TSP: Tutorial 1

Question 1

The equation $3n^2 + 5n$ is dominated by the n^2 component. For any input, the dominant component of the output will be bounded below by $\Omega(n^2)$ and above by $O(n^2)$, therefore $3n^2 + 5n = \Theta(n^2)$.

Question 2

- `ababab`
  - Prefixes: `a`, `ab`, `aba`, `abab`, `ababa`, `ababab`
  - Suffixes: `b`, `ab`, `bab`, `abab`, `babab`, `ababab`
  - Roots: `ab`, `ababab`
  - Primitive: no, because `ababab` $=$ `ab`^{3}
- `aaaaaa`
  - Prefixes: `a`, `aa`, `aaa`, `aaaa`, `aaaaa`, `aaaaaa`
  - Suffixes: `a`, `aa`, `aaa`, `aaaa`, `aaaaa`, `aaaaaa`
  - Roots: `a`, `aa`, `aaa`, `aaaaaa`
  - Primitive: no, because `aaaaaa` $=$ `a`^{6}
- `abcacb`
  - Prefixes: `a`, `ab`, `abc`, `abca`, `abcac`, `abcacb`
  - Suffixes: `b`, `cb`, `acb`, `cacb`, `bcacb`, `abcacb`
  - Roots: `abcacb`
  - Primitive: yes, because `abcacb` $=$ `abcacb`^{1}

Question 3

If the two strings x and y can be decomposed such that $x = uv$ and $y = vu$, then z can be selected such that $z = u$. This gives $xz = uvu = zy$. For example:

- $x =$ `goldfish`
- $y =$ `fishgold`
- $u =$ `gold`
- $v =$ `fish`
- $z = u =$ `gold`
- $xz = uvu = zy =$ `goldfishgold`

Question 4

Proof by contradiction.

Assume that the primitve word w has a third occurence in ww. Therefore two non-empty words u and v exist such that $ww = uwv$. It immediately follows that:

- u is a prefix of w [1]
- v is a suffix of w [2]
- $|ww| = |u| + |w| + |v|$ [3]
- $|w| = |u| + |v|$ [4]
- $w = uv$ [5]

It can now be stated that $ww = uwv = uvuv = uuvv$. [6]

By taking a factor of length $|w|$ (or length $|uv|$) starting from position $|u|$ [7], it can also be stated that $uv = vu$. [8]

By the periodicity lemma, it is known that u and v are both $gcd(|u|, |v|)$-periodic. Since one of u and v is a prefix of the other, it follows that they must both be powers of the same word.

Since $w = uv$ then w is also a power of that word, but because $|w| > |u|$ and $|w| > |v|$ this makes w non-primitive, thereby violating the assumption.

[1] Because u appears at the start of ww.
[2] Because v appears at the end of ww.
[3] Because of the assumption $ww = uwv$.
[4] By removing $|w|$ from both sides of $|ww| = |u| + |w| + |v|$.
[5] Because the lengths match, u is a prefix of w and v is a suffix of w.
[6] $ww = uwv$ is the assumption; $ww = uvuv$ and $uwv = uuvv$ both follow from $w = uv$.
[7] i.e. taking a w-length chunk from ww after skipping the first occurence of u.
[8] By taking $|uv|$ characters afer skipping the first u in $uvuv$ and $uuvv$.