Text Categorization using Naïve Bayes

Mausam

(based on slides of Dan Weld, Dan Jurafsky, Prabhakar Raghavan, Hinrich Schutze, Guillaume Obozinski, David D. Lewis, Fei Xia)

Categorization

• Given:

- A description of an instance, $x \in X$, where X is the *instance language* or *instance space*.
- A fixed set of categories: $C = \{c_1, c_2, ... c_n\}$

• Determine:

- The **category of** x: $c(x) \in C$, where c(x) is a categorization function whose domain is X and whose range is C.

County vs. Country?

article edit this page article discussion edit this page history · Ten things you didn't know about images on Wikipedia · Ten things you didn't know about V King County, Washington Kenya Coordinates: (a) 47.47, -121.84 From Wikipedia, the free encyclopedia From Wikipedia, the free encyclopedia "King County" redirects here. For other uses, see King County (disambiguation). This article needs additional references or sources for ve King County is located in the U.S. state of King County, Washington Please help improve this article by adding reliable references. Unverifiable m Washington. The population in the 2000 census be challenged and removed. was 1,737,034 and in 2006 was an estimated 1,835,300. By population, King is the largest The Republic of Kenya is a country in Jamhuri ya county in Washington, and the 12th largest in the King County Eastern Africa. It is bordered by Ethiopia to United States. As of 2006, the county had a Republic of the north, Somalia to the northeast, Tanzania Map population comparable to that of the state of to the south, Uganda to the west, and Sudan Nebraska. to the northwest, with the Indian Ocean The county seat is Seattle, which is the state's running along the southeast border. largest city. About two-thirds of the county's population lives in the city's suburbs. King County Contents [show] ranks among the 100 highest-income counties in Flag the United States. Motte History [edit] Location in the state of Washington Contents [show] "Harambee" "Let us all pull Main article: History of Kenya History [edit] Anthe Paleontologists have discovered many fossils Ee Mungu Ng "Oh God of All of prehistoric animals in Kenya. At one of the The county was formed out of territory within Thurston County on December 22, 1852, by the rare dinosaur fossil sites in Africa, two Oregon Territory legislature, and was named after hundred Cretaceous theropod and giant

Washington's location in the USA

Statistics

Seattle

December 22, 1852

Founded

Seat

crocodile fossils have been discovered in

Kenya, dating from the Mesozoic Era, over

in an excavation conducted by a team from

Mucoume of Konyo in July August 2004 at

the University of Utah and the National

200 million years ago. The fossils were found

King County originally extended to the Olympic Peninsula. According to historian Bill Speidel,

1853.[1] [3][2] 🚱

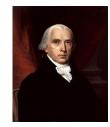
Alabama resident William Rufus King, vice

was made the county seat on January 11,

president under president Franklin Pierce. Seattle

Who wrote which Federalist papers?

- 1787-8: anonymous essays try to convince New York to ratify U.S Constitution: Jay, Madison, Hamilton.
- Authorship of 12 of the letters in dispute
- 1963: solved by Mosteller and Wallace using Bayesian methods



James Madison



Alexander Hamilton

Male or female author?

- The main aim of this article is to propose an exercise in stylistic analysis which can be employed in the teaching of English language. It details the design and results of a workshop activity on narrative carried out with undergraduates in a university department of English. The methods proposed are intended to enable students to obtain insights into aspects of cohesion and narrative structure; insights, it is suggested, which are not as readily obtainable through mor Female writers use
 - more first person/second person pronouns
- My aim in the transportation more gender laiden third person pronouns approach to utterance int (overall more personalization) anding of what some of these so-called apposition markers indicate. It will be argued that the decision to put something in other words is essentially a decision about style, a point which is, perhaps, anticipated by Burton-Roberts when he describes loose apposition as a rhetorical device. However, he does not justify this suggestion by giving the criteria for classifying a mode of expression as a rhetorical device.

S. Argamon, M. Koppel, J. Fine, A. R. Shimoni, 2003. "Gender, Genre, and Writing Style in Formal Written Texts," Text, volume 23, number 3, pp. 321–346

Positive or negative movie review?



unbelievably disappointing



 Full of zany characters and richly applied satire, and some great plot twists



• this is the greatest screwball comedy ever filmed



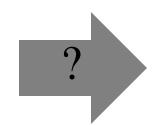
• It was pathetic. The worst part about it was the boxing scenes.

What is the subject of this article?

MeSH Subject Category Hierarchy

MEDLINE Article





- Antogonists and Inhibitors
- Blood Supply
- Chemistry
- Drug Therapy
- Embryology
- Epidemiology

•

Text Classification

- Assigning documents to a fixed set of categories, e.g.
- Web pages
 - Yahoo-like classification
 - Assigning subject categories, topics, or genres
- Email messages
 - Spam filtering
 - Prioritizing
 - Folderizing
- Blogs/Letters/Books
 - Authorship identification
 - Age/gender identification
- Reviews/Social media
 - Language Identification
 - Sentiment analysis

_ ...

Classification Methods: Hand-coded rules

- Rules based on combinations of words or other features
 - spam: black-list-address OR ("dollars" AND "have been selected")
- Accuracy can be high
 - If rules carefully refined by expert
- But building and maintaining these rules is expensive

Learning for Text Categorization

- Hard to construct text categorization functions.
- Learning Algorithms:
 - Bayesian (naïve)
 - Neural network
 - Logistic Regression
 - Nearest Neighbor (case based)
 - Support Vector Machines (SVM)

Bayesian Methods

- Learning and classification methods based on probability theory.
 - Bayes theorem plays a critical role in probabilistic learning and classification.
 - Uses *prior* probability of each category given no information about an item.
- Categorization produces a *posterior* probability distribution over the possible categories given a description of an item.

Bayes' Rule Applied to Documents and Classes

• For a document d and a class c

$$P(c \mid d) = \frac{P(d \mid c)P(c)}{P(d)}$$

Naïve Bayes Classifier (I)

$$c_{MAP} = \underset{c \mid C}{\operatorname{argmax}} P(c \mid d)$$

MAP is "maximum a posteriori" = most likely class

$$= \underset{c \mid C}{\operatorname{argmax}} \frac{P(d \mid c)P(c)}{P(d)}$$

Bayes Rule

$$= \underset{c \mid C}{\operatorname{argmax}} P(d \mid c) P(c)$$

Dropping the denominator

Naïve Bayes Classifier (II)

$$c_{MAP} = \underset{c \mid C}{\operatorname{argmax}} P(d \mid c) P(c)$$

$$= \underset{c \in C}{\operatorname{argmax}} P(x_1, x_2, \dots, x_n \mid c) P(c)$$
represented as features x1..xn

Document d

Naïve Bayes Classifier (IV)

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}} P(x_1, x_2, \dots, x_n \mid c) P(c)$$

 $O(|X|^n \bullet |C|)$ parameters

Could only be estimated if a very, very large number of training examples was available.

How often does this class occur?

We can just count the relative frequencies in a corpus

Multinomial Naïve Bayes Independence Assumptions

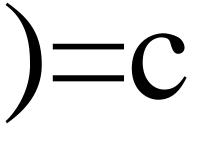
$$P(x_1, x_2, \ldots, x_n \mid c)$$

Bag of Words assumption: Assume position doesn't matter

The bag of words representation

γ(

I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun... It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet.



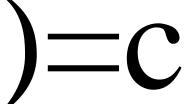




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The bag of words representation: using a subset of words

love xxxxxxxxxxxxxx sweet

The bag of words representation

Y(love 2 love 1 laugh 1 laugh 1 laupy 1 love 1

Bag of words for document classification

Test document

parser language label translation

. . .

'

Machine Learning

learning training algorithm shrinkage network... NLP

parser tag training translation language...

Garbage Collection

garbage collection memory

optimization region...

Planning

GUI

planning temporal reasoning

plan

language...

Multinomial Naïve Bayes Classifier

$$c_{MAP} = \underset{c \in C}{\operatorname{argmax}} P(x_1, x_2, \dots, x_n \mid c) P(c)$$

$$c_{NB} = \underset{c \mid C}{\operatorname{argmax}} P(c_j) \widetilde{O}_{X \mid X} P(x \mid c)$$

Multinomial Naïve Bayes Independence Assumptions

$$P(x_1, x_2, \ldots, x_n \mid c)$$

- Bag of Words assumption: Assume position doesn't matter
- Conditional Independence: Assume the feature probabilities $P(x_i | c_i)$ are independent given the class c.

$$P(x_1, , x_n | c) = P(x_1 | c) \bullet P(x_2 | c) \bullet P(x_3 | c) \bullet ... \bullet P(x_n | c)$$

Applying Multinomial Naive Bayes Classifiers to Text Classification

positions ← all word positions in test document

$$c_{NB} = \underset{c_{j} \cap C}{\operatorname{argmax}} P(c_{j}) \underbrace{\widetilde{O}}_{i \cap positions} P(x_{i} \mid c_{j})$$

Learning the Multinomial Naïve Bayes Model

- First attempt: maximum likelihood estimates
 - simply use the frequencies in the data

$$\hat{P}(c_{j}) = \frac{doccount(C = c_{j})}{N_{doc}}$$

$$\hat{P}(w_{i} | c_{j}) = \frac{count(w_{i}, c_{j})}{\overset{\circ}{a} count(w, c_{j})}$$

Parameter estimation

$$\hat{P}(w_i | c_j) = \frac{count(w_i, c_j)}{\mathop{a}\limits_{w \mid V}}$$
 fraction of times word w_i appears among all words in documents of topic c_j

- Create mega-document for topic *j* by concatenating all docs in this topic
 - Use frequency of w in mega-document

Problem with Maximum Likelihood

 What if we have seen no training documents with the word *fantastic* and classified in the topic positive (thumbs-up)?

$$\hat{P}(\text{"fantastic" | positive}) = \frac{count(\text{"fantastic", positive})}{\mathop{\aa}\limits_{w \mid V}} = 0$$

 Zero probabilities cannot be conditioned away, no matter the other evidence!

$$c_{MAP} = \operatorname{argmax}_{c} \hat{P}(c) \tilde{O}_{i} \hat{P}(x_{i} \mid c)$$

Laplace (add-1) smoothing for Naïve Bayes

$$\begin{split} \hat{P}(w_i \mid c) &= \frac{count(w_i, c) + 1}{\overset{\circ}{\partial} \left(count(w, c) \right) + 1} \\ &= \frac{count(w_i, c) + 1}{\overset{\circ}{\partial} \left(count(w_i, c) \right) + 1} \\ &= \frac{count(w_i, c) + 1}{\overset{\circ}{\partial} \left(count(w_i, c) \right) + 1} \\ &\stackrel{\circ}{\in} \overset{\circ}{\partial} count(w_i, c) + |V| \\ &\stackrel{\circ}{\partial}_{w_i} V \end{split}$$

Multinomial Naïve Bayes: Learning

- From training corpus, extract Vocabulary
- Calculate $P(c_i)$ terms
 - For each c_j in C do $docs_j \leftarrow \text{all docs with class} = c_j$ $P(c_j) \neg \frac{|docs_j|}{|\text{total # documents}|}$
- Calculate $P(w_k \mid c_i)$ terms
 - $Text_i \leftarrow single doc containing all <math>docs_i$
 - For each word w_k in *Vocabulary* $n_k \leftarrow \#$ of occurrences of w_k in $Text_j$

$$P(w_k | c_j) \neg \frac{n_k + \partial}{n + \partial |Vocabulary|}$$

Naïve Bayes Time Complexity

- Training Time: $O(|D|L_d + |C||V|)$ where L_d is the average length of a document in D.
 - Assumes V and all D_i , n_i , and n_{ij} pre-computed in $O(|D|L_d)$ time during one pass through all of the data.
 - Generally just $O(|D|L_d)$ since usually $|C||V| < |D|L_d$
- Test Time: $O(/C/L_t)$ where L_t is the average length of a test document.
- Very efficient overall, linearly proportional to the time needed to just read in all the data.

Easy to Implement

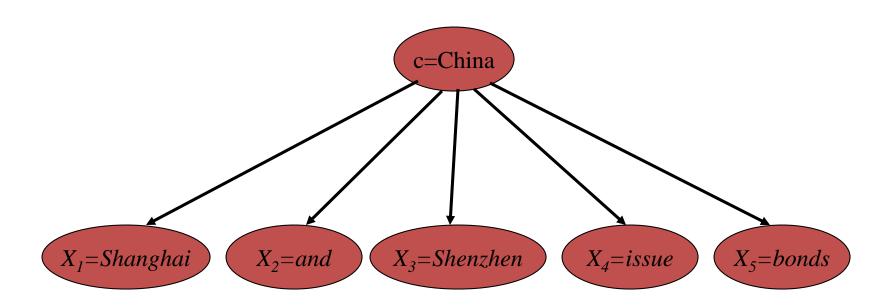
• But...

• If you do... it probably won't work...

Probabilities: Important Detail!

- We are multiplying lots of small numbers Danger of underflow!
 - $-0.5^{57} = 7 E 18$
- Solution? Use logs and add!
 - $p_1 * p_2 = e^{\log(p_1) + \log(p_2)}$
 - Always keep in log form

Generative Model for Multinomial Naïve Bayes



Naïve Bayes and Language Modeling

- Naïve bayes classifiers can use any sort of feature
 - URL, email address, dictionaries, network features
- But if, as in the previous slides
 - We use only word features
 - we use **all** of the words in the text (not a subset)
- Then
 - Naïve bayes has an important similarity to language modeling.

Each class = a unigram language model

- Assigning each word: P(word | c)
- Assigning each sentence: P(s|c)=Π P(word|c)

film

```
Class pos
0.1
                                            this
                                    love
                                                     fun
0.1
        love
                                           .05
                                                     0.01 0.1
                           0.1
                                    0.1
0.01
        this
        fun
0.05
        film
0.1
                                                 P(s \mid pos) = 0.0000005
```

Naïve Bayes as a Language Model

 Which class assigns the higher probability to s?

Model pos 0.1 I 0.1 love 0.01 this 0.05 fun 0.1 film

Model neg 0.2 I 0.001 love 0.01 this 0.005 fun 0.1 film

<u> </u>	love	this	fun	fi <u>lm</u>
0.1 0.2	0.1 0.001	0.01 0.01	0.05 0.005	0.1 0.1
P(s pos) > P(s neg)				

Example: Naïve Bayes in Spam Filtering

SpamAssassin Features:

- Mentions Generic Viagra
- Online Pharmacy
- Mentions millions of (dollar) ((dollar) NN,NNN,NNN.NN)
- Phrase: impress ... girl
- From: starts with many numbers
- Subject is all capitals
- HTML has a low ratio of text to image area
- One hundred percent guaranteed
- Claims you can be removed from the list
- Prestigious Non-Accredited Universities'
- http://spamassassin.apache.org/tests_3_3_x.html

Advantages

- Simple to implement
 - No numerical optimization, matrix algebra, etc
- Efficient to train and use
 - Easy to update with new data
 - Fast to apply
- Binary/multi-class
- Good in domains with many equally important features
 - Decision Trees suffer from fragmentation in such cases –
 especially if little data
- Comparatively good effectiveness with small training sets
- A good dependable baseline for text classification
 - But we will see other classifiers that give better accuracy

Disadvantages

- Independence assumption wrong
 - Absurd estimates of class probabilities
 - Output probabilities close to 0 or 1
 - Thresholds must be tuned; not set analytically

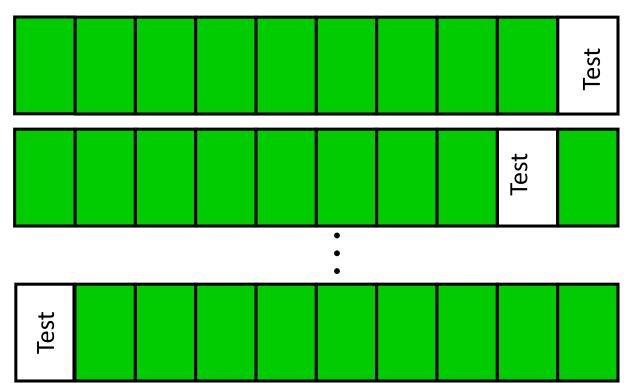
- Generative model
 - Generally lower effectiveness than discriminative techniques

Experimental Evaluation

- Question: How do we estimate the performance of classifier on unseen data?
- Can't just at accuracy on training data this will yield an over optimistic estimate of performance
- Solution: Cross-validation
- Note: this is sometimes called estimating how well the classifier will generalize

Evaluation: Cross Validation

- Partition examples into *k* disjoint sets
- Now create *k* training sets
 - Each set is union of all equiv classes except one
 - So each set has (k-1)/k of the original training data



Cross-Validation (2)

- Leave-one-out
 - Use if < 100 examples (rough estimate)
 - Hold out one example, train on remaining examples
- 10-fold
 - If have 100-1000's of examples

- M of N fold
 - Repeat M times
 - Divide data into N folds, do N fold cross-validation

Evaluation Metrics

- Accuracy: no. of questions correctly answered
- Precision (for one label): accuracy when classification = label
- Recall (for one label): measures how many instances of a label were missed.
- F-measure (for one label): harmonic mean of precision & recall.
- Area under Precision-recall curve (for one label): vary parameter to show different points on p-r curve; take the area

Precision & Recall

Two class situation

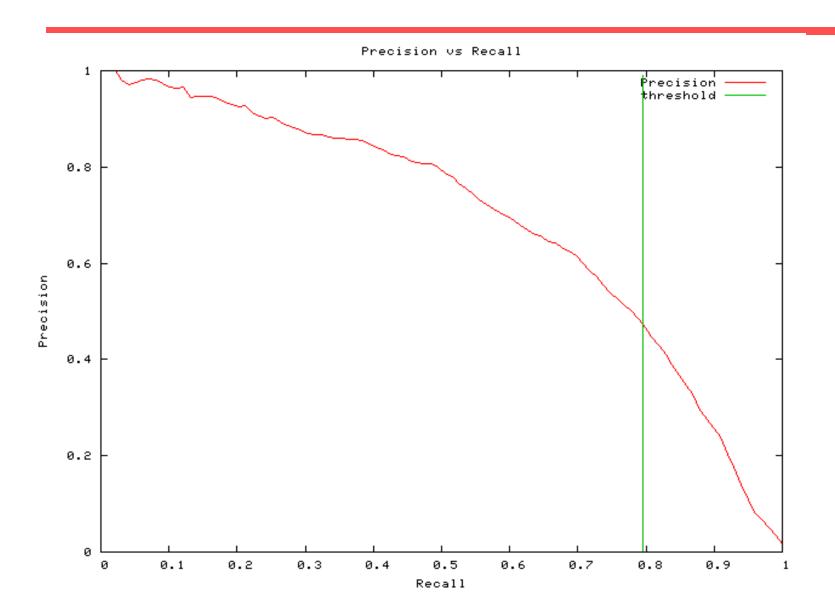
		Predicted											
ıal		"P"	"N"										
Actu	P	TP	FN										
7	N	FP	TN										

Precision = TP/(TP+FP)

Recall = TP/(TP+FN)

F-measure = 2pr/(p+r)

A typical precision-recall curve



Construct Better Features

 Key to machine learning is having good features

• In gen 2 ML, large effort devoted to constructing appropriate features

• Ideas??

Issues in document representation

Cooper's concordance of Wordsworth was published in 1911. The applications of full-text retrieval are legion: they include résumé scanning, litigation support and searching published journals on-line.

- Cooper's vs. Cooper vs. Coopers.
- Full-text vs. full text vs. {full, text} vs. fulltext.
- résumé vs. resume.

Punctuation

- *Ne'er*: use language-specific, handcrafted "locale" to normalize.
- State-of-the-art: break up hyphenated sequence.
- *U.S.A.* vs. *USA*
- a.out

Numbers

- 3/12/91
- Mar. 12, 1991
- 55 B.C.
- B-52
- 100.2.86.144
 - Generally, don't represent as text
 - Creation dates for docs

Possible Feature Ideas

 Look at capitalization (may indicated a proper noun)

- Look for commonly occurring sequences
 - E.g. New York, New York City
 - Limit to 2-3 consecutive words
 - Keep all that meet minimum threshold (e.g. occur at least 5 or 10 times in corpus)

Case folding

- Reduce all letters to lower case
- Exception: upper case in mid-sentence
 - e.g., General Motors
 - **Fed** vs. **fed**
 - SAIL vs. sail

Thesauri and Soundex

- Handle synonyms and spelling variations
 - Hand-constructed equivalence classes
 - e.g., *car* = *automobile*

Spell Correction

- Look for all words within (say) edit distance
 3 (Insert/Delete/Replace) at query time
 - − e.g., arfiticial inteligence
- Spell correction is expensive and slows the processing significantly
 - Invoke only when index returns zero matches?

Lemmatization

Reduce inflectional/variant forms to base form

```
- am, are, is \rightarrow be
```

- car, cars, car's, cars' \rightarrow car

the boy's cars are different colors

 \rightarrow

the boy car be different color

Stemming

- Are there different index terms?
 - retrieve, retrieving, retrieval, retrieved, retrieves...
- Stemming algorithm:
 - (retrieve, retrieving, retrieval, retrieved, retrieves) ⇒ retriev
 - Strips prefixes of suffixes (-s, -ed, -ly, -ness)
 - Morphological stemming
- Problems: sand / sander & wand / wander

Stemming Continued

- Can reduce vocabulary by ~ 1/3
- C, Java, Perl versions, python, c# www.tartarus.org/~martin/PorterStemmer
- Criterion for removing a suffix
 - Does "a document is about w₁" mean the same as
 - a "a document about w₂"
- Problems: sand / sander & wand / wander
- Commercial SEs use giant in-memory tables

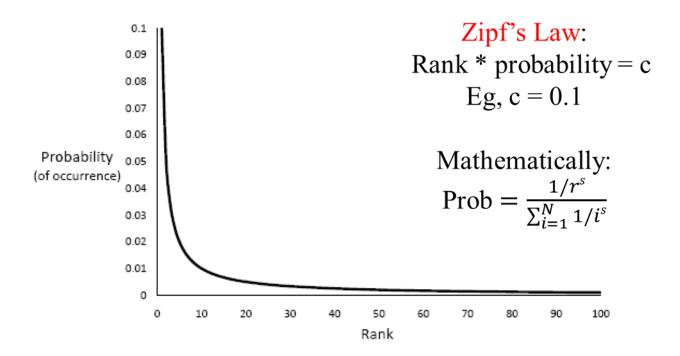
Features

 Domain-specific features and weights: very important in real performance

- Upweighting: Counting a word as if it occurred twice:
 - title words (Cohen & Singer 1996)
 - first sentence of each paragraph (Murata, 1999)
 - In sentences that contain title words (Ko et al, 2002)

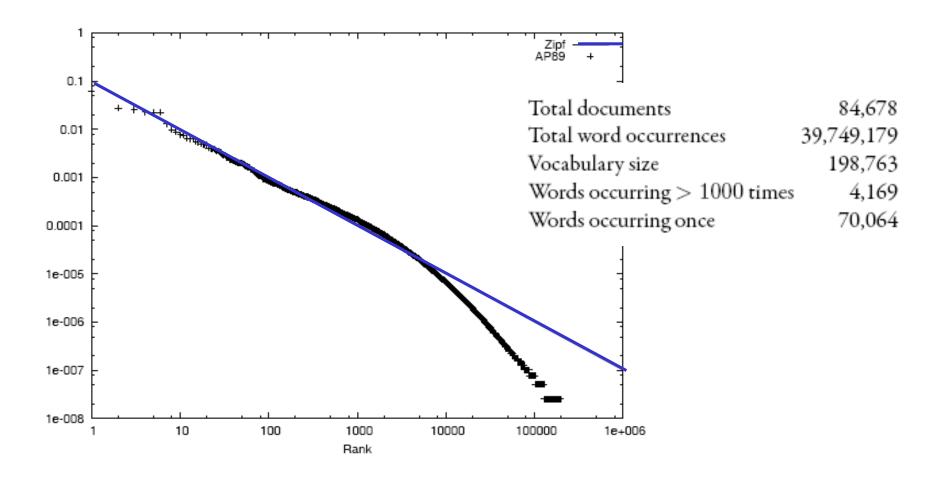
Properties of Text

- Word frequencies skewed distribution
- 'The' and 'of' account for 10% of all words
- Six most common words account for 40%



From [Croft, Metzler & Strohman 2010]

Associate Press Corpus 'AP89'



Middle Ground

- Very common words → bad features
 - Language-based stop list:
 words that bear little meaning
 20-500 words

http://www.dcs.gla.ac.uk/idom/ir_resources/linguistic_utils/stop_words

Subject-dependent stop lists

Very rare words also bad features
 Drop words appearing less than k times / corpus

Word Frequency

- Which word is more indicative of document similarity?
 - 'book,' or 'Rumplestiltskin'?
 - Need to consider "document frequency"--- how frequently the word appears in doc collection.
- Which doc is a better match for the query "Kangaroo"?
 - One with a single mention of Kangaroos... or a doc that mentions it 10 times?
 - Need to consider "term frequency"--- how many times the word appears in the current document.

TF x IDF

$$w_{ik} = tf_{ik} * \log(N/n_k)$$

 $T_k = term \ k \ in \ document \ D_i$ $tf_{ik} = frequency \ of \ term \ T_k \ in \ document \ D_i$ $idf_k = inverse \ document \ frequency \ of \ term \ T_k \ in \ C$ $idf_k = \log\left(\frac{N}{n_k}\right)$

N = total number of documents in the collection <math>C $n_k = the number of documents in <math>C$ that contain T_k

Inverse Document Frequency

 IDF provides high values for rare words and low values for common words

$$\log\left(\frac{10000}{10000}\right) = 0$$

$$\log\left(\frac{10000}{5000}\right) = 0.301$$

$$\log\left(\frac{10000}{20}\right) = 2.698$$

$$\log\left(\frac{10000}{1}\right) = 4$$

• Add 1 to avoid 0.

TF-IDF normalization

- Normalize the term weights
 - so longer docs not given more weight (fairness)
 - force all values to fall within a certain range: [0, 1]

$$w_{ik} = \frac{tf_{ik}(1 + \log(N/n_k))}{\sqrt{\sum_{k=1}^{t} (tf_{ik})^2 [1 + \log(N/n_k)]^2}}$$

Term-Weighted Complement Naïve Bayes

- Let $\vec{d} = (\vec{d}_1, \dots, \vec{d}_n)$ be a set of documents; d_{ij} is the count of word i in document j.
- Let $\vec{y} = (y_1, \dots, y_n)$ be the labels.
- TWCNB (\vec{d}, \vec{y})

$$d_{ij} = \log(d_{ij} + 1)$$
 (TF transform § 4.1)

$$d_{ij} = d_{ij} \log \frac{\sum_{k} 1}{\sum_{k} \delta_{ik}}$$
 (IDF transform § 4.2)

$$d_{ij} = \frac{d_{ij}}{\sqrt{\sum_{k} (d_{kj})^2}}$$
 (length norm. § 4.3)

$$\hat{\theta}_{ci} = \frac{\sum_{j:y_j \neq c} d_{ij} + \alpha_i}{\sum_{j:y_j \neq c} \sum_k d_{kj} + \alpha} \text{ (complement § 3.1)}$$

$$w_{ci} = \log \hat{\theta}_{ci}$$

Let $t = (t_1, \ldots, t_n)$ be a test document; let t_i be the count of word i.

Label the document according to

$$l(t) = \arg\min_{c} \sum_{i} t_i w_{ci}$$

Multi-class Problems

Evaluation: Classic Reuters-21578 Data Set

- Most (over)used data set, 21,578 docs (each 90 types, 200 tokens)
- 9603 training, 3299 test articles (ModApte/Lewis split)
- 118 categories
 - An article can be in more than one category
 - Learn 118 binary category distinctions
- Average document (with at least one category) has 1.24 classes
- Only about 10 out of 118 categories are large

Common categories (#train, #test)

- Earn (2877, 1087)
- Acquisitions (1650, 179)
- Money-fx (538, 179)
- Grain (433, 149)
- Crude (389, 189)

- Trade (369,119)
- Interest (347, 131)
- Ship (197, 89)
- Wheat (212, 71)
- Corn (182, 56)

Reuters Text Categorization data set (Reuters-21578) document

<REUTERS TOPICS="YES" LEWISSPLIT="TRAIN" CGISPLIT="TRAINING-SET" OLDID="12981" NEWID="798">

<DATE> 2-MAR-1987 16:51:43.42</DATE>

<TOPICS><D>livestock</D><D>hog</D></TOPICS>

<TITLE>AMERICAN PORK CONGRESS KICKS OFF TOMORROW</TITLE>

<DATELINE> CHICAGO, March 2 - </DATELINE><BODY>The American Pork Congress kicks off tomorrow, March 3, in Indianapolis with 160 of the nations pork producers from 44 member states determining industry positions on a number of issues, according to the National Pork Producers Council, NPPC.

Delegates to the three day Congress will be considering 26 resolutions concerning various issues, including the future direction of farm policy and the tax law as it applies to the agriculture sector. The delegates will also debate whether to endorse concepts of a national PRV (pseudorabies virus) control and eradication program, the NPPC said.

A large trade show, in conjunction with the congress, will feature the latest in technology in all areas of the industry, the NPPC added. Reuter

:</BODY></TEXT></REUTERS>

Precision & Recall

Two class situation

	Predicted										
tual		"P"	"N"								
Actu	P	TP	FN								
7	N	FP	TN								

Precision = TP/(TP+FP)

Recall = TP/(TP+FN)

F-measure = 2pr/(p+r)

Multi-class situation:

Classname		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
alt.atheism	1	251	6	1	3	32	1	1	2	1	2	0	0	0	0	0	0	0	0	0	0
soc.religion.christian	2	9	277	0	1	6	0	0	1	0	0	0	0	0	1	2	2	0	0	0	1
sci.space	3	3	1	273	1	0	1	2	0	1	1	9	0	0	1	2	3	0	0	1	1
talk.politics.misc	4	2	0	3	213	24	3	0	17	3	0	0	0	0	0	0	1	0	1	33	0
talk.religion.misc	5	88	36	2	23	132	0	1	0	0	0	0	0	0	0	0	2	0	1	15	0
rec.autos	6	0	0	0	3	1	272	0	0	0	7	1	2	1	Б	4	[1		0	2	0
comp.windows.x	7	1	1	2	1	0	1	246	0	2	2	30	5	3		7			1	0	0
talk.politics.mideast	8	0	3	1	18	0	0	0	275	0	1	0	0	0		Ò	J	0	1	1	0
sci.crypt	9	1	0	1	2	1	0	3	0	284	0	3	8	1	0	0	1	0	0	3	0
rec.motorcycles	10	0	0	0	1	0	4	1	0	0		P	2	0	1	2	1	0	0	1	0
comp.graphics	11	0	1	2	1	1	0	10	1	2	0	243	23	7	3	3	3	0	0	0	0
comp.sys.ibm.pc.hardware	12	0	0	0	0	0	2	7	0	1	0	5	243	23	12	3	1	3	0	0	0
comp.sys.mac.hardware	13	0	0	1	1	1	2	1		0	0	7	10	260	8	9	1	0	0	0	0
sci.electronics	14	1	0	1	0		7	2		2	0	7	13	13	245	6	3	0	1	0	0
misc.forsale	15	0	1	4	2	1	12	1	_0	0	4	1	19	10	8	233	1	0	1	1	2
sci.med	16	0	1	5	0	1	1	0	0	0	1	2	0	2	7	2	275	0	1	1	1
comp.os.mswindows.misc	17	1	0	2	0	1	1	58	1	3	0	38	71	17	3	6	0	97	1	0	0
rec.sport.baseball	18	2	1	1	0	0	0	0	0	0	0	4	0	0	0	1	1	0	282	1	7
talk.politics.guns	19	0	0	0	9	5	1	0	0	1	0	0	0	0	1	0	0	1	1	281	0
rec.sport.hockey	20	0	1	0	0	0	1	0	0	0	2	0	0	1	1	0	0	0	3	0	291

Micro-- vs. Macro--Averaging

• If we have more than one class, how do we combine multiple performance measures into one quantity?

- Macroaveraging
 - Compute performance for each class, then average.

- Microaveraging
 - Collect decisions for all classes, compute contingency table, evaluate

Precision & Recall

Multi-class Multi-label situation:

Classname		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
alt.atheism	1	251	6	1	3	32	1	1	2	1	2	0	0	0	0	0	0	0	0	0	0
soc.religion.christian	2	9	277	Λ	1	6	0	0	1	0	0	0	0	0	1	2	2	0	0	0	1
sci.space	3	3	1	273	1	0	1	2	0	1	1	9	0	0	1	2	3	0	0	1	1
talk.politics.misc	4	2	0	3	213	24	3	0	17	3	0	0	0	0	0	0	1	0	1	33	0
talk.religion.misc	5	88	36	2	23	132	Λ	1	0	0	0	0	0	0	0	0	2	0	1	15	0
rec.autos	δ	0	0	0	3	1	272	Λ	0	0	7	1	2	1	6	4	1	0	0	2	0
comp.windows.x	7	1	1	2	1	0	1	246	n	2	2	30	5	3	1	1	2	1	1	0	0
talk.politics.mideast	8	0	3	1	18	0	0	0	275	0	1	0	0	0	0	0	0	0	1	1	0
sci.crypt	g	1	0	1	2	1	0	3	0	284	0	3	0	1	0	0	1	0	0	3	0
rec.motorcycles	10	0	0	0	1	0	4	1	0	0	286	1	2	0	1	2	1	0	0	1	0
comp.graphics	11	0	1	2	1	1	0	10	1	2	0	243	23	7	3	3	3	0	0	0	0
comp.sys.ibm.pc.hardware	12	0	0	0	0	0	2	7	0	1	0	5	243	23	12	3	1	3	0	0	0
comp.sys.mac.hardware	13	0	0	1	1	0	2	1	0	0	0	7	10	260	8	9	1	0	0	0	0
sci.electronics	14	1	0	1	0	1	5	2	0	2	0	7	13	13	245	6	3	0	1	0	0
misc.forsale	15	0	1	4	2	0	12	1	0	0	4	1	19	10	8	233	1	0	1	1	2
sci.med	16	0	1	5	0	1	1	0	0	0	1	2	0	2	7	2	275	0	1	1	1
comp.os.mswindows.misc	17	1	0	2	0	1	1	58	1	3	0	38	71	17	3	6	0	97	1	0	0
rec.sport.baseball	18	2	1	1	0	0	0	0	0	0	0	4	0	0	0	1	1	0	282	1	7
talk.politics.guns	19	0	0	0	9	5	1	0	0	1	0	0	0	0	1	0	0	1	1	281	Λ
rec.sport.hockey	20	0	1	0	0	0	1	0	0	0	2	0	0	1	1	0	0	0	3	0	29

Aggregate

 $\begin{array}{ll} Average\ Macro\ Precision =\ \Sigma p_i/N \\ Average\ Macro\ Recall &= \Sigma r_i/N \\ Average\ Macro\ F\text{-measure} = 2p_M r_M/(p_M + r_M) \end{array}$

Average Micro Precision = $\Sigma TP_i/\Sigma_i Col_i$ Average Micro Recall = $\Sigma TP_i/\Sigma_i Row_i$ Average Micro F-measure = $2p_\mu r_\mu/(p_\mu + r_\mu)$

```
Precision(class i) = TP_i/(TP_i+FP_i)

Recall(class i) = TP_i/(TP_i+FN_i)
```

F-measure(class i) =
$$2p_i r_i/(p_i + r_i)$$

```
Precision(class 1) = 251/(\text{Column}_1)

Recall(class 1) = 251/(\text{Row}_1)

F-measure(class 1)) = 2p_i r_i/(p_i+r_i)
```

Multi-Class Classification

- What?
 - Converting a k-class problem to a binary problem.
- Why?
 - For some ML algorithms, a direct extension to the multiclass case may be problematic.
- How?
 - Many methods

Methods

One-vs-all

All-pairs

Error-correcting Output Codes (ECOC)

• • •

One-vs-all

- Idea:
- Create many 1 vs other classifiers
 - Classes = City, County, Country
 - Classifier 1 = {City} {County, Country}
 - Classifier 2 = {County} {City, Country}
 - Classifier 3 = {Country} {City, County}
- Training time:
 - For each class c_m , train a classifier $cl_m(x)$
 - replace (x,y) with (x, 1) if $y = c_m$ (x, -1) if $y != c_m$

An example: training

- x1 c1 ...
- x2 c2 ...
- x3 c1 ...
- x4 c3 ...

for c1-vs-all:

- x1 1 ...
- x2 -1 ...
- x3 1 ...
- x4 -1 ...

for c2-vs-all:

- x1 -1
- x2 1 ...
- x3 -1 ...
- x4 -1 ...

for c3-vs-all:

- x1 -1...
- x2 -1...
- x3 -1 ...
- x4 1 ...

One-vs-all (cont)

- Testing time: given a new example x
 - Run each of the k classifiers on x

- Choose the class c_m with the highest confidence score $cl_m(x)$:

$$c^* = arg max_m cl_m(x)$$

An example: testing

- x1 c1 ...
- x2 c2 ...
- x3 c1 ...
- x4 c3 ...
- → three classifiers

Test data:

for c1-vs-all:

x ?? 1 0.7 -1 0.3

for c2-vs-all

x ?? 1 0.2 -1 0.8

for c3-vs-all

x ?? 1 0.6 -1 0.4

 \Rightarrow what's the system prediction for x?

All-pairs (All-vs-All (AVA))

• Idea:

- For each pair of classes build a classifier
- {City vs. County}, {City vs Country}, {County vs.Country}
- $-C_k^2$ classifiers: one classifier for each class pair.

• Training:

- For each pair (c_m, c_n) of classes, train a classifier cl_{mn}
 - replace a training instance (x,y) with

$$(x, 1)$$
 if $y = c_m$
 $(x, -1)$ if $y = c_n$
otherwise ignore the instance

An example: training

- x1 c1 ...
- x2 c2 ...
- x3 c1 ...
- x4 c3 ...

for c1-vs-c2:

x1 1 ...

x2 -1 ...

x3 1 ...

for c2-vs-c3:

x2 1 ...

x4 -1 ...

for c1-vs-c3:

x1 1...

x3 1 ...

x4 -1 ...

All-pairs (cont)

- Testing time: given a new example x
 - Run each of the C_k^2 classifiers on x
 - Max-win strategy: Choose the class c_m that wins the most pairwise comparisons:
 - Other coupling models have been proposed:
 e.g., (Hastie and Tibshirani, 1998)

An example: testing

- x1 c1 ...
- x2 c2 ...
- x3 c1 ...
- x4 c3 ...
- → three classifiers

Test data:

x ?? f1 v1 ...

for c1-vs-c2:

x ?? 1 0.7 -1 0.3

for c2-vs-c3

x ?? 1 0.2 -1 0.8

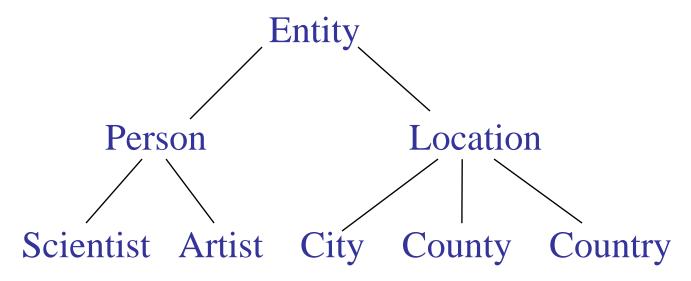
for c1-vs-c3

x ?? 1 0.6 -1 0.4

 \Rightarrow what's the system prediction for x?

Hierarchical Categorization

- Pick the category with max probability
- Create many OVA/AVA classifiers
- Use a hierarchical approach (wherever hierarchy available)



Error-correcting output codes (ECOC)

Proposed by (Dietterich and Bakiri, 1995)

• Idea:

- Each class is assigned a unique binary string of length n.
- Train n classifiers, one for each bit.
- Testing time: run n classifiers on x to get a n-bit string s,
 and choose the class which is closest to s.

An example: Digit Recognition

	Code Word									
Class	vl	hl	dl	cc	ol	or				
0	0	0	0	1	0	0				
1	1	0	0	0	0	0				
2	0	1	1	0	1	0				
3	0	0	0	0	1	0				
4	1	1	0	0	0	0				
5	1	1	0	0	1	0				
6	0	0	1	1	0	1				
7	0	0	1	0	0	0				
8	0	0	0	1	0	0				
9	0	0	1	1	0	0				

Meaning of each column

Column position	Abbreviation	Meaning
1	vl	contains vertical line
2	hl	contains horizontal line
3	dl	contains diagonal line
4	cc	contains closed curve
5	ol	contains curve open to left
6	or	contains curve open to right

Another example: 15-bit code for a 10-class problem

	Code Word														
Class	f_0	f_1	f_2	f_3	f_4	f_5	f_6	f ₇	f_8	f_9	f_{10}	f_{11}	f_{12}	f_{13}	f_{14}
0	1	1	0	0	0	0	1	0	1	0	0	1	1	0	1
1	0	0	1	1	1	1	0	1	0	1	1	0	0	1	0
2	1	0	0	1	0	0	0	1	1	1	1	0	1	0	1
3	0	0	1	1	0	1	1	1	0	0	0	0	1	0	1
4	1	1	1	0	1	0	1	1	0	0	1	0	0	0	1
5	0	1	0	0	1	1	0	1	1	1	0	0	0	0	1
6	1	0	1	1	1	0	0	0	0	1	0	1	0	0	1
7	0	0	0	1	1	1	1	0	1	0	1	1	0	0	1
8	1	1	0	1	0	1	1	0	0	1	0	0	0	1	1
9	0	1	1	1	0	0	0	0	1	0	1	0	0	1	1

Hamming distance

• Definition: the **Hamming distance** between two strings of equal length is the number of positions for which the corresponding symbols are different.

• Ex:

- 10111 and 10010
- 2143 and 2233
- Toned and roses

How to choose a good errorcorrecting code?

• Choose the one with large minimum Hamming distance between any pair of code words.

• If the min Hamming distance is d, then the code can correct at least (d-1)/2 single bit errors.

Two properties of a good ECOC

 Row separations: Each codeword should be well-separated in Hamming distance from each of the other codewords

 Column separation: Each bit-position function f_i should be uncorrelated with each of the other f_j.

Summary

- Different methods:
 - Direct multiclass, if possible
 - One-vs-all (a.k.a. one-per-class): k-classifiers
 - All-pairs: C_k² classifiers
 - ECOC: n classifiers (n is the num of columns)
 - Hierarchical classification (logC classifiers)

• Some studies report that All-pairs and ECOC work better than one-vs-all.