SURDS & INDICES

- KOUSTAV

CONCEPT

$(1024)^{-\frac{1}{10}} = (2^{10})^{-\frac{1}{10}} = 2^{10 \times -\frac{1}{10}} = 2^{-1} = \frac{1}{2} = 0.5$

1. Laws of Indices:

i. $a^m \times a^n = a^{m+n}$

ii. $\frac{a^m}{a^n} = a^{m-n}$

iii. $(a^m)^n = a^{mn}$

iv. $(ab)^n = a^n b^n$

v. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

vi. $a^0 = 1$

$64^{\frac{1}{3}}$

$\left(2^6\right)^{\frac{1}{3}}$

$2^{6 \times \frac{1}{3}}$

$= 2^2 = 4$

$a^{-1} = \frac{1}{a}$

2. Surds:

Let a be rational number and n be a positive integer such that $a^{(1/n)} = \sqrt[n]{a}$

Then, $\sqrt[n]{a}$ is called a surd of order n.

3. Laws of Surds:

$\sqrt{4} = 4^{\frac{1}{2}}$

i. $\sqrt[n]{a} = a^{(1/n)}$

ii. $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b} = (ab)^{\frac{1}{n}}$

iii. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \left(\frac{a}{b}\right)^{\frac{1}{n}}$

iv. $(\sqrt[n]{a})^n = a$ $= a^{n \times \frac{1}{n}}$

v. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ $\sqrt[2]{\sqrt[3]{1000}} = \sqrt[6]{1000}$

vi. $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

1. $(17)^{3.5} \times (17)^{?} = 17^{8}$

A. 2.29

B. 2.75

C. 4.25

✓ D. 4.5

$17^{3.5+x} = 17^{8}$

$3.5 + x = 8$

$x = 8 - 3.5 = 4.5$

2. If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$, then the value of x is:

A. $\frac{1}{2}$

B. 1

✓ C. 2

D. $\frac{7}{2}$

$a^{-1} = \frac{1}{a}$

$\left(\frac{a}{b}\right)^{x-1} = \left(\left(\frac{a}{b}\right)^{x-3}\right)^{-1} =$

$x - 1 = 3 - x$

$2x = 4$

$x = 2$

3. Given that $10^{0.48} = x$, $10^{0.70} = y$ and $x^z = y^2$, then the value of z is close to:

A. 1.45

B. 1.88

C. 2.9

D. 3.7

$(10^{0.48})^2 = (10^{0.70})^2$

$10^{0.48z} = 10^{0.7\times 2}$

$0.48z = 1.4$

$z = \frac{1.4}{0.48} = \frac{140}{48} = \frac{35}{12} \simeq \frac{36}{12} = 3$

4. If $5^a = 3125$, then the value of $5^{(a-3)}$ is:

A. 25

B. 125

C. 625

D. 1625

$5^a = 3125 = 5^5$

$a = 5$

$5^{(5-3)} = 5^2 = 25$

5. If $3^{(x-y)} = 27$ and $3^{(x+y)} = 243$, then x is equal to:

A. 0

B. 2

C. 4

D. 6

$3^{x-y} = 3^3$ $\quad 3^{x+y} = 3^5$

$x - y = 3$

$x + y = 5$ (+)

$2x = 3 + 5$

$= 8$

$x = 4$

6. $(256)^{0.16} \times (256)^{0.09} = ?$

A. 4

B. 16

C. 64

D. 256.25

$256^{0.16+0.09} = 256^{0.25}$

$= (256)^{1/4} = (2^8)^{1/4}$

$= 4$

7. The value of $[(10)^{150} \div (10)^{146}]$

A. 1000

B. 10000

C. 100000

D. 10^6

$10^{150-146} = 10^4$

8. $$\frac{1}{1 + x^{(b-a)} + x^{(c-a)}} + \frac{1}{1 + x^{(a-b)} + x^{(c-b)}} + \frac{1}{1 + x^{(b-c)} + x^{(a-c)}} = ?$$

A. 0

B. 1

C. x^{a-b-c}

D. None of these

$$\frac{1}{1 + \frac{x^b}{x^a} + \frac{x^c}{x^a}} = \frac{1}{\frac{x^a + x^b + x^c}{x^a}}$$

$$= \frac{x^a}{x^a + x^b + x^c} + \frac{x^b}{x^b + x^a + x^c} + \frac{x^c}{x^c + x^b + x^a}$$

$$= \frac{x^a + x^b + x^c}{x^a + x^b + x^c}$$

9. $(25)^{7.5} \times (5)^{2.5} \div (125)^{1.5} = 5^{?}$

A. 8.5

B. 13

C. 16

D. 17.5

E. None of these

$(5^2)^{7.5} \times 5^{2.5} \div (5^3)^{1.5}$

$2 \times 7.5 + 2.5 - 3 \times 1.5$

$15 + 2.5 - 4.5$

$= 13$

10. $(0.04)^{-1.5} = ?$

A. 25

B. 125

C. 250

D. 625

$\left(\frac{4}{100}\right)^{-\frac{15}{10}}$

$= \left(\frac{100}{4}\right)^{3/2} = (25)^{3/2}$

$= 5^{2 \times 3/2}$

$= 125$

11. $\dfrac{(243)^{n/5} \times 3^{2n+1}}{9^n \times 3^{n-1}} = ?$

A. 1

B. 2

C. 9

D. 3^n

$$\frac{\left(3^5\right)^{n/5} \times 3^{2n+1}}{3^{2n} \times 3^{n-1}} = 3^{5\times n/5 + 2n+1-2n-(n-1)} = 3^{n+2n+1-2n-n+1}$$

$$= 3^2 = 9$$

12. $\dfrac{1}{1 + a^{(n-m)}} + \dfrac{1}{1 + a^{(m-n)}} = ?$

A. 0

B. $\frac{1}{2}$

C. 1

D. a^{m+n}

$$\frac{1}{1+\frac{a^n}{a^m}} + \frac{1}{1+\frac{a^m}{a^n}}$$

$$= \frac{a^m}{a^m+a^n} + \frac{a^n}{a^n+a^m} =$$

13. If m and n are whole numbers such that $m^n = 121$, the value of $(m - 1)^{n+1}$ is:

A. 1

B. 10

C. 121

D. 1000 ✓

$121 = 11^2 = m^n$

$m = 11$

$n = 2$

10^3

$= 1000$

14. $\left(\frac{x^b}{x^c}\right)^{(b+c-a)} \cdot \left(\frac{x^c}{x^a}\right)^{(c+a-b)} \cdot \left(\frac{x^a}{x^b}\right)^{(a+b-c)} = ?$

A. x^{abc}

B. 1 ✓

C. $x^{ab+bc+ca}$

D. x^{a+b+c}

$x^{b^2+bc-ba+c^2+ca-cb+a^2+ab-ac}$

$x^{cb+c^2-ca+ca+a^2-ab+ba+b^2-bc}$

$= \frac{x^{b^2+c^2+a^2}}{x^{c^2+a^2+b^2}}$

ANSWER KEY

QUESTION	ANSWER	QUESTION	ANSWER
1	D	8	B
2	C	9	B
3	C	10	B
4	A	11	C
5	C	12	C
6	A	13	D
7	B	14	B