

Optimal Control Design of a Reparable Multistate system

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Abstract:

Keywords:

1. PROBLEM DESCRIPTION

Carefully provide a mathematical description of the problem discussed in this report.

Using the new notations boundary conditions and initial values:

Initial conditions:

2. METHODOLOGY

$$w_j^{20} = 0 \quad (13)$$

$$v_0 = 1 \quad (14)$$

$$\frac{dp_0}{dt} = -\lambda_0 p_0(t) + \int_0^1 \mu_1(x) p_1(x, t) dx + \int_0^1 u^*(x, t) dx \quad (1)$$

Boundary Conditions:

$$w_j^0 = \lambda v_j \quad (15)$$

$$\frac{\partial p_1(x, t)}{\partial t} + \frac{\partial p_1(x, t)}{\partial x} = -\mu_1(x) p_1(x, t) - u^*(x, t) \quad (2)$$

Also condensing,

Given Initial Conditions:

$$p_1(1, t) = 0 \quad (3)$$

$$p_0(0) = 1 \quad (4)$$

$$I_j^* = u^*(x_i, t_j) = g^i b_j \quad (16)$$

$$\int_0^1 u^*(x, t_j) dx = G b_j \quad (17)$$

where $g^i = (0.3 + 0.1 \sin(x_i))$

Given Boundary Conditions:

$$p_1(0, t) = \lambda_0 p_0(t) \quad (5)$$

And,

$$u^*(x, t) = (0.3 + 0.1 \sin(x)) b(t) \quad (6)$$

$$b(t) + \int_0^1 \mu_1(x) f(x, t) dx - 0.3 p_0^*(t) = c(t)$$

$$b_j = c_j - f_j \quad (18)$$

And,

$$b(t) + \int_0^1 \mu_1(x) f(x, t) dx - 0.3 p_0^*(t) = c(t) \quad (7)$$

$$\text{where } f_j = \int_0^1 \mu_1(x) f(x, t_j) dx - 0.3 p_0^*(t_j) \quad (19)$$

$$f(x, t) = 0.1 \cos(\pi t) \sin^2(1 - x) \quad (8)$$

$$p_0^*(t) = 0.85 + 0.05 \cos(2\pi t) \quad (9)$$

Discretizing (1)

We make couple of substitutions, following the notation that z_i^j refers to the value of z evaluated at time point i and at position j

$$\frac{v_{j+1} - v_j}{\tau} = -\lambda v_j + I_j + I_j^* \quad (21)$$

So

$$p_0(t_j) = v_j \quad (10)$$

$$p_1(x_i, t_j) = w_j^i \quad (11)$$

$$\mu_1(x_i) = \mu^i \quad (12)$$

$$\lambda = \lambda_0 \quad (12)$$

Discretizing (2)

$$I_j = h \left[\frac{\mu^0 w_j^0}{2} + \sum_{k=1}^{19} \mu^k w_j^k + \frac{\mu^{20} w_j^{20}}{2} \right] \quad (22)$$

$$= h \left[\frac{\mu^0 w_j^0}{2} + \sum_{k=1}^{19} \mu^k w_j^k \right]$$

$$I_j^* = \alpha b_j \quad (23)$$

$$\frac{w_{j+1}^i - w_j^i}{\tau} + \frac{w_j^{i+1} - w_j^{i-1}}{2h} = -\mu^i w_j^i - g^i b_j$$

$$w_{j+1}^i = w_j^i - \frac{\tau}{2h}(w_j^{i+1} - w_j^{i-1}) - \tau\mu^i w_j^i - \tau g^i b_j \quad (24)$$

Applying LAX scheme $w_j^i = \frac{w_j^{i-1} + w_j^{i+1}}{2}$ we get,

$$w_{j+1}^i = \frac{1}{2} \left(1 - \mu^i \tau + \frac{\tau}{h} \right) w_j^{i-1} + \frac{1}{2} \left(1 - \mu^i \tau - \frac{\tau}{h} \right) w_j^{i+1} - \tau g^i b_j$$

Under an appropriately defined matrix A , we can re-write the above equation to read

$$\mathbf{w}_{j+1} = A\mathbf{w}_j - b_j \tau \mathbf{g} + \mathbf{e}_1 v_{j+1} \quad (25)$$

$$= (A)^{j+1} \mathbf{w}_0 - \left[\sum_{k=0}^j b_k (A)^{j-k} \right] \mathbf{g} \tau \quad (26)$$

$$+ \left[\sum_{k=0}^j v_{k+1} (A)^{j-k} \right] \mathbf{e}_1$$

where \mathbf{e}_1 is an $m \times 1$ matrix given by

$$\mathbf{e}_1 = [\lambda, 0, \dots, 0]^T \quad (27)$$

From (21)

$$v_{j+1} = (1 - \lambda \tau) v_j + \tau I_j + \tau I_j^* \quad (28)$$

$$= (1 - \lambda \tau + \frac{h\tau}{2}) v_j + h\tau \boldsymbol{\mu}^T \mathbf{w}_j + \alpha b_j \tau \quad (29)$$

Substitute the expression for the time evolution for \mathbf{w} in the above to obtain,

$$v_{j+1} = (1 - \lambda \tau + \frac{h\tau}{2}) v_j + h\tau \boldsymbol{\mu}^T (A)^j \mathbf{w}_0 - \boldsymbol{\mu}^T \left[\sum_{k=0}^{j-1} b_k (A)^{j-1-k} \right] \mathbf{g} \tau + \alpha b_j \tau$$

Let's define

$$\beta_k = \boldsymbol{\mu}^T (A)^{j-1-k} \mathbf{g} \quad (30)$$

$$\omega_j = \boldsymbol{\mu}^T (A)^j \mathbf{w}_0 \quad (31)$$

$$\gamma = (1 - \lambda \tau + \frac{h\tau}{2}) \quad (32)$$

$$v_{j+1} = \gamma v_j + h\tau \omega_j - \tau \sum_{k=0}^{j-1} \beta_k b_k + \alpha b_j \tau \quad (33)$$

$$= \gamma v_j + h\tau \omega_j - \tau \boldsymbol{\beta}_j^T \mathbf{b} \quad (34)$$

$$= \gamma \sum_{k=0}^j \gamma^{j-k} v_k + h\tau \sum_{k=0}^j \gamma^{j-k} \omega_k - \left(\sum_{k=0}^j \gamma^{j-k} \beta_k \right)^T \mathbf{b} \quad (35)$$

Under appropriately defined strictly lower triangular matrices G and B ,

$$\mathbf{v} = \gamma G \mathbf{v} + h\tau G \boldsymbol{\omega} - B \mathbf{b} \quad (36)$$

where row $j+1$ of matrix B is given by

$$\left(\sum_{k=0}^j \gamma^{j-k} \beta_k \right)^T$$

Consequently, we have,

$$(\gamma G - I) \mathbf{v} = B \mathbf{b} - h\tau G \boldsymbol{\omega} \quad (37)$$

$$\mathbf{v} = (\gamma G - I)^{-1} B \mathbf{b} - h\tau (\gamma G - I)^{-1} G \boldsymbol{\omega} \quad (38)$$

$$= B' \mathbf{b} - \mathbf{c} \quad (39)$$

for appropriate matrix B' and vector \mathbf{c} . Note that \mathbf{c} is a known quantity. Hence our optimization problem reduces to finding \mathbf{b} that best fits the equation

$$B' \mathbf{b} = \mathbf{v}^* + \mathbf{c} \quad (40)$$

3. RESULTS

Illustrate the results using the methodology proposed in Section. 2

4. OBSERVATION AND CONCLUSIONS

State your observation from the results and make conclusions.

REFERENCE

All materials (books, papers, and websites) mentioned in your reports.

APPENDIX

Attach the Matlab code here.