Fall 2023

CSE250 - Circuits and Electronics

AC Fundamentals, AC Circuits, and AC Power

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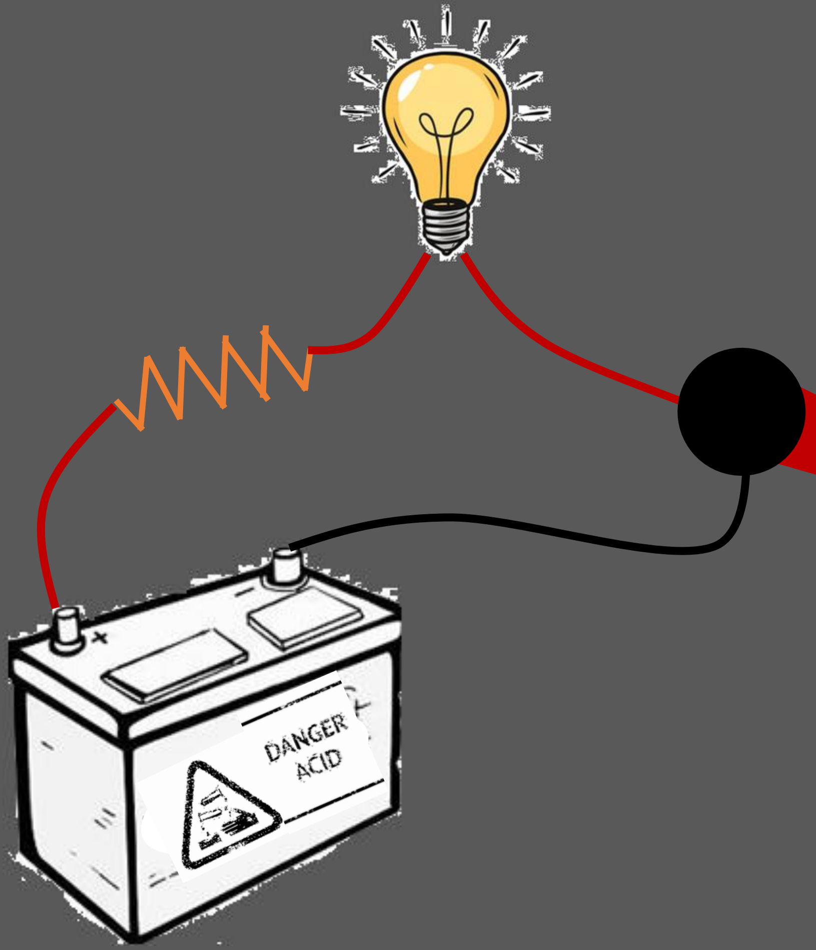
Course Syllabi

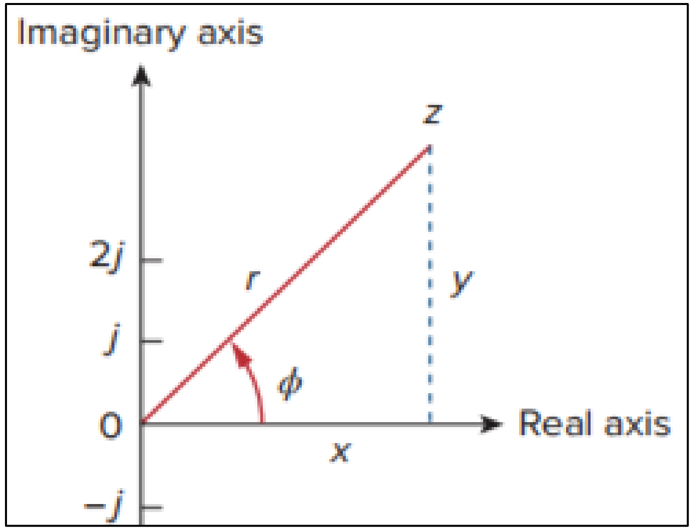


- Circuit Laws
- Methods of Analysis
- Circuit Theorems
- First Order Circuits
- AC Circuits

Fundamentals of AC

Complex Number

- A complex number z can be written as,
- $z = x + jy$ Rectangular form
- $z = r\angle\varphi$ Polar form $\quad j = \sqrt{-1} = -\frac{1}{j}$
- $z = re^{j\varphi}$ Exponential form
- The relationship between the rectangular and the polar form can be written from the figure as,
- $r = \sqrt{x^2 + y^2}$
- $\phi = \tan^{-1}\frac{y}{x};$
- $x = r\cos\theta$
- $y = r\sin\theta$



- The complex exponential can be expanded by a Taylor's series expansion as,
- $e^{j\varphi} = 1 + j\varphi + \frac{(j\varphi)^2}{2!} + \frac{(j\varphi)^3}{3!} + \frac{(j\varphi)^4}{4!} + \cdots$
- Separating the real and imaginary parts,

$$\Rightarrow e^{j\varphi} = \left(1 - \frac{\varphi^2}{2!} + \frac{\varphi^4}{4!} - \cdots\right) + j(x - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \cdots)$$

- The real and imaginary parts are the Taylor's series expansion of a cosine and a sine respectively.

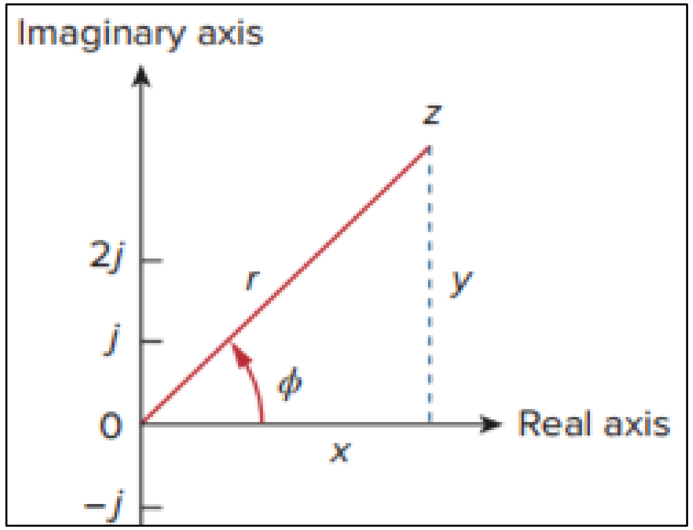
$$\Rightarrow e^{j\varphi} = cos\varphi + jsin\varphi$$

$$\rightarrow re^{j\varphi} = rcos\varphi + jrsin\varphi$$

Operations of Complex Numbers

- Rectangular form $z = x + jy$
- Polar form $z = r\angle\varphi$ where, $j = \sqrt{-1} = -\frac{1}{j}$
- Exponential form $z = re^{j\varphi}$

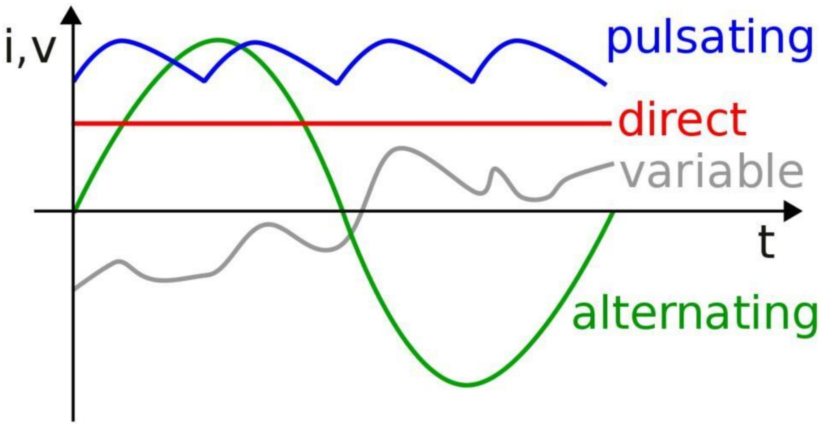
- Let, $z_1 = x_1 + jy_1 = r_1\angle\varphi_1 = r_1 e^{j\varphi_1}$ and $z_1 = x_2 + jy_2 = r_2\angle\varphi_2 = r_2 e^{j\varphi_2}$
- Addition: $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
- Subtraction: $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$
- Multiplication: $z_1 z_2 = r_1 r_2 \angle(\varphi_1 + \varphi_2)$
- Division: ${z_1}/{z_2} = \frac{r_1}{r_2}\angle(\varphi_1 - \varphi_2)$
- Reciprocal: $\frac{1}{z} = \frac{1}{r}\angle - \varphi$
- Square root: $\sqrt{z} = \sqrt{r}\angle\frac{\varphi}{2}$

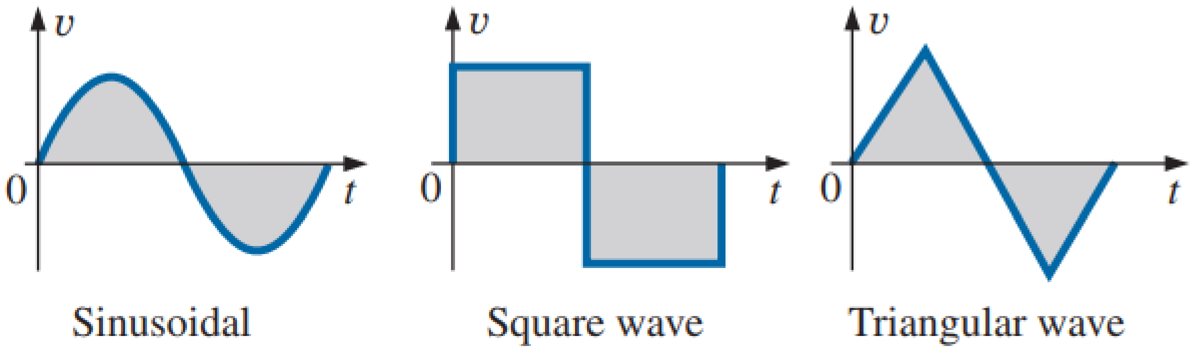


- Complex Conjugate: $z^* = x - jy = r\angle{-\varphi} = re^{-j\varphi}$

Alternating Current or Voltage

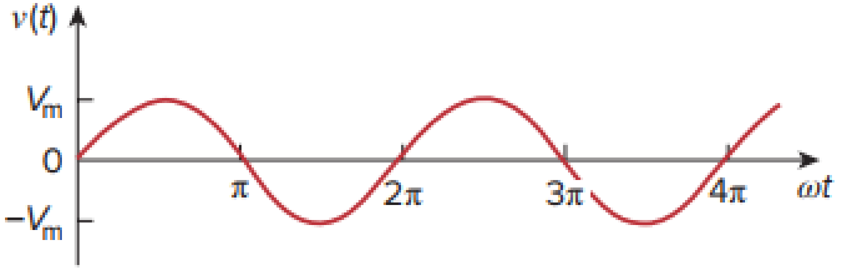
- *Alternating current (AC)* is a flow of electric charge that periodically reverses its direction, in contrast to direct current (DC) which only flows in a single direction. It starts, say, from zero, grows to a maximum, decreases to zero, reverses, reaches a maximum in the opposite direction, returns again to the original value, and repeats this cycle indefinitely.
- The interval of time between the attainment of a definite value on two successive cycles is called the *period*, the number of cycles or periods per second is the *frequency*, and the maximum value in either direction is the *amplitude* of the alternating current.

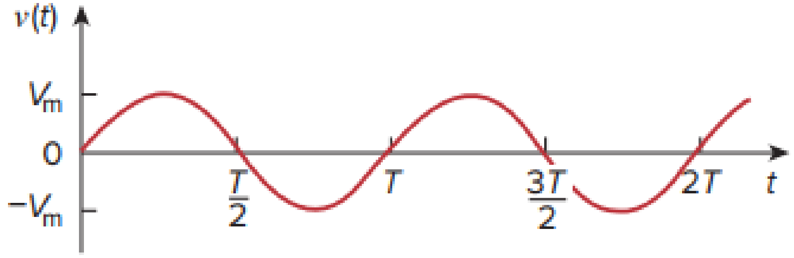




Alternating waveforms.

Sinusoid





- Among different types of ac waveforms, the pattern of particular interest is the sinusoidal because it is the voltage generated by utilities throughout the world and supplied to homes, factories, laboratories, and so on.
- A *sinusoid* is a signal that has the form of the sine or cosine function. A sinusoidal current is usually referred to as *alternating current (ac)*. Circuits driven by sinusoidal current sources or voltage sources are called *ac circuits*.
- The basic mathematical format for the sinusoidal waveform is $A_m sin\omega t$ or $A_m cos\omega t$
- For electrical quantities such as current and voltage, the general format is,

$$v(t) = V_m sin(\omega t) \text{ or } V_m cos(\omega t) \qquad and \qquad i(t) = I_m sin(\omega t) \text{ or } I_m cos(\omega t)$$

where, V_m, I_m = amplitudes of the sinusoids; ω = the angular frequency in radians/s; ωt = the argument of the sinusoid in deg or rad; time period, $T = \frac{2\pi}{\omega} = \frac{1}{f}$, f = frequency

Leading and Lagging Sinusoids

- The equation for a sinusoid in more general form,

$$v(t) = V_m\, sin(\omega t + \varphi)$$

where $(\omega t\ +\ \varphi)$ is the argument and φ is the initial phase. Both argument and phase can be in radians or degrees.

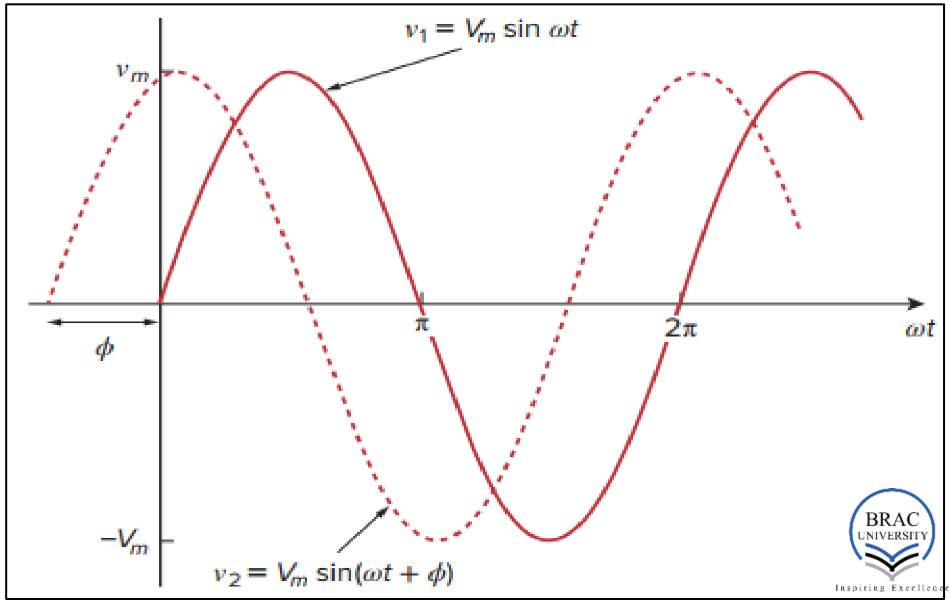
- Let's examine two sinusoids $v_1(t) = V_m sin\omega t$ and $v_2(t) = V_m sin(\omega t + \varphi)$

- The starting point of v_2 occurs first in time. Or v_2 passes the zero-crossing line first if compared between two same phase points of v_1 and v_2.

- Therefore, we say that v_2 leads v_1 by φ or that v_1 lags v_2 by φ.

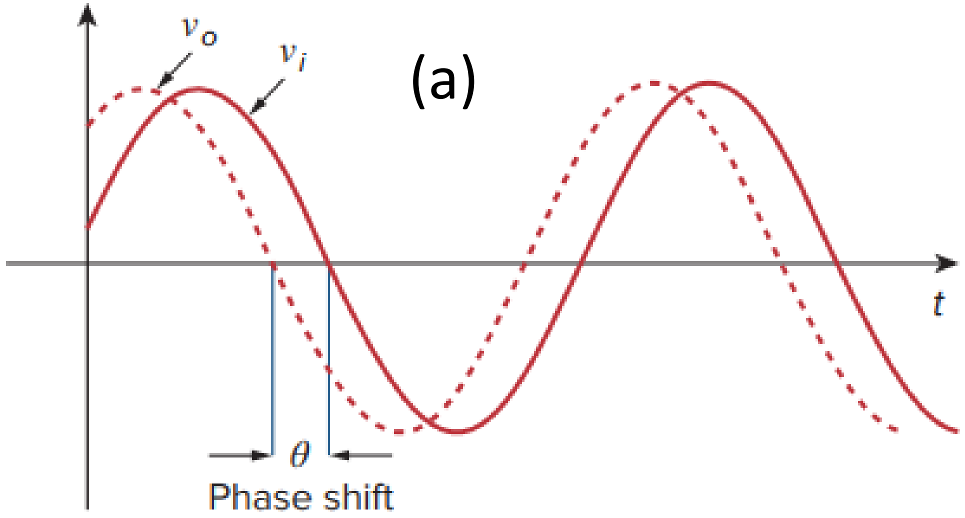
- If $\varphi \neq\ 0$, v_1 and v_2 are out of phase.

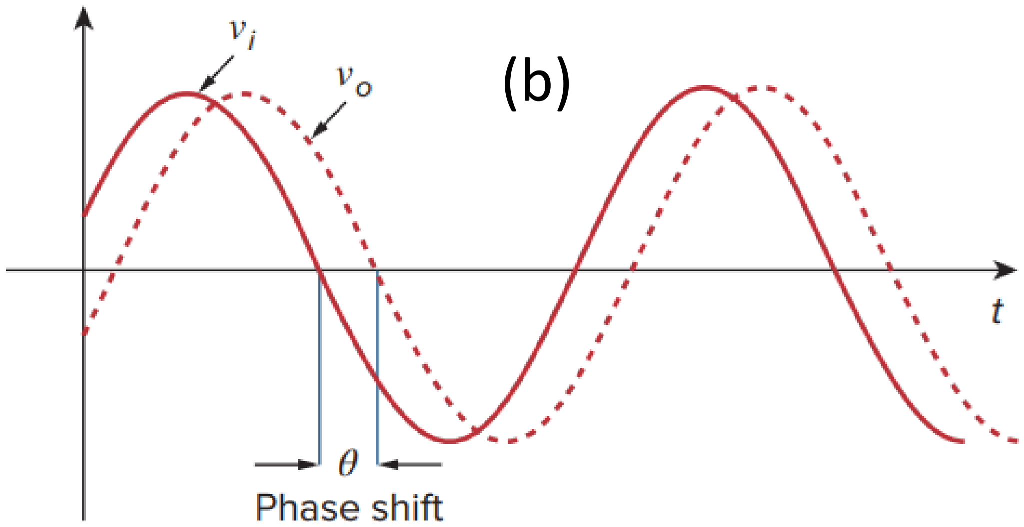
- If $\varphi =\ 0$, v_1 and v_2 are in phase



Problem 1

- Determine for each of the plots, which one is leading/lagging.



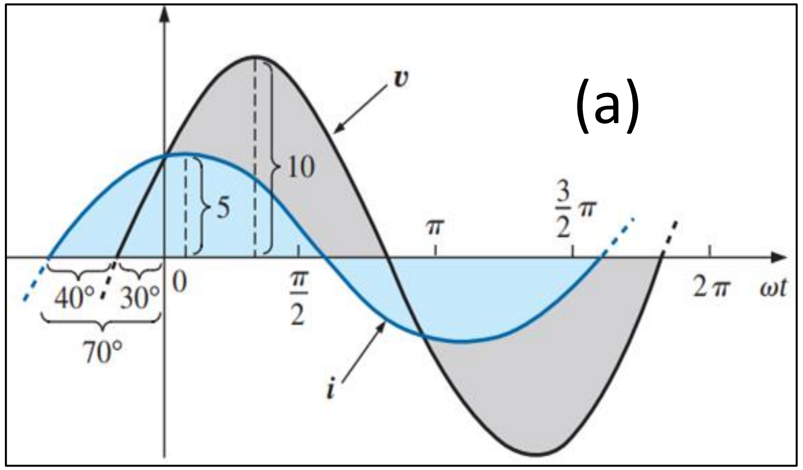


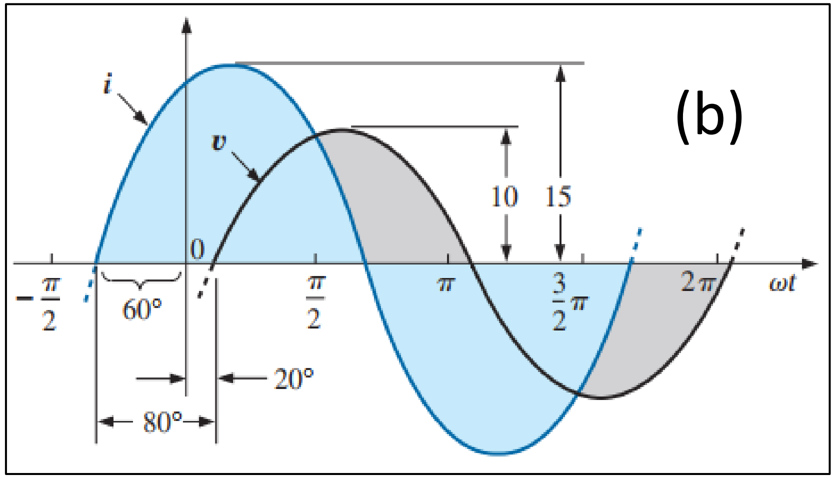
Ans: (a) v_0 leading; (b) v_i leading

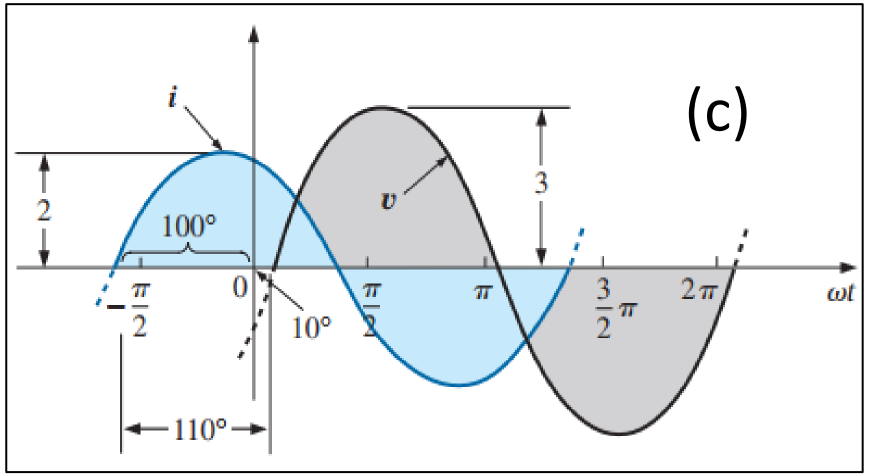
Problem 2

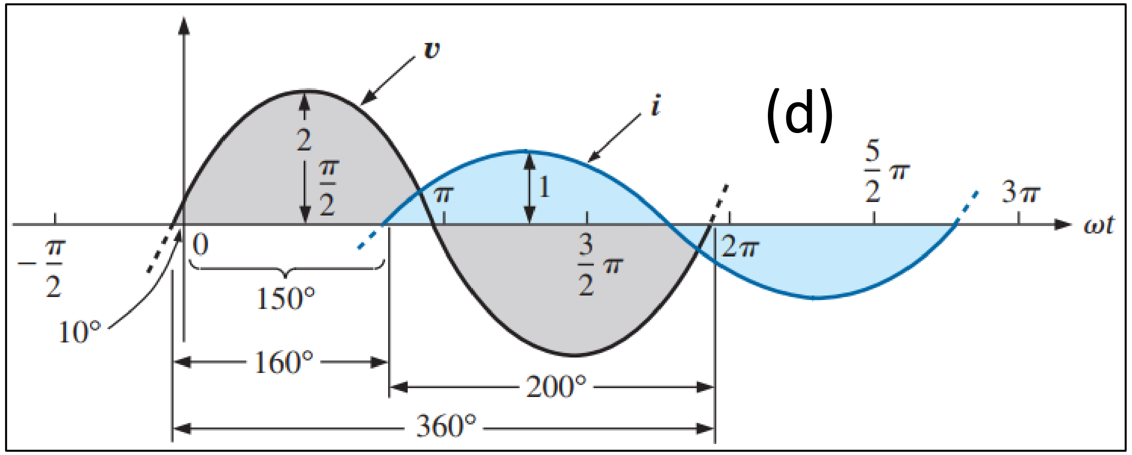
Ans: (a) i leads v by 40°; (b) i leads v by 80°; (c) i leads v by 110°; (d) v leads i by 160°; (e) v and i are in phase

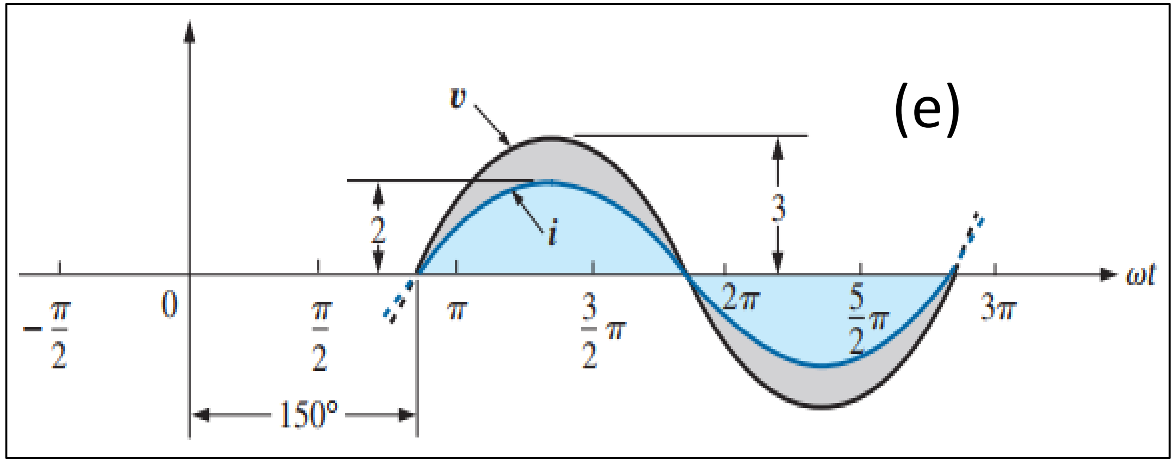
- Determine for each of the plots, which one (v or i) is leading/lagging and by how much.











Sine-Cosine Conversion

- A sinusoid can be expressed in either sine or cosine form. When comparing two sinusoids, it is expedient to express both as either sine or cosine with positive amplitudes.

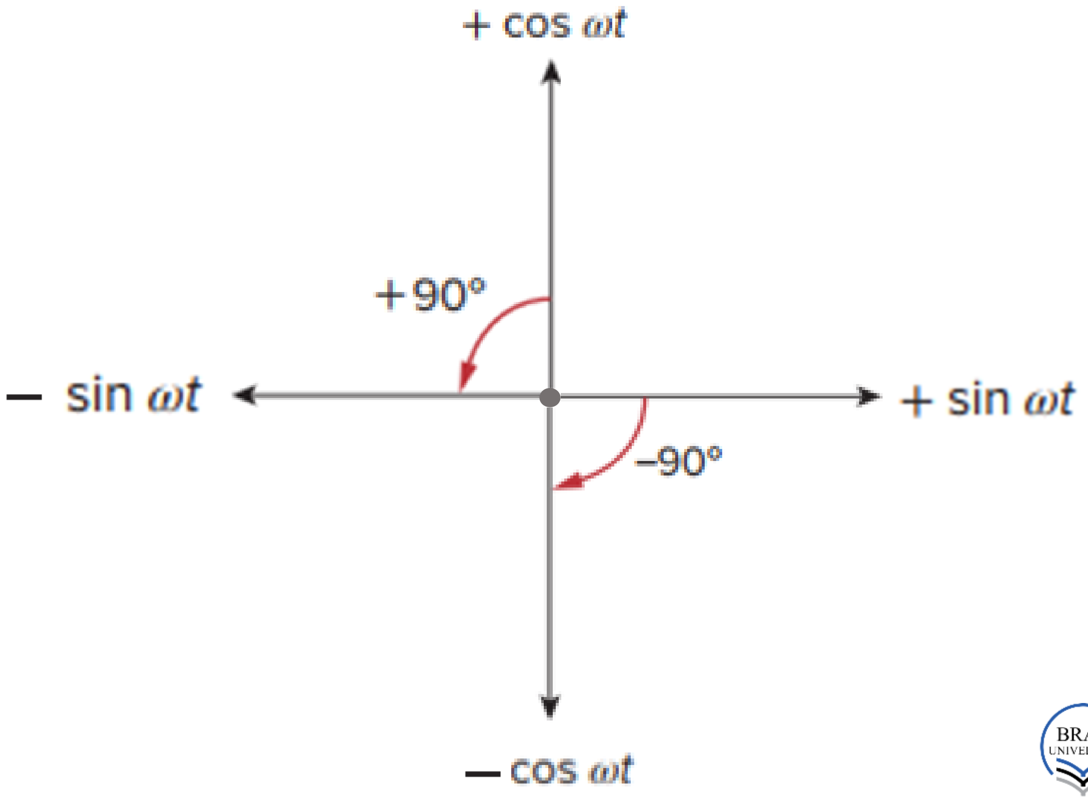
- The following trigonometric identities can be used to convert from sine to cosine or vice versa.

- $\sin(\omega t \pm 180°) = -\,sin\omega t$

- $\cos(\omega t \pm 180°) = -cos\omega t$

- $\sin(\omega t \pm 90°) = \pm\, cos\omega t$

- $\cos(\omega t \pm 90°) = \mp\, sin\omega t$



Problem 3

- Calculate amplitude, initial phase ($0 \le \varphi \le 180°$), angular frequency, period, and frequency for the following sinusoids. What are the values of V_S and I_s at $t = 20\ ms$? *[Both sin and cosine forms can be employed]*

  I. $V_s = 45\cos(5\pi t + 36°)\ (V)$

  II. $I_s = 15\cos(25\pi t + 25°)\ (A)$

  III. $I_s = -20\cos(314t - 30°)\ (A)$

  IV. $V_s = -4\sin(628t + 55°)\ (V)$

<u>Ans</u>: $I.\ 45\,V;\ 36°\ or\ 126°;\ 5\pi\ {}^{rad}/_{s};\ 0.4\,s;\ 2.5\,Hz;\ 26.45\,V$

$II.\ 15\,A;\ 25°\ or\ 115°;\ 25\pi\ {}^{rad}/_{s};\ 80\,ms;\ 12.5\,Hz; -6.34\,A$

$III.\ 20\,A;\ 150°\ or\ 60°;\ 314\ {}^{rad}/_{s};\ 20\,ms;\ 50\,Hz; -17.29\,A$

$IV.\ 4\,V; -130°\ or\ 140°;\ 628\ {}^{rad}/_{s};\ 10\,ms;\ 100\,Hz; -3.26\,V$

Problem 4

- For the following pairs of sinusoids, determine which one leads and by how much $(0 \leq \theta \leq 180°)$.

  I. $v(t) = 10\cos(4t - 60°)$ & $i(t) = 4\sin(4t + 50°)$

  II. $v_1(t) = 4\cos(377t + 10°)$ & $v_2(t) = -20\cos 377t$

  III. $v_1(t) = 45\sin(\omega t + 30°)\,V$ & $v_2(t) = 50\cos(\omega t - 30°)$

  IV. $i_1(t) = -4\sin(377t + 55°)$ & $i_2(t) = 5\cos(377t - 65°)$

  V. $x(t) = \underbrace{(13\cos 2t + 5\sin 2t)}$ & $y(t) = 15\cos(2t - 11.8°)$

Hint: convert both the sinusoids into sine or cosine form → convert them into phasors → add them in frequency domain → convert them back in the time domain and compare

<u>Ans</u>: I. i leads v by $20°$

II. v_2 leads v_1 by $170°$

III. v_2 leads v_1 by $30°$

IV. i_1 leads i_2 by $155°$

V. y leads x by $9.24°$

Phasor

- A *phasor* is a complex number that represents the amplitude and phase of a sinusoid. It is a useful notion for solving ac circuits excited with sinusoids.
- Let $z = x \pm jy = r\angle\varphi = r(cos\varphi \pm jsin\varphi) = re^{\pm j\varphi} = r(cos\varphi \pm jsin\varphi)$
- We can write, $cos\varphi = Re\{e^{\pm j\varphi}\}$ and $sin\varphi = \pm\, Im\{e^{\pm j\varphi}\}$
- Given a sinusoid, $v(t) = V_m cos(\omega t + \varphi) = Re\{V_m e^{j(\omega t+\varphi)}\} = Re\{V_m e^{j\omega t} e^{j\varphi}\} = Re\{\mathbf{V} e^{j\omega t}\}$
- where $\boxed{\mathbf{V} = V_m e^{j\varphi} = V_m\angle\varphi}$ is the phasor representation of $v(t)$
- $v(t) = \underbrace{V_m cos(\omega t + \varphi)}_{\text{Time domain representation (cosine)}} \Leftrightarrow \underbrace{\mathbf{V} = V_m\angle\theta}_{\text{Phasor domain representation}} \Leftrightarrow \underbrace{V_m sin(\omega t + \varphi)}_{\text{Time domain representation (sine)}}$

** *from now on, phasors will be represented in bold letters*

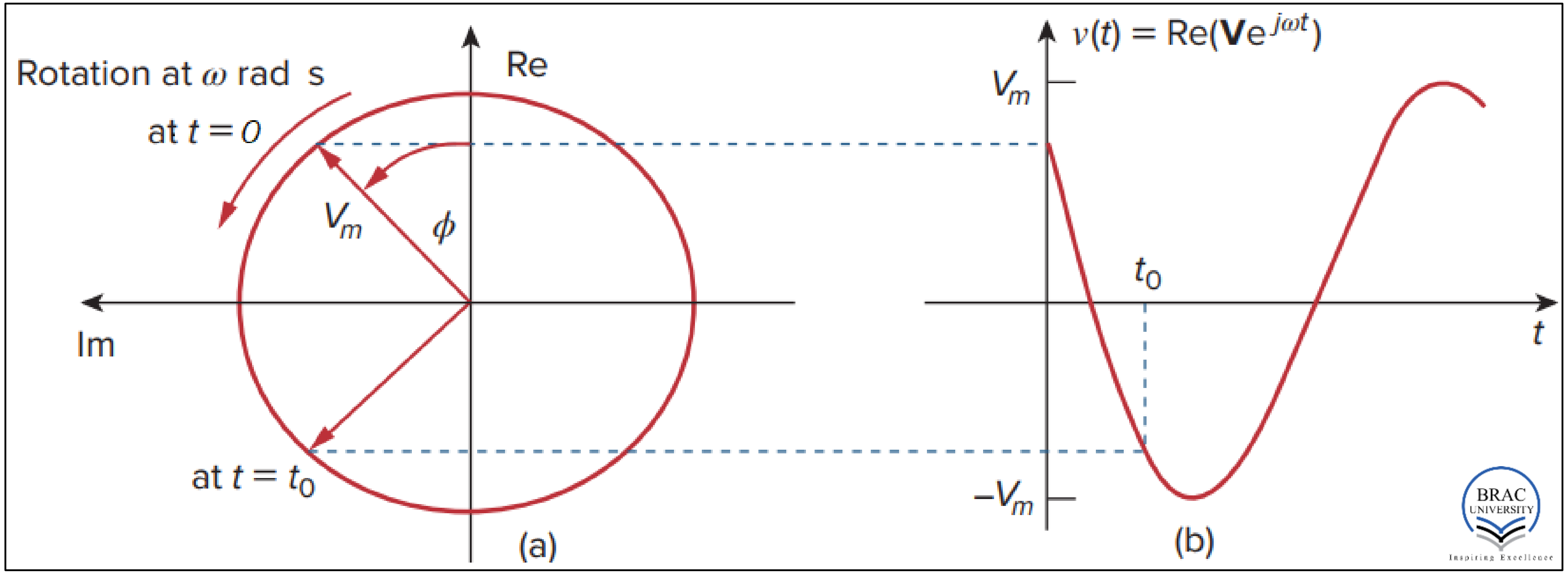
Time-domain vs Phasor-domain

$$\underbrace{v(t) = V_m cos(\omega t + \varphi)}_{\text{Time domain representation (cosine)}} \Leftrightarrow \underbrace{\boldsymbol{V} = V_m e^{j\varphi} = V_m \angle \varphi}_{\text{Phasor domain representation}}$$

- $v(t)$ is the instantaneous or time domain representation, while $\boldsymbol{V}$ is the frequency or phasor domain representation.
- $v(t)$ is time dependent, while $\boldsymbol{V}$ is not.
- $v(t)$ is always real with no complex term, while $\boldsymbol{V}$ is generally complex.
- Phasor analysis applies only when frequency is constant; it applies in manipulating two or more sinusoidal signals only if they are of the same frequency.

Phasor: graphical representation

- A sinusoidal voltage of the form $V_m cos(\omega t + \varphi)$ or $V_m sin(\omega t + \varphi)$ can be expressed as $\boldsymbol{V}e^{j\omega t}$, where $\boldsymbol{V} = V_m e^{j\varphi} = V_m\angle\varphi$ is the phasor representation of $v(t)$.
- Consider the plot of the *sinor* $\boldsymbol{V}e^{j\omega t} = V_m e^{j(\omega t+\varphi)}$ on the complex plane in figure (a). As time increases, the sinor rotates on a circle of radius V_m at an angular velocity ω in the CCW direction.
- We may regard $v(t)$ as the projection of the sinor $\boldsymbol{V}e^{j\omega t}$ on the real axis, as shown in (b). The value of the sinor at time $t = 0$ is the phasor $\boldsymbol{V}$ of the sinusoid $v(t)$. The sinor may be regarded as a rotating phasor.
- Thus, whenever a sinusoid is expressed as a phasor, the term $e^{j\omega t}$ is implicitly present. It is therefore important to keep in mind the frequency ω of the phasor.



Problem 5

- Transform the following sinusoids to phasors. Specify if they are sin or cosine phasors. *[Answers are given in such a way that the phase angle is between* $0°$ & $180°$)

I. $-14\sin(5t - 22°)$

II. $-8\cos(16t + 15°)$

III. $-20\cos(314t - 30°)$

IV. $-4\sin(628t + 55°)$

<u>Ans</u>: $I.$ $\mathbf{14\angle 158°}$ (sin)

$II.$ $\mathbf{8\angle -165°}$ (cos)

$III.$ $\mathbf{20\angle 150°}$ (cos)

$IV.$ $\mathbf{4\angle -125°}$ (sin)

Problem 6

- Find the sinusoids (in sin or cosine form as specified) corresponding to these phasors:

I. $-25\angle 40°$ to sine

II. $j(12 - j5)$ to cosine

III. $-10\angle -30°$ to sine

IV. $20\angle 45°$ to cosine

V. $If\ v_1(t) = -10\sin(\omega t - 30°)\,V\ and$
$v_2(t) = 20\cos(\omega t + 45°)\,V,$
find $v(t) = v_1(t) + v_2(t)$

Ans: I. $25\sin(\omega t - 140°)$
II. $13\sin(\omega t - 67.38°)$
III. $10\sin(\omega t + 120°)$
IV. $29.77\sin(\omega t + 140°)$
Or, $29.77\cos(\omega t + 50°)$

Phasor relationship for a Resistor

- If current through a resistor R is,

$$i(t) = I_m cos(\omega t + \varphi),$$

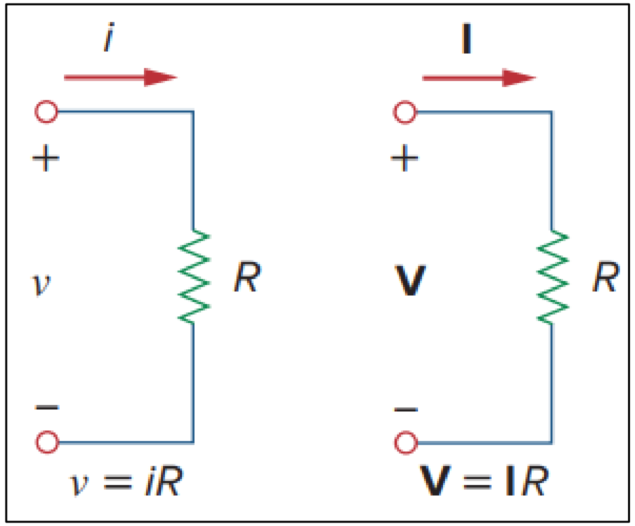
the voltage across it is given by Ohm's law as,

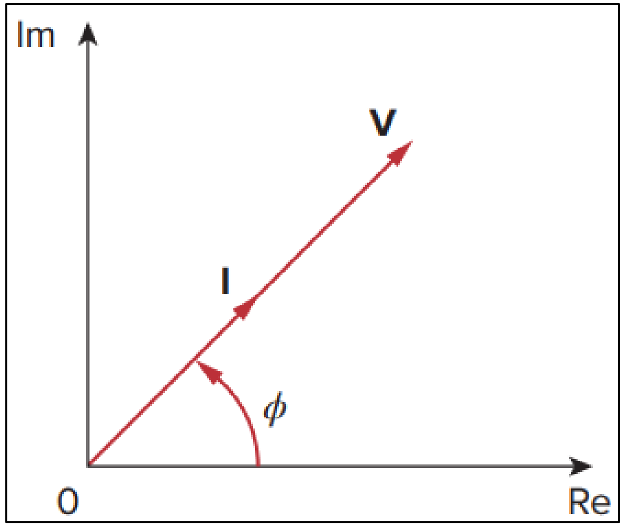
$$v(t) = Ri(t) = RI_m cos(\omega t + \varphi).$$

In phasor form, $\boldsymbol{V} = RI_m \angle\varphi = \boldsymbol{RI}$

- Hence, the voltage-current relation for the resistor in the phasor domain continues to be Ohm's law, as in the time domain.

- The angles of the voltage and current of a resistor are identical, hence, current and voltage of a resistor are always in phase.





Phasor relationship for an Inductor

- For an inductor L, assume the current through it is

$$i(t) = I_m cos(\omega t + \varphi)$$

- The voltage across the inductor is,

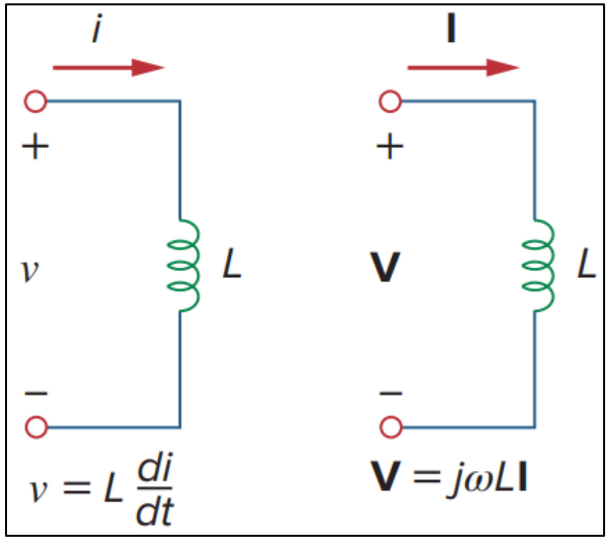
$$v(t) = L\frac{di}{dt} = -\,\omega L I_m sin(\omega t + \varphi) = \omega L I_m cos(\omega t + \varphi + 90°)$$

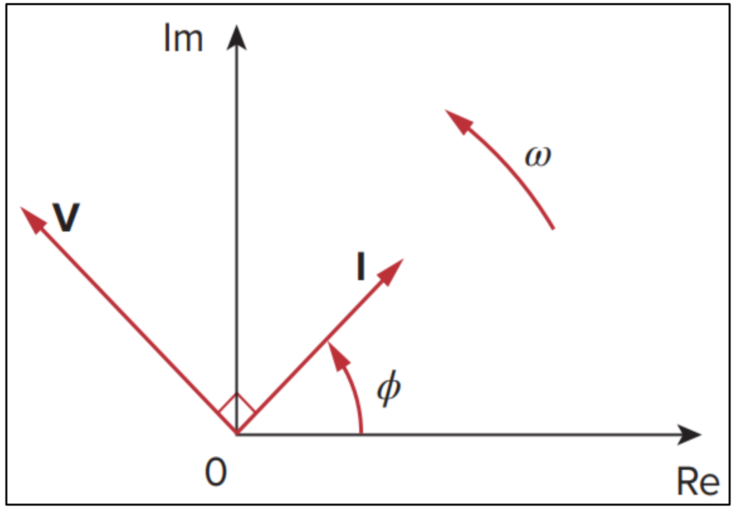
- In phasor form, $\boldsymbol{V} = \omega L I_m e^{j(\varphi+90^{\circ})} = \omega L I_m e^{j\varphi} e^{j90}$

$$= \omega L I_m e^{j\varphi}(cos90° + jsin90°) = j\omega L I_m e^{j\varphi} = j\omega L I_m \angle\varphi$$

$\Rightarrow \boldsymbol{V} = j\omega L\boldsymbol{I}$

- The voltage has a magnitude of $\omega L I_m$ and a phase of φ + 90°. Voltage and current of an inductor are 90° out of





Phasor relationship for a Capacitor

- For a capacitor C, assume the voltage across it is

$$v(t) = V_m cos(\omega t + \varphi)$$

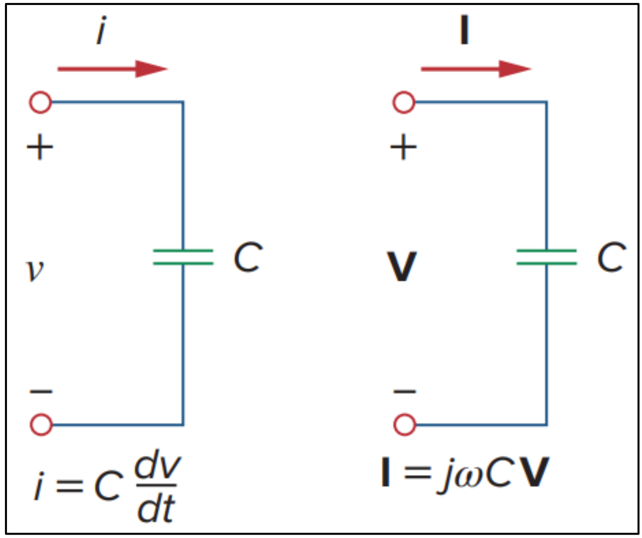
- The current through the capacitor is,

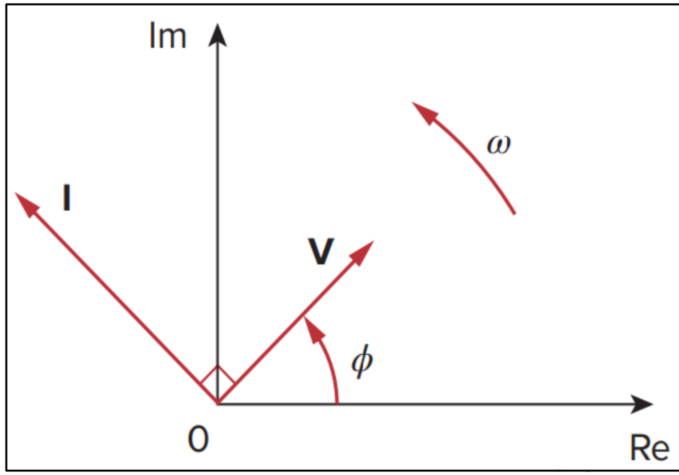
$$i(t) = C\frac{dv}{dt} = -\omega CV_m sin(\omega t + \varphi) = \omega LC cos(\omega t + \varphi + 90°)$$

- In phasor form, $\boldsymbol{I} = \omega CV_m e^{j(\varphi+90)} = \omega CV_m e^{j\varphi} e^{j90°}$
$= \omega CV_m e^{j\varphi}(cos90° + jsin90°) = j\omega CV_m e^{j\varphi} = j\omega CV_m \angle\varphi$

$\Rightarrow$ $\boldsymbol{I} = j\omega C\boldsymbol{V}$ $\Rightarrow$ $\boldsymbol{V} = \frac{1}{j\omega C}\boldsymbol{I}$

- The current has a magnitude of ωCV_m and a phase of $\phi + 90°$. Voltage and current of a capacitor are 90° out of





Impedance

- In the preceding section, we obtained the voltage-current relations for the three passive elements as,

$$\boldsymbol{V} = \boldsymbol{RI} \qquad \boldsymbol{V} = j\omega L\boldsymbol{I} \qquad \boldsymbol{V} = \frac{1}{j\omega C}\boldsymbol{I}$$

$$\frac{\boldsymbol{V}}{\boldsymbol{I}} = \boldsymbol{R} \qquad \frac{\boldsymbol{V}}{\boldsymbol{I}} = j\omega L \qquad \frac{\boldsymbol{V}}{\boldsymbol{I}} = \frac{1}{j\omega C}$$

- From these three expressions, we obtain Ohm's law in phasor form for any type of element as, $\boldsymbol{Z} = \frac{\boldsymbol{V}}{\boldsymbol{I}} \Rightarrow \boldsymbol{V} = \boldsymbol{ZI}$

  where $\boldsymbol{Z}$ is a frequency-dependent quantity known as impedance, measured in $ohms$.

- The *impedance* $\boldsymbol{Z}$ of a circuit is the ratio of the phasor voltage $\boldsymbol{V}$ to the phasor current $\boldsymbol{I}$, measured in ohms (Ω).

Summary of voltage-current relationships.

Element	Time domain	Frequency domain
R	$v = Ri$	$\mathbf{V} = R\mathbf{I}$
L	$v = L\frac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$
C	$i = C\frac{dv}{dt}$	$\mathbf{V} = \frac{\mathbf{I}}{j\omega C}$

Impedances of passive elements

Element	Impedance
R	$\mathbf{Z} = R$
L	$\mathbf{Z} = j\omega L$
C	$\mathbf{Z} = \frac{1}{j\omega C}$

Impedance: frequency dependency

- *The impedance represents the opposition that the circuit exhibits to the flow of sinusoidal current.*

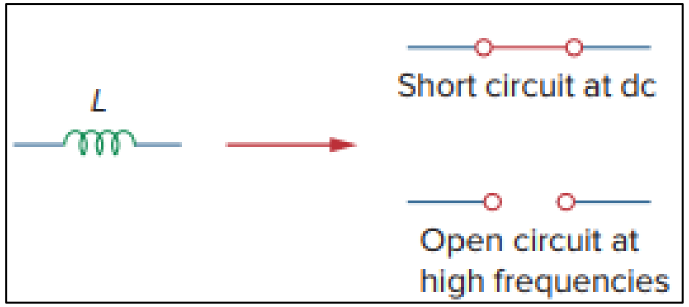
- Although the impedance is the ratio of two phasors, it is not a phasor, because it does not correspond to a sinusoidally varying quantity.

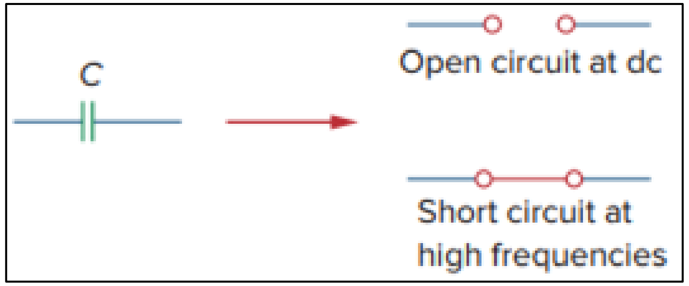
- When $\omega = 0\ (for\ dc\ sources)$ $\quad \boldsymbol{Z_L} = j\omega L = 0 \quad and\ \boldsymbol{Z_C} = \frac{\mathbf{1}}{\boldsymbol{j\omega C}} \to \infty$, confirming what we already know—that the inductor acts like a short circuit, while the capacitor acts like an open circuit.

- When $\omega \to \infty\ (for\ high\ frequencies)$ $\quad \boldsymbol{Z_L} \to \infty \quad and\ \boldsymbol{Z_C} = 0$, indicating that the inductor is an open circuit to high frequencies, while the capacitor is a short circuit.

Impedances of passive elements.

Element	Impedance
R	$\mathbf{Z} = R$
L	$\mathbf{Z} = j\omega L$
C	$\mathbf{Z} = \frac{1}{j\omega C}$





Impedance: resistance & reactance

- As a complex quantity, the impedance may be expressed in rectangular form as,

$$\boldsymbol{Z} = R \pm jX$$

where, $R = Re\{\boldsymbol{Z}\}$ (Ω) is the *resistance* and $X = Im\{Z\}$ (Ω) is the *reactance*

- The reactance, X, is just a magnitude, a positive value, but when used as a vector, a j is associated with inductance and a $-j$ is associated with capacitance.

- Impedance $\boldsymbol{Z} = R + jX$ is said to be inductive or lagging since current lags voltage, while impedance $\boldsymbol{Z} = R - jX$ is capacitive or leading because current leads voltage.

- In polar form, $\boldsymbol{Z} = R \pm jX = |\boldsymbol{Z}|\angle \pm \varphi$ where, $|\boldsymbol{Z}| = \sqrt{R^2 + X^2}$; $\theta = \tan^{-1}\frac{\pm X}{R}$

$$R = |\boldsymbol{Z}|cos\theta; \qquad X = |\boldsymbol{Z}|sin\theta$$

Admittance

- It is sometimes convenient to work with the reciprocal of impedance, known as admittance.

- The *admittance* $\boldsymbol{Y}$ is the reciprocal of impedance, measured in siemens (S).

$$\boldsymbol{Y} = \frac{\boldsymbol{I}}{\boldsymbol{V}} = \frac{1}{\boldsymbol{Z}}$$

- As a complex quantity, we may write $\boldsymbol{Y}$ as,

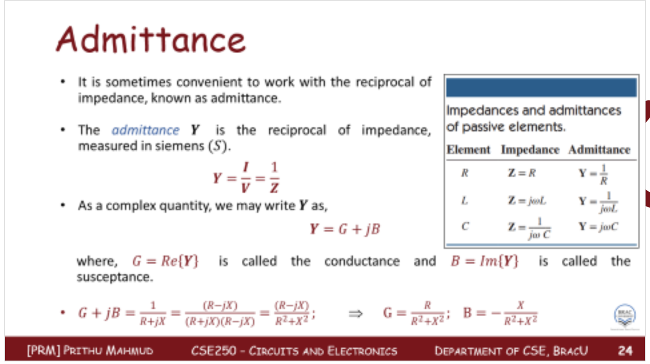
$$\boldsymbol{Y} = G + jB$$

where, $G = Re\{\boldsymbol{Y}\}$ is called the conductance and $B = Im\{\boldsymbol{Y}\}$ is called the susceptance.

Impedances and admittances of passive elements.

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \frac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \frac{1}{j\omega L}$
C	$\mathbf{Z} = \frac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

- $G + jB = \frac{1}{R+jX} = \frac{(R-jX)}{(R+jX)(R-jX)} = \frac{(R-jX)}{R^2+X^2}; \quad \Rightarrow \quad \mathrm{G} = \frac{R}{R^2+X^2}; \quad \mathrm{B} = -\frac{X}{R^2+X^2}$



Admittance

- It is sometimes convenient to work with the reciprocal of impedance, known as admittance.
- The *admittance* $\boldsymbol{Y}$ is the reciprocal of impedance, measured in siemens (S).

$$\boldsymbol{Y} = \frac{\boldsymbol{I}}{\boldsymbol{V}} = \frac{1}{\boldsymbol{Z}}$$

- As a complex quantity, we may write $\boldsymbol{Y}$ as,

$$\boldsymbol{Y} = G + jB$$

where, $G = Re\{\boldsymbol{Y}\}$ is called the conductance and $B = Im\{\boldsymbol{Y}\}$ is called the susceptance.

- $G + jB = \frac{1}{R+jX} = \frac{(R-jX)}{(R+jX)(R-jX)} = \frac{(R-jX)}{R^2+X^2}; \quad \Rightarrow \quad G = \frac{R}{R^2+X^2}; \quad B = -\frac{X}{R^2+X^2}$

Impedances and admittances of passive elements.

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CL in phasor domain

- Let $v_1, v_2, \ldots, v_n$ be the voltages around a closed loop. Then, $v_1 + v_2 + \ldots + v_n = 0$.
- In the sinusoidal steady state, each voltage may be written in cosine form, so that

$\Rightarrow$ $V_{m_1}\cos(\omega t + \theta_1) + V_{m_2}\cos(\omega t + \theta_2) + \cdots + V_{m_n}\cos(\omega t + \theta_n) = 0$

$\Rightarrow$ $Re\{V_{m_1}e^{j\theta_1}e^{j\omega t}\} + Re\{V_{m_2}e^{j\theta_2}e^{j\omega t}\} + \cdots + Re\{V_{m_n}e^{j\theta_n}e^{j\omega t}\} = 0$

$\Rightarrow$ Let $\boldsymbol{V_k} = V_{mk}e^{\theta_k}$, then $Re\{(\boldsymbol{V_1} + \boldsymbol{V_2} + \cdots + \boldsymbol{V_n})e^{j\omega t}\} = 0$

$\Rightarrow$ Because $e^{j\omega t} \neq 0$, $\boldsymbol{V_1} + \boldsymbol{V_2} + \cdots + \boldsymbol{V_n} = \boldsymbol{0}$, indicating that KVL holds for phasors

- ***KCL***

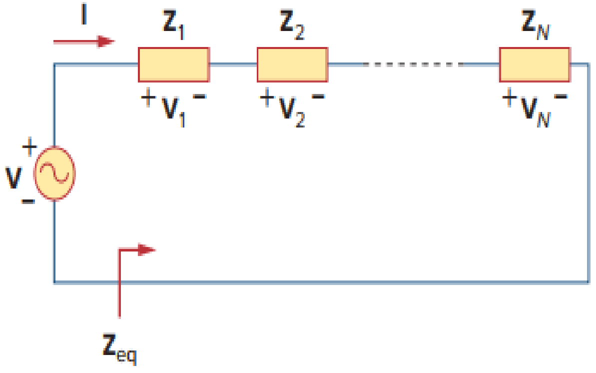
$\Rightarrow$ By following a similar procedure, we can show that KVL holds for phasors.

$\Rightarrow$ $i_1 + i_2 + \cdots + i_n = 0 \quad \leftrightarrow \quad \boldsymbol{I_1} + \boldsymbol{I_2} + \cdots + \boldsymbol{I_n} = \boldsymbol{0}$

Series and Parallel Impedances

- ***Series:*** Consider the N series-connected impedances. The same current $\boldsymbol{I}$ flows through the impedances. Applying KVL around the loop gives,

$$\boldsymbol{V_1} + \boldsymbol{V_2} + \cdots + \boldsymbol{V_N} = \boldsymbol{I}(\boldsymbol{Z_1} + \boldsymbol{Z_2} + \cdots + \boldsymbol{Z_N})$$

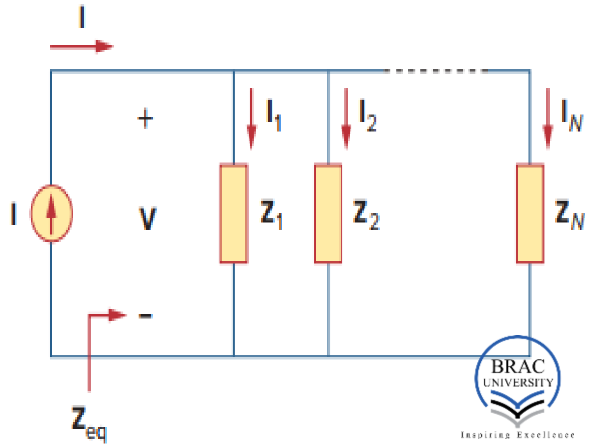


$\Rightarrow$ The equivalent impedance at the input terminals is,

$$\boldsymbol{Z_{eq}} = \frac{\boldsymbol{V}}{\boldsymbol{I}} = \boldsymbol{Z_1} + \boldsymbol{Z_2} + \cdots + \boldsymbol{Z_N}$$

- ***Parallel:*** In the same manner , we can obtain the equivalent impedance or admittance of the N parallel-connected impedances. The voltage across each impedance is the same. Applying KCL at the top node,

$$\boldsymbol{I} = \boldsymbol{I_1} + \boldsymbol{I_2} + \cdots + \boldsymbol{I_N} = \boldsymbol{V}\left(\frac{\boldsymbol{1}}{\boldsymbol{Z_1}} + \frac{\boldsymbol{1}}{\boldsymbol{Z_2}} + \cdots + \frac{\boldsymbol{1}}{\boldsymbol{Z_N}}\right)$$



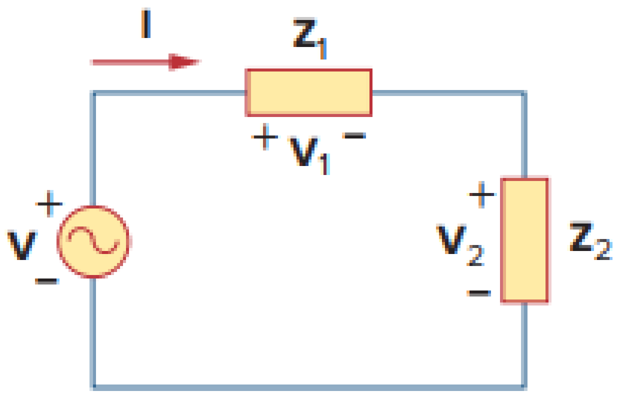
$\Rightarrow$ The equivalent impedance at the input terminals is,

$$\frac{\boldsymbol{1}}{\boldsymbol{Z_{eq}}} = \frac{\boldsymbol{1}}{\boldsymbol{Z_1}} + \frac{\boldsymbol{1}}{\boldsymbol{Z_2}} + \cdots + \frac{\boldsymbol{1}}{\boldsymbol{Z_N}}$$

Voltage Division Rule

- For ac circuits in phasor domain, the voltage division rule applies exactly as they do in dc circuits.

$$I = \frac{V}{Z_1 + Z_2}$$



- Because $V_1 = IZ_1$ *and* $V_2 = IZ_2$

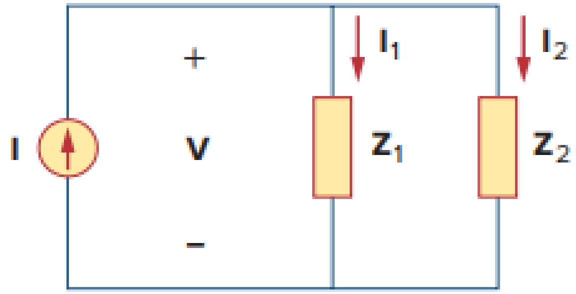
$$V_1 = \frac{Z_1}{Z_1 + Z_2} V \qquad V_2 = \frac{Z_2}{Z_1 + Z_2} V$$

- In general, for any number of impedances connected in series to a supply voltage, the voltage across any particular impedance Z_x is,

$$\boxed{V_x = \frac{Z_x}{Z_1 + Z_2 + Z_3 + \ \dots \dots + Z_N} \times V}$$

Current Division Rule

- For ac circuits in phasor domain, the current division rule applies exactly as they do in dc circuits.



$$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

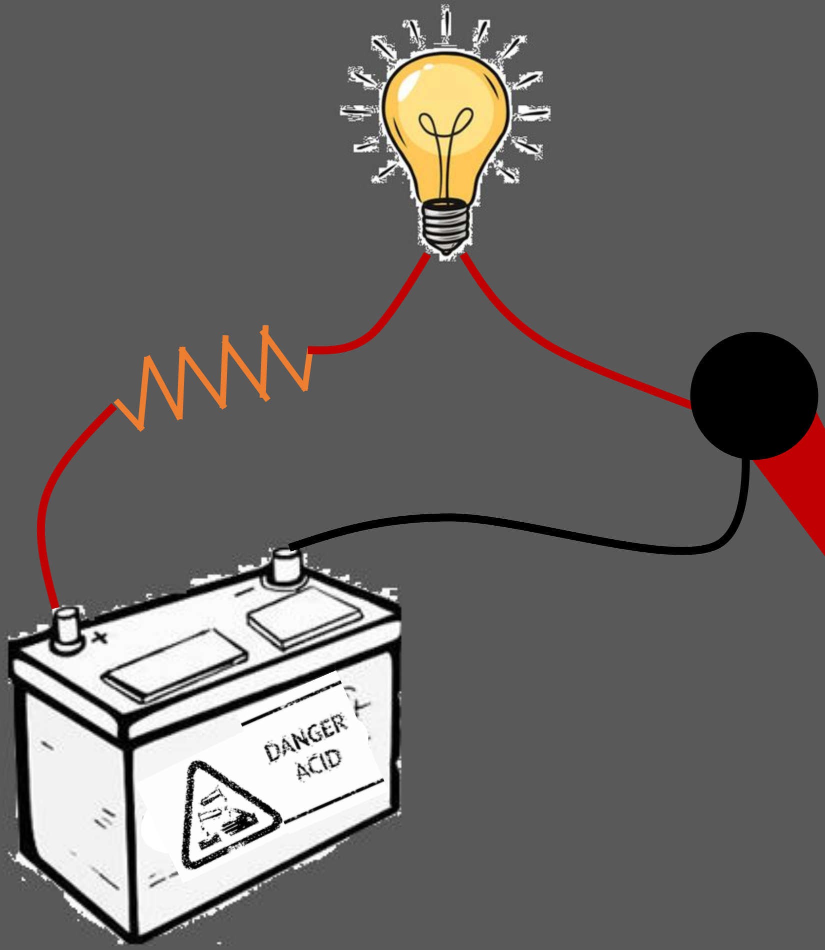
- Because $V = IZ_{eq} = Z_1 I_1 = Z_2 I_2$

$$I_1 = \frac{Z_2}{Z_1 + Z_2} \times I = \frac{(Z_1)^{-1}}{(Z_1)^{-1} + (Z_2)^{-1}} \times I \qquad I_2 = \frac{Z_1}{Z_1 + Z_2} \times I = \frac{(Z_2)^{-1}}{(Z_1)^{-1} + (Z_2)^{-1}} \times I$$

- In general, for any number of impedances connected in parallel to a supply current, the current through any particular impedance Z_x is,

$$\boxed{I_x = \frac{(Z_x)^{-1}}{(Z_1)^{-1} + (Z_2)^{-1} + (Z_3)^{-1} + \ldots\ldots + (Z_N)^{-1}} \times I}$$

Course Syllabi



Circuit Laws

Methods of Analysis

Circuit Theorems

First Order Circuits

AC Circuits

Fundamentals of AC Analysis

Nodal Analysis

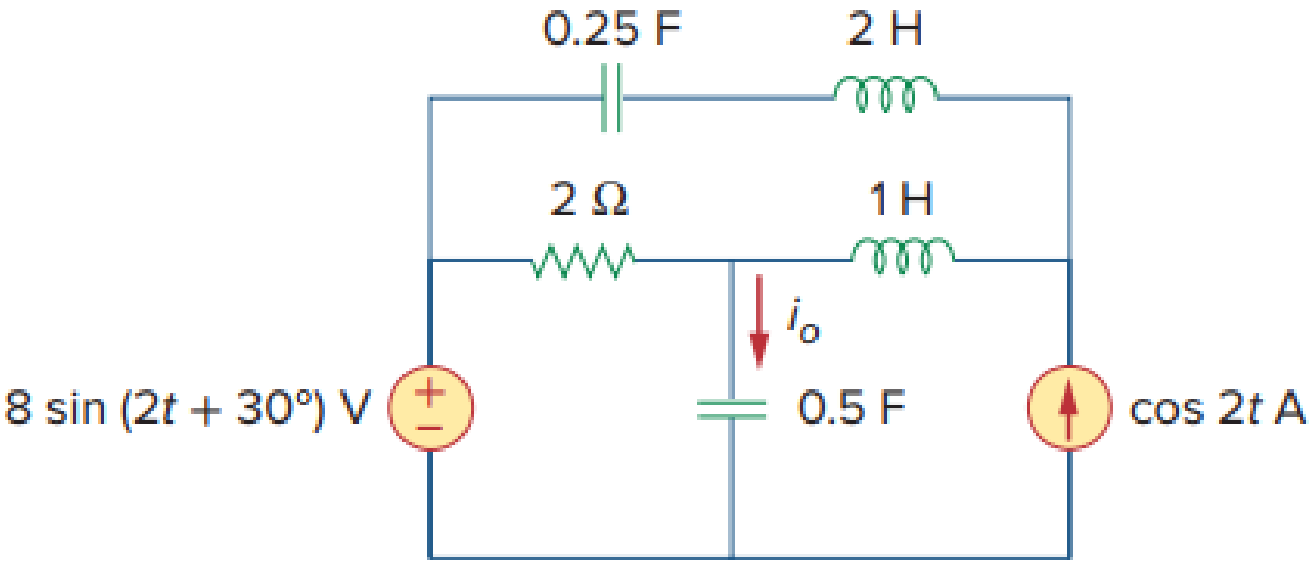
Superposition Principle (AC)

- Since ac circuits are linear, the superposition theorem applies to ac circuits the same way it applies to dc circuits.
- *The theorem becomes important if the circuit has sources operating at different frequencies.*
- In this case, since the impedances depend on frequency, we must have a different frequency domain circuit for each source of different frequency.
- The total response must be obtained by adding the individual responses.
- In which domain should we add the responses from individual independent sources?

$\Rightarrow$ *Because the exponential factor $e^{j\omega t}$ is implicit in sinusoidal analysis, and that factor would change for every angular frequency ω. It would therefore not make sense to add responses at different frequencies in the phasor domain. Thus, when a circuit has sources operating at different frequencies, one must add the responses due to the individual frequencies in the time domain.*

Problem 8

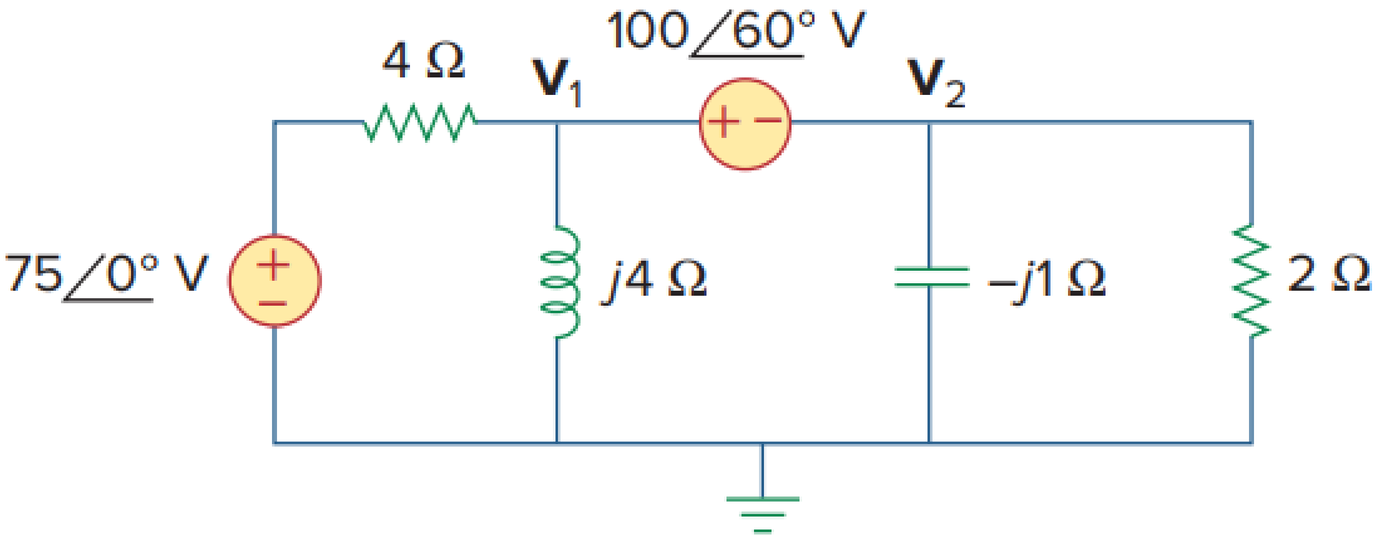
- Use Nodal Analysis to find $i_o(t)$ in the circuit.



Ans: $\mathbf{5.02\cos(\omega t - 46.55°)}$

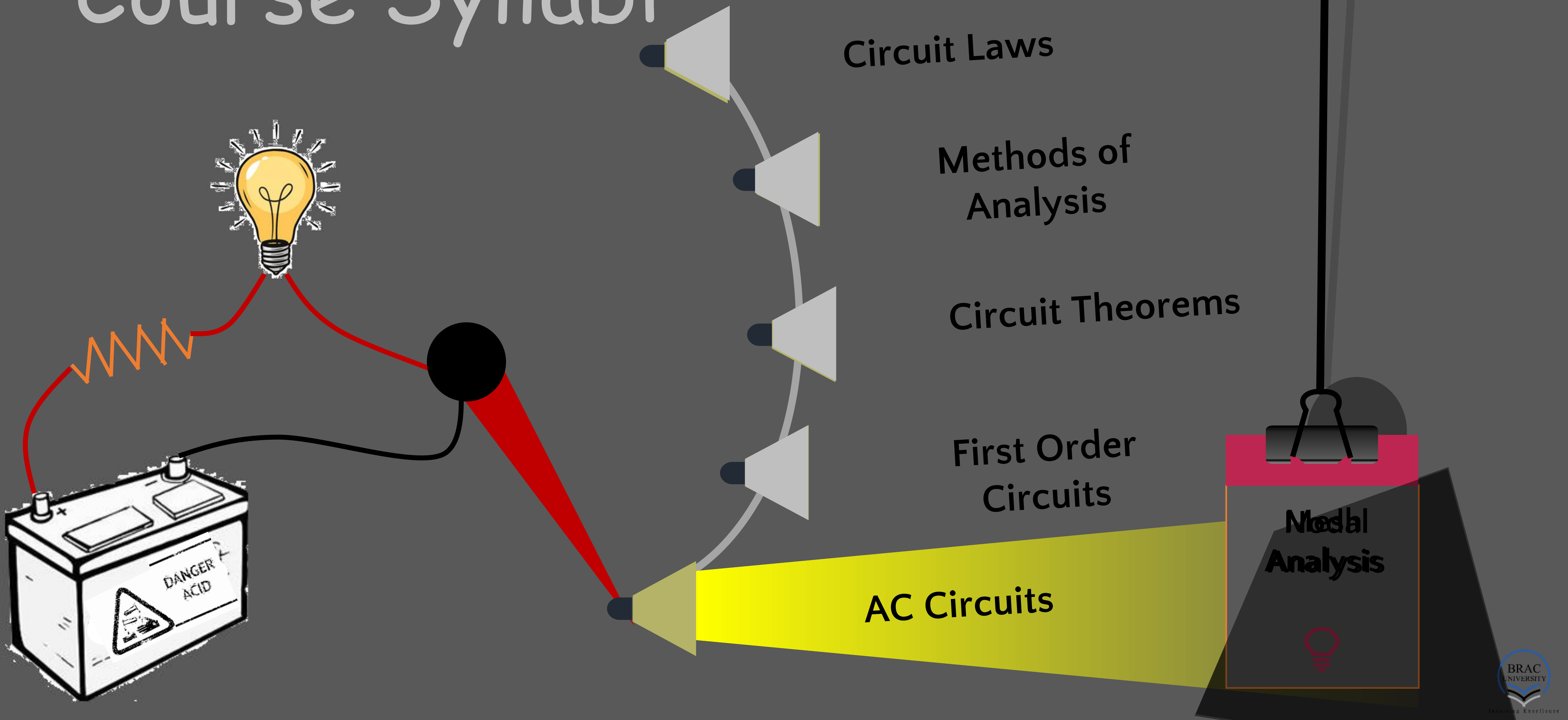
Problem 12

- Evaluate $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$ using Nodal Analysis.



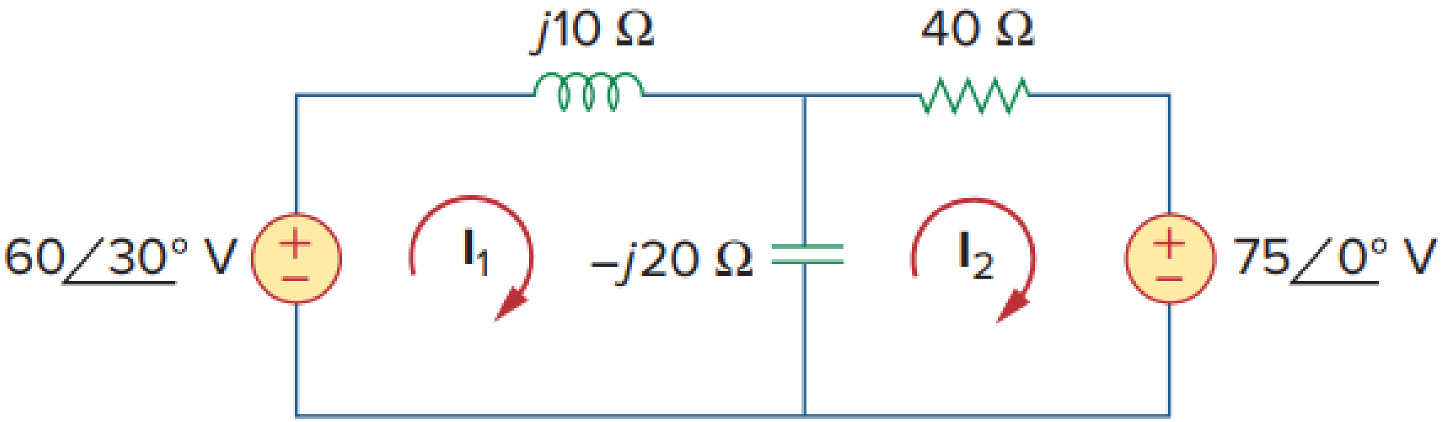
Ans: $\boldsymbol{V_1 = 96.8\angle{-69.66^\circ}\,V;\ V_2 = 16.88\angle 165.72^\circ\,V}$

Course Syllabi



Problem 13

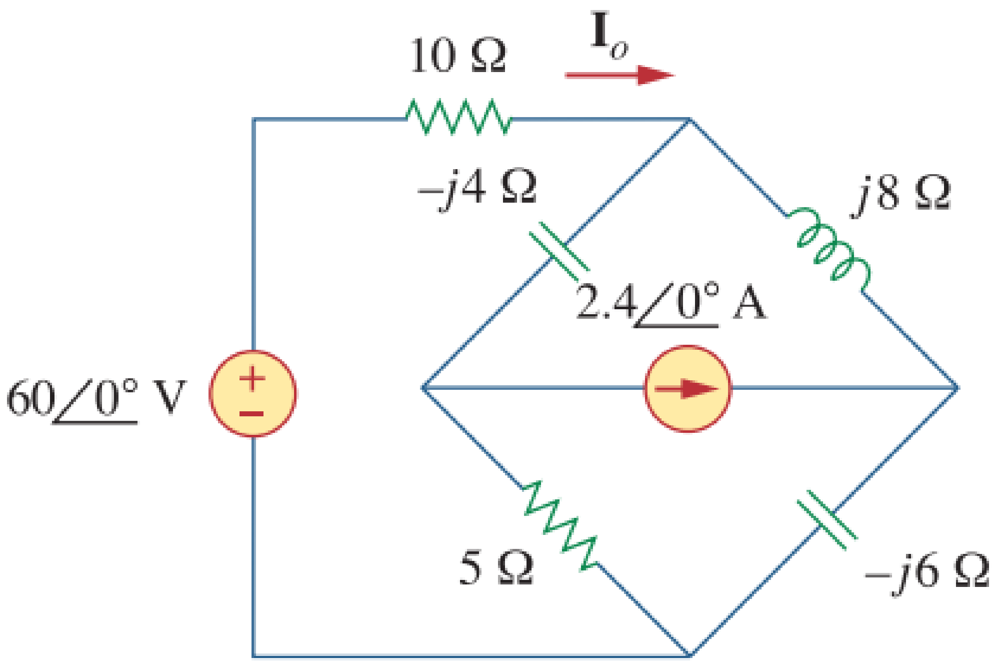
- Using Mesh Analysis, find $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$



Ans: $\boldsymbol{I_1 = 4.698\angle 95.24^\circ\ A;\ I_2 = 0.9928\angle 37.71^\circ\ A}$

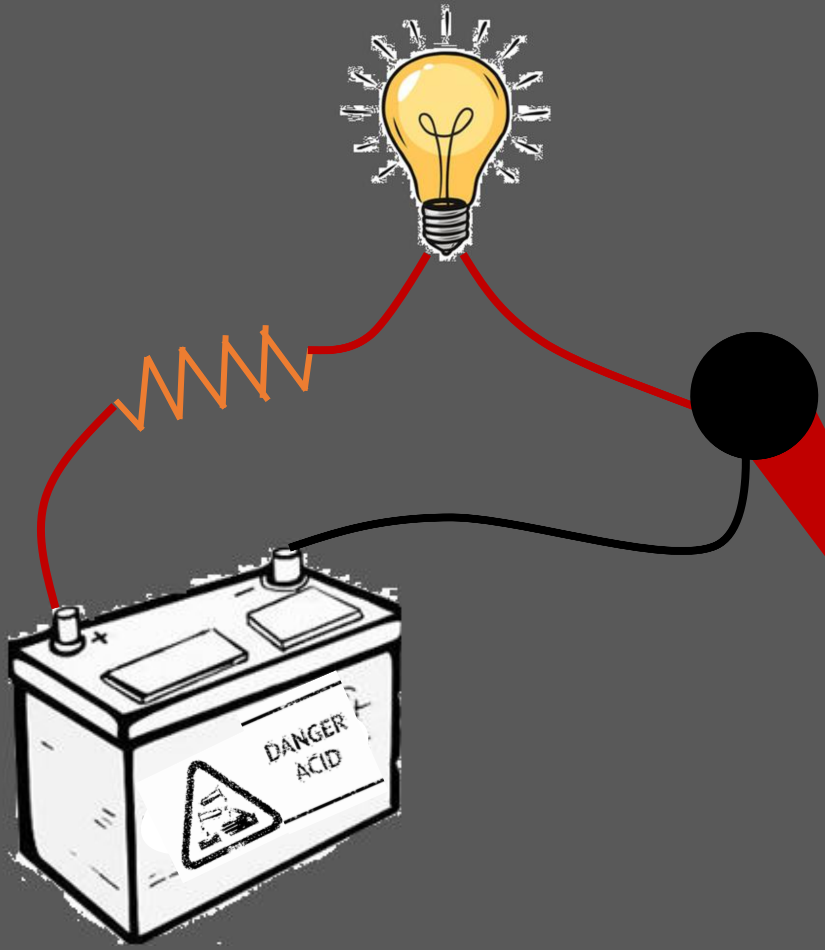
Problem 16

- Solve for the current $\boldsymbol{I_0}$ using Mesh Analysis.



Ans: $\boldsymbol{I_0} = \mathbf{6.089\angle 5.94°}\ \boldsymbol{A}$

Course Syllabi



- Circuit Laws
- Methods of Analysis
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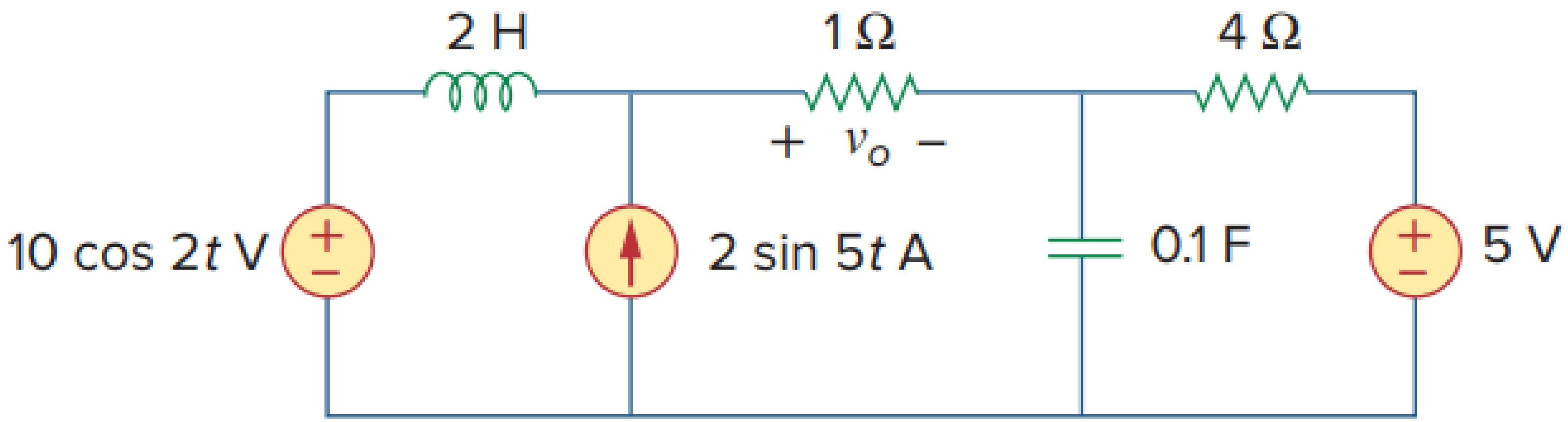
- AC Circuits

Superposition Principle

Mesh Analysis

Superposition Principle: an example

- Find v_0 using the Superposition Theorem.



Example 3

$\Rightarrow$ Because the circuit operates at three different frequencies ($\omega = 0$ for the dc voltage source), one way to obtain a solution is to use superposition.

$\Rightarrow$ So let, $v_0 = v' + v'' + v'''$; where v' is due to the $10 \cos 2t\ V$ voltage source, and v'' is due to the $2 \sin 5t\ A$ current source, and v''' is due to the $5\ V$ dc voltage source

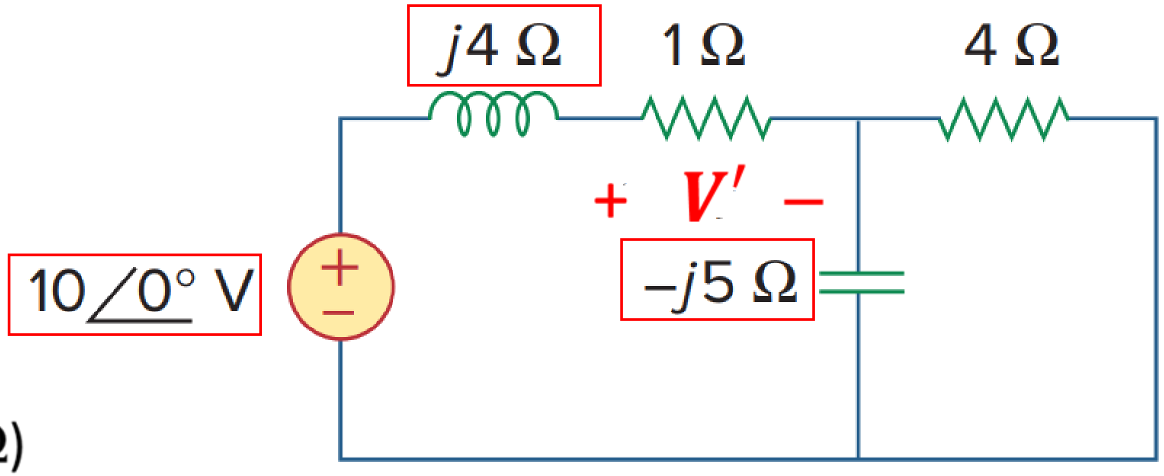
Only the 10cos(2t) V source is active

Both the $5\,V$ source and the $2\sin 5t$ current source are set to zero. Transforming the circuit to the frequency domain:

$$10\cos(2t) \Rightarrow 10\angle 0^\circ \qquad \omega = 2\,rad/s$$

$$2\text{ H} \Rightarrow j\omega L = j \times 2 \times 2 = j4 \quad (\Omega)$$

$$0.1\text{ F} \Rightarrow \frac{1}{j\omega C} = \frac{-j}{2 \times 0.1} = -j5 \quad (\Omega)$$



$$\boldsymbol{Z_1} = (-j5)||(4) = \frac{(-j5)\times 4}{4-j5} = 2.439 - j1.951\ (\Omega)$$

By voltage division, $\boldsymbol{V'} = \dfrac{1}{j4+1+\boldsymbol{Z_1}} \times 10\angle 0^\circ$

$$= 2.498\angle -30.79^\circ$$

In time domain, $v''(t) = 2.498\cos(2t - 30.79^\circ)\ (V)$

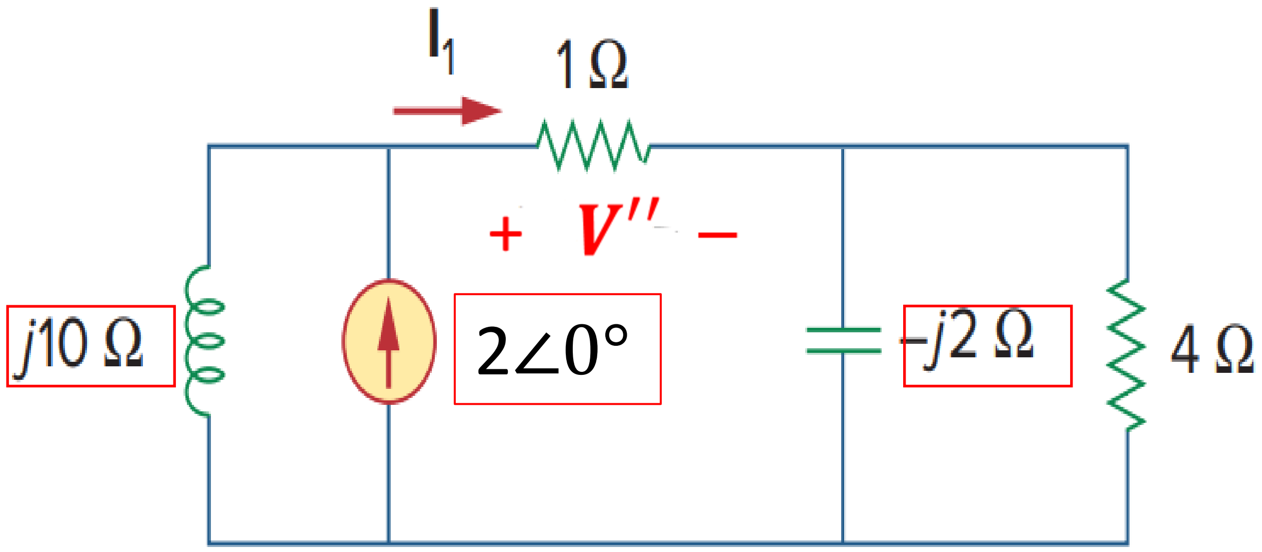
Only the 2sin(5t) A source is active

Both the $5\,V$ source and the $10\cos 2t$ current source are set to zero. Transforming the circuit to the frequency domain:

$2\sin(5t) \Rightarrow 2\angle 0° \qquad \omega = 2\,rad/s$

$$2\text{ H} \Rightarrow j\omega L = j \times 5 \times 2 = j10 \quad (\Omega)$$

$$0.1\text{ F} \Rightarrow \frac{1}{j\omega C} = \frac{-j}{5 \times 0.1} = -j2 \quad (\Omega)$$



$$\boldsymbol{Z_1} = (-j2)||(4) = \frac{(-j2)\times 4}{4-j2} = 0.8 - j1.6$$

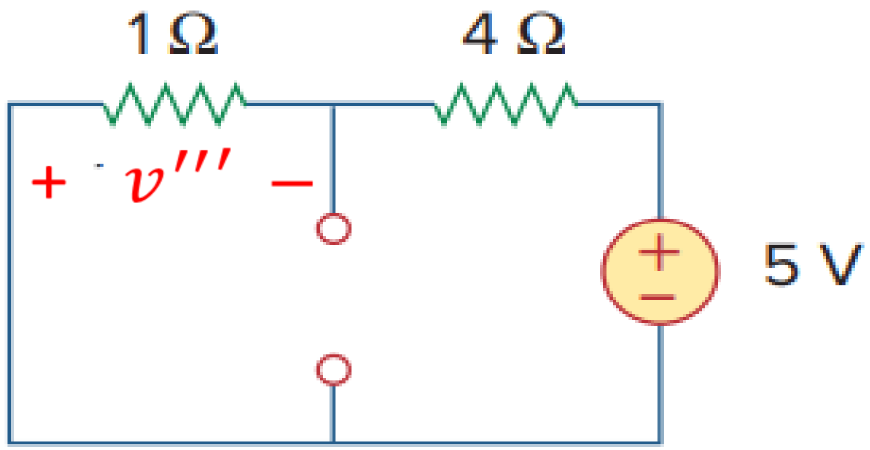
By current division, $\mathbf{I}_1 = \dfrac{j10}{j10+1+\boldsymbol{Z}_1} \times 2\angle 0° = 2.276 + j0.488\ (A)$

$$\mathbf{V}''' = \boldsymbol{I}_1 \times 1 = 2.328\angle 12.10°$$

In time domain, $v'''(t) = 2.328\sin(5t + 12.10°)\ (V)$

Only the 5 V dc source is active

As in dc, the capacitor and inductor have been replaced with open and short circuits, respectively.



Because $\omega = 0\ (dc)$,

$$j\omega L = 0, \qquad \frac{1}{j\omega C} \rightarrow \infty$$

v''' can be found by voltage division as,

$$v''' = \frac{1}{1+4} \times (-5)$$

$$\Rightarrow v''' = -1\ V$$

According to the superposition principle,

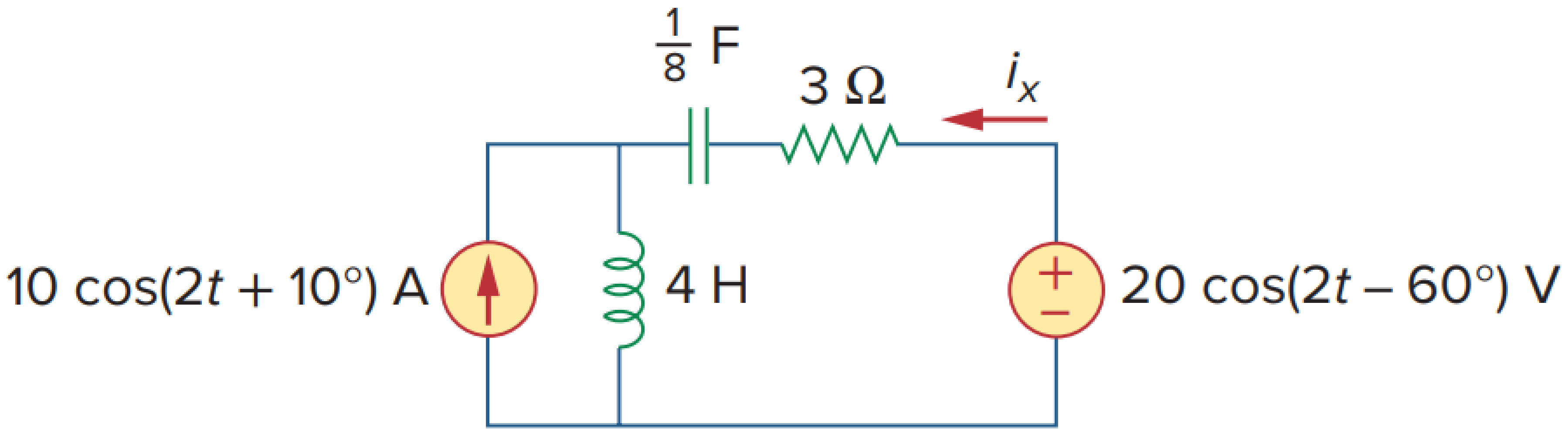
$$v_0(t) = v'(t) + v''(t) + v'''(t)$$

$$= 2.498\cos(2t - 30.79°) + 2.328\sin(5t + 12.10°) + (-1)$$

$$= \boxed{-1 + 2.498\cos(2t - 30.79°) + 2.328\sin(5t + 12.10°)\ V}$$

Problem 17

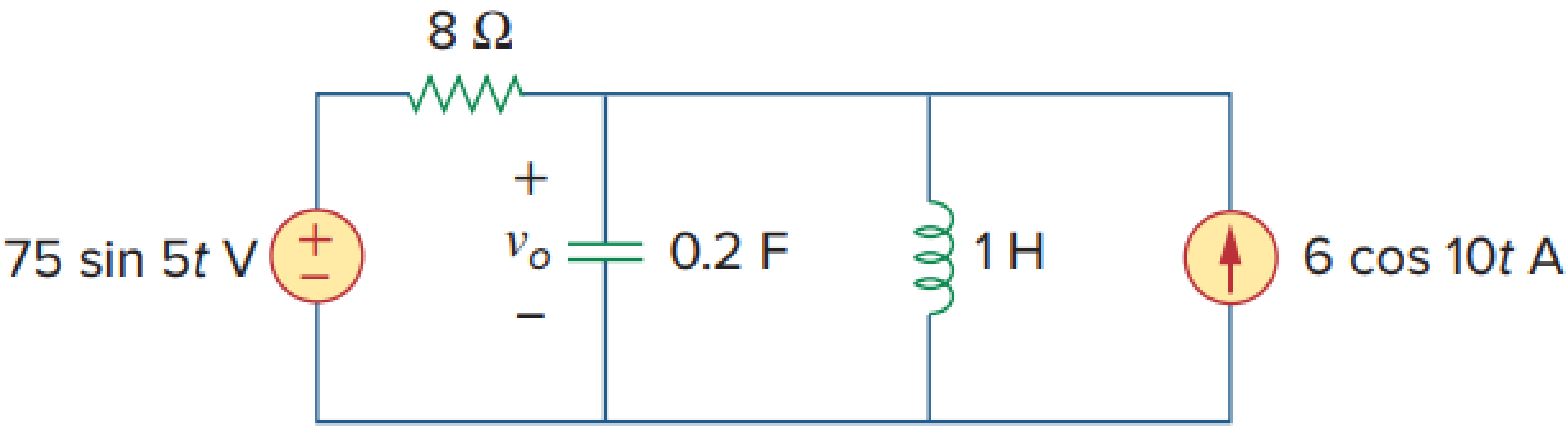
- Using the Superposition Principle, find $i_x(t)$ in the following circuit.



Ans: $\mathbf{19.804}\cos(\mathbf{2}\boldsymbol{t} - \mathbf{129.17}°)\ \boldsymbol{A}$

Problem 18

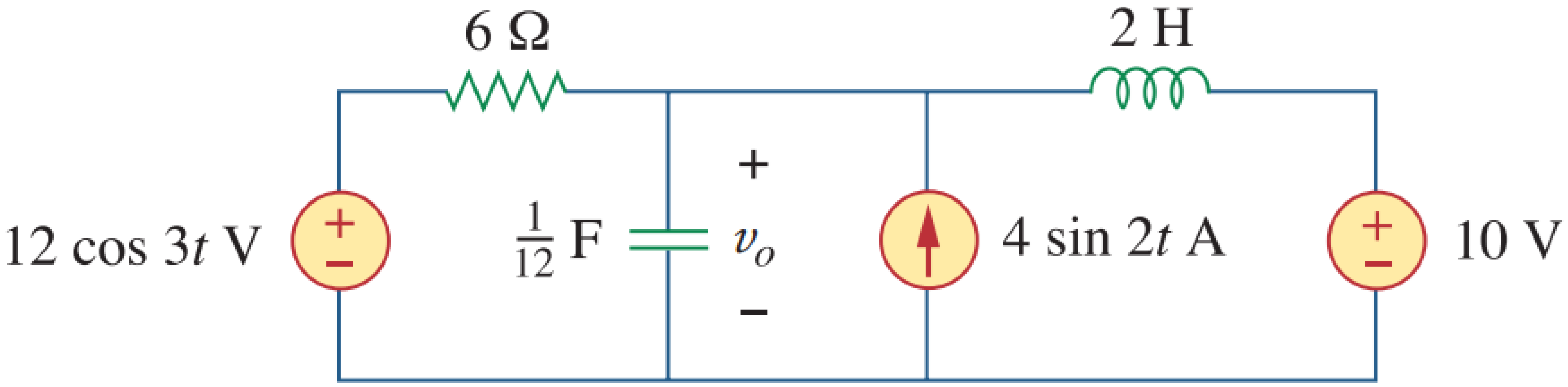
- Calculate $v_0(t)$ in the following circuit using the Superposition Principle.



Ans: $\mathbf{11.577}\sin(\mathbf{5}\boldsymbol{t} - \mathbf{81.12}°) + \mathbf{3.154}\sin(\mathbf{10}\boldsymbol{t} - \mathbf{86.24}°)\ \boldsymbol{V}$

Problem 19

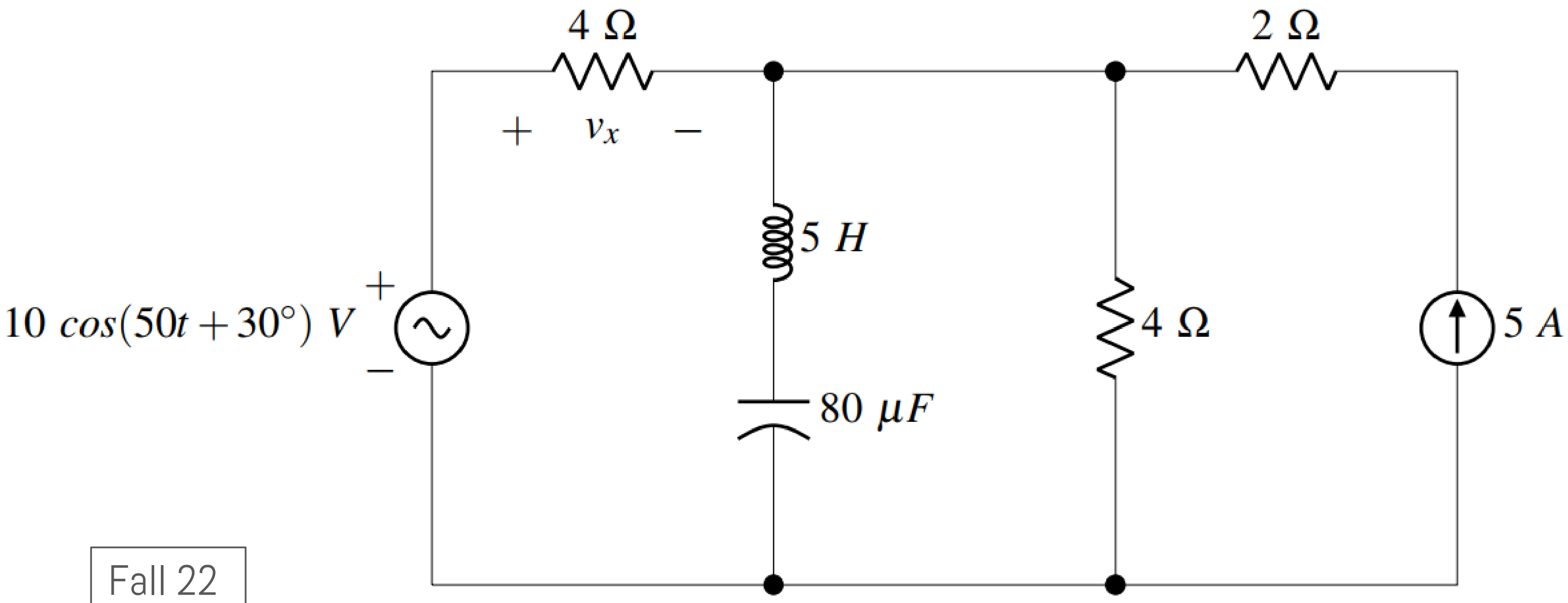
- Solve for the voltage $v_0(t)$.



Ans: $\mathbf{10} + \mathbf{21.47}\sin(\mathbf{2}\boldsymbol{t} + \mathbf{26.57}°) + \mathbf{10.73}\cos(\mathbf{3}\boldsymbol{t} - \mathbf{26.57}°)\ \boldsymbol{V}$

Problem 20

- Solve for the voltage $v_x(t)$.

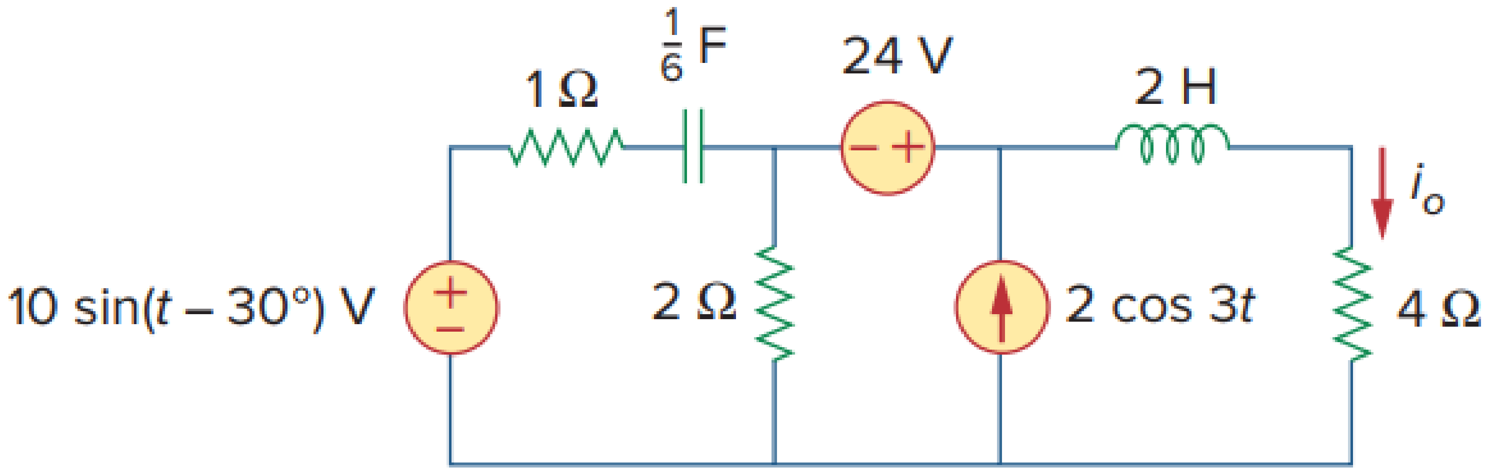


Fall 22

Ans: $\mathbf{10\cos(50t + 30^{\circ}) - 10\ V}$

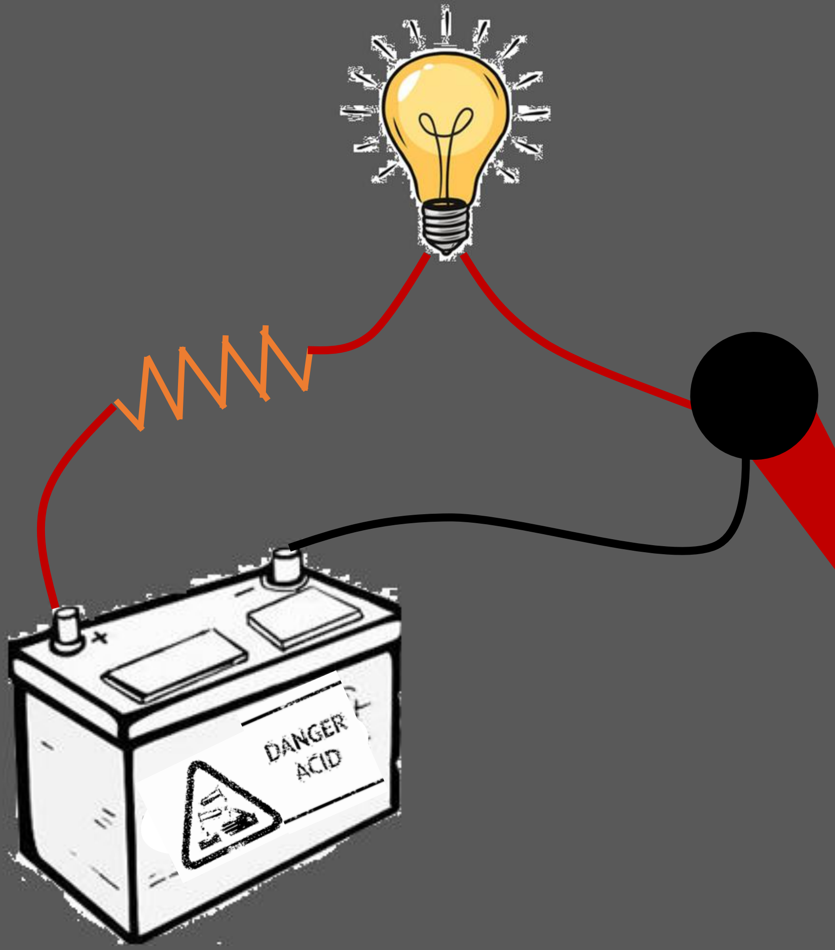
Problem 21

- Determine $i_0(t)$.



Ans: $\mathbf{4 + 0.504}\sin(\boldsymbol{t} + \mathbf{19.1}^\circ) + \mathbf{0.3352}\cos(\mathbf{3}\boldsymbol{t} - \mathbf{76.43}^\circ)\ \boldsymbol{V}$

Course Syllabi



- Circuit Laws
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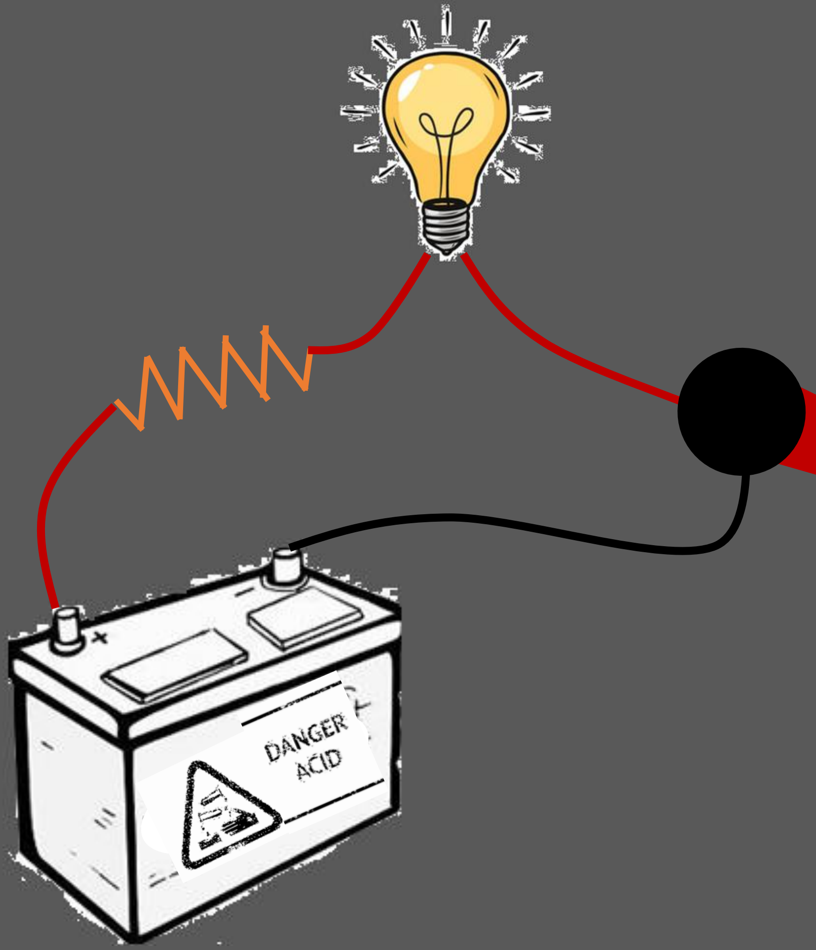
Superposition Principle
AC Power Analysis

Course Syllabi



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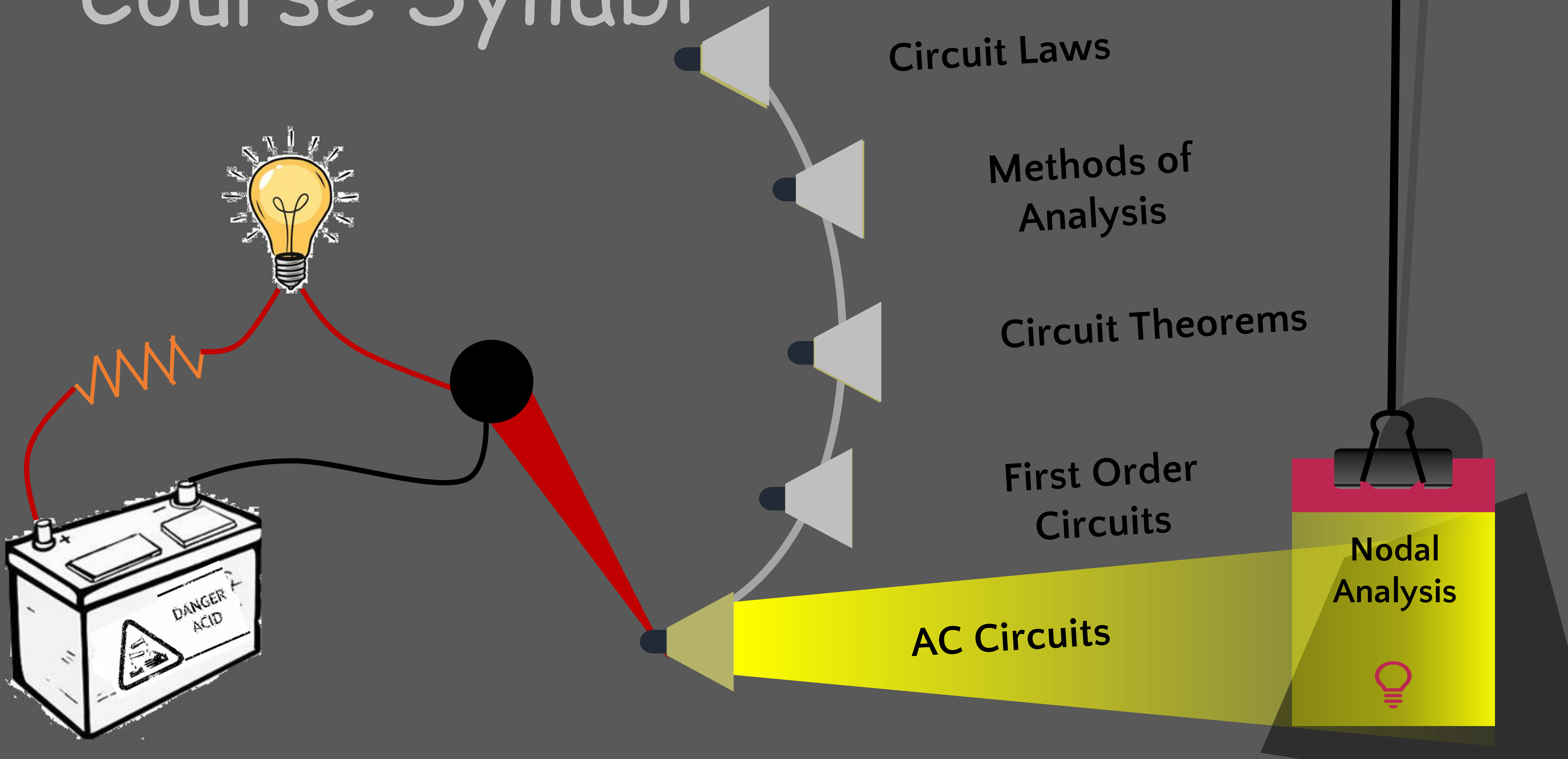
Fundamsentals of AC

Course Syllabi



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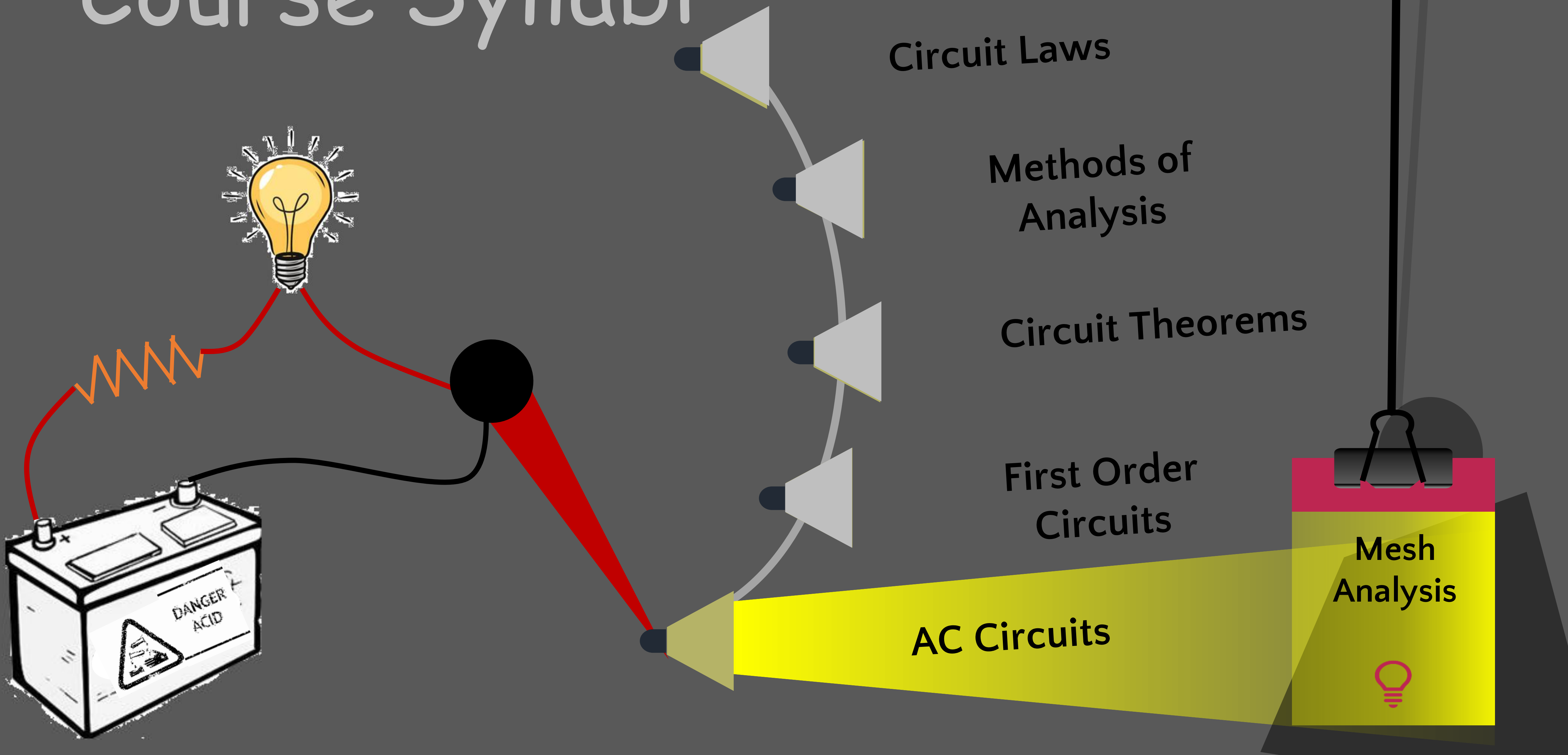

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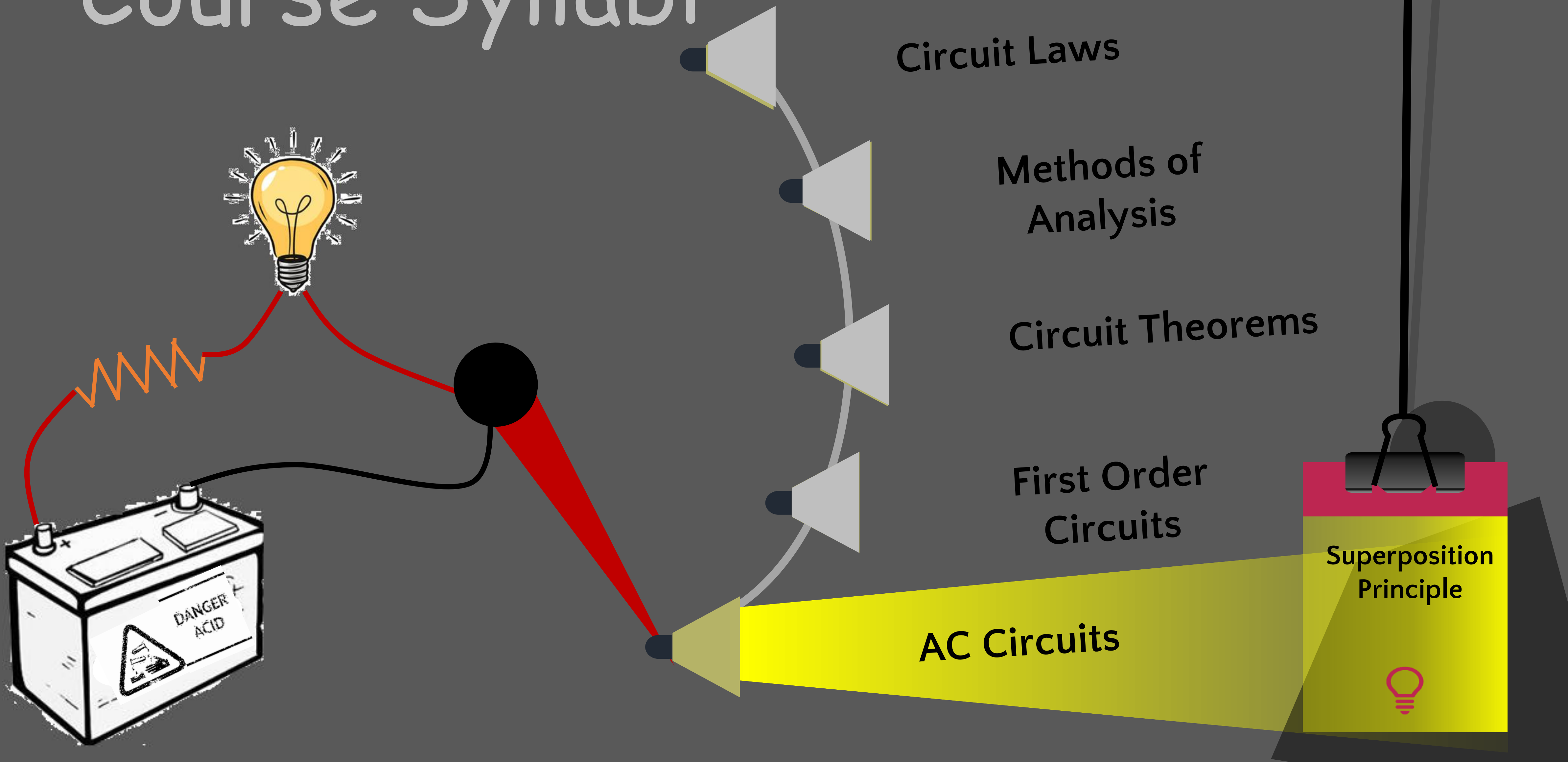

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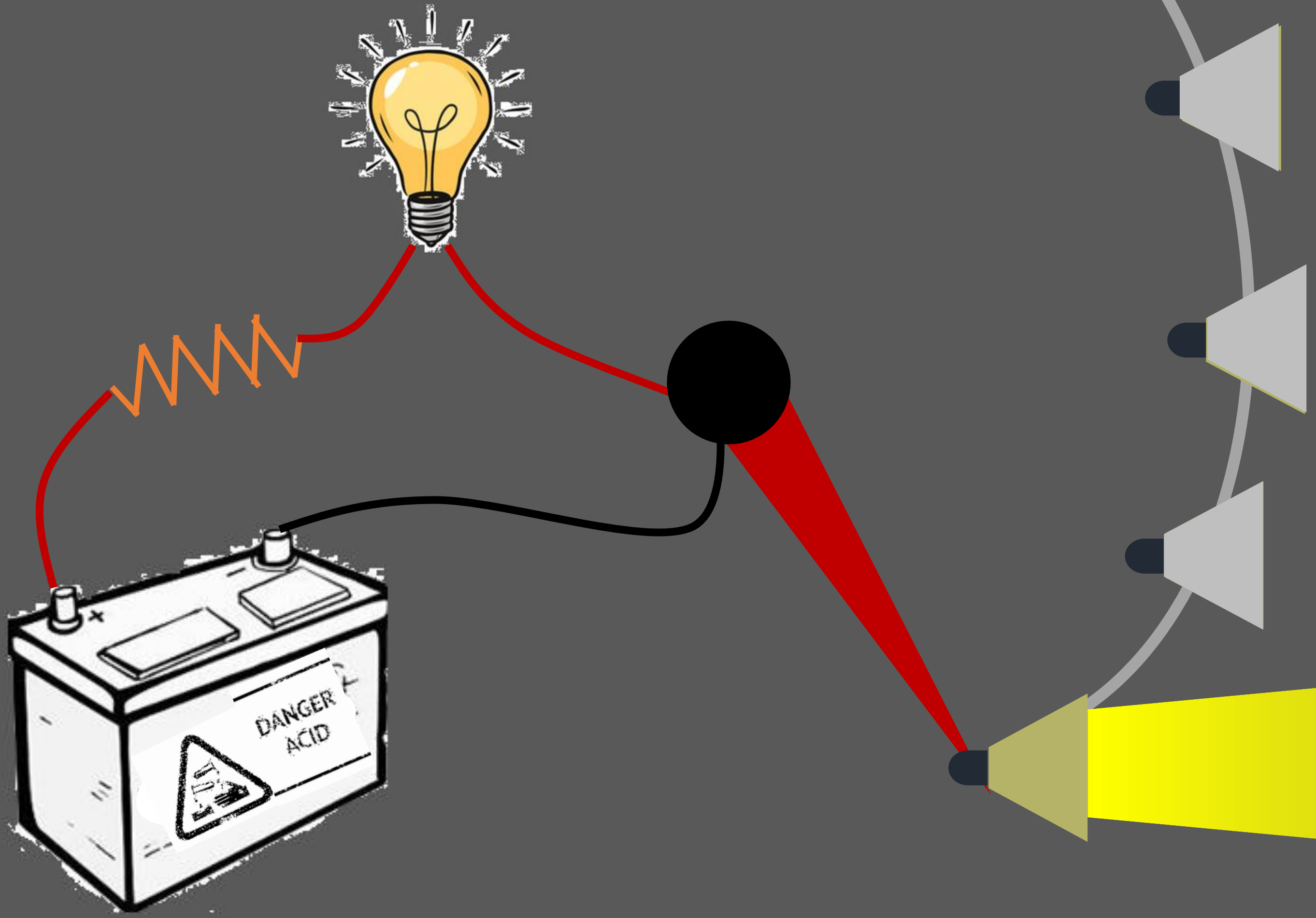

Circuit Laws

Methods of Analysis

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AC Power Analysis