DIGITAL ELECTRONICS CIRCUIT(BCA 103)

DEPARTMENT OF COMPUTER SCIENCE
PROGRAMME: BCA

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DIGITAL LOGIC

- Since Boolean functions are expressed in terms of AND, OR, and NOT operations, it is easier to implement a Boolean function with these type of gates.
- Factors to be weighed in considering the construction of other types of logic gates are:

(1) the feasibility and economy of producing the gate with physical components,

(2) the possibility of extending the gate to more than two inputs,

(3) the basic properties of the binary operator, such as commutativity and associativity, and

(4) the ability of the gate to implement Boolean functions alone or in conjunction with other gates.

- There are 2^{2n} functions for *n* binary variables.
- Thus, for two variables, $n = 2$, and the number of possible Boolean functions is 16.
- The 16 functions listed can be subdivided into three categories: **1. Two** functions that produce a constant 0 or 1. **2. Four** functions with unary operations: complement and transfer. **3. Ten** functions with binary operators that define eight different operations: AND, OR, NAND, NOR, exclusive-OR, equivalence, inhibition, and implication.
- Of the 16 functions, two are equal to a constant and four are repeated.
- There are only 10 functions left to be considered as candidates for logic gates.
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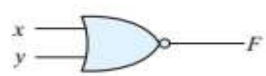
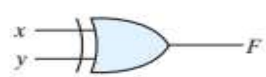
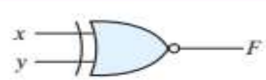
- Two—inhibition and implication—are not commutative or associative and thus are impractical to use as standard logic gates.
- The other eight—complement, transfer, AND, OR, NAND, NOR, exclusive-OR, and equivalence—are used as standard gates in digital design.

DIGITAL LOGIC GATES

Name	Graphic symbol	Algebraic function	Truth table
AND		$F = x \cdot y$	x y | F 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1
OR		$F = x + y$	x y | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1
Inverter		$F = x'$	x | F 0 | 1 1 | 0
Buffer		$F = x$	x | F 0 | 0 1 | 1
NAND		$F = (xy)'$	x y | F 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0
NOR		$F = (x + y)'$	x y | F 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0
Exclusive-OR (XOR)		$F = xy' + x'y$ $= x \oplus y$	x y | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0
Exclusive-NOR or equivalence		$F = xy + x'y'$ $= (x \oplus y)'$	x y | F 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 1

- Each gate has one or two binary input variables, designated by x and y, and one binary output variable, designated by F.
- The inverter circuit inverts the logic sense of a binary variable, producing the NOT, or complement, function.
- The small circle in the output of the graphic symbol of an inverter (referred to as a *bubble*) designates the logic complement.
- The triangle symbol by itself designates a buffer circuit.
- A buffer produces the *transfer* function, but does not produce a logic operation, since the binary value of the output is equal to the binary value of the input.
- This circuit is used for power amplification of the signal and is equivalent to two inverters connected in cascade.

- The NAND function is the complement of the AND function, as indicated by a graphic symbol that consists of an AND graphic symbol followed by a small circle.
- The NOR function is the complement of the OR function and uses an OR graphic symbol followed by a small circle.
- NAND and NOR gates are used extensively as standard logic gates and are in fact far more popular than the AND and OR gates.
- This is because NAND and NOR gates are easily constructed with transistor circuits and because digital circuits can be easily implemented with them.
- The exclusive-OR gate has a graphic symbol similar to that of the OR gate, except for the additional curved line on the input side.
- The equivalence, or exclusive-NOR, gate is the complement of the exclusive-OR, as indicated by the small circle on the output side of the graphic symbol.

EXTENSION TO MULTIPLE INPUTS

- The gates, except for the inverter and buffer—can be extended to have more than two inputs.
- A gate can be extended to have multiple inputs if the binary operation it represents is commutative and associative.
- The AND and OR operations, defined in Boolean algebra, possess these two properties.
- For the OR function, we have

  $x + y = y + x$ (commutative) and

  $(x + y) + z = x + (y + z) = x + y + z$ (associative)

  which indicates that the gate inputs can be interchanged and that the OR function can be extended to three or more variables.
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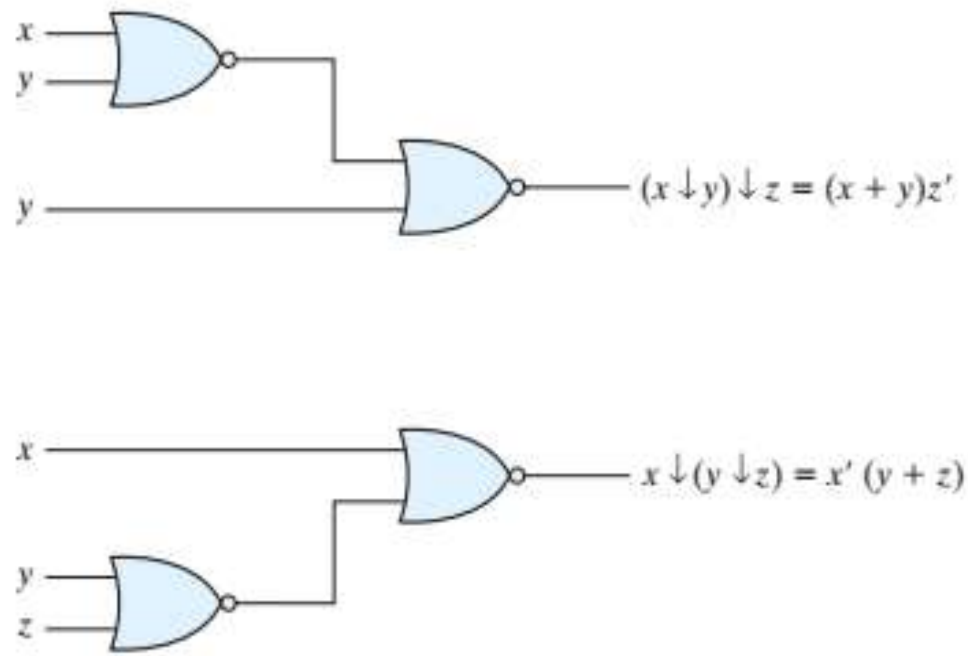
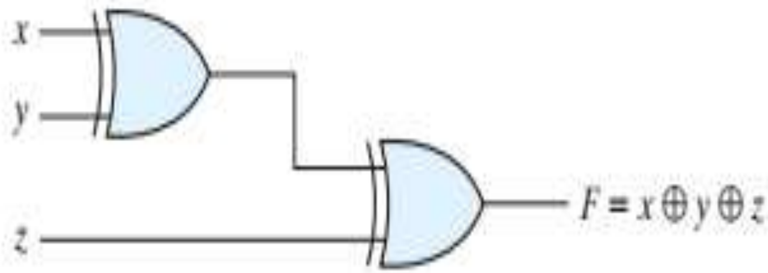
- The NAND and NOR functions are commutative, and their gates can be extended to have more than two inputs, provided that the definition of the operation is modified slightly.
- The difficulty is that the NAND and NOR operators are not associative (i.e., $(x \downarrow y) \downarrow z \neq x \downarrow (y \downarrow z)$)
- i.e. $(x \downarrow y) \downarrow z = [(x + y)' + z]' = (x + y)z' = xz' + yz'$
  $x \downarrow (y \downarrow z) = [x + (y + z)']' = x'(y + z) = x'y + x'z$
- To overcome this difficulty, we define the multiple NOR (or NAND) gate as a complemented OR (or AND) gate. Thus, by definition, we have:
  NOR : $x \downarrow y \downarrow z = (x + y + z)'$
  NAND: $x \uparrow y \uparrow z = (xyz)'$
- In writing cascaded NOR and NAND operations, one must use the correct parentheses to signify the proper sequence of the gates.

Demonstrating the non-associativity of the NOR operator

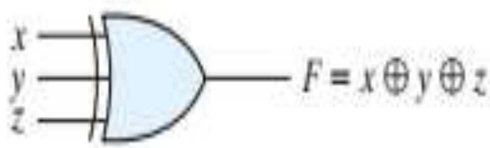


- The exclusive-OR and equivalence gates are both commutative and associative and can be extended to more than two inputs.
- However, multiple-input exclusive-OR gates are uncommon from the hardware standpoint.
- The definition of the function must be modified when extended to more than two variables.
- Exclusive-OR is an *odd* function (i.e., it is equal to 1 if the input variables have an odd number of 1’s).
- The output (Exclusive-OR)F is equal to 1 if only one input is equal to 1 or if all three inputs are equal to 1 (i.e., when the total number of 1’s in the input variables is *odd*).

Three-input exclusive-OR gate



(a) Using 2-input gates



(b) 3-input gate

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

(c) Truth table

Gate-Level Minimization

- *Gate-level minimization* is the design task of finding an optimal gate-level implementation of the Boolean functions describing a digital circuit.
- This task is well understood, but is difficult to execute by manual methods when the logic has more than a few inputs.
- Fortunately, computer-based logic synthesis tools can minimize a large set of Boolean equations efficiently and quickly.
- The complexity of the digital logic gates that implement a Boolean function is directly related to the complexity of the algebraic expression from which the function is implemented.
- Although the truth table representation of a function is unique, when it is expressed algebraically it can appear in many different, but equivalent, forms.

The Map Method

- The map method provides a simple, straightforward procedure for minimizing Boolean functions.
- This method may be regarded as a pictorial form of a truth table. The map method is also known as the *Karnaugh map* or *K-map*.
- A K-map is a diagram made up of squares, with each square representing one minterm of the function that is to be minimized.
- Since any Boolean function can be expressed as a sum of minterms, it follows that a Boolean function is recognized graphically in the map from the area enclosed by those squares whose minterms are included in the function.
- In fact, the map presents a visual diagram of all possible ways a function may be expressed in standard form.

- The simplified expressions produced by the map are always in one of the two standard forms: sum of products or product of sums.
- It will be assumed that the simplest algebraic expression is an algebraic expression with a minimum number of terms and with the smallest possible number of literals in each term.
- This expression produces a circuit diagram with a minimum number of gates and the minimum number of inputs to each gate.
- The simplest expression is not unique: It is sometimes possible to find two or more expressions that satisfy the minimization criteria.
- In that case, either solution is satisfactory.

Two-Variable K-Map

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x \ y	0	1
0	m_0	m_1
1	m_2	m_3

x \ y	0	1
0	x'y'	x'y
1	xy'	xy

Two-Variable K-Map

- Ex: xy

x \ y	0	1
0	m_0	m_1
1	m_2	m_3

x \ y	0	1
0	**x'y'**	**x'y**
1	xy'	xy

x \ y	0	1
0	**0**	**0**
1	0	1

Two-Variable K-Map

- Ex: x + y

  These squares are found from the minterms of the function:

$$m_1 + m_2 + m_3 = x'y + xy' + xy = x'y + x(y + y') = x'y + x = (x + y)(x + x') = x + y$$

x \ y	0	1
0	m_0	m_1
1	m_2	m_3

x \ y	0	1
0	x'y'	x'y
1	xy'	xy

x \ y	0	1
0	0	1
1	1	1

Three Variable K-Map

x \ y	00	01	11	10
0	**x'y'z'**	**x'y'z**	**x'yz**	**x'yz'**
1	xy'z'	xy'z	xyz	xyz'

x \ y	00	01	11	10
0	$\mathbf{m_0}$	$\mathbf{m_1}$	$\mathbf{m_3}$	$\mathbf{m_2}$
1	m_4	m_5	m_7	m_6

- There are eight minterms for three binary variables; therefore, the map consists of eight squares.
- Note that the minterms are arranged, not in a binary sequence, but in a sequence similar to the Gray code.
- The characteristic of this sequence is that **only one bit changes in value from one adjacent column to the next.**

- To understand the usefulness of the map in simplifying Boolean functions, we must recognize the basic property possessed by adjacent squares:

Any two adjacent squares in the map differ by only one variable, which is primed in one square and unprimed in the other.

- From the postulates of Boolean algebra, the sum of two minterms in adjacent squares can be simplified to a single product term consisting of only two literals.
- Any two minterms in adjacent squares (vertically or horizontally, but not diagonally, adjacent) that are ORed together will cause a removal of the dissimilar variable.
- **Ex:** $m_5 + m_7 = xy'z + xyz = xz(y' + y) = xz$
Here, the two squares differ by the variable y, which can be removed when the sum of the two minterms is formed.

Three Variable K-Map

- **Simplify the Boolean function $F\ (x, y, z) = \sum(2, 3, 4, 5)$**

x \ yz	00	01	11	10
0	**x'y'z'**	**x'y'z**	**x'yz**	**x'yz'**
1	xy'z'	xy'z	xyz	xyz'

x \ yz	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

x \ yz	00	01	11	10
0	0	0	1	1
1	1	1	0	0

x \ yz	00	01	11	10
0	0	0	1	1
1	1	1	0	0

- $m_3 + m_2 = x'yz + x'yz' = x'y$
- $m_4 + m_5 = xy'z' + xy'z = xy'$ Hence, $F\ (x, y, z) = \sum(2, 3, 4, 5) = x'y + xy'$

Three Variable K-Map

- **Simplify the Boolean function *F* (*x*, *y*, *z*) = ∑(3, 4, 6, 7)**

x \ y	00	01	11	10
0	**x’y’z’**	**x’y’z**	**x’yz**	**x’yz’**
1	xy’z’	xy’z	xyz	xyz’

x \ y	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

x \ y	00	01	11	10
0	**0**	**0**	**1**	**0**
1	1	0	1	1

x \ y	00	01	11	10
0	**0**	**0**	**1**	**0**
1	1	0	1	1

- $m_3 + m_7 =$ x’yz + xyz = yz
- $m_4 + m_6 =$ xy’z’ + xyz’ = xz’ Hence, ***F* (*x*, *y*, *z*) = ∑(3, 4, 6, 7) = yz + xz’**

- The number of adjacent squares that may be combined must always represent a number that is a power of two, such as 1, 2, 4, and 8.
- As more adjacent squares are combined, we obtain a product term with fewer literals.

- One square represents one minterm, giving a term with three literals.
- Two adjacent squares represent a term with two literals.
- Four adjacent squares represent a term with one literal.
- Eight adjacent squares encompass the entire map and produce a function that is always equal to 1.

Three Variable K-Map

- **Simplify the Boolean function** $\boldsymbol{F\,(x, y, z) = \sum(0, 2, 4, 5, 6)}$

x \ y	00	01	11	10
0	x'y'z'	x'y'z	x'yz	x'yz'
1	xy'z'	xy'z	xyz	xyz'

x \ y	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

x \ y	00	01	11	10
0	1	0	0	1
1	1	1	0	1

x \ y	00	01	11	10
0	1	0	0	1
1	1	1	0	1

- $m_0 + m_2 + m_4 + m_6 = x'y'z' + x'yz' + xy'z' + xyz' = z'$
- $m_4 + m_5 = xy'z' + xy'z = xy'$ Hence, $\boldsymbol{F\,(x, y, z) = \sum(0, 2, 4, 5, 6) = z' + xy'}$

- If a function is not expressed in sum-of-minterms form, it is possible to use the map to obtain the minterms of the function and then simplify the function to an expression with a minimum number of terms.
- It is necessary, however, to make sure that the algebraic expression is in sum-of-products form.
- Each product term can be plotted in the map in one, two, or more squares. The minterms of the function are then read directly from the map.

Three Variable K-Map

- For the Boolean function $F = A'C + A'B + AB'C + BC$

  (a) Express this function as a sum of minterms. $F = A'C + A'B + AB'C + BC = \sum(1, 2, 3, 5, 7)$

  (b) Find the minimal sum-of-products expression.

x \ y	00	01	11	10
0	A'B'C'	A'B'C	A'BC	A'BC'
1	AB'C'	AB'C	ABC	ABC'

x \ y	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

x \ y	00	01	11	10
0	0	1	1	1
1	0	1	1	0

x \ y	00	01	11	10
0	0	1	1	1
1	0	1	1	0

- $m_1 + m_3 + m_5 + m_7 = A'B'C + A'BC + AB'C + ABC = C$
- $m_3 + m_2 = A'BC + A'BC' = A'B$ Hence, $F(A, B, C) = \sum(1, 2, 3, 5, 7) = C + A'B$

Four Variable K-Map

wx \ yz	00	01	11	10
00	**w'x'y'z'**	**w'x'y'z**	**w'x'yz**	**w'x'yz'**
01	w'xy'z'	w'xy'z	w'xyz	w'xyz'
11	wxy'z'	wxy'z	wxyz	wxyz'
10	wx'y'z'	wx'y'z	wx'yz	wx'yz'

wx \ yz	00	01	11	10
00	$\mathbf{m_0}$	$\mathbf{m_1}$	$\mathbf{m_3}$	$\mathbf{m_2}$
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

Four Variable K-Map

- **Simplify the Boolean function *F* (*w, x, y, z*) = ∑(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)**

wx \ yz	00	01	11	10
00	**w'x’y’z’**	**w'x’y’z**	**w'x’yz**	**w'x’yz’**
01	w'xy’z’	w'xy’z	w'xyz	w'xyz’
11	wxy'z’	wxy'z	wxyz	wxyz'
10	wx'y’z’	wx'y’z	wx'yz	wx'yz’

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

wx \ yz	00	01	11	10
00	**1**	**1**	**0**	**1**
01	1	1	0	1
11	1	1	0	1
10	1	1	0	0

wx \ yz	00	01	11	10
00	**1**	**1**	**0**	**1**
01	1	1	0	1
11	1	1	0	1
10	1	1	0	0

Four Variable K-Map

- **Simplify the Boolean function *F* (*w*, *x*, *y*, *z*) = ∑(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)**

wx \ yz	00	01	11	10
00	**w'x'y'z'**	**w'x'y'z**	**w'x'yz**	**w'x'yz'**
01	w'xy'z'	w'xy'z	w'xyz	w'xyz'
11	wxy'z'	wxy'z	wxyz	wxyz'
10	wx'y'z'	wx'y'z	wx'yz	wx'yz'

m_0	**m_1**	**m_3**	**m_2**
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

wx \ yz	00	01	11	10
00	**1**	**1**	**0**	**1**
01	1	1	0	1
11	1	1	0	1
10	1	1	0	0

wx \ yz	00	01	11	10
00	**1**	**1**	**0**	**1**
01	1	1	0	1
11	1	1	0	1
10	1	1	0	0

Four Variable K-Map

- **Simplify the Boolean function *F* (*w*, *x*, *y*, *z*) = ∑(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)**

wx \ yz	**00**	**01**	**11**	**10**
00	**w'x'y'z'**	**w'x'y'z**	**w'x'yz**	**w'x'yz'**
01	w'xy'z'	w'xy'z	w'xyz	w'xyz'
11	wxy'z'	wxy'z	wxyz	wxyz'
10	wx'y'z'	wx'y'z	wx'yz	wx'yz'

m_0	**m_1**	**m_3**	**m_2**
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

wx \ yz	**00**	**01**	**11**	**10**
00	**1**	**1**	**0**	**1**
01	1	1	0	1
11	1	1	0	1
10	1	1	0	0

wx \ yz	**00**	**01**	**11**	**10**
00	**1**	**1**	**0**	**1**
01	1	1	0	1
11	1	1	0	1
10	1	1	0	0

- i) $m_0 + m_1 + m_4 + m_5 + m_8 + m_9 + m_{12} + m_{13}$

  $= w'x'y'z' + w'x'y'z + w'xy'z' + w'xy'z + wx'y'z' + wx'y'z + wx'y'z' + wx'y'z$

  $= y'$

- ii) $m_0 + m_2 + m_4 + m_6 = w'x'y'z' + w'x'yz' + w'xy'z' + w'xyz'$

  $= w'z'$

- iii) $m_4 + m_6 + m_{12} + m_{14} = w'xy'z' + w'xyz' + wxy'z' + wxyz'$

  $= xz'$

- Hence, $\boldsymbol{F\ (w, x, y, z) = \sum(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)}$

  $= y' + w'z' + xz'$

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