KARNAUGH MAP basics & Key-points.

- Developed by Karnaugh in 1953.
- It is used to minimize boolean equations.
- It is build based on gray code.

- Two Variable K-map.

Total Cells $= 2^n = 2^2 = 4$

B \ A	$\bar{A}$ 0	A 1
$\bar{B}$ 0	1_{0}	$_{2}$
B 1	$_{1}$	1_{3}

$Y(A,B) = \Sigma m(0,3)$

- Three Variable K-map

Total Cells $= 2^n = 2^3 = 8$

C \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$ 0	1_{0}	$_{2}$	1_{6}	1_{4}
C 1	1_{1}	$_{3}$	$_{7}$	$_{5}$

$Y(A,B,C) = \Sigma m(0,1,4,6)$

- Four Variable K-map

Total Cells $= 2^n = 2^4 = 16$

CD \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}\bar{D}$ 00	1_{0}	1_{4}	1_{12}	$_{8}$
$\bar{C}D$ 01	1_{1}	$_{5}$	$_{13}$	$_{9}$
CD 11	$_{3}$	$_{7}$	$_{15}$	$_{11}$
$C\bar{D}$ 10	$_{2}$	1_{6}	$_{14}$	1_{10}

$Y(A,B,C,D) = \Sigma m(0,1,4,6,10,12)$

K-map rules for grouping.

1) Group should not contain zero and cells contain '1' must be grouped.
2) We can group 1, 2, 4, 8, ..., 2^n cells.
3) Each group should be as large as possible.
4) Group may overlap.
5) Opposite grouping and corner grouping is allowed.
6) There should be as few groups as possible.

CD \ AB	00	01	11	10
00	1_0	1_4	0_{12}	1_8
01	0_1	0_5	0_{13}	0_9
11	0_3	0_7	0_{15}	0_{11}
10	1_2	1_6	0_{14}	1_{10}

1st, 2nd

$$Y = \underset{1^{st}}{\overline{B}\,\overline{D}} + \underset{2^{nd}}{\overline{A}\,\overline{D}}$$

CD \ AB	00	01	11	10
00	1_0	1_4	0_{12}	0_8
01	0_1	1_5	0_{13}	1_9
11	0_3	1_7	0_{15}	1_{11}
10	1_2	1_6	0_{14}	0_{10}

1st, 2nd, 3rd

$$Y = \underset{1^{st}}{\overline{A}\,\overline{D}} + \underset{2^{nd}}{\overline{A}B} + \underset{3^{rd}}{A\overline{B}D}$$

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Implicants, Prime Implicants and Essential Prime Implicants in K-map.

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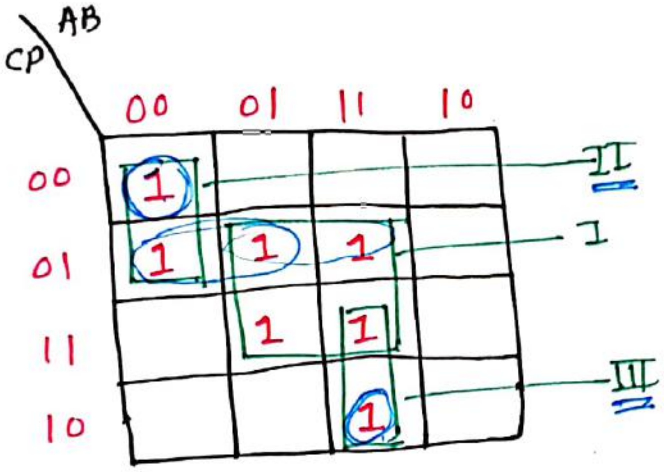
Implicants – The group's of 1's is Implicants.

Prime Implicants – It largest possible group of 1's.

Essential prime Implicants – At least, There is single 1 which can not be combined in other way.



- group I, II, III are prime Implicants.
- group II & III are Ess. prime Implicants.

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K-map Examples 7h

1] In the sum of products function is

$$f(x, y, z) = \Sigma m(2, 3, 4, 5)$$. The prime implicants are

✓ a] $\bar{x}y$, $x\bar{y}$

b] $\bar{x}y$, $x\bar{y}\bar{z}$, $x\bar{y}z$

c] $\bar{x}y\bar{z}$, $\bar{x}yz$, $x\bar{y}$

d] $\bar{x}y\bar{z}$, $\bar{x}yz$, $x\bar{y}z$

Z \ XY	00	01 ($\bar{X}Y$)	11	10 ($X\bar{Y}$)
0	0	1 (2)	6	1 (4)
1	1	1 (3)	7	1 (5)

1st ($\bar{X}Y$) 2nd ($X\bar{Y}$)

2] The K-map for a boolean function is shown in figure. The number of essential prime implicants for this function is

✓ a] 4

b] 5

c] 6

d] 8

CD \ AB	00	01	11	10
00	1	1	0	1
01	0	0	0	1
11	1	0	0	0
10	1	0	0	1

1st, 2nd, 3rd, 4th

3] Solve given boolean expression using K-map

$$y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} = \boxed{\bar{A}\bar{C} + \bar{A}B + B\bar{C}}$$

C \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$ 0	1	1	1	
C 1		1		

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Examples on K- Map 75.

1] Find the Boolean expression for K-map given below.

$CD \backslash AB$	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}\bar{D}$ 00	1	0	0	1
$\bar{C}D$ 01	0	1	1	1
CD 11	0	1	1	1
$C\bar{D}$ 10	1	0	0	1

III $(A\bar{B})$

III $(A.D)$

II (BD)

I $(\bar{B}\bar{D})$

$= \bar{B}\bar{D} + BD + AD$ or

$= \bar{B}\bar{D} + BD + A\bar{B}$

2] $y = \underline{AB} + \underline{\bar{A}BC} + \underline{A\bar{B}} + \underline{C}$. Solve boolean expression by K-map.

y

$C \backslash AB$	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}$ 0			1	1
C 1	1	1	1	1

I (A)

II (C)

$\boxed{y = A + C}$

3] Find The Boolean expression for K-map given below.

F

$CD \backslash AB$	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}\bar{D}$ 00	0	1	1	0
$\bar{C}D$ 01	1	1	0	1
CD 11	1	1	0	1
$C\bar{D}$ 10	0	1	1	0

II $(B\bar{D})$

I $(\bar{B}D)$

III $(\bar{A}B)$

III $(\bar{A}D)$

$F = \bar{B}D + B\bar{D} + \bar{A}D$

or

$F = \bar{B}D + B\bar{D} + \bar{A}B$

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K-map examples on don't care 76

1) The numbers of product term in the minimized Sum of product expression obtained through the following K-map is ______ [x is don't care].

a) 2 ✓
b) 3
c) 4
d) 5

$Y = \bar{B}\bar{D} + ACD$

or

$Y = \bar{B}\bar{D} + A\bar{B}C$

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	X	0	0
11	0	0	X	1
10	1	0	0	1

1st ($\bar{B}\bar{D}$)

2nd (ACD)

or

2nd ($A\bar{B}C$)

2) Solve given boolean K-map [x is don't care]

CD \ AB	00	01	11	10
00	1	X	0	1
01	0	0	0	0
11	0	0	0	1
10	X	1	0	1

1st

2nd

3rd

$Y = \bar{B}\bar{D} + \bar{A}\bar{D} + A\bar{B}C$

3) Solve given boolean K-map [x is don't care]

CD \ AB	00	01	11	10
00	X	0	1	1
01	1	0	0	1
11	X	0	0	0
10	X	0	0	X

2nd

1st

1st

2nd

$Y = \bar{B}\bar{C} + A\bar{C}\bar{D}$

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K-map Examples with don't care. 77

1) Solve K-map

F

CD \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
($\bar{C}\bar{D}$) 00	1	0	0	0
($\bar{C}D$) 01	1	X	1	0
(CD) 11	X	1	1	X
($C\bar{D}$) 10	1	0	0	0

I (BD)

II ($\bar{A}\bar{B}$)

$F = BD + \bar{A}\bar{B}$

2) $y_{ABCD} = \Sigma_m (1, 2, 5, 9, 13) + \Sigma_d (3, 6, 11, 15)$

y

CD \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}\bar{D}$ 00	0	4	12	8
$\bar{C}D$ 01	1 1	1 5	1 13	1 9
CD 11	X 3	7	X 15	X 11
$C\bar{D}$ 10	1 2	X 6	14	10

I ($\bar{C}D$)

II ($\bar{A}\bar{B}C$)

II ($\bar{A}C\bar{D}$)

$y = \bar{C}D + \bar{A}\bar{B}C$

or

$y = \bar{C}D + \bar{A}C\bar{D}$

3) Solve K-map

F

CD \ AB	$\bar{A}\bar{B}$ 00	$\bar{A}B$ 01	AB 11	$A\bar{B}$ 10
$\bar{C}\bar{D}$ 00	1	0	0	1
$\bar{C}D$ 01	1	0	X	X
CD 11	X	X	1	1
$C\bar{D}$ 10	1	0	0	1

II (AD)

II (CD)

I ($\bar{B}$)

$F = \bar{B} + AD$

or

$F = \bar{B} + CD$

K-Map for POS expression

Steps for POS expression

- take grouping of 0
- find function (f_d)
- Put compliment of all variables (f)

★ If boolean function is given by $Y = \Sigma m(3,6)$ then

a) $Y = B(A+C)(\bar{A}+\bar{C})$ ✓

b) $Y = B(A+\bar{C})(\bar{A}+\bar{C})$

c) $Y = \bar{B}(A+\bar{C})(\bar{A}+C)$

d) $Y = \bar{B}(A+C)(\bar{A}+\bar{C})$

2nd $(\bar{A}+\bar{C})$

C \ AB	00	01	11	10
0	0 (0)	0 (2)	1 (6)	0 (4)
1	0 (1)	1 (3)	0 (7)	0 (5)

1st

3rd

$Y_d = \bar{B}.(\bar{A}+\bar{C}).(A+C)$

$$\boxed{Y = B.(A+C).(\bar{A}+\bar{C})}$$

★ If boolean function is given by.

A B C	Y
000	1
001	0
010	0
011	1
100	0
101	1
110	1
111	0

→ function of Y in terms of POS

$Y =$ ____________

C \ AB	00	01	11	10
0	1 (0)	0 (2)	1 (6)	0 (4)
1	0 (1)	1 (3)	0 (7)	1 (5)

1st 2nd 3rd 4th

$Y_d = (\bar{A}+\bar{B}+C).(\bar{A}+B+\bar{C}).(A+B+C).(A+\bar{B}+\bar{C})$

$$\boxed{Y = (A+B+\bar{C}).(A+\bar{B}+C).(\bar{A}+\bar{B}+\bar{C}).(\bar{A}+B+\bar{C})}$$

5 Variable K-Map 79

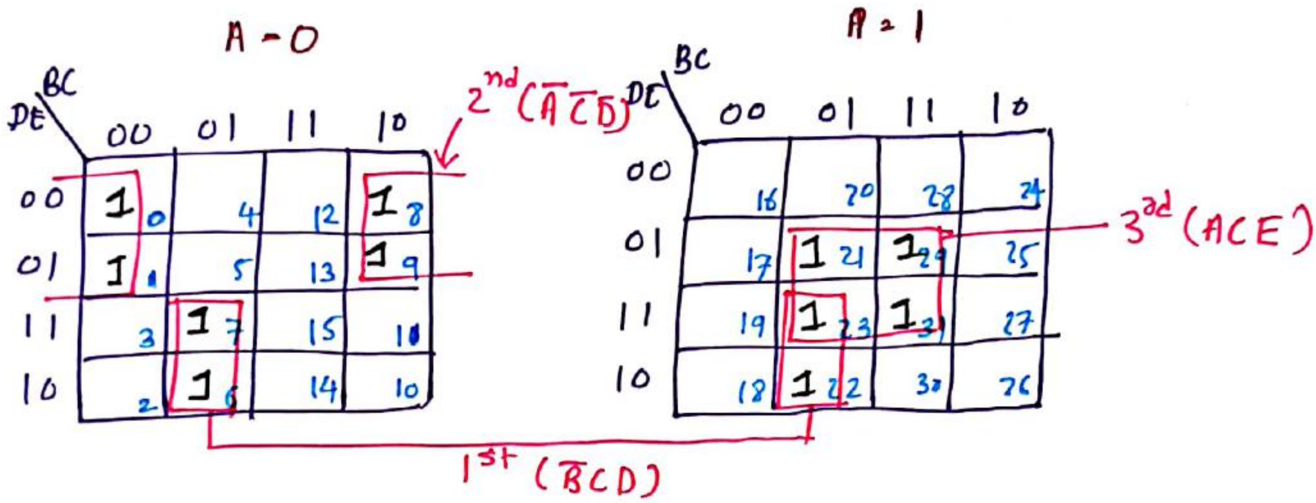
$f(\underline{A}, B, C, D, E) = \Sigma_m (0, 1, 6, 7, 8, 9, 21, 22, 23, 29, 31)$

$= \bar{B}CD + \bar{A}\bar{C}\bar{D} + ACE$

Total cell $= 2^n$

$= 2^5$

$= 32$



6 Variable K-Map. 80

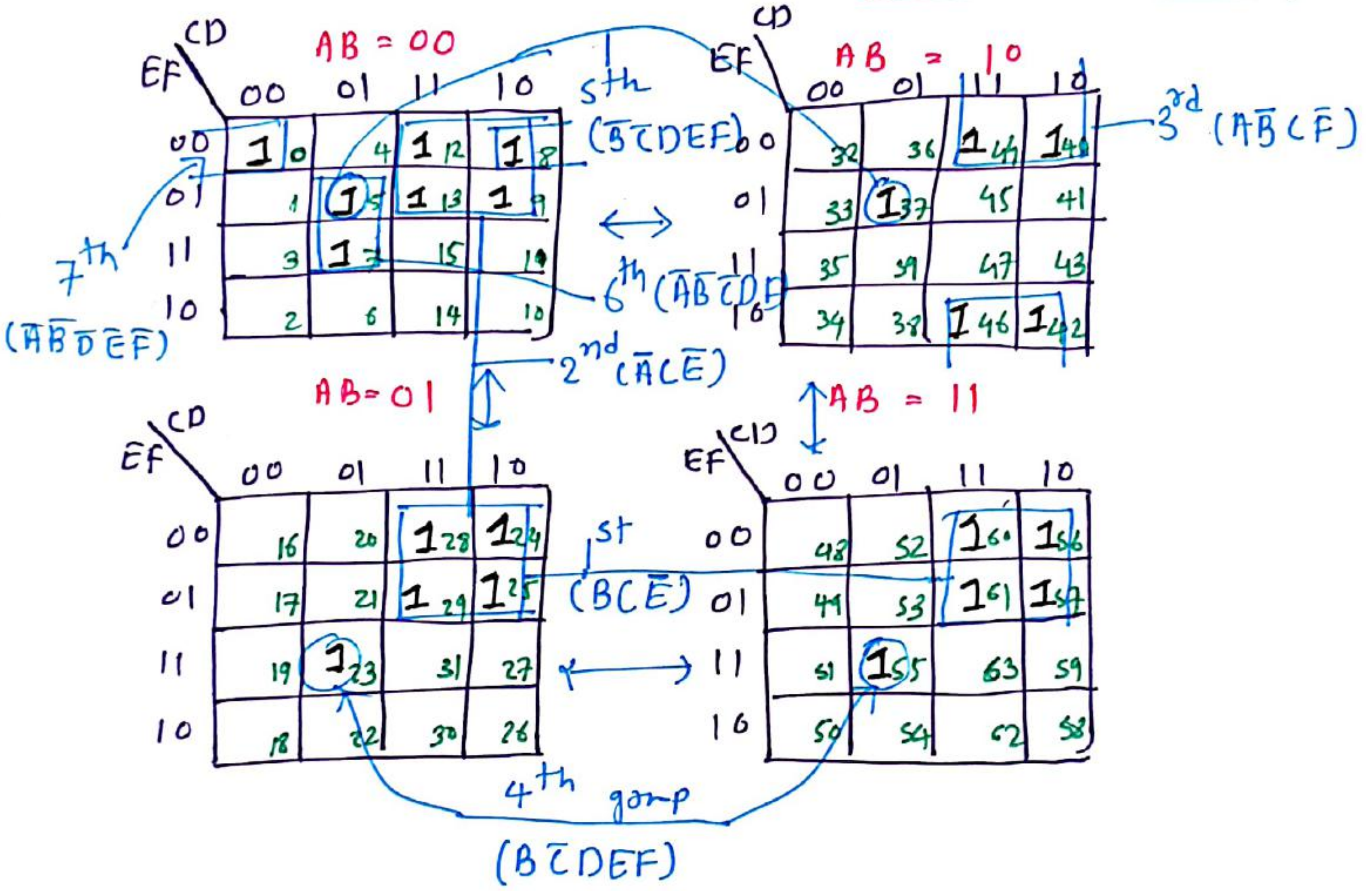
$f(A,B,C,D,E,F) = \Sigma m(0,5,7,8,9,12,13,23,24,25,28,29,37,40,42,44,46,55,56,57,60,61)$

$\downarrow$

$n = 6$

$\rightarrow$ total $= 2^n = 2^6 = 64$

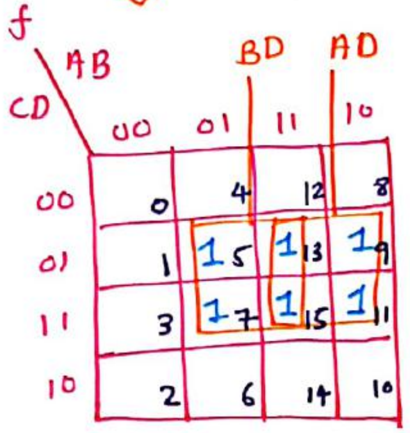
$$= \boxed{BC\bar{E} + \bar{A}C\bar{E} + A\bar{B}C\bar{F} + B\bar{C}DEF + \bar{B}\bar{C}DEF + \bar{A}B\bar{C}DF + \bar{A}\bar{B}\bar{D}\bar{E}\bar{F}}$$



K - map examples of GATE, DRDO & ISRO Examination 81

1] The function $f(A, B, C, D) = \Sigma_m(5, 7, 9, 11, 13, 15)$ is independent of Variables

a] B   b] C   c] A & C   d] D



$f = BD + AD$

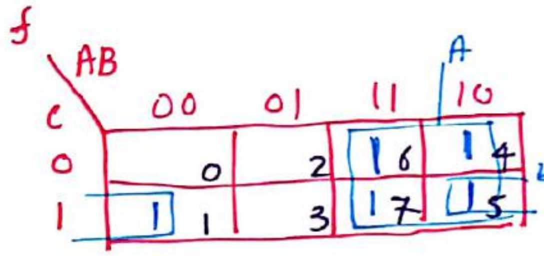
2] The Standard Sum of Product of the function $f = A + \bar{B}C$ is expressed as

(a) $\Sigma_m(1, 4, 5, 6, 7) + d(2, 3)$

(b) $\Sigma_m(1, 4, 5, 6, 7)$

(c) $\Sigma_m(0, 2, 3) + d(1, 4, 5, 6, 7)$

(d) $\pi M(1, 4, 5, 6, 7)$



$f = \Sigma_m(1, 4, 5, 6, 7)$

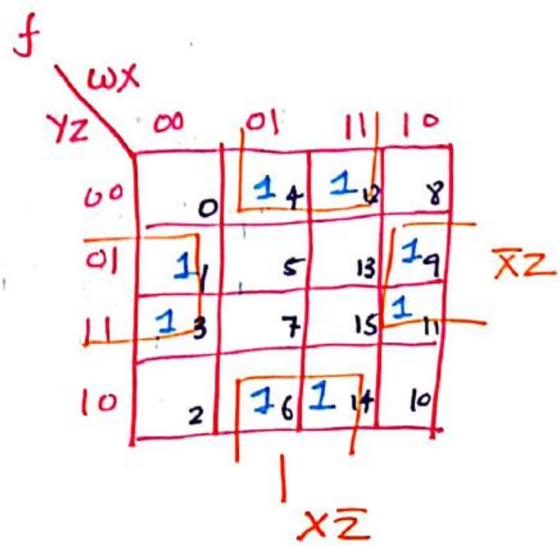
③ Consider the following Boolean function of four variables
$f(w, x, y, z) = \Sigma_m(1, 3, 4, 6, 9, 11, 12, 14)$, the function

(a) Independent of One Variable

(b) Independent of two Variable

(c) Independent of three Variable

(d) Dependent of all Variable.



$f = \bar{X}Z + \bar{Z}X$

$\boxed{f = X \oplus Z}$

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Quine McCluskey Minimization Technique

$Y(A,B,C,D) = \Sigma_m(0,1,3,7,8,9,11,15)$

Step-1 – Represent given Number in Binary.

N	A B C D
0	0 0 0 0 ✓
1	0 0 0 1 ✓
3	0 0 1 1 ✓
7	0 1 1 1 ✓
8	1 0 0 0 ✓
9	1 0 0 1 ✓
11	1 0 1 1 ✓
15	1 1 1 1

Step-2 – Form a group based on Number of 1's

group	Term	A B C D
group-0	(0)	0 0 0 0 ✓
group-1	(1)	0 0 0 1 ✓
	(8)	1 0 0 0 ✓
group-2	(3)	0 0 1 1 ✓
	(9)	1 0 0 1 ✓
group-3	(7)	0 1 1 1 ✓
	(11)	1 0 1 1 ✓
group-4	(15)	1 1 1 1 ✓

Step-3 Find matched Pair with One bit Difference & Mark location

group	Pair	A B C D
group-0	(0-1)	0 0 0 – ✓
	(0-8)	– 0 0 0 ✓
group-1	(1-3)	0 0 – 1 ✓
	(1-9)	– 0 0 1 ✓
	(8-9)	1 0 0 – ✓
group-2	(3-7)	0 – 1 1 ✓
	(3-11)	– 0 1 1 ✓
	(9-11)	1 0 – 1 ✓
group-3	(7-15)	– 1 1 1 ✓
	(11-15)	1 – 1 1 ✓

Step-4 – Repeat Step 3 till end.

group	Pair	A B C D
group-0 $\bar{B}\bar{C}$	(0-1-8-9)	– 0 0 –
	(0-8-1-9)	– 0 0 –
group-1 $\bar{B}D$	(1-3-9-11)	– 0 – 1
	(1-9-3-11)	– 0 – 1
group-2 CD	(3-7-11-15)	– – 1 1
	(3-11-7-15)	– – 1 1

Step-5 Implicant Table.

PI	Minterms	0	1	3	7	8	9	11	15
$\bar{B}\bar{C}$	0-1-8-9	X	X			X	X		
$\bar{B}D$	1-3-9-11		X	X			X	X	
CD	3-7-11-15			X	X			X	X

$\boxed{Y = \bar{B}\bar{C} + CD}$

Y

CD \ AB	00	01	11	10
00	1 (0)	4	12	1 (8)
01	1 (1)	5	13	1 (9)
11	1 (3)	1 (7)	1 (15)	1 (11)
10	2	6	14	10

$\bar{C}\bar{B}$

CD