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A Reconfigurable Hardware Emulator of MEMS Gyroscopes with Built-in Error Source Models

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***Abstract*—The paper presents a hardware emulator of MEMS vibratory gyroscopes that can be used for accelerating the characterization and verification of control electronics, by decoupling these tasks from the MEMS design and fabrication cycles. The hardware emulator is synthesized on a field-programmable gate array (FPGA). In addition to modeling the Coriolis effect, the emulator also models typical error sources in MEMS gyroscopes, namely quadrature error, Duffing non-linearity, thermo-mechanical noise etc. Preliminary results based on a prototype with user-controllable device parameters synthesized on a Xilinx Zynq-7000 FPGA (Digilent Zybo Z7) are presented in this paper.**

***Index Terms*—MEMS inertial sensor; hardware emulator; FPGA; gyroscope error sources**

I. Introduction

Conventional MEMS gyroscope products comprise of a micromechanical sensing element (MEMS chip), and an application specific integrated circuit (ASIC) that implements the interface circuits for the drive and sense resonator channels and control-loops for maintaining drive amplitude and mitigating errors in the integrated system. Developing MEMS prototypes typically requires longer design cycles than the ASIC design cycle, and therefore in-the-loop characterization and verification of the ASIC and examination of errors that are hard to simulate (e.g. drift) is often delayed. While some compact systems have been reported that could be used to emulate the performance of an ASIC for rapid prototyping and testing of the MEMS chip [1], [2], a corresponding reconfigurable system for emulating the MEMS performance for rapid prototyping and testing of the ASIC design is missing. Limited reports on MEMS emulators using field-programmable gate arrays (FPGAs) are available, that implement the Coriolis coupling between the drive and sense modes, without considering error sources of interest in practical designs [3], [4].

Extensive numerical models describing complete behavior of MEMS gyroscopes are available in literature [5]. Equivalent analog electrical circuit implementations of MEMS resonators are not possible due to the typically impractical values of capacitances and inductances. The numerical models could instead be discretized and implemented in digital hardware. We demonstrate a method to discretize the differential equations assuming them to be linear and incorporate small perturbations due to the non-idealities (e.g. Duffing non-linearity). The result is a system of linear equations that accounts for the error sources in a gyroscope. The digital implementation on a Xilinx Zynq-7000 FPGA (Digilent Zybo Z7) thus enables hardware-in-the-loop emulation of a gyroscope, that can be tested in real-time, unlike a simulation based solution of the differential equations. The emulator design is easily customized to model any MEMS gyroscope implementation. The highly reconfigurable emulator presented here is thus a powerful medium for testing and instrumentation of electronics for MEMS inertial sensors.

II. Methods and results

We consider a tuning-fork gyroscope configuration with angular gain $= 1$, represented by proof mass M, stiffness K_x and K_y, and damping coefficients D_x and D_y along the drive (x) and sense (y) axes respectively. Additionally, we consider that the system experiences angular rate Ω_z along the orthogonal (z) axis. The equations of motion for the drive and sense axes are expressed as:

$$M\ddot{x} + D_x\dot{x} + \left(K_x + k_1 x + k_2 x^2\right) x = f_x \qquad (1)$$

$$M\ddot{y} + D_x\dot{y} + K_y y = f_y + 2M\Omega_z\dot{x} + K_{xy}x + f_N^* \qquad (2)$$

The motion amplitude along the drive axis is maintained by the drive force f_x. The sense force f_y is absent when we consider open-loop sense configuration without force feedback. The term of interest is the Coriolis force component $(2M\Omega_z\dot{x})$ that produces a displacement proportional to the angular rate. The system of equations also includes errors due to Duffing non-linearity (k_1, k_2), cross-axial stiffness (K_{xy}) that leads to quadrature error, and noise on the sense axis (f_N^*). The displacement along sense axis is typically much smaller than the drive axis, and hence the equations above consider only the dominant terms of interest in a gyroscope. The two differential equations are represented as discrete time filters using bilinear transform on the analog system function $T(s(z)) \longrightarrow H(z) = \frac{b_0+b_1z^{-1}+b_2z^{-2}}{a_0+a_1z^{-1}+a_2z^{-2}}$ where the bilinear transform is given by $s(z) = \frac{z-1}{z+1}$. The non-linear behavior is approximated as perturbations to the filter coefficients. This system is realized using a $32-$bit floating point arithmetic unit implementing multipliers, adders and delay registers. The input force f_x and angular velocity Ω_z are external signals provided to the emulator using $10-$bit high speed analog-to-digital converters (Maxim Integrated MAX-1426), followed

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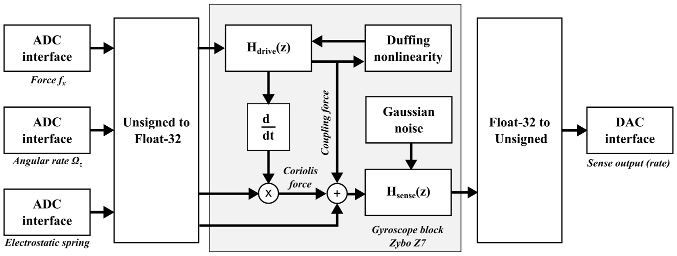

Fig. 1. Block diagram of the MEMS gyroscope emulator.

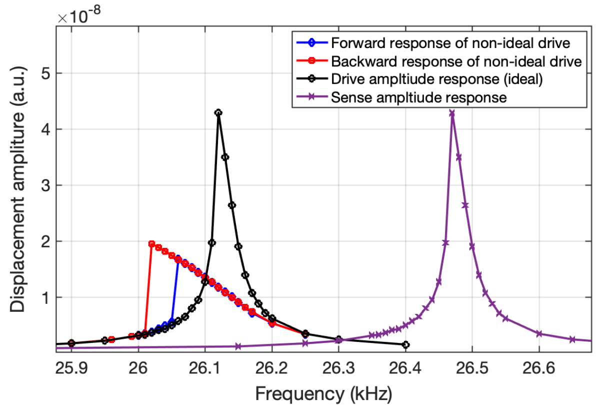

Fig. 2. Frequency response of drive and sense axis modes, including an example of Duffing nonlinearity in drive mode.

by the conversion of the digitized samples to 32−bit Floating Point representation. Figure 1 shows a block diagram and photograph of the system. The thermo-mechanical noise [6], [7] is internally generated in the emulator using a white Gaussian noise generator implemented using pseudo-random number generation by parallely operating linear feedback shift registers (LFSRs) with different seeds. The average of the outputs of these LFSRs is scaled in accordance with the desired variance and converted to floating point numbers. The displacement of the sense and drive axes (typically implemented using capacitive transduction in physical MEMS chips) are scaled and obtained as outputs of a digital-to-analog converter, so that the output analog signal can be connected to the ASIC electronics for control. The output of the sense axis is an amplitude-modulated wave whose carrier is the drive axis excitation signal and the envelope contains information about the angular rate. Additionally, it also contains a quadrature component due to the cross-axial stiffness. To maximize the sensitivity of the gyroscope, the drive axis is excited at a frequency very close to resonance. We consider a typical configuration of commercial MEMS gyroscopes i.e. mode mismatched configuration, wherein the drive and sense axes have the same effective mass, but different stiffness.

The example that we implemented comprises of a gyroscope with resonance frequency $26.1\,\mathrm{kHz}$ and $26.4\,\mathrm{kHz}$ for drive and sense axes respectively, and quality factor = $10\,000$ for both modes. The quadrature coupling parameter, K_{xy} is set to $100\,\mathrm{kN/m}$ and the proof mass is $1\,\mu\mathrm{g}$. The response of

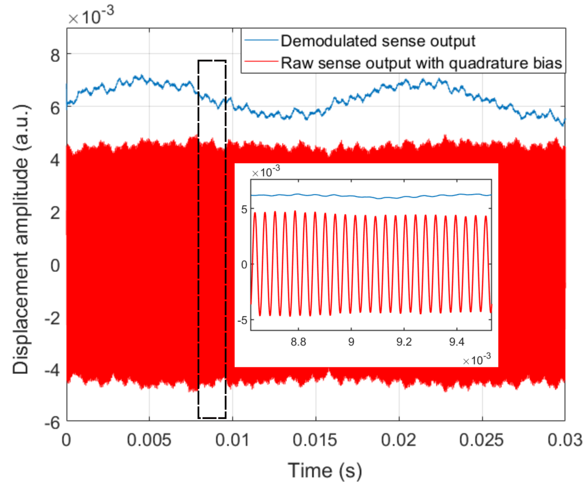

Fig. 3. Sense axis output: (red) prior to demodulation; (blue) after demodulation. The model includes quadrature and noise error sources.

the drive and sense axes (Figure 2) are obtained with $1\,\mathrm{MHz}$ sampling rate. The bandwidth of the gyroscope is measured by varying the frequency of a low-frequency sinusoid as input Ω_z, and observing the amplitude of the demodulated sense axis displacement. As expected, the bandwidth is observed to be approximately equal to the difference in the resonance frequencies of the drive and the sense axis modes [5]. The output of the gyroscope with thermal noise and quadrature error, in presence of sinusoidal angular rate is shown in Figure 3.

III. Conclusion

In summary, we present a complete hardware-in-the-loop platform to emulate a MEMS gyroscope for rapid prototyping and testing of ASIC implementations of control electronics. The architecture presented in our work is readily scalable to all commonly investigated MEMS gyroscope architectures.

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