Assignment1

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1

To Prove: There does not exist $n \in \mathbb{Z}^+$ such that $n^2 + n^3 = 100$.

Since n is positive, we can say that n^2 and n^3 will be positive and increasing. Let us assume $n^2 + n^3 = 100$. Then $\Leftrightarrow n^2(n+1) = 100$.

Either:-

$$n^2 = 100 \Rightarrow n = \sqrt{100} \tag{1}$$

This implies that n is not an integer, hence $n^2 \neq 100$.

Or:-

$$n + 1 = 100 \Rightarrow n = 99 \quad \text{but} \quad n^2 = 9801 > 100 \tag{2}$$

Hence, our assumption is false and we can say that there does not exist $n \in \mathbb{Z}^+$ such that $n^2 + n^3 = 100$.

2

To Prove: $n^2 + 1 \geq 2^n$ when n is a positive integer with $1 \leq n \leq 4$.

Since n is a positive integer and $1 \leq n \leq 4$, we can easily compute and verify.

- For $n = 1$:
$$1^2 + 1 \geq 2^1 \quad \Rightarrow \quad 1 + 1 \geq 2 \quad \Rightarrow \quad 2 \geq 2 \tag{3}$$

- For $n = 2$:
$$\begin{aligned} 2^2 + 1 &= 4 + 1 = 5 \\ 2^2 &= 4 \\ 5 &\geq 4 \end{aligned}$$

- For $n = 3$:
$$\begin{aligned} 3^2 + 1 &= 9 + 1 = 10 \\ 2^3 &= 8 \\ 10 &\geq 8 \end{aligned}$$

- For $n = 4$:

$$4^2 + 1 \geq 2^4 \quad \Rightarrow \quad 16 + 1 \geq 16 \quad \Rightarrow \quad 17 \geq 16 \tag{4}$$

Since we have considered all the cases and our assumption holds true in all, we can say that $n^2 + 1 \geq 2^n$ when n is a positive integer with $1 \leq n \leq 4$.

3

Let us assume cases:-

3.1 p,q,r are true and s is false

$$(p \land q \land r \land \neg s)$$

3.2 p,q,s are true and r is false

$$(p \land q \land s \land \neg r)$$

3.3 p,s,r are true and q is false

$$(p \land s \land r \land \neg q)$$

3.4 s,q,r are true and p is false

$$(s \land q \land r \land \neg p)$$

COMBINING ALL :

$$(p \land q \land r \land \neg s) \lor (p \land q \land s \land \neg r) \lor (p \land s \land r \land \neg q) \lor (s \land q \land r \land \neg p) \tag{5}$$

4

To Prove: $\exists x(P(x) \to Q(x)) \equiv \forall x P(x) \to \exists x Q(x)$

$$(\exists x(P(x) \to Q(x))) = \exists x(\neg P(x) \lor Q(x)) \quad and \quad (\forall x P(x) \to \exists x Q(x)) = \neg(\forall x P(x)) \lor \exists x Q(x) \tag{6}$$

The first expression means that there exist some x for which eith P is false or Q is true or both could happen. The second expression says that there exist some x for which P is false or there exist some x for which Q is true. Let say P(x) is false hence ¬P(x) is true and there exist some x for which ¬P(x) is true hence the first expression is true as ¬P(x)∨Q(x) is true.

Let say P(x) is true so ¬P(x) is false, hence it must always be false and if ¬ P(x) ∨ Q(x) is true hence Q(x) must be true for some x, hence, by which we can say that second expression will also be true.

5

To Prove: Power set of A is a subset of the power set of B implies that A is a subset of B. The given statement is true as the power set includes all the subset of A even the empty set hece since it is itself a subset of the power set of B hence it must contain all the elements of A which are therefore subset of the power set of B and therefore subset of B as power set of B also contains all the subsets of B. Let there be a element 'e' in A which is not present in B. Hence the power set of A will contain that element 'e' but it will not be there in B hence power set of A is not a subset of power set of B. But this is a contradiction hence our assumotion is correct.

6

To Prove : $A \subseteq B$ iff $A \cap B = A$

Proof : Let say $A \subseteq B$ means that all elemnet of A must be in B and B can have other elements. Hence it implies that $A \cap B$ will contain all the elements of A. Now, let say, $A \cap B = A$, this means all the elements of A are inside B hence we can say that A is a proper subset of B. As the definition of proper subset states that all elemenst of A must be in B.