Optimal Control Design of a Reparable Multistate system

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* *University of Southern California, LA, CA 90089 USA (e-mail: XXX@ usc.edu).*
** *University of Southern California, LA, CA 90089 USA (e-mail: XXX@ usc.edu).*

Abstract:

Keywords:

1. PROBLEM DESCRIPTION

Carefully provide a mathematical description of the problem discussed in this report.

2. METHODOLOGY

$$\frac{dp_0}{dt} = -\lambda_0 p_0(t) + \int_0^1 \mu_1(x)p_1(x,t)dx + \int_0^1 u^*(x,t)dx \quad (1)$$

$$\frac{\partial p_1(x,t)}{\partial t} + \frac{\partial p_1(x,t)}{\partial x} = -\mu_1(x)p_1(x,t) - u^*(x,t) \quad (2)$$

Given Initial Conditions:

$$p_1(1,t) = 0 \quad (3)$$
$$p_0(0) = 1 \quad (4)$$

Given Boundary Conditions:

$$p_1(0,t) = \lambda_0 p_0(t) \quad (5)$$

$$u^*(x,t) = (0.3 + 0.1sin(x))b(t) \quad (6)$$

And,

$$b(t) + \int_0^1 \mu_1(x)f(x,t)dx - 0.3p_0^*(t) = c(t) \quad (7)$$
$$f(x,t) = 0.1\cos(\pi t)\sin^2(1-x) \quad (8)$$
$$p_0^*(t) = 0.85 + 0.05\cos(2\pi t) \quad (9)$$

We make couple of substitutions, following the notation that z_i^j refers to the value of z evaluated at time point i and at position j

So

$$p_0(t_j) = v_j$$
$$p_1(x_i,t_j) = w_j^i \quad (10)$$
$$\mu_1(x_i) = \mu^i \quad (11)$$
$$\lambda_= \lambda_0 \quad (12)$$

Using the new notations boundary conditions and initial values:

Initial conditions:

$$w_j^{20} = 0 \quad (13)$$
$$v_0 = 1 \quad (14)$$

Boundary Conditions:

$$w_j^0 = \lambda v_j \quad (15)$$

Also condensing,

$$u^*(x_i,t_j) = g^i b_j \quad (16)$$
$$\int_0^1 u^*(x,t_j)dx = Gb_j \quad (17)$$
$$where\ g^i = (0.3 + 0.1sin(x_i))$$

And,

$$b(t) + \int_0^1 \mu_1(x)f(x,t)dx - 0.3p_0^*(t) = c(t)$$
$$b_j = c_j - F_j \quad (18)$$
$$where\ F_j = \int_0^1 \mu_1(x)f(x,t_j)dx - 0.3p_0^*(t_j) \quad (19)$$
$$(20)$$

Discretizing 1

$$\frac{v_{j+1} - v_j}{\tau} = -\lambda v_j + I_j + I_j^* \quad (21)$$

$$I_j = h[\frac{\mu^0 w_j^0}{2} + \sum_{k=1}^{19} \mu^k w_j^k + \frac{\mu^{20} w_j^{20}}{2}] \quad (22)$$

$$= h[\frac{\mu^0 w_j^0}{2} + \sum_{k=1}^{19} \mu^k w_j^k]$$
$$I_j^* = Gb_j \quad (23)$$

Discretizing 2

$$\frac{w_{j+1}^i - w_j^i}{\tau} + \frac{w_j^{i+1} - w_j^{i-1}}{2h} = -\mu_j w_j^i - g^i b_j$$

$$w_{j+1}^i = w_j^i - \frac{\tau}{2h}(w_j^{i+1} - w_j^{i-1}) - \tau\mu_j w_j^i - \tau g^i b_j \quad (24)$$

Applying LAX scheme $w_j^i = \frac{w_j^{i-1} + w_j^{i+1}}{2}$????

From 21:

$$v_{j+1} = (1 - \lambda\tau)v_j + \tau I_j + \tau I_j^* \quad (25)$$

$$v_{j+1} = av_j + \tau I_j + \tau I_j^* \quad (26)$$

$$v_j = av_{j-1} + \tau I_{j-1} + \tau I_{j-1}^* \quad (27)$$

$$v_{j+1} = a^2 v_{j-1} + \tau I_j + a\tau I_{j-1} + \tau I_j^* + a\tau I_{j-1}^* \quad (28)$$

For any v_{j+1} expansion the sum of subscripts of all possible terms should sum to j except the leading term which sums to $j+1$ (trivial) .

Hence:

$$v_{j+1} = a^{n+1} v_0 + \tau I_j + a\tau I_{j-1} + a^2 \tau I_{j-2} + \ldots + \tau I_j^* + a\tau I_{j-1}^* \quad (29)$$

NOTE: I_j^* has b_j term hidden inside. But basically this can be written as a matrix form

$$\begin{pmatrix} a^n\tau & a^{n-1}\tau & \cdots & a^2\tau & a\tau & \tau \end{pmatrix} . \begin{pmatrix} I_0^* \\ I_1^* \\ \cdots \\ I_{j-2}^* \\ I_{j-1}^* \\ I_j^* \end{pmatrix} \quad (30)$$

3. RESULTS

Illustrate the results using the methodology proposed in Section. 2

4. OBSERVATION AND CONCLUSIONS

State your observation from the results and make conclusions.

REFERENCE

All materials (books, papers, and websites) mentioned in your reports.

APPENDIX

Attach the Matlab code here.