Answer

$$\oint \frac{e^{3z}}{z-\pi i}\,dz \qquad (a)\ |z-1|=4$$

$z=\pi i$ verify $|\pi i-1| = \sqrt{\pi^2+1} = 3.29 < 4$

$$\oint f(z)\,dz = 2\pi i\,[\text{Residue}]$$

$$\oint \frac{e^{3z}}{z-\pi i}\,dz = 2\pi i\left[\lim_{z\to\pi i} \cancel{(z-\pi i)}\,\frac{e^{3z}}{\cancel{(z-\pi i)}}\right]$$

$$= 2\pi i\left[e^{3\pi i}\right] \qquad \boxed{e^{i\theta} = \cos\theta + i\sin\theta}$$

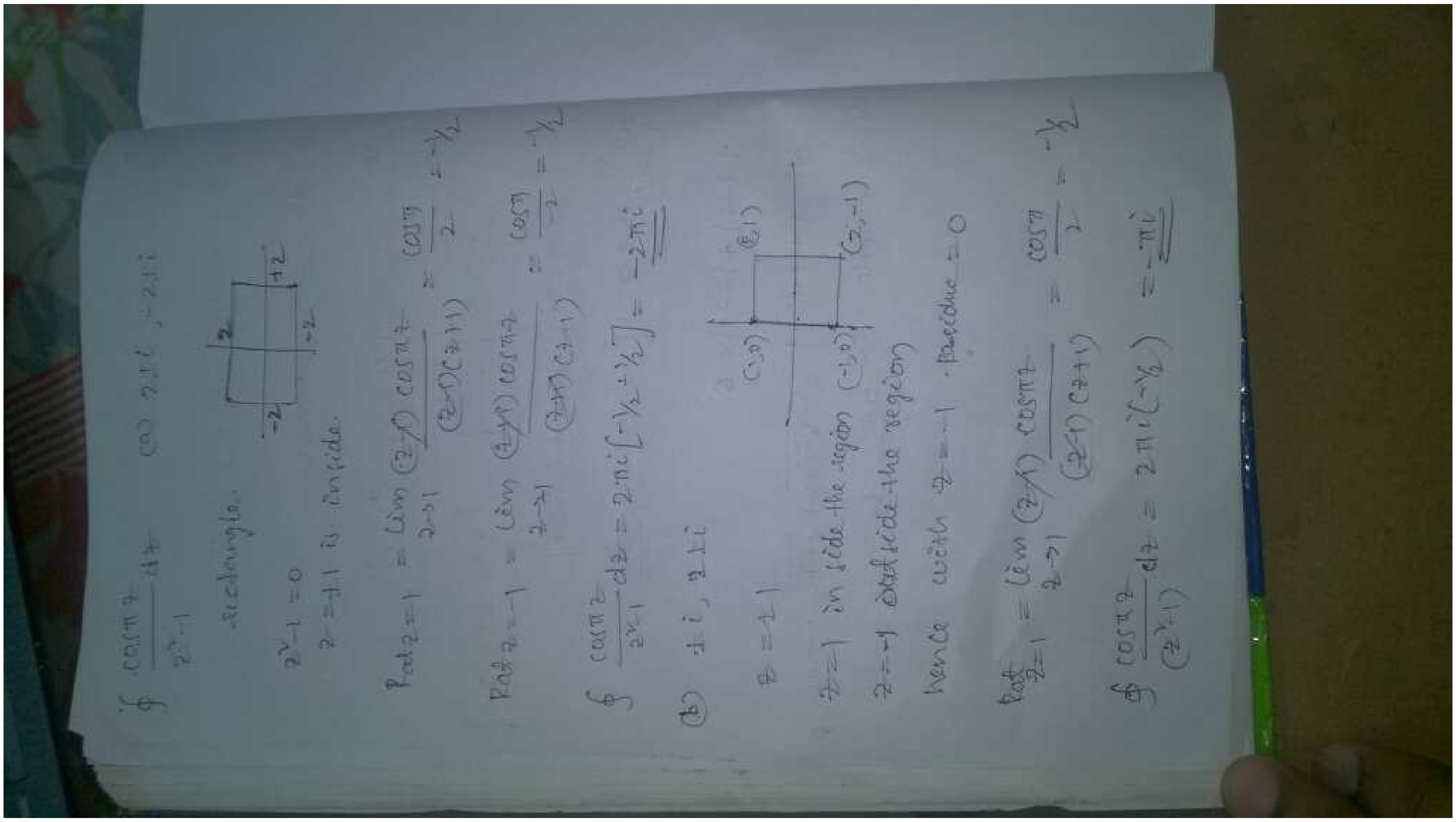
$$= 2\pi i\left[\cos 3\pi + i\sin 3\pi\right]$$

$$\oint \frac{e^{3z}}{(z-\pi i)}\,dz = 2\pi i\left[-1+0\right] = -2\pi i$$

(b) $:|z-2|+|z+2|=6$

$|\pi i-2|+|\pi i+2| \Rightarrow \sqrt{\pi^2+4}+\sqrt{\pi^2+4} = 7.44 > 6$

it is outside so $\oint \frac{e^{3z}}{z-\pi i}\,dz = 0$



$\oint \frac{\cos \pi z}{z^2-1} dz$ (a) $2 \pm i$, $-2 \pm i$

rectangle.

$z^2 - 1 = 0$

$z = \pm 1$ is inside.

$Res_{z=1} = \lim_{z \to 1} \frac{(z-1)\cos \pi z}{(z-1)(z+1)} = \frac{\cos \pi}{2} = -\frac{1}{2}$

$Res_{z=-1} = \lim_{z \to -1} \frac{(z+1)\cos \pi z}{(z+1)(z-1)} = \frac{\cos \pi}{-2} = -\frac{1}{2}$

$\oint \frac{\cos \pi z}{z^2-1} dz = 2\pi i\left[-\frac{1}{2} - \frac{1}{2}\right] = \underline{\underline{-2\pi i}}$

(b) $\pm i$, $2 \pm i$

$z = \pm 1$

$z = 1$ inside the region $(-1,0)$

$z = -1$ outside the region

hence with $z = -1$ residue $= 0$

$Res_{z=1} = \lim_{z \to 1} \frac{(z-1)\cos \pi z}{(z-1)(z+1)} = \frac{\cos \pi}{2} = -\frac{1}{2}$

$\oint \frac{\cos \pi z}{(z^2-1)} dz = 2\pi i\left(-\frac{1}{2}\right) = \underline{\underline{-\pi i}}$

Likes: 0 **Dislikes: 0**