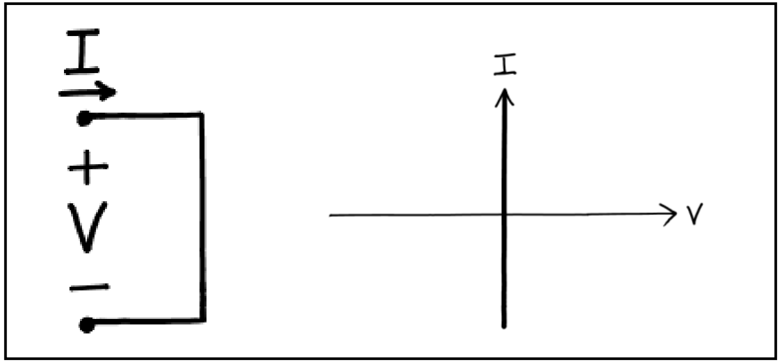
I-V characteristics





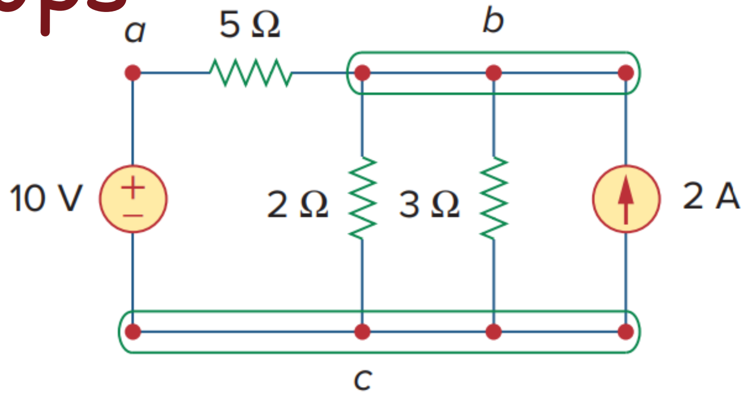






Nodes, Branches, & Loops

- A *branch* represents a single element such as a voltage source or a resistor. In other words, a branch represents a two-terminal element.
- A *node* is the point of connection between two or more branches.
- A *loop* is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.
- A loop is said to be *independent* if no other loops can be formed within it.



☞ 5 branches: $10\ V$ source, $2\ \Omega$, $3\ \Omega$, and $5\ \Omega$ resistors, $2\ A$ current source

☞ 3 nodes (n): a, b, c

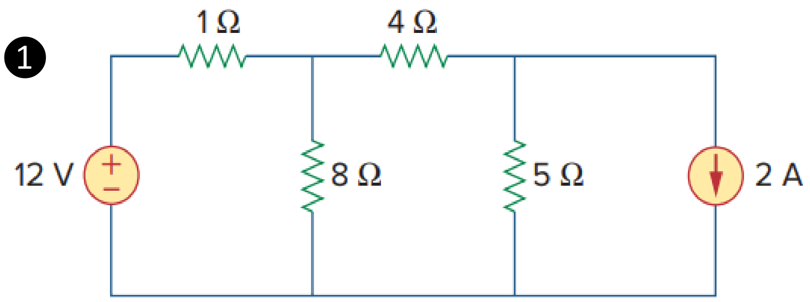
☞ 3 independent loops (l)

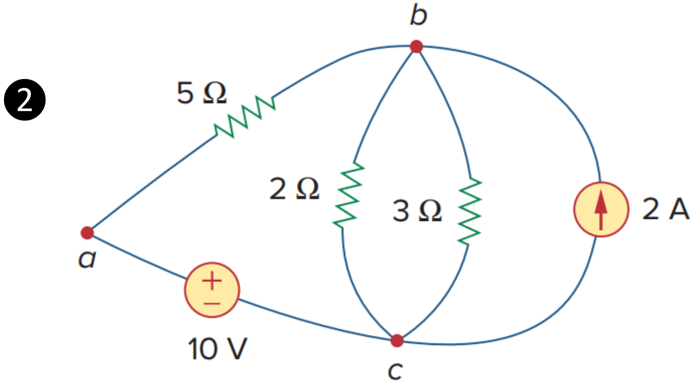
☞ 3 dependent loops (l_T)

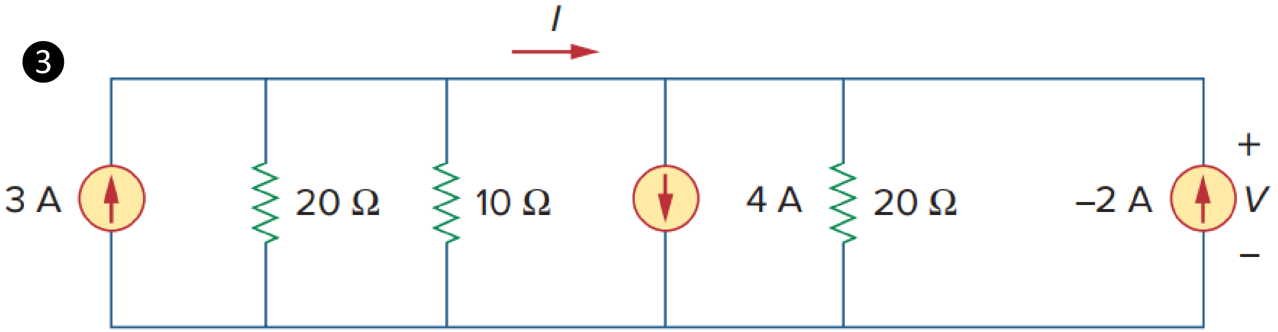
Problem 5

Ans:
1. $b = 6;\ n = 4;\ l_T = 3 + 3$
2. $b = 5;\ n = 3;\ l_T = 3 + 3$
3. $b = 6;\ n = 2;\ l_T = 5 + 10$

- Determine the number of **branches**, **nodes,** and **loops** in the following circuits.



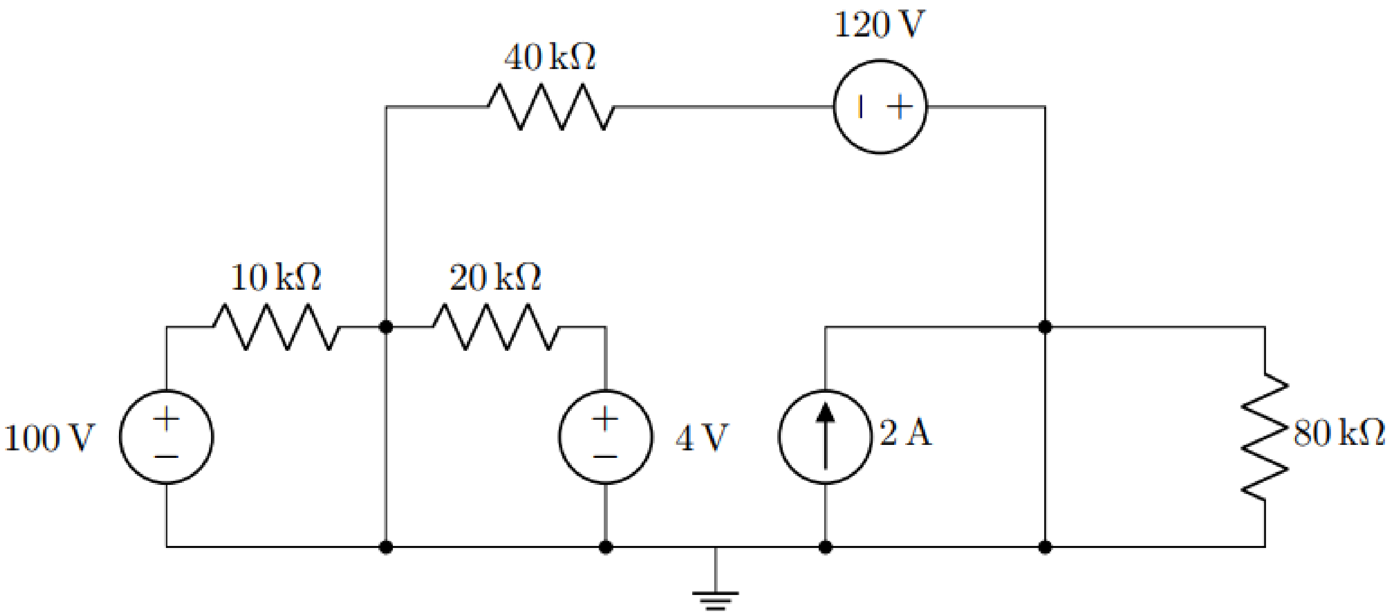




The three can be related as, $l = b - n + 1$, where l is the number of independent loops

Problem 6

- Determine the number of **nodes** in the following circuit.

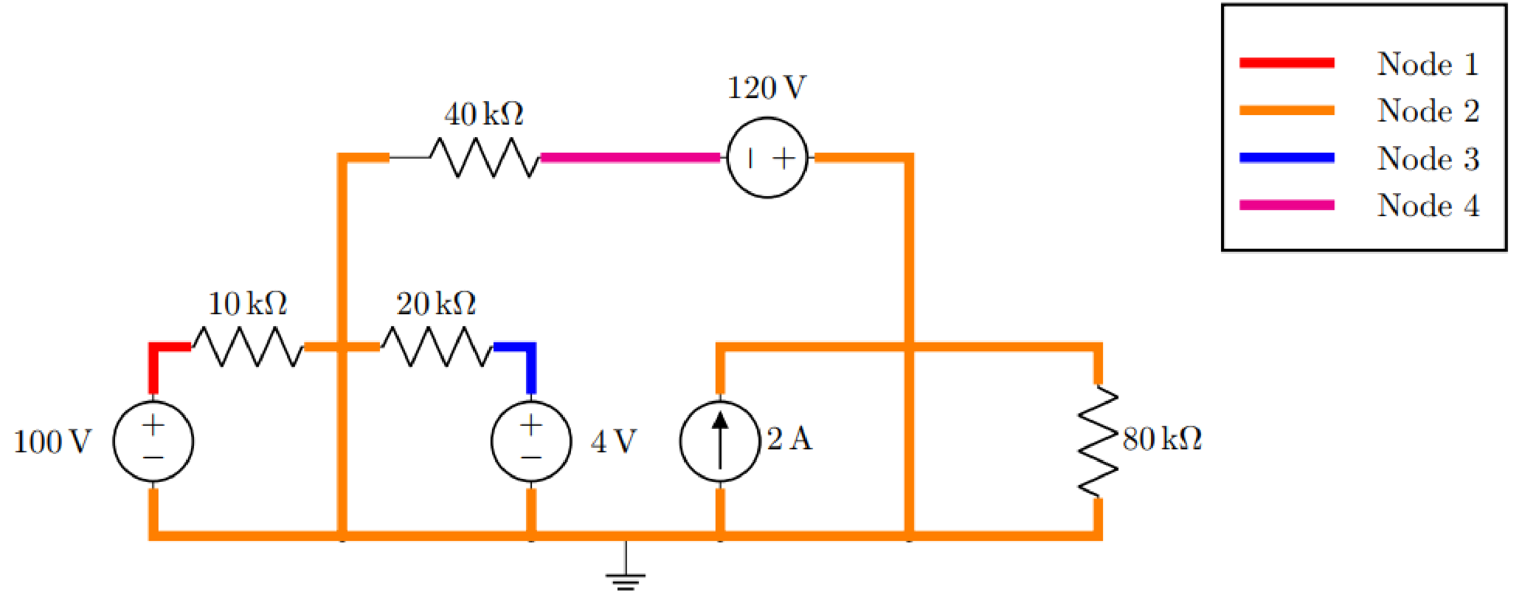


Ans: **Try yourself**

Problem 6: Solution

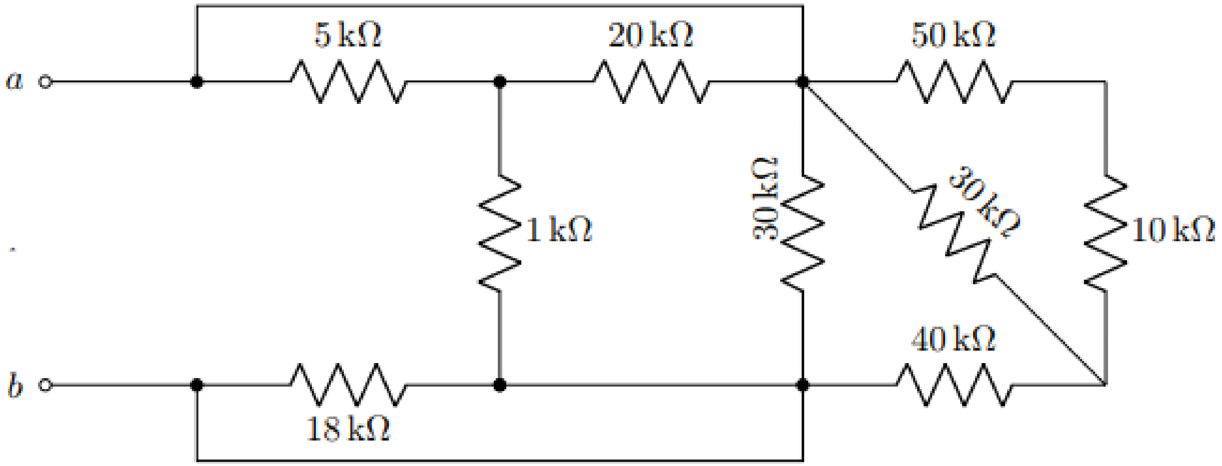
- Question: Determine the number of **nodes** in the following circuit.

Solution:



Problem 7

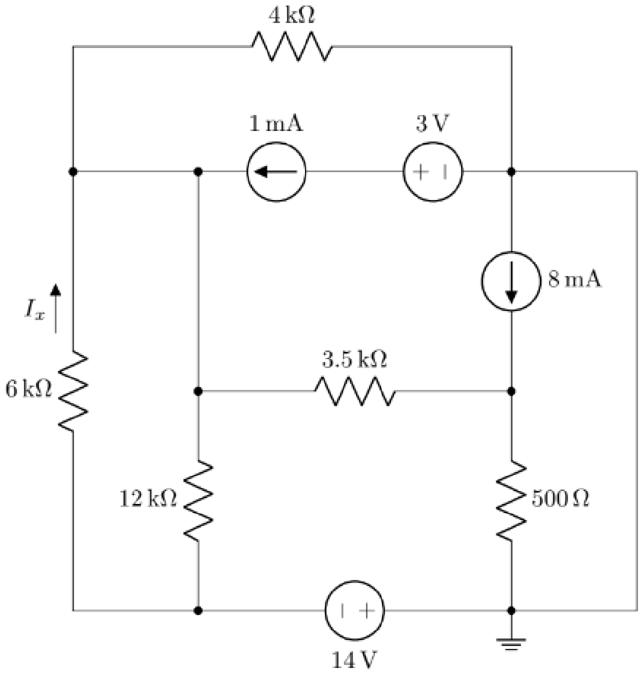
- Determine the number of **nodes** in the following circuit.



Ans: **Try yourself**

Problem 8

- Determine the number of **nodes** in the following circuit.



Ans: **Try yourself**

Problem 8: solution

- Question: Determine the number of **nodes** in the following circuit.

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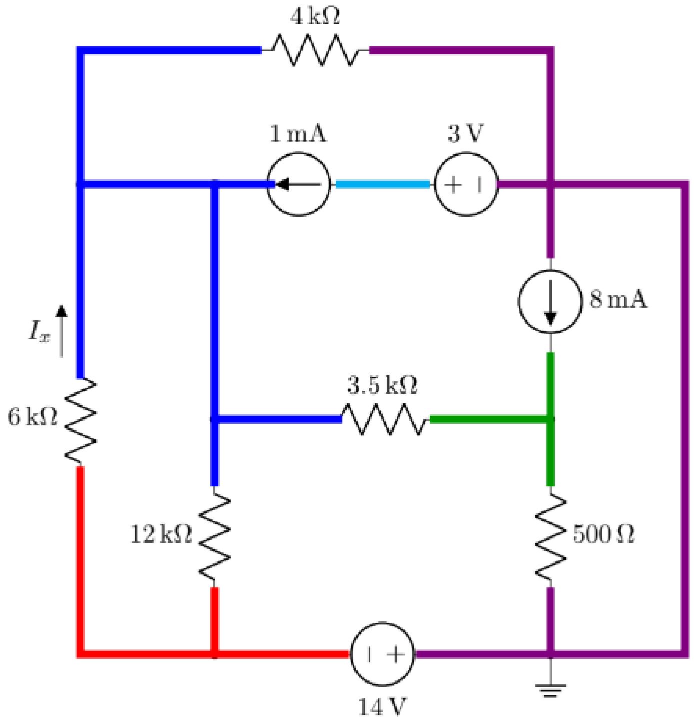
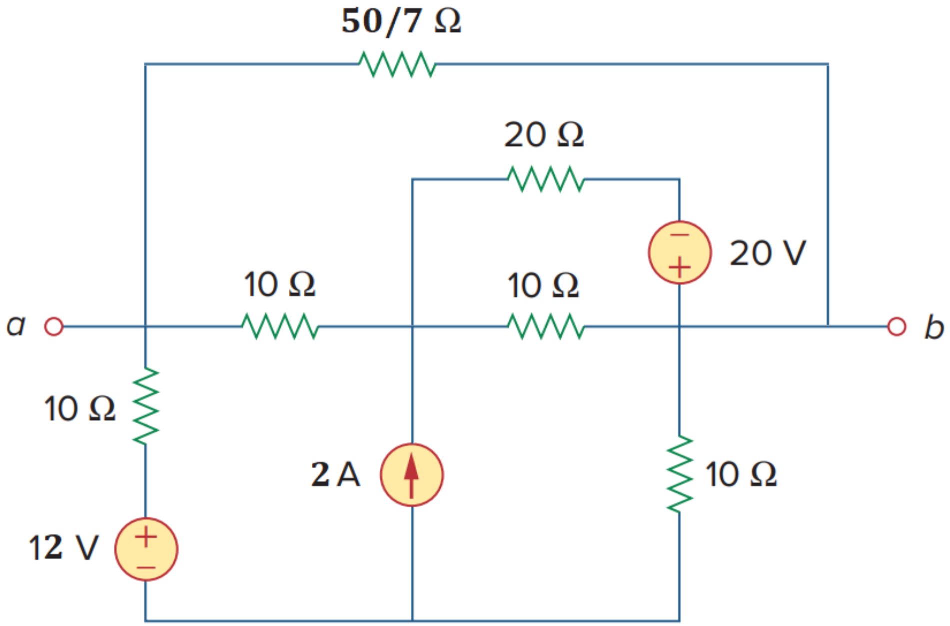


Ans: **4 nodes**

Problem 9

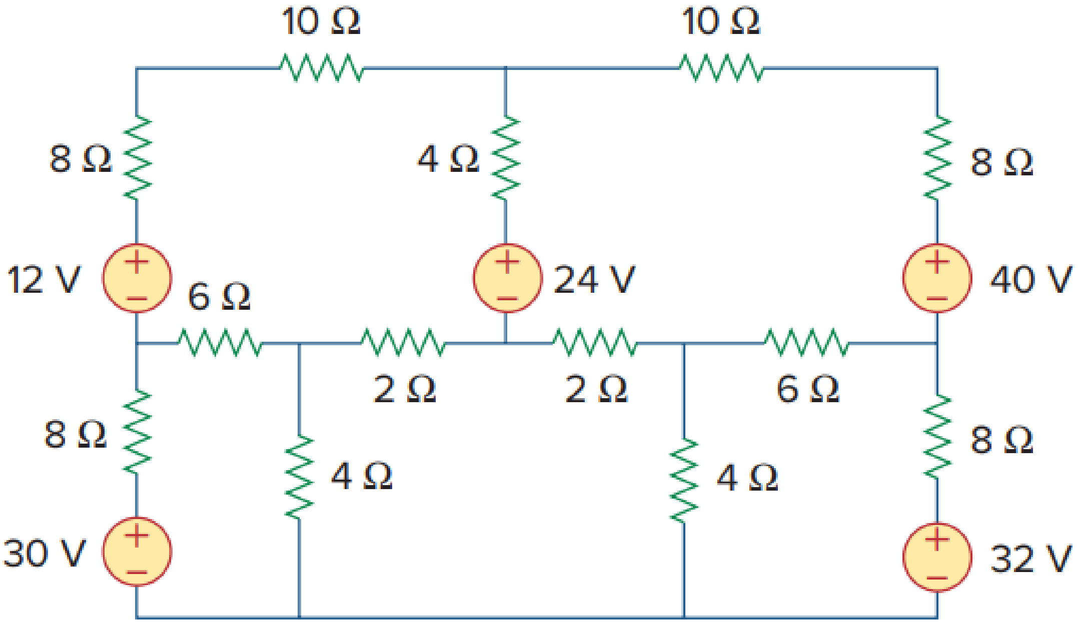
- Determine the number of **nodes** in the following circuit.



Ans: **Try yourself**

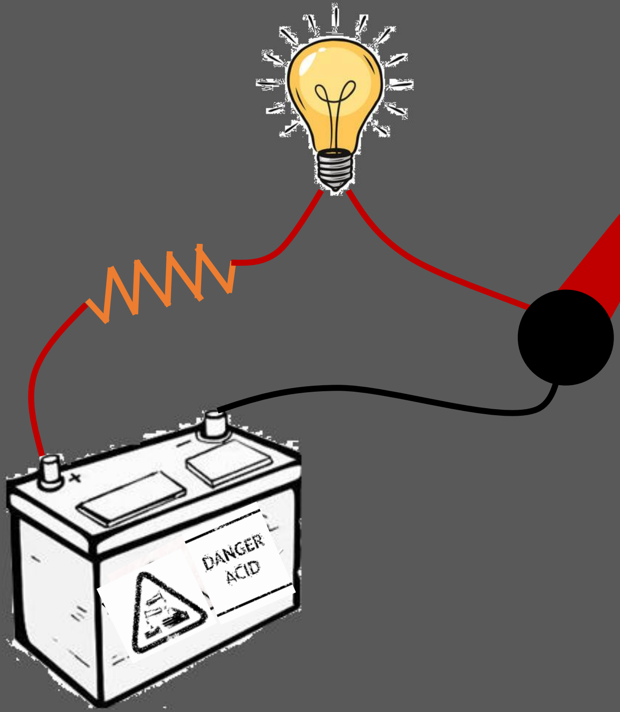
Problem 10

- How many **nodes** are there in the following circuit.



Ans: 14

Course Outlines: broad themes



- Circuit Laws
  - Kirchhoff's Current Law
- Methods of Analysis
- Circuit Theorems
- First Order Circuits
- AC Circuits

Kirchhoff's Current Law (KCL)

- Kirchhoff's current law (KCL) the algebraic sum of the currents entering a node is equal to the algebraic sum of the currents leaving the node.

- Mathematically, $\sum_{n=1}^{N} i_n = 0$, where N is the number of branches connected to the node and I_n is the nth current entering (or leaving) the node.

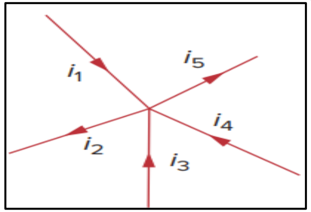
- Assume a set of currents $i_k(t), k = 1, 2, \ldots,$ flow into a node. The algebraic sum of currents at the node is, $i_{total}(t) = i_1(t) + i_2(t) + i_3(t) + \ \ldots \ldots$

- Integrating both sides, $q_{total}(t) = q_1(t) + q_2(t) + q_3(t) + \ \ldots \ldots, \ [q_k(t) = \int i_k(t)\, dt]$

- The *law of conservation of electric charge* requires that the algebraic sum of electric charges at the node must not change; that is, the node stores no net charge. Thus, $q_{Total}(t) = 0 \rightarrow i_T(t) = 0$, confirming the validity of KCL.

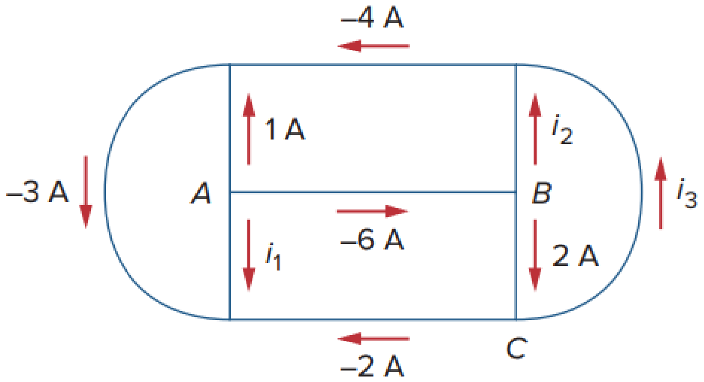
- For the node shown beside, $i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$

$$or, i_1 + i_3 + i_4 = i_2 + i_5$$



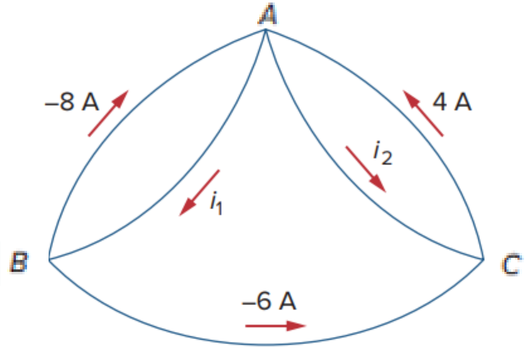
Example 2

(i) Find i_1, i_2, and i_3



Note that, in both the circuits A, B, and C are the same nodes. It is more appropriate to call them junctions in this case.

(ii) Find $i_1, and\ i_2$



KCL at junction A,

$$i_1 + 1 + (-6) = 0$$
$$\Rightarrow \quad i_1 = 5\ A$$

KCL at junction B,

$$i_2 + 2 = -6$$
$$\Rightarrow \quad i_2 = -8\ A$$

KCL at junction C,

$$2 = (-2) + i_3$$
$$\Rightarrow \quad i_3 = 4\ A$$

KCL at junction B,

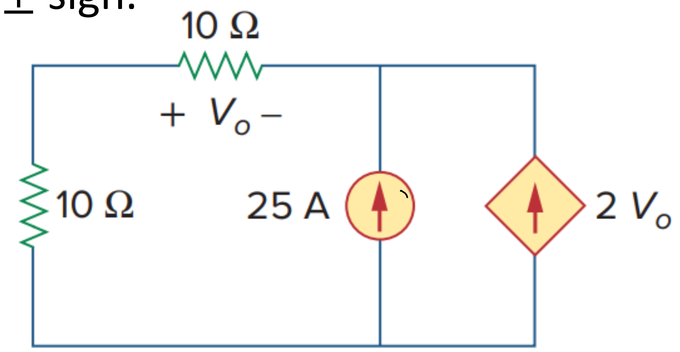
$$i_1 = (-8) + (-6)$$
$$\Rightarrow \quad i_1 = -14\ A$$

KCL at junction C,

$$i_2 + (-6) = 4$$
$$\Rightarrow \quad i_2 = 10\ A$$

Example 3

- Find V_0 and power absorbed/supplied by the dependent source with appropriate $\pm$ sign.



Current through the series resistances = $25 + 2V_0$

According to the Ohm's law,

$$V_0 = -10 \times (25 + 2V_0)$$
$$V_0 = -11.9\ V$$

The voltage across the dependent source is,

$$V_x = (10 + 10) \times (25 + 2V_0) = 24\ V$$

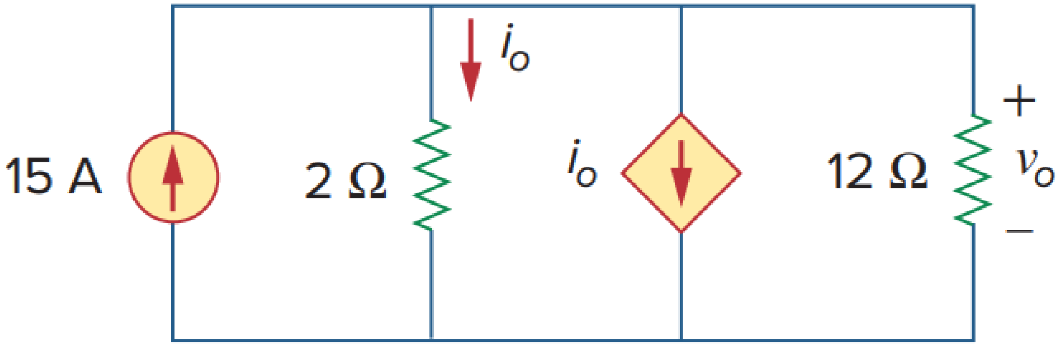
With the polarity of V_x and the direction of the current $(2V_0)$ given, according to the passive sign convention, the dependent source is supplying power. So,

$$p = -\,24 \times 2V_0 = 571.2\ W$$

The power is positive, hence, the dependent source is actually absorbing power. This is true as V_0 is negative, the current $2V_0$ is actually flowing in the opposite direction.

Problem 11

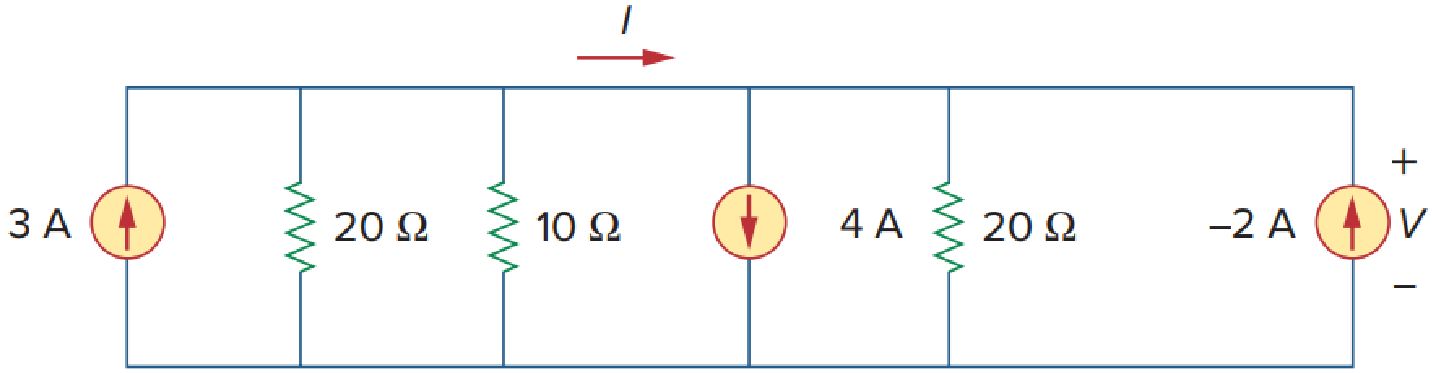
- Determine v_0 and i_0.



Ans: $\boldsymbol{v_0 = 13.85\,V;\ i_0 = 6.92\,A.}$

Problem 12

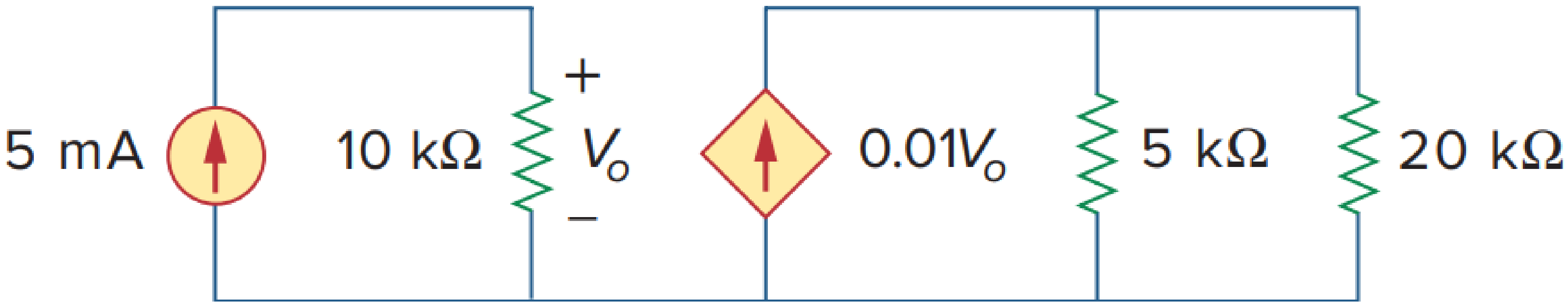
- Find the I and V shown in the following circuit.



Ans: $\boldsymbol{V = -15\,V;\ I = 5.25\,A.}$

Problem 13

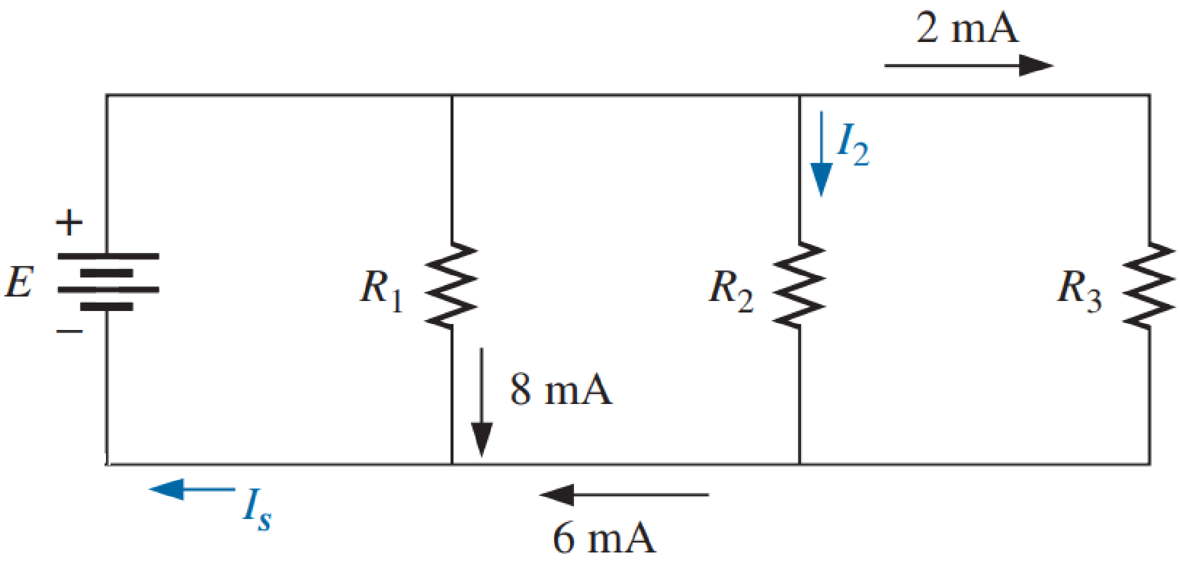
- For the network shown below, find the current, voltage, and power associated with the 20 $k\Omega$ resistor.



Ans: $\mathbf{0.1\ mA, 2\ V, 0.2\ mW}$

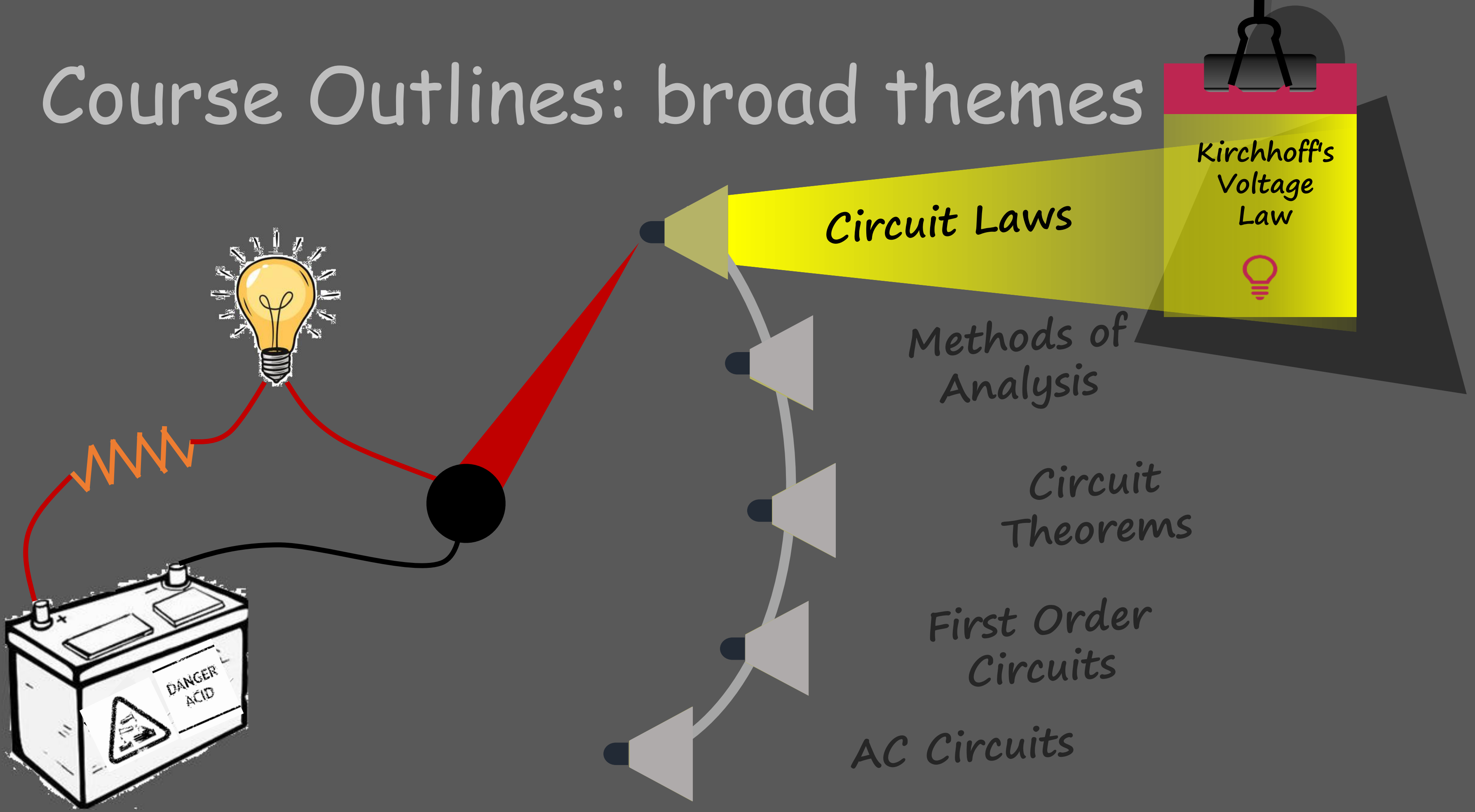
Problem 14

- Using KCL, determine the unknown currents.



Ans: $I_2 = 4\ mA, I_s = 14\ mA$

Course Outlines: broad themes



Kirchhoff's Voltage Law (KVL)

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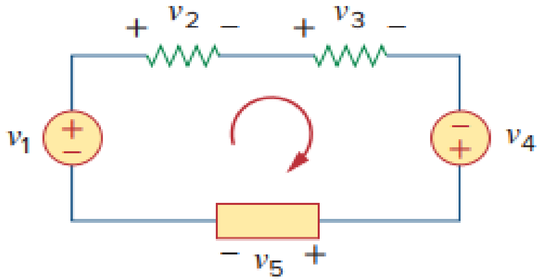
- Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.
- Mathematically, $\sum_{m=1}^{M} v_m = 0$, where M is the number of voltages (or branches) in the loop and v_m is the m^{th} voltage.
- To illustrate KVL, consider the circuit shown. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop.
- If we start with the voltage source and go clockwise around the loop as shown; then voltages would be $-v_1$, $+v_2$, $+v_3$, $-v_4$, and $+v_5$, in that order. For example, as we reach branch 3, the positive terminal is met first; hence, we have $+v_3$. For branch 4, we reach the negative terminal first; hence, $+v_4$. Thus, KVL yields



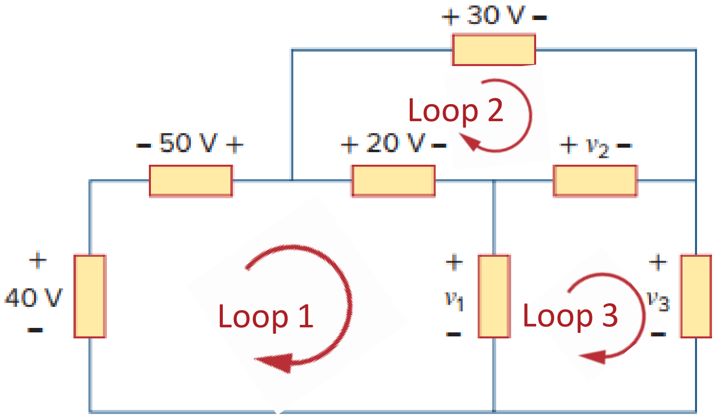
$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

$$or, v_2 + v_3 + v_5 = v_1 + v_4$$

Sum of voltage drops = Sum of voltage rises

Example 4

- Determine v_1, v_2, v_3 using KVL



KVL at loop 1,

$$-40 - 50 + 20 + v_1 = 0$$
$$v_1 = 70\ V$$

KVL at loop 2,

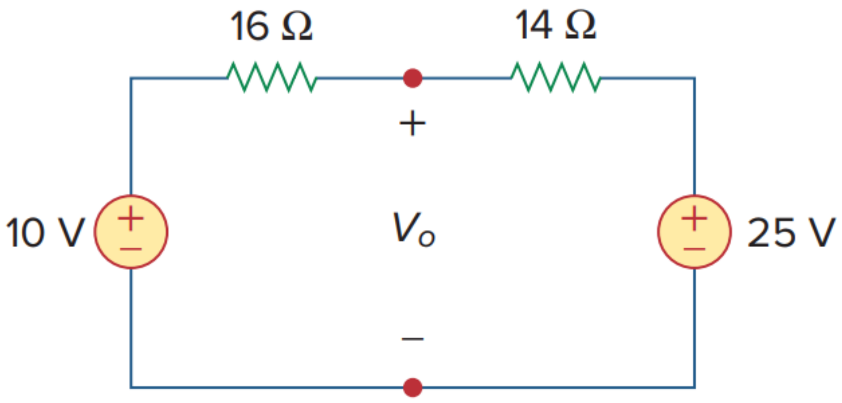
$$-20 + 30 - v_2 = 0$$
$$v_2 = 10\ V$$

KVL at loop 3,

$$-v_1 + v_2 + v_3 = 0$$
$$-70 + 10 + v_3 = 0$$
$$v_3 = 60\ V$$

Example 5

- Determine V_0 using KVL.



Let's assume that the current through the series circuit is i.

Applying KVL around the loop,

$$-10 + 16i + 14i + 25 = 0$$

$$i = -0.5\ A$$

V_0 can be found either by applying KVL through the loop consisting of V_0, 14 Ω, and 25 V or applying KVL through the loop consisting of V_0, 16 Ω, and 10 V. That is,

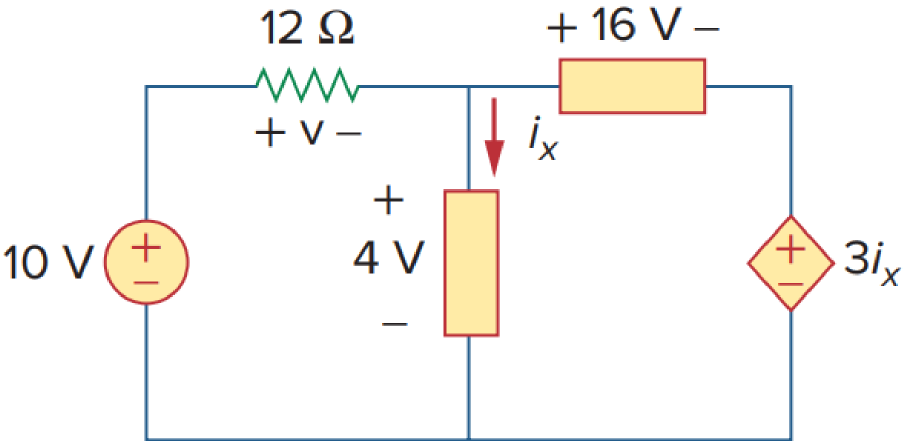
$-V_0 + 14i + 25 = 0$, or $V_0 = 18\ V$

Or,

$-10 + 16i + V_0 = 0$, or $V_0 = 18\ V$

Problem 15

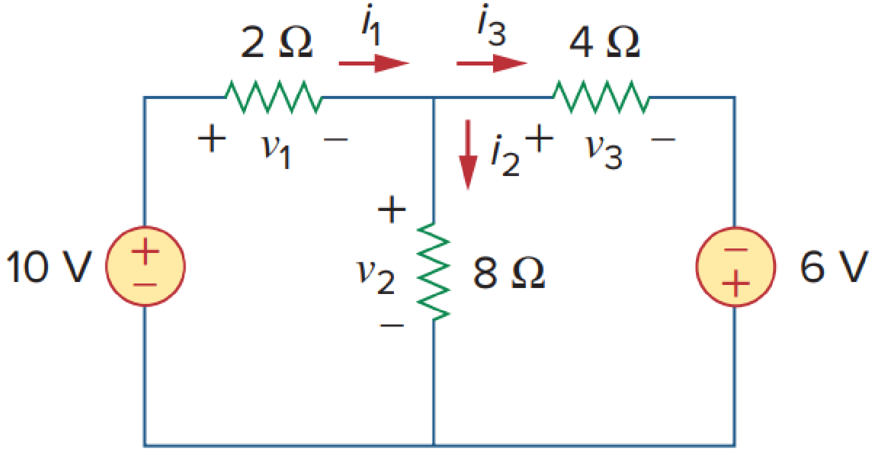
- Find v and i_x in the following circuit.



Ans: $v = 6\ V;\ i_x = -4\ A.$

Problem 16

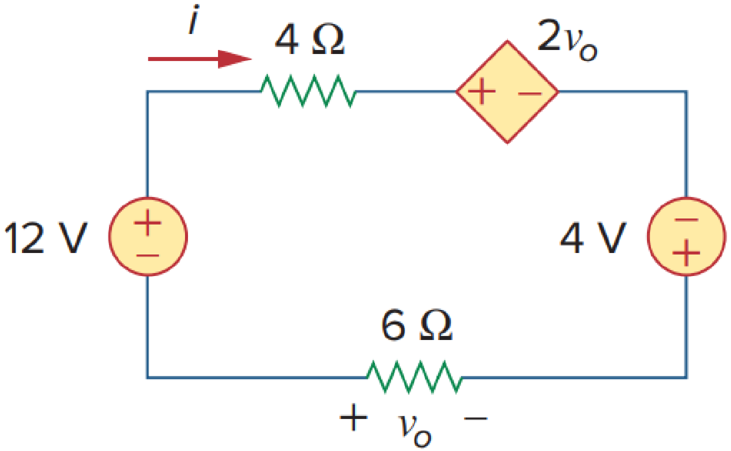
- Find the voltages and currents shown in the following circuit.



<u>Ans</u>: $v_1 = 6\,V$; $v_2 = 4\,V$; $v_3 = 10\,V$.
$i_1 = 3\,A$; $i_2 = 0.5\,A$; $i_3 = 2.5\,A$

Problem 17

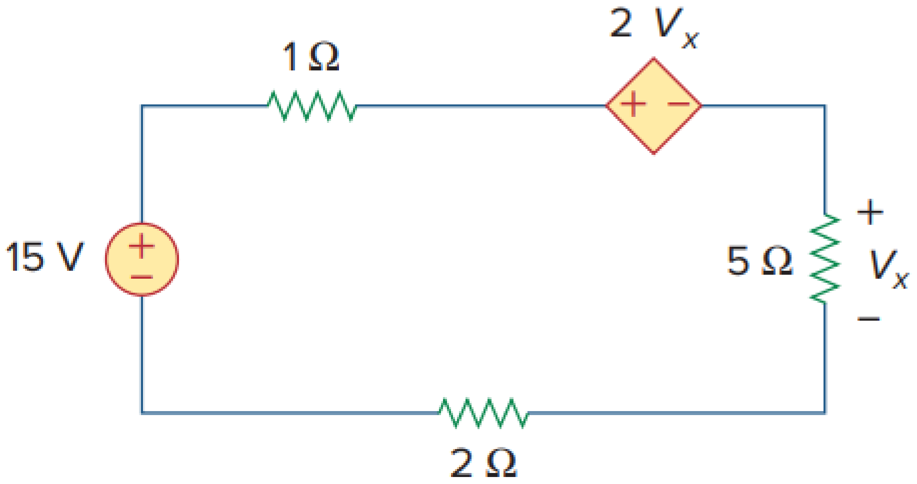
- Find v_0 and i in the circuit



Ans: $v_0 = 48\ V$; $I = -8\ A$.

Problem 18

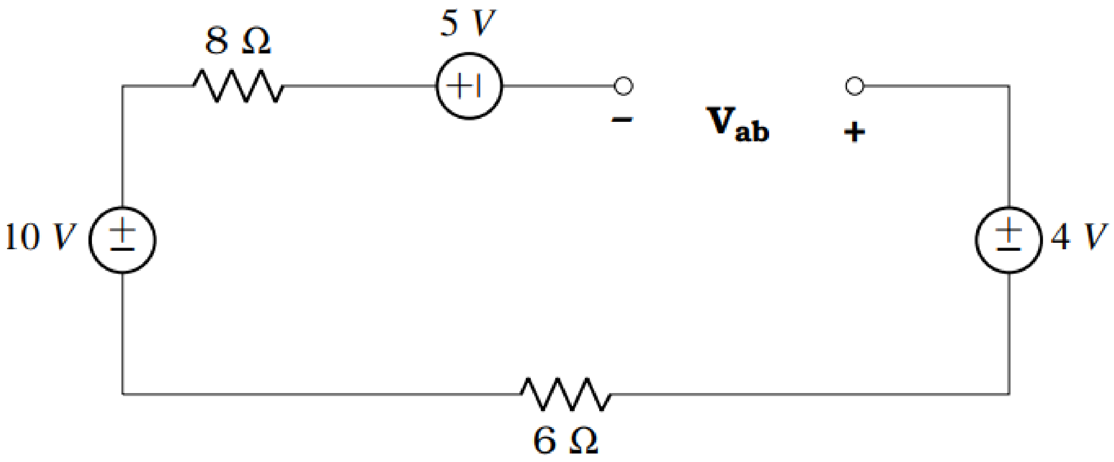
- Find V_x



Ans: $\boldsymbol{V_x = 4.167\ V.}$

Problem 19

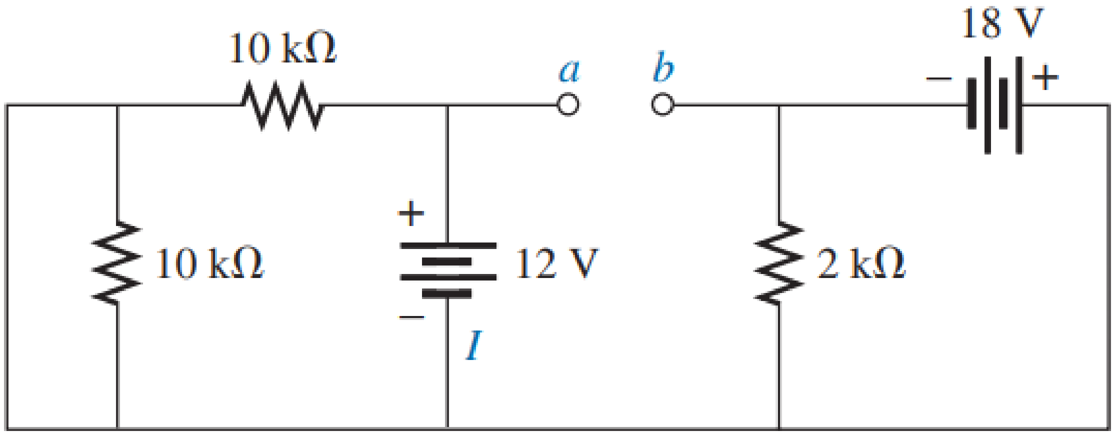
- Determine the voltage V_{ab} as indicated.



Ans: $\boldsymbol{V_{ab} = -1\ V}$.

Problem 20

- Determine the voltage between terminals a and b and the current I for the network shown below.



Ans: $\boldsymbol{V_{ab} = 30\ V; I = 1.2\ A}$.

Practice Problems

- Additional recommended practice problems: here
- Other suggested problems from the text book: here

Thank you for your attention