

Machine learning Assignment no -7

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$$SBAF = \frac{1}{1 + kx^\alpha(1-x)^{1-\alpha}}$$

(1)

Show that SBAF is the solution to the first order differential equation:

$$f(x, y) = dy/dx = \frac{y(1-y)}{x(1-x)} * (\alpha - x) \quad (2)$$

solution:

$$y = \frac{1}{1 + kx^\alpha(1-x)^{1-\alpha}} \quad (3)$$

$$\ln y = \ln 1 - \ln(1 + kx^\alpha(1-x)^{1-\alpha}) \quad (4)$$

$$= -\ln(1 + kx^\alpha(1-x)^{1-\alpha}) \quad (5)$$

$$1/y \cdot dy/dx = -\frac{1}{(1 + kx^\alpha(1-x)^{1-\alpha})} \cdot [k\alpha x^{\alpha-1}(1-x)^{1-\alpha} - kx^\alpha(1-\alpha)(1-x)^{1-\alpha-1}] \quad (6)$$

$$= -\frac{k}{(1 + kx^\alpha(1-x)^{1-\alpha})} \cdot [\alpha x^{\alpha-1}(1-x)^{1-\alpha} - (1-\alpha)x^\alpha(1-x)^{-\alpha}] \quad (7)$$

$$dy/dx = y \left[\frac{\alpha}{x} - (1-\alpha) \frac{1}{1-x} \right] kx^\alpha(1-x)^{1-\alpha} \quad (8)$$

$$= y \left[\frac{\alpha(1-x) - (1-\alpha)x}{x(1-x)} \right] kx^\alpha(1-x)^{1-\alpha} \quad (9)$$

$$= y^2 \left[\frac{\alpha - x}{x(1-x)} \right] kx^\alpha(1-x)^{1-\alpha} \quad (10)$$

$$= y^2 \left[\frac{\alpha - x}{x(1-x)} \right] kx^\alpha (1-x)^{1-\alpha}$$

(11)

$$\text{We know, } y = \frac{1}{1 + kx^\alpha (1-x)^{1-\alpha}} \quad (12)$$

$$\Rightarrow kx^\alpha (1-x)^{1-\alpha} = \frac{1-y}{y}$$

(13)

$$\text{Substituting the value from equation, } dy/dx = y^2 \cdot \frac{\alpha - x}{x(1-x)} \cdot \frac{1-y}{y} \quad (14)$$

$$= \frac{y(1-y)}{x(1-x)} \cdot (\alpha - x) \quad (15)$$