Rank Modulation in Flash Memories

Course Project, EE 605

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Error Correcting Codes, Autumn 2021

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Introduction

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Designing codes where k bits are stored using a block of n cells with q levels each, with the additional requirement of addressing the challenges above.

One such scheme is Rank Modulation.

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- The only allowed programming operation is a 'push-to-the-top' operation, raising the charge level of one of the cells to above the current highest one
- Additionally, 'block-deflation', decrease of all the charge levels in a block by a constant amount smaller than the lowest, maintaining relative values is allowed

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The traversal of states by the Gray Code is mapped to the increase of cell level in the multi-level flash cell.

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Notation

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- Let $[n]$ denote the set $\{1, 2, \ldots n\}$. An ordered set of n memory cells labelled $1, 2 \ldots n$ which distinct charge levels induce permutations of $[n]$ by representing the cell names as vectors $[a_1, a_2, \ldots a_n]$ in descending order of charge levels.
- State space would be all possible permutations on $[n]$.

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  $t_i([a_1, a_2, \ldots, \boldsymbol{a_i}, a_{i+1}, \ldots a_n]) = [\boldsymbol{a_i}, a_1, a_2, \ldots, a_{i-1}, a_{i+1}, \ldots a_n]$

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- Let T be the set of all push-to-the-top transitions. That is, $s_{i+1} = t(s_i)$ for some $t \in T, s_i, s_{i+1} \in S$.

Definitions

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- If the code is such that $s_1 = t(s_m)$ for some $t \in T$, the code is **cyclic**.
- If the code spans all of S, it is considered **complete**.

Example

Example 2

A 3-RMGC is given below:

$$\begin{array}{cccccc} 1 & 2 & 3 & 1 & 3 & 2 \\ 2 & 1 & 2 & 3 & 1 & 3 \\ 3 & 3 & 1 & 2 & 2 & 1 \end{array}$$

Here, the permutations are the different columns.

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Here, the permutations are the different columns.This code is defined by the transitions $t_2, t_3, t_3, t_2, t_3, t_3$. Note that the last t_3 transition gives back $1, 2, 3$, making the code cyclic.

Construction of n-RMGC

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- The $t_{i_{(n-1)!}} = t_2$ condition is not very restrictive, since it only necessitates a t_2 transition somewhere in the transition sequence, and we can always rotate the sequence to make t_2 the last one.
- We further assume t_2 appears at least twice.

Construction

- Without loss of generality, assume the first permutation is $[1, 2, \ldots, n]$.
- Using $t_{i_1}, t_{i_2}, \ldots, t_{i_{(n-1)!-1}}$ from C_{n-1} (and thus effectively keeping the last element fixed), we get the first *block* of $(n-1)!$ elements of our construction.

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- Since $t_{i_{(n-1)!}} = t_2$, the last element of the first block is $[2, 1, 3, 4, \ldots, n-1, n]$
- For the first element of the second block, we first use t_n, and use $t_{i_1}, t_{i_2}, \ldots, t_{i_{(n-1)!-1}}$ again.

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- This time, $(n-1)$ would be the fixed last element while all the other elements create $(n-1)!$ more permutations.

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- This time, $(n-1)$ would be the fixed last element while all the other elements create $(n-1)!$ more permutations.
- Repeat this process $(n-1)$ times to get $(n-1)!$ permutations each time.

Construction

Lemma 3

Second element in first permutation to every block is 2. It follows that first element of last permutation in every block is also 2.

Proof.

We use the transitions $t_{i_1}, t_{i_2}, \ldots, t_{i_{(n-1)!-1}}$ in this particular order. If we use $t_{i_{(n-1)!}} = t_2$, we get the first permutation of the block back, which has 2 in the second position. Therefore, after $t_{i_{(n-1)!-1}}$, we get the last permutation of the block in which 2 must have been the first element. $\square$

Construction

Lemma 4

In any block, last element of all permutations is always constant, and the sequence of last elements starting from $[1, 2, \ldots, n]$ *is always* $n, n-1, \ldots, 4, 3, 1$. *2 is never the last element.*

Proof.

- The first claim is justified since we use transitions that define C_{n-1} which only operate on the first $(n-1)$ elements.
- 2 is the first element for the last permutation of each block from Lemma 3, so it can't simultaneously the last element as well.

$\square$

Construction

- Let the set of $(n-1)$ blocks we constructed be C'.
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- From the lemmas above, we can see that C' is not complete yet since it doesn't have any permutations ending in 2.
- We provide an alternate construction to build C'', the set of all permutations with 2 as last element.

Construction of C''

- Start by rotating $t_{i_1}, t_{i_2}, \ldots, t_{i_{(n-1)!}}$ so that the last transition is t_{n-1}.

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- We start with the first permutation in the block as $[a_1, a_2, \ldots, a_{n-1}, 2]$. Set the permutations of C'' as those formed by $\tau_{i_1}, \tau_{i_2}, \ldots, \tau_{i_{(n-1)!-1}}$ starting from this first permutation.

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- We start with the first permutation in the block as $[a_1, a_2, \ldots, a_{n-1}, 2]$. Set the permutations of C'' as those formed by $\tau_{i_1}, \tau_{i_2}, \ldots, \tau_{i_{(n-1)!-1}}$ starting from this first permutation.
- The last permutation of C'' will therefore be $[a_2, a_3, \ldots, a_{n-1}, a_1, 2]$

Construction

- In C', we look for a transition of the form $[a_2, a_3, \ldots, a_{n-1}, 2, a_1] \to [2, a_2, \ldots, n-1, a_1]$

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- Such a transition would surely exist; while C' doesn't have any permutations in which 2 is the last element, it does have permutations in which 2 is the penultimate element and others in which 2 is the first element. Since C' is cyclic, such a transition would surely exist.

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- Whenever such a transition occurs (note that there can be multiple such occurrences, so we can choose any one), we split C' and insert C'' to create a complete RMGC as follows:

$$\begin{array}{c} a_2\\ a_3\\ \vdots\\ a_{n-1}\\ 2\\ a_1 \end{array} \underset{t_n}{\longrightarrow} \boxed{\begin{array}{ccc} a_1 & & a_2\\ a_2 & & a_3\\ \vdots & \rightsquigarrow & \vdots\\ a_{n-2} & & a_{n-1}\\ a_{n-1} & & a_1\\ 2 & & 2 \end{array}} \underset{t_n}{\longrightarrow} \begin{array}{c} 2\\ a_2\\ \vdots\\ a_{n-2}\\ a_{n-1}\\ a_1 \end{array}$$

Summary

Theorem 5

For every integer $n \geq 2$, there exists a cyclic and complete n-RMGC.

Example

Example 6

As considered in Example 2, we start the 3-RMGC we obtained. However, this had the transition sequence $t_2, t_3, t_3, t_2, t_3, t_3$, but we want it to end in t_2

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$$\boxed{\begin{array}{cccccc} 1 & 3 & 2 & 3 & 1 & 2 \\ 2 & 1 & 3 & 2 & 3 & 1 \\ 3 & 2 & 1 & 1 & 2 & 3 \\ 4 & 4 & 4 & 4 & 4 & 4 \end{array}} \boxed{\begin{array}{cccccc} 4 & 1 & 2 & 1 & 4 & 2 \\ 2 & 4 & 1 & 2 & 1 & 4 \\ 1 & 2 & 4 & 4 & 2 & 1 \\ 3 & 3 & 3 & 3 & 3 & 3 \end{array}} \boxed{\begin{array}{cccccc} 3 & 4 & 2 & 4 & 3 & 2 \\ 2 & 3 & 4 & 2 & 4 & 3 \\ 4 & 2 & 3 & 3 & 2 & 4 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{array}}$$

Example

To construct C'', we look for a transition in which 2 goes from the penultimate position to the top. Multiple such transitions exist, so we choose $[1, 3, 2, 4] \rightarrow [2, 1, 3, 4]$. Inserting C'' between C', we get:

$$\begin{matrix}
1 & 3 & 2 & 3 & 1 & 4 & 1 & 3 & 4 & 3 & 1 & 2 & 4 & 1 & 2 & 1 & 4 & 2 & 3 & 4 & 2 & 4 & 3 & 2 \\
2 & 1 & 3 & 2 & 3 & 1 & 4 & 1 & 3 & 4 & 3 & 1 & 2 & 4 & 1 & 2 & 1 & 4 & 2 & 3 & 4 & 2 & 4 & 3 \\
3 & 2 & 1 & 1 & 2 & 3 & 3 & 4 & 1 & 1 & 4 & 3 & 1 & 2 & 4 & 4 & 2 & 1 & 4 & 2 & 3 & 3 & 2 & 4 \\
4 & 4 & 4 & 4 & 4 & 2 & 2 & 2 & 2 & 2 & 2 & 4 & 3 & 3 & 3 & 3 & 3 & 3 & 1 & 1 & 1 & 1 & 1 & 1
\end{matrix}$$

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Definitions

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We define $c_i : \mathbb{N} \mapsto \mathbb{N}$ where $c_i(p)$ is the charge level of the i^{th} cell after the p^{th} programming cycle.

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- $c_i(p) > \max_k c_k(p-1)$
- for $k \neq i, c_k(p) = c_k(p-1)$
- In an optimal setting with no overshoots, or 'gaps', $c_i(p) = \max_k c_k(p-1) + 1$

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- **Jump Cost** of an n-RMGC is defined as the maximum jump over all transitions of the code
- An n-RMGC is said to be optimal if it's jump cost is not larger than any other n-RMGC
- In the above definition we assume that there are no degenerate cells, i.e. every cell level is raised at-least once

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- However we cannot apply t_n n times consecutively, as that would lead us to the first permutation after just n steps
- Hence our set of transitions must at-least one other transition $t_i, i \neq n$
- This would cause a 'gap' in the charge levels, and the first t_n to be used after it will have a jump of magnitude at-least $n + 1$

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- We observe that the transition t_n is the only one that does not introduce gaps between charge levels.
- Hence we base our construction around t_n and try to use it as often as possible

Balanced n-RMGC Construction

Theorem 9

Given a cyclic and complete $(n-1)$-RMGC C_{n-1} defined by the transitions $t_{i_1}, \ldots, t_{i_{(n-1)!}}$, the following transitions define an n-RMGC that is cyclic, complete and balanced

$$t_{j_k} = \begin{cases} t_{n-i_{\lceil k/n \rceil}+1} & k \equiv 1 (mod n) \\ t_n & otherwise \end{cases}$$

for all $k \in \{1, \ldots, n\}$

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Proof.

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- $\bar{t}_i([a_1, a_2, \ldots, \boldsymbol{a_{n-i+1}}, a_{i+1}, \ldots a_n]) = [a_1, a_2, \ldots, a_{i-1}, a_{i+1}, \ldots a_n, \boldsymbol{a_{n-i+1}}]$

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- Since C_{n-1} is cyclic and complete, using $\{\bar{t}_i\}_{(n-1)!}$ we can traverse all permutations of $[n-1]$

Balanced n-RMGC Construction

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- Consider the abstract transition $\bar{t}_i, 2 \leq i \leq n$ that pushes the i^{th} element from the bottom to the bottom of a permutation
- $\bar{t}_i([a_1, a_2, \ldots, \boldsymbol{a_{n-i+1}}, a_{i+1}, \ldots a_n]) = [a_1, a_2, \ldots, a_{i-1}, a_{i+1}, \ldots a_n, \boldsymbol{a_{n-i+1}}]$
- Since C_{n-1} is cyclic and complete, using $\{\bar{t}_i\}_{(n-1)!}$ we can traverse all permutations of $[n-1]$
- Using the above with $[n]$ will thus generate all $(n-1)!$ permutations of [n] with first element fixed

Balanced n-RMGC Construction

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- It is not hard to see that

$$\bar{t}_i(\sigma) = (t_n \circ \ldots (n - 1 times) \circ t_n \circ t_{n-i+1})(\sigma), \forall \sigma \in S_n$$

Balanced n-RMGC Construction

contd.

$$\underbrace{t_{n-i_1+1}, \overbrace{t_n, \dots, t_n}^{n-1 \text{ times}}}_{\overrightarrow{t_{i_1}}}, \dots, \underbrace{t_{n-i_{(n-1)!}+1}, \overbrace{t_n, \dots, t_n}^{n-1 \text{ times}}}_{\overrightarrow{t_{i_{(n-1)!}}}}.$$

Figure: Transition sequence

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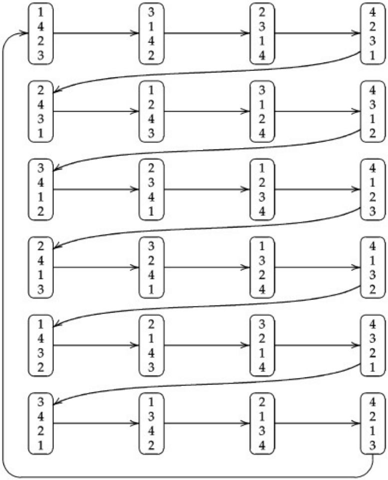
- $n-i$ transitions of type t_n with a jump of n

Balanced n-RMGC - Example

We now demonstrate the above theorem by ways of an example.



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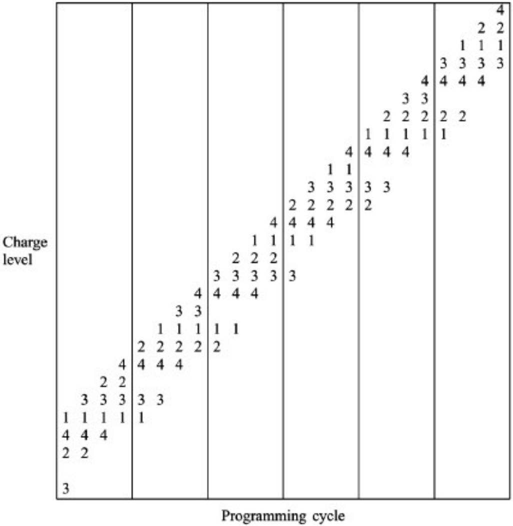

Ranking Permutations

For completing the design of a multi-level flash cell, we must define the correspondence between a permutation and its rank in the balanced n-RMGC.

Ranking Permutations

For completing the design of a multi-level flash cell, we must define the correspondence between a permutation and its rank in the balanced n-RMGC. We use the B-Factoradic Number System for representation.

B-Factoradic Number System

A number $m \in \{0, \ldots, n! - 1\}$ can be uniquely represented by the digits $b_{n-1} b_{n-2} \ldots b_1 b_0$ where $b_i \in \{0, \ldots, n - i - 1\}$ and the weight of b_i is $\frac{n!}{(n-i)!}$.

Ranking Permutations - Construction

We now describe an algorithm by way of an example to ascertain the rank of a permutation in the B-Factoradic Number System.

Example 10

- Let $n = 4$ and the current permutation be $\sigma = [1, 4, 3, 2]$, let $pos(k), k \in \{1, \ldots, n\}$ be the position of k in σ from the left. For example, $pos(4) = 2$

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- $\sigma_2 = \{2, 1\}$ and $b_2 = \{pos(2) - 2\}(mod 2) = 1$
- $\sigma_3 = \{1\}$ and $b_3 = \{pos(1) - 2\}(mod 1) = 0$

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- $\sigma_3 = \{1\}$ and $b_3 = \{pos(1) - 2\}(mod1) = 0$
- The B-Factoradic representation is therefore $0_3 1_2 1_1 0_0$

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Conclusion

- We started by looking at what rank modulation is, and why it is needed
- We have looked at a simple construction of cyclic and complete n-RMGC codes
- We then moved to a more efficient balanced n-RMGC construction
- The final mapping from the permutations to the symbols is done by using the B-Factoradic number system

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References

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