

So, as  $\lambda \uparrow$   $m \downarrow$  due to this  $b$  will also affected &  $b$  will also change.

$$\therefore (b = \bar{y} - m\bar{x})$$

## Ridge Regression for nD data.

$$\begin{aligned} \text{Loss} &= \sum_{i=1}^n (y_i - \hat{y}_i)^2 \\ &= (XW - Y)^T (XW - Y) \end{aligned}$$

$$\begin{array}{ccccccc} x_1 & x_2 & \dots & \dots & x_n & y \\ w_1 & w_2 & \dots & \dots & w_n & \vdots \end{array}$$

same  $w$

where,

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad (m \text{ values}), \quad W = \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_n \end{bmatrix} \quad (n \text{ values}), \quad X = \begin{bmatrix} 1 & x_{11} & x_{12} & \dots & x_{1n} \\ \vdots & x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix}$$

Loss function with regularization term

$$L = (XW - Y)^T (XW - Y) + \lambda \|W^2\|$$

$$\frac{\partial L}{\partial W} ?$$

$$\{ \lambda w_0^2 + \lambda w_1^2 + \lambda w_2^2 + \dots + \lambda w_n^2 \}$$

$$\therefore \|W^2\| \approx W^T W$$

$$[w_0 \ w_1 \ \dots \ w_n] \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_n \end{bmatrix}$$

$$L = [(XW)^T - Y^T] (XW - Y) + \lambda W^T W$$

$\therefore (a-b)^T = a^T - b^T$

$$L = [(W^T X^T - Y^T) (XW - Y) + \lambda W^T W$$

$\therefore (ab)^T = b^T a^T$

$$= W^T X^T X W - \underbrace{W^T X^T Y - Y^T X W}_{\text{same.}} + Y^T Y + \lambda W^T W$$

$$L = W^T X^T X W - 2W^T X^T Y + Y^T Y + \lambda W^T W$$

$$\frac{\partial L}{\partial W} \Rightarrow 2X^T X W - 2X^T Y + 0 + 2\lambda W = 0$$

$$\Rightarrow X^T X W + \lambda W = X^T Y$$

$$\Rightarrow (X^T X + \lambda I) W = X^T Y$$

$$W = (X^T X + \lambda I)^{-1} X^T Y$$

$$W = (X^T X)^{-1} X^T Y$$

normal without regularization.

extra due to regularization.

