Question no. 41

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1. Problem
2. Solution
    - equations of motion
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Question

Given the combined translational and rotational system shown in Figure, find the transfer function, *G(s)=X(s)/T(s).*

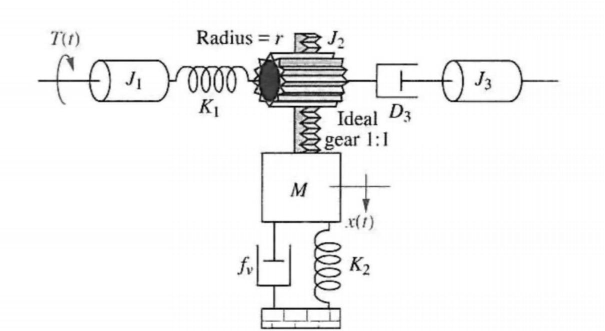

Figure: Rotational System

equations of motion

The equations of motion for the given figure are,
for the left part of the system,

$$(J_1 s^2 + K_1)\theta_1(s) - K_1\theta_2(s) = T(s)$$

for middle one (J_2),

$$-K_1\theta_1(s) + (J_2 s^2 + D_3 s + K_1)\theta_2(s) + F(s)r - D_3 s\theta_3(s) = 0$$

and for right part of the system,

$$-D_3 s\theta_2(s) + (J_3 s^2 + D_3 s)\theta_3(s) = 0$$

Here F(s) in second equation of motion is the opposing force on J_2 due to translation and r is the radius of J_2.

Solving equations

Here F(s) is ,

$$F(s) = (Ms^2 + f_s s + K_2)X(s) = (Ms^2 + f_s s + K_2)r\theta(s)$$

Substituting F(s) in second equation of motion we get,

$$-K_1\theta_1(s) + (J_2 s^2 + D_3 s + K_1 + Ms^2 r^2 + f_v s r^2 + K_2 r^2)\theta_2(s) - D_3 s\theta_3(s) = 0$$

Now here we get three equations with three variables i.e. θ_1,θ_2 and θ_3.By using Cramer's Rule we can find the values for θ_1,θ_2 and θ_3.

The value of θ_i(s) is

$$\theta_i(s) = \frac{\Delta_{\theta_i(s)}}{\Delta} \quad \text{where i=1,2,3}$$

Here Δ is formed by the coefficients of the three equations of motion.$\Delta_{\theta_i(s)}$ is similar to Δ but coefficients of θ_i(s) will be replaces by constants of the three equations of motion.

In order to find θ_2(s) first we need to find $\Delta_{\theta_2(s)}$ and Δ.

$$\Delta_{\theta_2(s)} = \begin{vmatrix} (J_1 s^2 + K_1) & T(s) & 0 \\ -K_1 & 0 & -D_3 s \\ 0 & 0 & (J_3 s^2 + D_3 s) \end{vmatrix}$$

$$\Delta_{\theta_2(s)} = -T(s)\left([-K_1 \times (J_3 s^2 + D_3 s)] - [0 \times (-D_3 s)\right)]$$

$$\Delta_{\theta_2(s)} = K_1 (J_3 s^2 + D_3 s) T(s)$$

$$\therefore \theta_2(s) = \frac{K_1 (J_3 s^2 + D_3 s) T(s)}{\Delta}$$

We know that transfer function is G(s)=$\frac{X(s)}{T(s)}$

Where X(s)=rθ_2(s)

Transfer function

Transfer function

$$G(s) = \frac{X(s)}{T(s)}$$

$$G(s) = \frac{r\theta_2(s)}{T(s)}$$

$$G(s) = \frac{K_1(J_3s^2 + D_3s)rT(s)}{\Delta T(s)} \implies G(s) = \frac{K_1(J_3s^2 + D_3s)r}{\Delta}$$

$\therefore$ The transfer function for the given rotational system is

$$\boxed{G(s) = \frac{K_1(J_3s^2 + D_3s)r}{\Delta}}$$

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