```
for (i=0; i<n; i++)      → (n+1)  }
{   st                   → n      }  O(n)
}
```

```
for (i=n; i>0; i--)      → n+1    }
{   st;                  → n      }  O(n)
}
```

```
for (i=1; i<n; i=i+2)    → (n/2 + 1)  }
{   st                   → (n/2)      }  O(n)
}
```

```
for (i=0; i<n; i=i+20)   → (n/20 + 1)  }
{   st;                  → n/20        }  O(n)
}
```

```
for (i=0; i<n; i++)             → n+1      }
{   for (j=0; j<n; j++)         → n(n+1)   }  O(n^2)
    {   st;                     → n x n    }
    }
}
```

$$f(n) = n+1+n^2+n+n^2$$
$$= \cancel{2}n^2 + \cancel{2n+1}$$
$$= O(n^2)$$

Ex

```
for (i=0; i<n; i++)          —     (i = 1, 2, 3)
{   for (j=0; j<i; j++)      —
    {   st;
    }
}
```

No execut

i	j	No. of times
0	0 x	0 x
1	0 1 x	1
2	0	2

$\boxed{1 + 2 + 3 + 4 + 5 + \cdots n}$ LHS

~~Natural no~~ $\Rightarrow \dfrac{n(n+1)}{2}$ RHS

	1	
	2x	
3	0	3
	1	
	2	
	3x	
⋮		
n		n

LHS = RHS

$$\boxed{\underbrace{1 + 2 + 3 + 4 \cdots n} = \frac{n(n+1)}{2}}$$

$$f(n) = \frac{n^2 + n}{2}$$

$$= \boxed{O(n^2)}$$

Ex:

```
p=0;
for (i=1; p<=n; i++)
{
    p=p+i;
}
```

2 3 4 5

Assume, $\boxed{p > n}$ Assumption

$$\boxed{1+2+3+4+\cdots n = \frac{n(n+1)}{2}}$$

$\boxed{p > n}$ — (1)

$p = 1+2+3+4 \cdots n$

$$\boxed{p = \frac{k(k+1)}{2}}$$

Sub. in Eq(1)

$$\frac{k(k+1)}{2} > n$$

$$\frac{k^2 + \cancel{k}}{\cancel{2}} > n$$

$$k^2 > n$$

$$k > \sqrt{n} \rightarrow O(\sqrt{n})$$

i	p
1	0+1=1
2	1+2=3
3	$\underbrace{1+2}_{p}+\underbrace{3}_{+i}=6$
4	$\underbrace{1+2+3}_{p}+\underbrace{4}_{i}$
5	$\underbrace{1+2+3+4}_{p}+\underbrace{5}_{i}$
⋮	
n (k Cond^n)	$\underbrace{1+2+3+\cdots n}_{p}$

k

$1+2+\cdots k$

$$\frac{k(k+1)}{2}$$

Time complexity

```
p=0
for (i=1; p<=n; i++)
{
    p=p+i;
}
```

Assume $p > n$ ($p <= n$) opposite

i	p
1	$0+1=1$
2	$1+2$
3	$1+2+3$
4	$1+2+3+4$
⋮	
k	$1+2+3+\cdots k$

$k \Rightarrow \frac{k(k+1)}{2}$

p

$$1+2+3+\cdots n = \frac{n(n+1)}{2}$$

$$p > n \quad \text{--- eq } ①$$

p value substituted in eq ①

$$\frac{k(k+1)}{2} > n$$

$$\frac{k^2+\cancel{k}}{\cancel{2}} > n$$

$$k^2 > n$$

$$k > \sqrt{n}$$

Cond^n → Time complexity

$O(\sqrt{n})$

```
for (i=1; i<n; i=i*2)
{
```

i
$1 = 2^0$
$1 \times 2 = 2 = 2^1$

st;

}

Assume $i \geq n$ — Eqⁿ (1)

$i = 2^k$

$2^k \geq n$

$2^k = n$

Apply logs both the sides

$k = \log_2 n$

→ Time Complexity

$O(\log_2 n)$

$2 \times 2 = 2^2$

$2^2 \times 2 = 2^3$

$2^3 \times 2 = 2^4$

$\vdots$

$2^k \Rightarrow i$

```
for (i=1; i<n; i=i*2)
{ st;
}
```

Imp

$i = 1 \times 2 \times 2 \times 2 \times 2 \cdots n$

$2^k = n$

$k = \log_2 n$

$O(\log_2 n)$

```
for (i=1; i<=n; i++)
{
    st;
}
```

$O(n)$

$i = 1+1+1+1 \cdots n$

$k = n$ $O(n)$

```
for (i=1; i<n; i=i*2)    →  O(log_2 n)
{                              ≈ log n
}
```
base 2

```
for (i=n; i>=1; i=i/2)   →  (log_2 (n))
{   st;
}
```

$i/2^0$

$1/2^1$

$1/2^2$

$1/2^3$

$\frac{n}{2^k}$

Assume $i < 1$

$$\frac{n}{2^k} < 1$$

$$n = 2^k$$

$$k = \log_2 n$$

$$O(\log n)$$

```
for (i=0; i*i<n; i++)
{   st;
}
```

condⁿ

$*2 \quad /2$

$\log_2 n$

$i*i < n$

$i*i >= n$

$i^2 = n$

$i = \sqrt{n} \longrightarrow O(\sqrt{n})$

```
p=0
for (i=1; i<n; i=i*2)
```

log n

{ p++; }

$p = \log n$

```
for (j=1; j<p; j=j*2)
{
    st;
}
```

$(\log p)$

$\log p$

$\log(\log n)$

$O(\log \log n)$

Summarize

for (i=0; i<n; i++) $\longrightarrow$ $O(n)$

for (i=0; i<n; i=i+2) $\longrightarrow$ $\frac{n}{2} \rightarrow O(n)$

for (i=n; i>1; i--) $\longrightarrow$ $O(n)$

for (i=1; i<n; i=i*2) $\longrightarrow$ $O(\log_2 n)$

for (i=1; i<n; i=i*3) $\longrightarrow$ $O(\log_3 n)$

for (i=n; i>1; i=i/2) $\longrightarrow$ $O(\log_2 n)$

Types of Time functions -

$O(1)$ — Constant

$O(\log n)$ — Logarithmic

$O(n)$ — Linear

$O(n^2)$ — Quadratic

$O(n^3)$ — Cubic

$O(2^n)$ — Exponential

$\left.\begin{array}{l} f(n) = 2 \\ f(n) = 5 \\ f(n) = 5000 \end{array}\right\} O(1)$

$O(3^n)$ = Exponential

$O(n^n)$ = Exponential

$f(n) = 2n+3 \Rightarrow O(n)$

$f(n) = 5000n + \underline{700}$

$O(n)$

$f(n) = \frac{n}{5000} + 6$

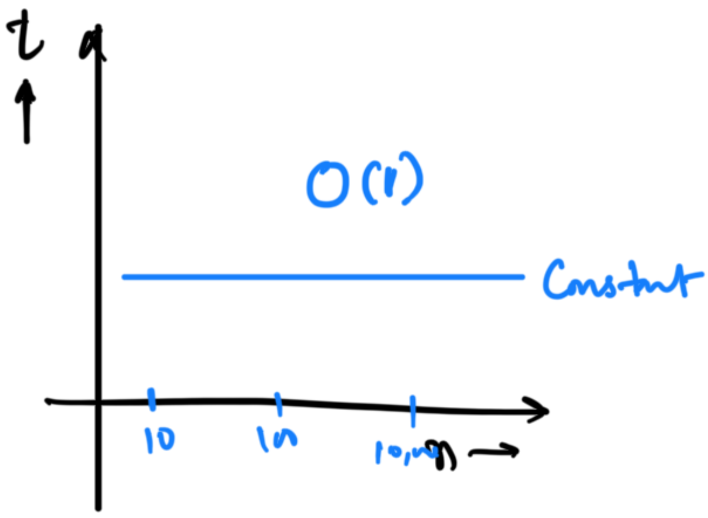
$= O(n)$

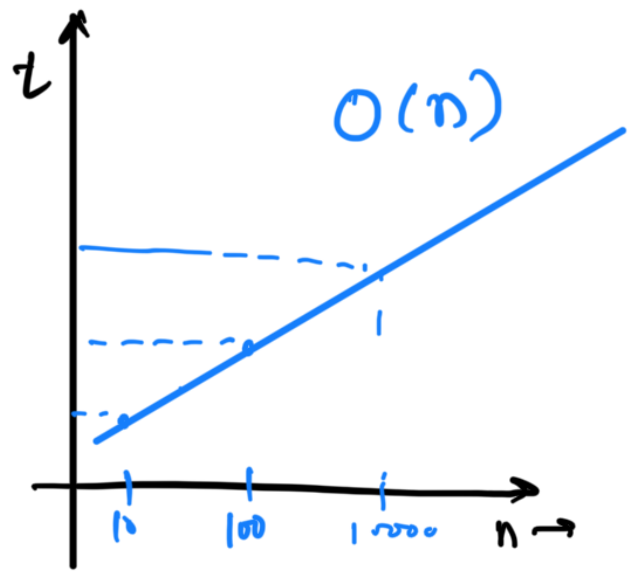
Order of Time complexity

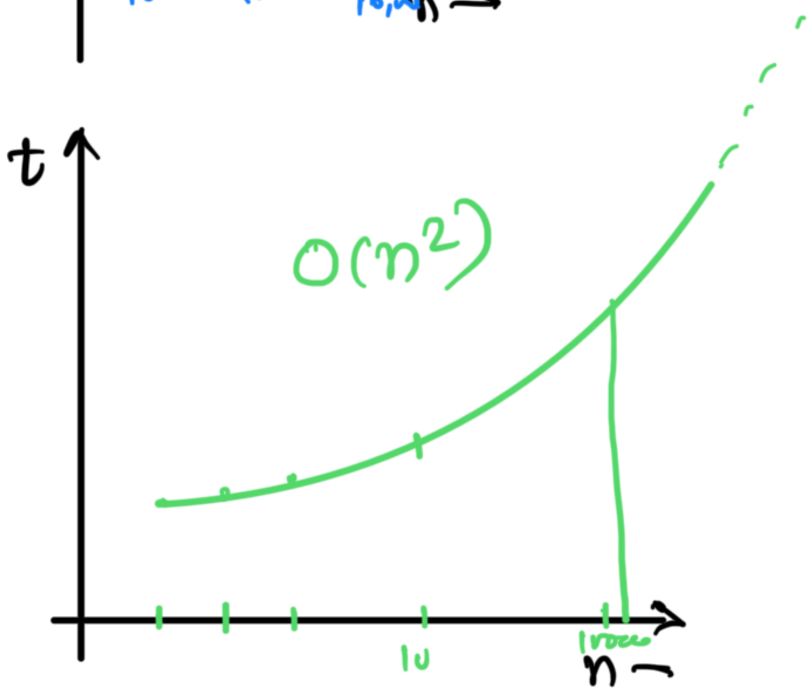
$$\boxed{1 < \log n < \sqrt{n} < n < n\log n < n^2 < n^3 < \cdots < 2^n < 3^n \cdots < n^n}$$

$2^n, \log_2 n, n^3, \log_3 n, n\log n$

Ans $\rightarrow \log_2 n, \log_3 n, n\log n, n^3, 2^n$





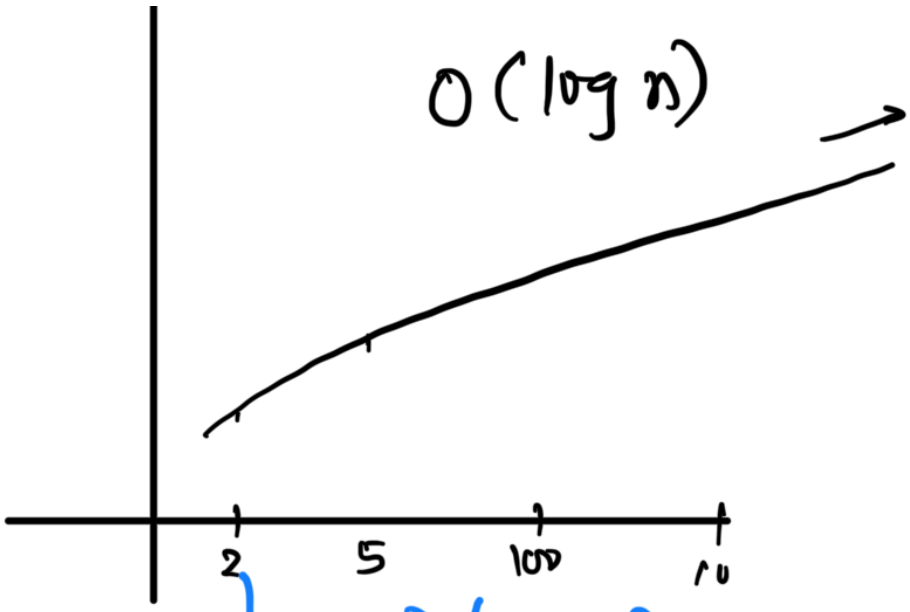


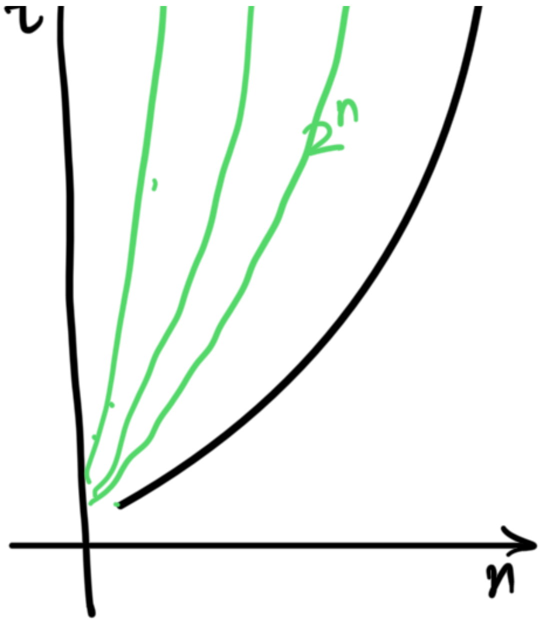
I/P	Time
n	n^2
1	1
2	4
3	9
4	16
⋮	⋮
10	100
100	10000

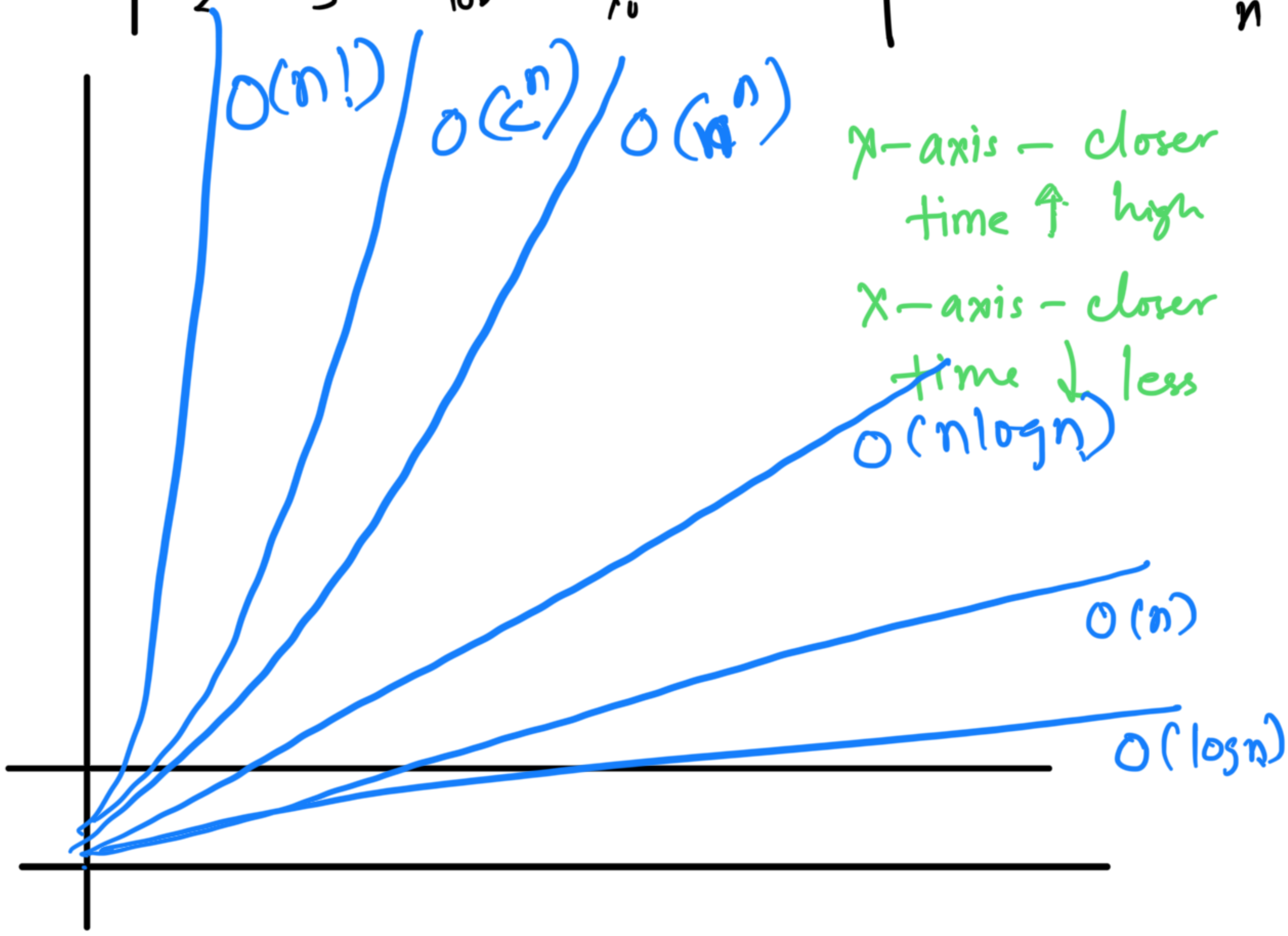
$O(n)$ & $O(n^2)$

↓

<u>linear</u> exponentially







x-axis – closer time ↑ high

x-axis – closer time ↓ less

Best → O(1)

Good → O(logn)

Fair → O(n)

optimizatn

Bad → O(nlogn)

Worst → O(n!) $O(c^n)$ $O(n^c)$

Asympotic Notatic

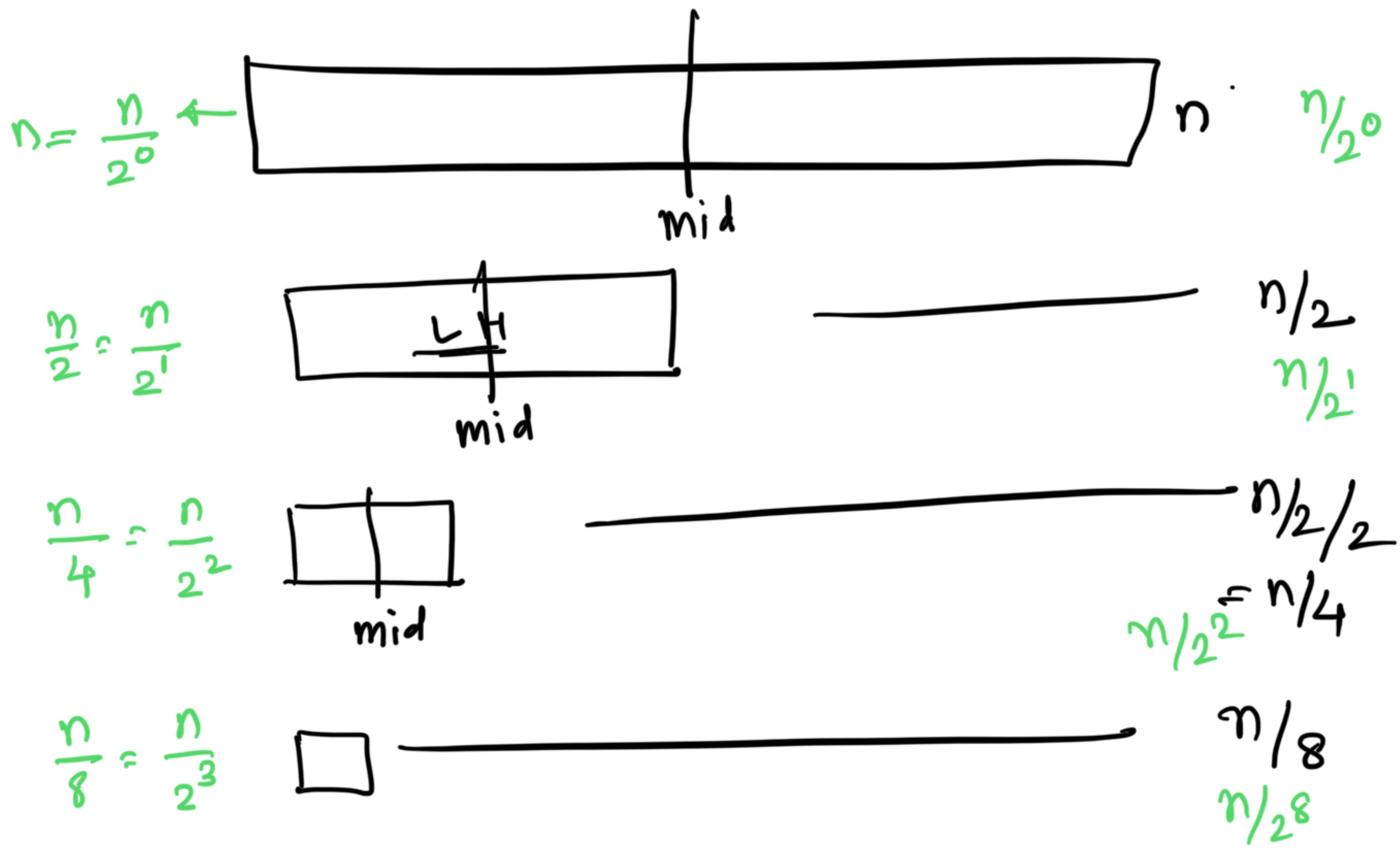
Best case	→	Ω	lower bound
Average case	→	Θ	Average bound
→ Worst case	→	O	Upper bound

8	6	12	15	19	23	27

Best case $\rightarrow$ key $\Rightarrow$ 8 $\rightarrow$ (1) $\rightarrow$ $O(1)$

Worst case $\rightarrow$ key $\Rightarrow$ 2;

29



$\vdots$

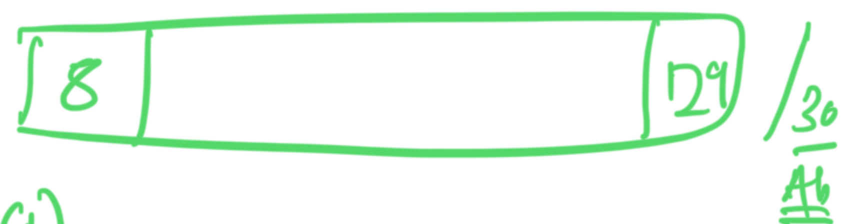
$\frac{n}{2^n}$ $\qquad O(\log_2 n)$

Imp Best case $\rightarrow$ First mid element $\rightarrow$ $O(1)$

Worst case $\rightarrow$ $\rightarrow$ $\log(n)$

Average $\rightarrow$ $\rightarrow$ $\log(n)$

<u>Linear search</u>



Best case — $O(1)$

Worst case $\rightarrow$ $O(n)$

Average case $\rightarrow$ $\frac{1+2+3+\cdots n}{n}$

$$= \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

$$= O(n)$$

Best case $= O(1)$, $\Theta(1)$, $\Omega(1)$

8	3	5	7	6