Factors Affecting Robustness in Financial Systems

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Abstract

Financial Systems have evolved to be increasingly complex and interconnected. Emergent network properties such as resilience and robustness need to be viewed as a result of a large number of individuals interacting with each other. Most of the financial risk assessment models do not take into account the structural properties of financial networks. To have an enhanced understanding of the changing nature of the financial network, we seek to explore the factors affecting the robustness of financial banking networks using various network models. We found that although financial networks tend to have small-world properties, they tend to be much more vulnerable to externalities and macroeconomic shocks. We found that as financial systems are coupled and modular in nature, they tend to lack robustness. Further, we try to suggest ways of structuring the present financial network system.

Index Terms

Robustness, Resilience, Macroeconomic Shocks, Network Science, Economy, Financial Networks, Modularity

I. Introduction

The Financial Crisis of 2008 prompted researchers to consider network interconnectivity to diversity firms' risk exposures and vulnerability [1]. Numerous studies have showcased the relationship between the stability of systems and network interconnectivity. The crisis of 2008 witnessed the failure of big institutions like AIG corporation and Lehman Brothers. Reports by the Financial Crisis Inquiry Commission (FCIC) have detailed transactions made by the institutions that seemed to have caused the breakdown, leading to the crisis.

According to the FCIC, the fallen standards of mortgage lending and securitisation added fire to the flame for the financial contagion [2]. In this paper, we study the existing complex and opaque nature of the models of financial systems that pose the challenge in analysing systemic resilience [3]. The efforts of the Central Bank - fire sale externalities and increasing liquidity - strengthens the statements above. Central banks could utilize the structural property of the network to minimize the impact of macroeconomic shocks.

Modern day banking systems have brought in deeper inter-banking relationships, enabling better monitoring of the financial system by the Central Bank. However, the Central Bank's purview is constrained to leave out externalities affecting organisations outside the core banking system, which leaves out important players in the economy. Learning about inter-bank linkages from the balance sheets, could potentially help in enhancing the resilience of the financial banking systems [4]. The argument is followed up with a study of correlations among various network structural coefficients.

The following paper is highly influenced by the literature on financial system resilience to external shocks. Here, we aim to develop a framework to simulate systemic risks with a macroeconomic shock affecting the financial nodes in the model. The network model attempts to establish the underlying importance of network structures in understanding and simulating complex economic models. In applying the concepts of complexity to problems in economics, we view the economy and its components not in static but in dynamic equilibrium. Using computational and mathematical analysis can help explore the reformation of economic structures according to the adaptive behaviour of the 'agents' of the economy [5]. In general, complex economic models avoid assumptions of the representative agent method – which attributes outcomes as a simplified summation of rational actions of agents in collective systems. In contrast, these models showcase the ability of simple interactions at micro-levels to cause non-intuitive results at the system's macro-levels [6].

The applications of network science in systems biology shows resilience in organismic subjects and ecological systems [7], [8], [9], [10], [11]. This paper takes great inspiration from these fields to apply these concepts, majorly discussing the resilience of the structure with reference to market structures. We further elaborate on modularity, clustering and feedback loops.

Neeraj et al. have shown the essential role played by modularity of network structure in increasing the robustness of biological systems [9]. Additionally, Heer et al. have shown that clustering could be a good indicator for network

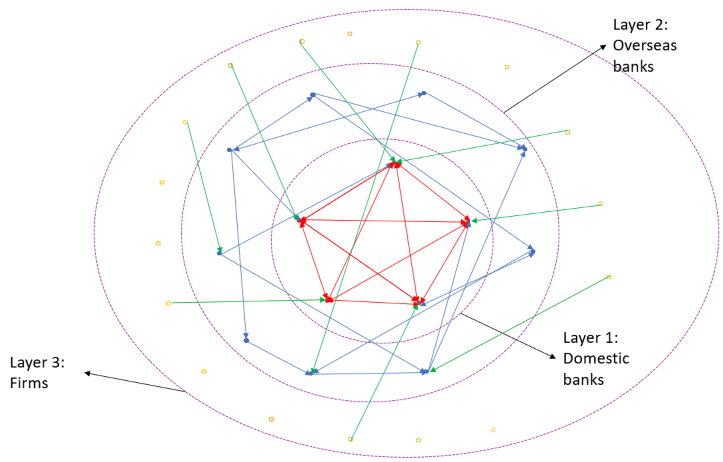


Fig. 1: An outline of the Network Model
(Bidirectional edges indicate that both forward and backward edges are present between the nodes)

robustness [8]. Further, it has also been shown by Kwon et al. that many evolved biological networks are more resilient and contain more coupled feedback loops than the networks obtained using a preferential attachment model [10]. It has also been noted by Hari et al. that higher number of feedback loops adds more plasticity to the network and allows it to be more resilient [7]. In 1956, Wroe Alderson addressed the nature of markets from being to becoming through human effort. In the following decades, the theory had developed to become a subject of a social phenomenon. The dynamic approach to studying the market's subject included product life cycles, the evolutionary process of markets model inspired by population ecology, and descriptions of specific sub processes influencing market dynamics, such as the socio-cognitive model of product market dynamics. In recent studies, the study of market dynamics has been in the form of developing and understanding the characteristics of the market. In a paper about a new perspective on market dynamics, the concept of market plasticity emerged as a potential new way of looking at market dynamics [12].

II. Methods

A. Network Structure

Our model of financial systems can be represented as a graph as shown in Fig 1 where nodes represent domestic banks, overseas banks and firms. The links (directed edges) between nodes represent equity or credit. An incoming edge would represent an asset, while outgoing edge represents a liability. Let's take a look at each of the institutions:

1) Domestic banks (red nodes) : Each domestic bank interacts (in form of loans) with all other domestic banks ie., the network of domestic banks is complete. (red edges)
2) Overseas (International) banks (blue nodes) : These banks form a non-complete network. The network exhibits small world properties (ie. high clustering and short path length). These networks are generated using the watts-strogatz method [13]. The International banks also interact with domestic banks.
3) Firms (yellow nodes) : Firms are assumed not to interact with each other for easy management (ie. the firms do not hold equity in other firms). Hence there are no edges between firms. The banks can own equity or offer loans to the firms.

The domestic banks, overseas banks and firms are considered to interact with each other with some probability. The properties and dynamics of the network are discussed in detail in the Appendix A at the end of this paper.

The comparative model introduced as a control in the paper is the Erdos-Renyi network [14]. The evolution of a random network is a dynamical process in which a set of disconnected nodes are connected by adding links between randomly selected pairs of nodes that leads to the emergence of a giant component with striking consequences on the topology of the network. In the Erdos - Renyi model, a graph is constructed by connecting the labeled nodes in the graph by adding an edge between them with a probability p, independent of other edges. This parameter p is generally taken to be proportional to the edge weights. This implies that as the values of the weights increase, the likelihood of graphs with more edges increases and vice-versa. The Erdos-Renyi model fulfils the requirement that the parameters that we would consider are close to those of an ideally generated network system [14].

We choose Erdos-Renyi network as our control to test our hypothesis if the structural properties of the network influence the robustness of the banking network. The Erdos-Renyi model is created such that the number of domestic banks, international banks and firms remains the same, along with the number of edges. Therefore, we derived a formula for the parameter p, which is given below.

$$p = \frac{(N_d \cdot (N_d - 1)/2 + k \cdot N_i/2 + 0.225 \cdot N_i \cdot N_d)}{((N_i + N_d) \cdot (N_i + N_d - 1)/2)} = \frac{{}^{N_d}C_2 + k \cdot N_i/2 + 0.225(N_i)(N_d)}{{}^{N_i+N_d}C_2} \tag{1}$$

B. Estimation of number of communities and Modularity

To estimate the modularity, we seek to divide the network in communities and then estimate the modularity. For this purpose, we used the greedy modularity maximization algorithm as given in Clauset et al.[15]. This approach does not consider the edge weights and the directionality of the edge. It begins with each node as an individual community and joins the communities until a point is reached where joining two communities does not increase the modularity.

Modularity is defined as [15],

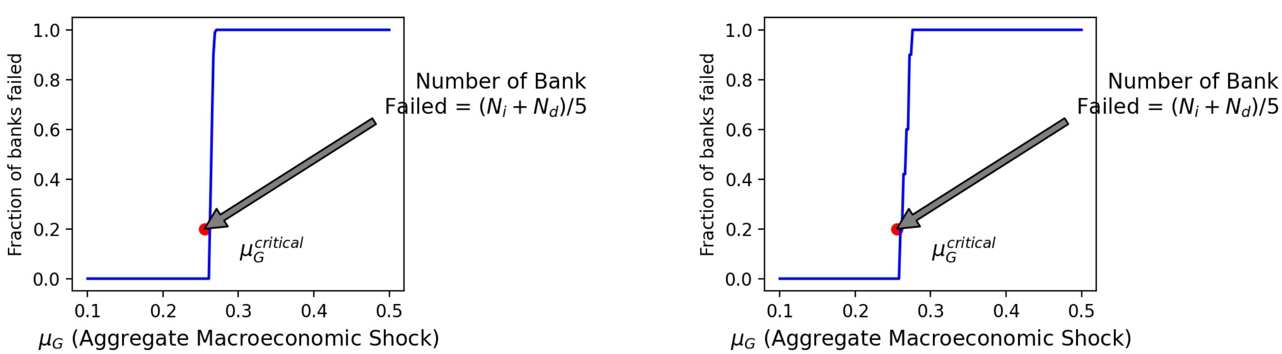
$$Q = \frac{1}{2m} \sum_{ij} (A_{ij} - \frac{k_i k_j}{2m}) \delta(c_i, c_j) \tag{2}$$

where, k_i is the degree of i, m is the number of edges, $\delta(c_i, c_j)$ is 1 if both nodes i and j are in the same community and 0 otherwise, A is the adjacency matrix.

III. Results and Observations

A. The μ_G^c parameter and robustness

The calibrated model in the work by Anand et al. represents the aggregate macroeconomic shocks in an economy that characterizes to have an adverse effect on the firms in it, is labeled as $\mu^{critical}$. The rise in the value represents the impacts of the firm defaults [3]. As seen in Fig. 2, there is a huge change in the fraction of failed banks at some critical value of μ_G, called as μ_G^c, at which there is a catastrophic failure of the whole banking system. Similar behavior is also seen in the Erdos Renyi Graph. However, the switch is a little less steep than it is observed in the Banking Graph. Therefore, we can introduce a value μ_G^c, which can be labeled as a threshold of aggregate macroeconomic shock - determining the current state of the network G, and conveying how far the network is from complete bankruptcy. A higher μ_G^c conveys higher robustness of a network for absorbing greater economic shocks. It is an inherent property of the network and its parameters.



(a) For a Banking Graph

(b) For an ER Graph

Fig. 2: Variation of μ_G with fraction of failed banks for graphs with 17 domestic banks (N_d), 240 overseas banks (N_i) and 40000 firms (N_f) as described in [3]

B. Some network parameters

Fig. 3 proposes the derived relations of different network parameters calculated on the networks. These include average shortest path length, average clustering coefficient, number of loops, the average length of loops, the small world measure (ω), modularity, and the number of communities. The fig. 3 shows the variation in the network parameters. Since, the number of loops in a large network of size ($N_d = 17, N_i = 240, N_f = 40000$), which is used in work of Anand et al, is huge, we calculated feedback loops only for the smaller networks. These small networks have ($N_d = 2-5$, $N_i = 10-13$, $N_f = 2000-4500$) nodes.

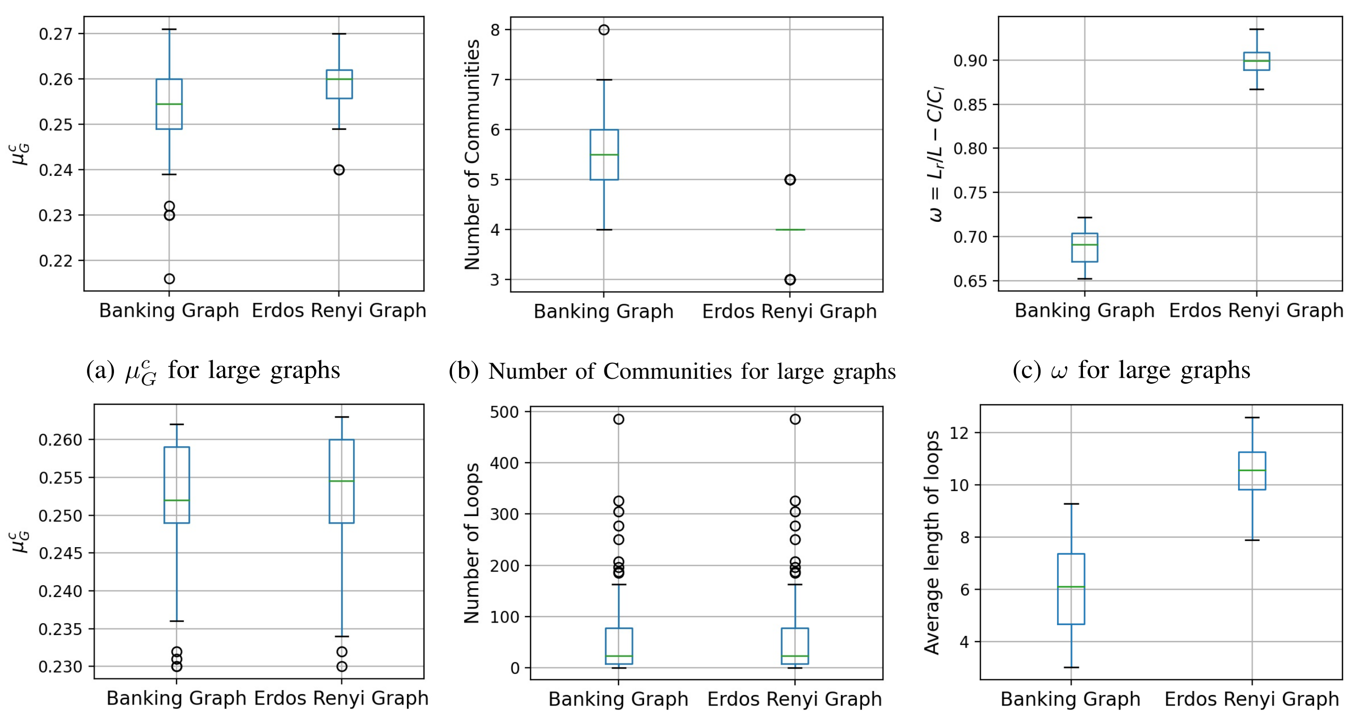
It is seen that the banking graph seems to have a lower mean μ_G^c compared to the ER Graph. The banking graph also has a higher spread towards the lower values of μ_G^c, suggesting that the robustness of the banking graph is marginally lower than the Erdos-Renyi graph. Additionally, the number of communities found in the Banking graph is higher than the number of communities found in Erdos Renyi graphs, suggesting higher clustering and modularity, which is verified in Appendix B.

Small worldness (ω) is defined as [16],

$$\omega = \frac{L_r}{L} - \frac{C}{C_i} \tag{3}$$

If $\omega = 1$, then the network is close to a random graph, if $\omega = -1$, then the network is close to a perfect lattice. An ideal small world network would have ω close to 0. It is a robust measure to quantify the 'small-worldness' of networks. It is seen that the small-worldness of the large banking graph is centered around 0.66 while that for ER graph is centered near 0.90, suggesting that the banking graph has stronger small-worldness compared to ER graph (from 3c).

For the smaller networks, it is seen that there is hardly any difference between the μ_G^c of Banking graph and the ER graph (from 3d). However, the mean of the Banking graph is less than the mean of the ER graph. The number of loops in a Banking graph is almost equal to the number of loops in the ER graph. However, it is seen that ER graphs tend to have longer loops compared to Banking graphs.



(a) μ_G^c for large graphs (b) Number of Communities for large graphs (c) ω for large graphs

(d) μ_G^c for small graphs (e) Number of loops (Cycles) for small graphs (f) Average cycle length for small graphs

Fig. 3: Box plots for comparison between parameters of Banking Graph model and ER Graph model

C. Relation between μ_G^c and the network parameters

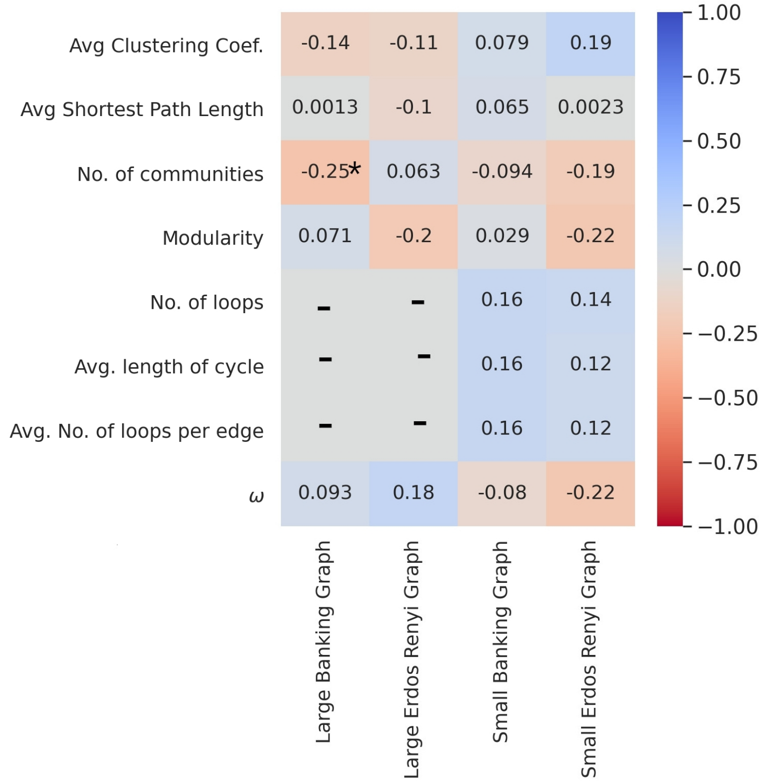


Fig. 4: The heat map of correlations of μ_G^c with different network parameters for different network models $\star$ represents a significant correlation between μ_G^c and the parameter (p-value less than 5%)

Using the network model, we have compared the different network parameters. The correlations between μ_G^c and different network parameters and different network models are shown in the Fig. 4. We found the structural properties of the network to have a definite empirical relation, but we could not establish a significant correlation in most of the cases. However, it is seen that number of communities and μ_G^c does have a statistically significant negative correlation ($p < 0.05$). Fig. 5 shows the regression line and the scatter plot for the number of communities in the Large banking graphs and μ_G^c.

Therefore, we observe that although among the different types of networks, modularity, small worldness (ω), number of loops, the average length of loops, clustering coefficient, average shortest path length, number of communities might individually not be good indicators of systemic robustness μ_G^c, ER graph which tends to have higher robustness and smaller spread are less modular and do not exhibit small-world properties. This result is also noted by Peng et al. in their study [17].

IV. Discussion and Conclusion

A. Remarks

It is established from the Fig. 3, that the banking graph tends to exhibit small-world properties to a greater extent compared to the ER graph. However, it is also seen that the Banking graph tends to be less robust compared to the ER graph. Such a result is also observed by Peng et al. in 2016 [17]. The authors observed a conflicting relation between robustness and small-world effect. Netotea et al. also observed that while optimizing the yeast genetic network for higher robustness, the small-world property was lost [18]. It has also been noted that modularity results in higher robustness [19], [20], [9]. However, most of the modular networks are prone to cascading failure [21]. This might result in a seemingly counter-intuitive paradox.

This is due to different ways of defining robustness. While modular networks may be robust to targetted node attacks, they would not be as robust to systemic cascading failures. Modularity improves the robustness of the

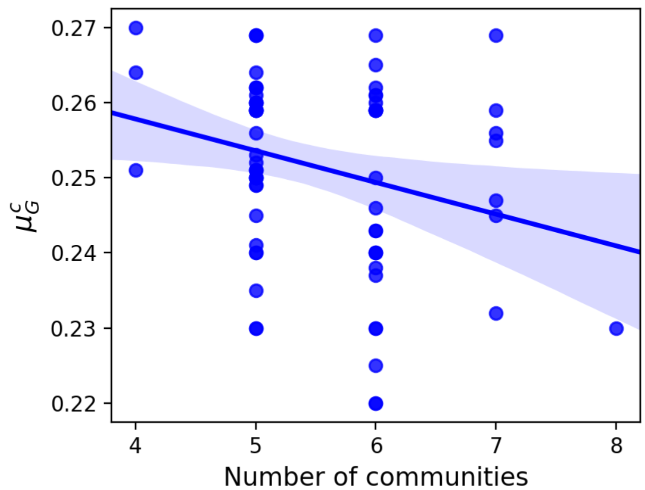


Fig. 5: Plot between μ_G^c and the number of communities for a large banking graph

system only when the effects of the modules are de-coupled [11]. In case of cascading failures, where the failure of one node affects the performance of the other nodes adversely, modularity could result in a far more vulnerable network, as shown by Bagrow et al. [22].

Such a cascading event must be taken into account while evaluating systematic risks of the banking network by the central banks and the governments. Banks will default if the firms default in case of high μ_G^c. If the default rate is high, the banks will opt for a firesale of assets, which would adversely affect the other banks too. Therefore, the banking system is highly coupled. The small-world property of the banking system would have an adverse effect. Risk assessment is crucial for the stable functioning of the banking ecosystem. Central banks and governments could factor-in the role of network structure for risk evaluation. As we have successfully shown that small-world properties adversely affect the robustness of the banking ecosystem, central banks could consciously make efforts to break short-range connections and encourage more random connections, which would reduce the modularity and small worldness of the financial networks. A robust financial system resonates with the state of the economy, serving as one of the primitive drivers in the world. It directly affects the foundational problems posed by economics.

B. Future works

Further assessment of our results could also be done by trying different models such as preferential attachment, power-law networks, etc. Further, our results could be verified from the real-world data.

V. Code Availability

The codes are available at https://github.com/saketh3121/robustness-of-financial-systems.

VI. Acknowledgements

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Appendix A[1]

In this section, we explore the dynamics of our model. The domestic banks and overseas banks can be considered as the set of banks in general because they only differ in their individual networks but their features behave in the same manner except for the extent of exposure. So, from this point we can consider two types of entities in the network - the banks and the firms.

[1]This section is a derivative of [3] with some notations changed and more assumptions taken. It is presented in a relatively simpler way for the reader to understand

The Network Links

We consider that all the bank-bank links from bank i to bank j present in the network will carry a weight equivalent to the loan held by bank j that of bank i. We also assume that the firms do not hold any assets or equities against any entity in the network (In other words, the in-degree of any firm is 0). Further there can be two types of firm-bank links, whose weights denote loans and equities of the firms held by the banks. The following table represents the network links more clearly.

Types of links → : link destination ↓ : link source	Bank	Firm
Bank	Edge weight denotes the loan	No edges
Firm	Edge weight can denote loan or equity	No edges

For every edge from entity i to entity j, let

1) L_{ij} denote the loan bank j holds that of entity i,
2) E_{ij} denote the equity bank j holds that of firm i and
3) W_{ij} denote the total weight of edge from entity i to entity j.

Bank Balance Sheets

Assuming no external liabilities and government bonds, if A_j denotes the total assets and B_j denote the total liabilities of bank i, we have
$A_i = \sum_{all\ incoming\ edges\ (j,i)} W_{ji}$ amd
$B_i = \sum_{all\ outgoing\ edges\ (i,j)} W_{ij} + \pi A_i$
where $K_i = \pi A_i$ is initial capital buffer with fraction $\pi \in (0, 1)$.

The Dynamics[2]

We perform simulations with performing updates to some parameters at each timestamp. Let $\mu_f(t)$ denote whether firm f is solvent ($\mu_f(t)$ = 0) or default ($\mu_f(t)$ = 1) at time t and $\nu_b(t)$ denote whether bank b is solvent ($\nu_b(t)$ = 0) or default ($\nu_b(t)$ = 1) at time t. At time $t = 0$, we assume all firms are solvent and start our simulations.

For a bank b, there can be losses from a firm f becoming default or another bank b' becoming default. If $DL_b(t)$ is the direct loss of the bank at time t, we have

$$DL_b(t) = \sum_{edges(f,b)\ with\ \mu_f(t)=1} ((1-\beta)L_{fb} + E_{fb}) + \sum_{edges(b',b)\ with\ \nu_{b'}(t)=1} L_{b'b}$$

where β is the firm loan recovery rate

Let $\phi_b(t)$ denote whether a bank b performs fire sales of its equities (= 1) or abstains (= 0). Then if α is the fire sale fraction criteria $\in (0, 1)$,

$$\phi_b(t) = \theta(DL_b(t) - \alpha K_j)$$

where $\theta(x) = 1$ if $x \geq 0$ and 0 otherwise.

If $Q(t)$ be the total equity of the banks at time t, (sum of equities of fire sale banks) performing fire sale and Q be the total equity of the banks, we have the indirect loss of bank b due to mark-to-market loss on equities given by

$$IDL_b(t) = \sum_{edges(f,b)\ with\ \mu_f(t)=0} E_{fb}\left(1 - \prod_{t'=0}^{t}\left(1 - \frac{\lambda Q(t')}{Q - Q(t')}\right)\right)$$

[2]For more detailed equations and explanations, refer [3]

where λ is the fire sale impact parameter.

Now, update equations for bank solvency and firm solvency parameter are as follows:

$$\nu_b(t+1) = \theta(DL_b(t) + IDL_b(t) - K_b)$$

$$\mu_f(t+1) = \theta(R_f + \mu_G + \psi \left(\sum_{edges(f,b)} \phi_b(t) \right) - 0.5)$$

where $R_f \in (0,1)$ is drawn from a normal (gaussian) distribution of firm defaulting probability, μ_G is the firm shock parameter and ψ being the increase in the firm defaulting probability for each fire sale performed by adjacent bank. Here we considered 0.5 as the threshold for which a firm defaults.

Model Calibration[3]

We have performed simulations on several graphs generated as described in the previous sections and in each simulation, we have varied the μ_G value uniformly between 0 and 1 increasing at steps of 0.001 and considered 1000 timestamps to update the parameters and have obtained the μ_G^c, the critical value of μ_G where the defaulting of some bank starts. We then plotted the μ_G^c of different graphs with their properties discussed in section III.

We have also split the firms into two categories - Speculative grade firms (with probability 0.3) and investment grade firms (with probability 0.7). The distribution of probability of default (PD) of firms is different for different grades. We fixed the values of some parameters such as α to 0.5, λ to 0.67 and β to 0.35. We assumed π to be chosen uniformly at random from $[0.04, 0.24]$ rounded up to two decimal places.

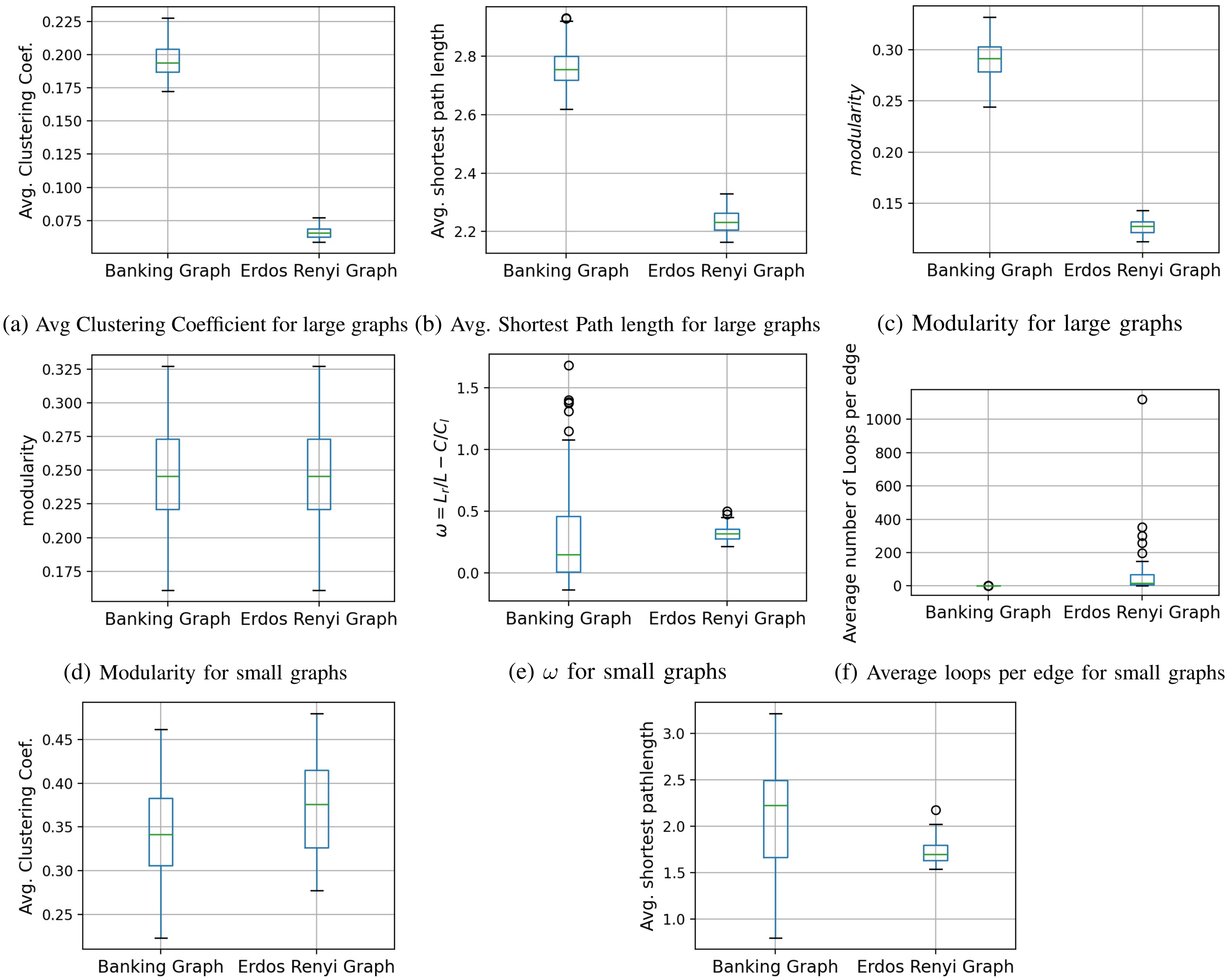
The equity from the firms is assumed to follow a poisson distribution with mean $2 \cdot 10^5$, while loans are assumed to follow a poisson distribution with mean $1.6 \cdot 10^5$.

ψ is taken to be equal to 0.4 divided by the number of firm-bank loan edges. If N_f is the total number of firms, then the following are chosen for the firm default probability distribution.

- For investment grade firms, $PD \sim \mathcal{N}(8.65 \cdot 10^{-3}, 2.1625 \cdot 10^{-3})$
- For speculative grade firms, $PD \sim \mathcal{N}(2 \cdot 10^{-1}, 2 \cdot 10^{-2})$ where $\mathcal{N}(\mu, \sigma^2)$ is the gaussian distribution.

[3]Most of the values used in computing μ_G^c are inspired from [3], a few of them have been modified accordingly

APPENDIX B



(a) Avg Clustering Coefficient for large graphs (b) Avg. Shortest Path length for large graphs (c) Modularity for large graphs

(d) Modularity for small graphs (e) ω for small graphs (f) Average loops per edge for small graphs

(g) Average clustering coefficient for small graphs (h) Average shortest path length for small graphs

Fig. 6: Additional Box plots for comparison between parameters of Banking Graph model and ER Graph model

REFERENCES

[1] P. Glasserman and H. P. Young, "Contagion in financial networks," *Journal of Economic Literature*, vol. 54, no. 3, pp. 779–831, 2016. [Online]. Available: http://www.jstor.org/stable/43932476

[2] F. C. I. Commission *et al.*, *The financial crisis inquiry report: The final report of the National Commission on the causes of the financial and economic crisis in the United States including dissenting views*. Cosimo, Inc., 2011.

[3] K. Anand, P. Gai, S. Kapadia, S. Brennan, and M. Willison, "A network model of financial system resilience," *Journal of Economic Behavior Organization*, vol. 85, pp. 219–235, 2013, financial Sector Performance and Risk. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0167268112000868

[4] F. Caccioli, P. Barucca, and T. Kobayashi, "Network models of financial systemic risk: a review," *Journal of Computational Social Science*, vol. 1, no. 1, pp. 81–114, Jan 2018. [Online]. Available: https://doi.org/10.1007/s42001-017-0008-3

[5] J. Foster, "From simplistic to complex systems in economics," *Cambridge Journal of Economics*, vol. 29, no. 6, pp. 873–892, 11 2005. [Online]. Available: https://doi.org/10.1093/cje/bei083

[6] J. B. Rosser, "On the complexities of complex economic dynamics," *Journal of Economic Perspectives*, vol. 13, no. 4, pp. 169–192, December 1999. [Online]. Available: https://www.aeaweb.org/articles?id=10.1257/jep.13.4.169

[7] K. Hari, B. Sabuwala, B. V. Subramani, C. A. M. La Porta, S. Zapperi, F. Font-Clos, and M. K. Jolly, "Identifying inhibitors of epithelial–mesenchymal plasticity using a network topology-based approach," *npj Systems Biology and Applications*, vol. 6, no. 1, p. 15, May 2020. [Online]. Available: https://doi.org/10.1038/s41540-020-0132-1

[8] H. Heer, L. Streib, R. B. Schäfer, and S. Ruzika, "Maximising the clustering coefficient of networks and the effects on habitat network robustness," *PLOS ONE*, vol. 15, no. 10, pp. 1–16, 10 2020. [Online]. Available: https://doi.org/10.1371/journal.pone.0240940

[9] N. Pradhan, S. Dasgupta, and S. Sinha, "Modular organization enhances the robustness of attractor network dynamics," *EPL (Europhysics Letters)*, vol. 94, no. 3, p. 38004, apr 2011. [Online]. Available: https://doi.org/10.1209/0295-5075/94/38004

[10] Y.-K. Kwon and K.-H. Cho, "Analysis of feedback loops and robustness in network evolution based on boolean models," *BMC Bioinformatics*, vol. 8, no. 1, p. 430, Nov 2007. [Online]. Available: https://doi.org/10.1186/1471-2105-8-430

[11] H. Kitano, "Biological robustness," *Nature Reviews Genetics*, vol. 5, no. 11, pp. 826–837, Nov 2004. [Online]. Available: https://doi.org/10.1038/nrg1471

[12] S. Nenonen, H. Kjellberg, J. Pels, L. Cheung, S. Lindeman, C. Mele, L. Sajtos, and K. Storbacka, "A new perspective on market dynamics: Market plasticity and the stability–fluidity dialectics," *Marketing Theory*, vol. 14, no. 3, pp. 269–289, 2014.

[13] D. J. Watts and S. H. Strogatz, "Collective dynamics of 'small-world' networks," *Nature*, vol. 393, no. 6684, pp. 440–442, Jun 1998. [Online]. Available: https://doi.org/10.1038/30918

[14] E. N. Gilbert, "Random graphs," *Ann. Math. Statist.*, vol. 30, no. 4, pp. 1141–1144, 12 1959. [Online]. Available: https://doi.org/10.1214/aoms/1177706098

[15] A. Clauset, M. E. J. Newman, and C. Moore, "Finding community structure in very large networks," *Physical Review E*, vol. 70, no. 6, Dec 2004. [Online]. Available: http://dx.doi.org/10.1103/PhysRevE.70.066111

[16] Q. K. Telesford, K. E. Joyce, S. Hayasaka, J. H. Burdette, and P. J. Laurienti, "The ubiquity of small-world networks," *Brain Connectivity*, vol. 1, no. 5, pp. 367–375, 2011, pMID: 22432451. [Online]. Available: https://doi.org/10.1089/brain.2011.0038

[17] G.-S. Peng, S.-Y. Tan, J. Wu, and P. Holme, "Trade-offs between robustness and small-world effect in complex networks," *Scientific Reports*, vol. 6, no. 1, p. 37317, Nov 2016. [Online]. Available: https://doi.org/10.1038/srep37317

[18] S. Netotea and S. Pongor, "Evolution of robust and efficient system topologies," *Cellular Immunology*, vol. 244, no. 2, pp. 80–83, 2006, international Conference on Immunogenomics and Immunomics, Budapest, Hungary, October 8-12, 2006. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0008874907000548

[19] T.-D. Tran and Y.-K. Kwon, "The relationship between modularity and robustness in signalling networks," *Journal of The Royal Society Interface*, vol. 10, no. 88, p. 20130771, 2013. [Online]. Available: https://royalsocietypublishing.org/doi/abs/10.1098/rsif.2013.0771

[20] M. Ramos-Robles, E. Andresen, and C. Díaz-Castelazo, "Modularity and robustness of a plant-frugivore interaction network in a disturbed tropical forest," *Écoscience*, vol. 25, no. 3, pp. 209–222, 2018. [Online]. Available: https://doi.org/10.1080/11956860.2018.1446284

[21] M. Babaei, H. Ghassemieh, and M. Jalili, "Cascading failure tolerance of modular small-world networks," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 58, no. 8, pp. 527–531, 2011.

[22] J. P. BAGROW, S. LEHMANN, and Y.-Y. AHN, "Robustness and modular structure in networks," *Network Science*, vol. 3, no. 4, p. 509–525, 2015.