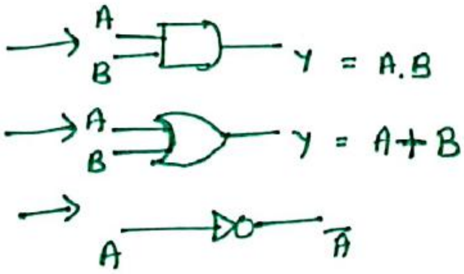
De Morgen's Theorm 40

- Digital Circuit can be implemented by many ways.
- De Morgen's theorm is used to simplify boolean equation.
- Digital expression is simply made by three basic operation

1] Boolean AND → $Y = A.B$

2] Boolean OR → $Y = A+B$

3] Boolean NOT →



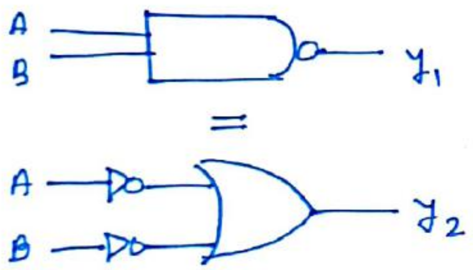
- De Morgen's Law

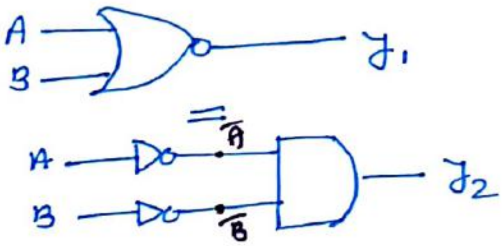
$$\overline{A.B} = \overline{A} + \overline{B} \quad (1)$$

- Compliment of product is Sum of Compliment

$$\overline{A+B} = \overline{A}.\overline{B} \quad (2)$$

- Compliment of sum is Product of Compliment





Truth Table.

A	B	y_1	$\overline{A}$	$\overline{B}$	y_2
0	0	1	1	1	1
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	0

A	B	y_1	$\overline{A}$	$\overline{B}$	y_2
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	0

Ex

$$\overline{X + \overline{Y.Z}} = \overline{X}.\overline{P}$$
$$= \overline{X}.\overline{\overline{Y.Z}}$$
$$= \overline{X}.Y.Z$$

Boolean Rules [Boolean Algebra Rules]

AND gate.

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

- $A.A = A$
- $A.0 = 0$
- $A.1 = A$
- $A.\bar{A} = 0$

NOT Gate

A	Y
0	1
1	0

- $\bar{\bar{A}} = A$

OR gate

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

- $A + A = A$
- $A + 0 = A$
- $A + 1 = 1$
- $A + \bar{A} = 1$

Distributive Law

- $A.(B+C) = A.B + A.C$

Imp - $A + (B.C) = (A+B).(A+C)$

Imp
- $A + \bar{A}B = (A+\bar{A}).(A+B) = A+B$
- $\bar{A} + AB = (\bar{A}+A).(\bar{A}+B) = \bar{A}+B$

Commutative Law

$A + B = B + A$

&

$A.B = B.A$

Associative law.

$(A+B)+C = A+(B+C)$

&

$(A.B).C = A.(B.C)$.

De Morgan's Law

$\overline{A+B} = \bar{A}.\bar{B}$ —— (1)

$\overline{A.B} = \bar{A} + \bar{B}$ —— (2)

- Priority → NOT | Highest
  → AND
  → OR ↓ Lowest

Ex

$P = XY\bar{Z} + X\bar{Y}\bar{Z} + Y\bar{Z}$

$= X\bar{Z}(Y+\bar{Y}) + Y\bar{Z}$

$= X\bar{Z} + Y\bar{Z}$

$= (X+Y).\bar{Z}$

$A.1 = A$ 1 | 0

Boolean Algebra Rules

NOT

$$\overline{\overline{A}} = A$$

AND

$$A.A = A$$
$$A.0 = 0$$
$$A.1 = A$$
$$A.\overline{A} = 0$$

OR

$$A + A = A$$
$$A + 0 = A$$
$$A + 1 = 1$$
$$A + \overline{A} = 1$$

Distributive Law

$$A + \overline{A}B = A + B$$
$$\overline{A} + AB = \overline{A} + B$$
$$A.(B + C) = A.B + A.C$$

De-Morgen's Law

$$\overline{A + B} = \overline{A} \cdot \overline{B}$$
$$\overline{A.B} = \overline{A} + \overline{B}$$

Boolean Algebra Examples 43

Consensus Theorm 42

Theorm.1 $AB + \bar{A}C + BC = AB + \bar{A}C$

$LHS = AB + \bar{A}C + BC(A+\bar{A})$

$= AB + \bar{A}C + BCA + BC\bar{A}$

$= AB(1+C) + \bar{A}C(1+B)$

$= AB + \bar{A}C$

$= RHS$

Theorm.2 $(A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$

$LHS = (A+B)(\bar{A}+C)(B+C)$

$= (A\bar{A} + AC + B\bar{A} + BC)(B+C)$ $[A\bar{A} = 0]$

$= (AC + B\bar{A} + BC)(B+C)$

$= ACB + B\bar{A}B + BCB + ACC + B\bar{A}C + BCC$

$= ABC + B\bar{A} + BC + AC + \bar{A}BC + BC$

$= ABC + B\bar{A} + BC + AC + \bar{A}BC$

$= BC[A + 1 + \bar{A}] + B\bar{A} + AC$

$= BC + B\bar{A} + AC$

$RHS = (A+B)(\bar{A}+C)$

$= A\bar{A} + AC + B\bar{A} + BC$

$= AC + B\bar{A} + BC$

Boolean Algebra Examples

1] $\overline{\overline{AB} + \bar{A} + AB}$

$= \overline{\bar{X} + \bar{A} + X}$ $\quad [\because X = A.B]$

$= \overline{\bar{A} + 1}$

↓ You may not see that sometimes.

$= \bar{1}$

$= 0$

$= \overline{\overline{AB} + \bar{A} + AB}$

$= \overline{\overline{AB}} \cdot \bar{\bar{A}} \cdot \overline{AB}$

$= AB \cdot A \cdot \overline{AB}$

$= AB \cdot \overline{AB}$

$= 0$

2] $A(B + \bar{C}\,\overline{(AB + A\bar{C})})$

$= A.(B + \bar{C}[\overline{AB} \cdot \overline{A\bar{C}}])$

$= A.(B + \bar{C}[(\bar{A} + \bar{B}).(\bar{A} + \bar{\bar{C}})])$

$= A.(B + \bar{C}[(\bar{A} + \bar{B}).(\bar{A} + C)])$

$= A.(B + \bar{C}(\bar{A}.\bar{A} + \bar{A}C + \bar{B}\bar{A} + \bar{B}C))$

$= A.(B + \bar{C}(\bar{A} + \bar{A}C + \bar{A}\bar{B} + \bar{B}C))$

$= A.(B + \bar{C}(\bar{A}(1 + C + \bar{B}) + \bar{B}C))$

$= A.(B + \bar{C}(\bar{A} + \bar{B}C))$

$= A.(B + \bar{C}\bar{A} + \bar{B}C\bar{C})$

$= A.(B + \bar{C}\bar{A})$

$= AB + \bar{C}\bar{A}A$

$= AB$

3] If $\bar{A} + AB = 0$, then find values of A & B.

$\Rightarrow \bar{A} + AB = 0$

$\Rightarrow \bar{A} + B = 0$

$\boxed{A = 1}$ & $\boxed{B = 0}$

Boolean Algebra Examples hh

1) $A\bar{B}C(C+BD)+\bar{A}\bar{B}$, Simplify given boolean eq^n

$= A\bar{B}C(C+BD)+\bar{A}\bar{B}$

$= A\bar{B}C + A\bar{B}BC + \bar{A}\bar{B}$

$= A\bar{B}C + \bar{A}\bar{B}$

$= \bar{B}(\bar{A}+AC)$

$= \boxed{\bar{B}(\bar{A}+C)}$

2) $\overline{ABC}\,(\overline{A+B+C})$

$= (\bar{A}+\bar{B}+\bar{C}).(\bar{A}.\bar{B}.\bar{C})$

$= \bar{A}\bar{A}\bar{B}\bar{C} + \bar{B}\bar{A}\bar{B}\bar{C} + \bar{C}\bar{A}\bar{B}\bar{C}$

$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}$

$= \bar{A}\bar{B}\bar{C}$

3) $\overline{(A+\overline{BC})}\,(A\bar{B}+\overline{ABC})$

$= (\bar{A}.\overline{\overline{BC}})(A\bar{B}+\bar{A}+\bar{B}+\bar{C})$

$= (\bar{A}.BC)(A\bar{B}+\bar{A}+\bar{B}+\bar{C})$

$= (\bar{A}.BC)(\bar{B}(A+1)+\bar{A}+\bar{C})$

$= \bar{A}BC(\bar{A}+\bar{B}+\bar{C})$

$= \bar{A}\bar{A}BC + \bar{B}\bar{A}BC + \bar{C}\bar{A}BC$

$= \bar{A}BC$

4) $A+\bar{B}C(A+\overline{BC})$

$= A+\bar{B}C(A+\bar{\bar{B}}+\bar{C})$

$= A+\bar{B}C(A+B+\bar{C})$

$= A+\bar{B}CA+\bar{B}CB+\bar{B}C\bar{C}$

$= A+\bar{B}CA$

$= A(1+\bar{B}C)$

$= A$

Boolean Algebra Examples 45

1] If $x = 1$ in the logic eq.^{n}

$$[x + z[\bar{y} + [\bar{z} + x\bar{y}]]]\,[\bar{x} + \bar{z}(x+y)] = 1$$

then

A] $y = z$ B] $y = \bar{z}$ C] $z = 1$ D] $z = 0$

$\Rightarrow [x + z[\bar{y} + [\bar{z} + x\bar{y}]]]\,[\bar{x} + \bar{z}(x+y)] = 1$

$\Rightarrow [1 + z[\bar{y} + [\bar{z} + 1.\bar{y}]]]\,[0 + \bar{z}(1+y)] = 1$

$[1 + A] = 1$ $[0 + A = A]$

$\Rightarrow 1.[\bar{z}.1] = 1$

$\Rightarrow \bar{z} = 1$

$\Rightarrow \boxed{z = 0}$

2] If we have 3 variables A, B & C. Find the output $Y = 1$ for majority of "1" in A, B & C. also minimise the function   Hw "0"

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1 ←	1
1	0	0	0
1	0	1 ←	1
1	1	0 ←	1
1	1	1 ←	1

$Y = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$

$= \bar{A}BC + A\bar{B}C + AB(\bar{C} + C)$

$[\bar{C} + C = 1]$

$= \bar{A}BC + A\bar{B}C + AB$

$= \bar{A}BC + A[\bar{B}C + B]$

[As distributive law $\bar{B}C + B = B + C$]

$= \bar{A}BC + A(B + C)$

$= \bar{A}BC + AB + AC$

$= B[\bar{A}C + A] + AC$

$[\bar{A}C + A = A + C]$

$= B(A + C) + AC$

$= \boxed{BA + BC + AC}$

Dual and Self Dual of Boolean expression

- To get dual of given expression, we need to replace
  - * OR with AND
  - * AND with OR
  - * 1 with 0
  - * 0 with 1

<u>e.g. 1</u> Find out dual of

$\Rightarrow (A+B).(\bar{A}+C).(B+C) = (A+B).(\bar{A}+C)$

$\Rightarrow \boxed{A.B + \bar{A}.C + B.C = A.B + \bar{A}.C}$

<u>e.g. 2</u> $F = AB + A\bar{B}C + A\bar{C}$, Then find dual of F.

$F_d = (A+B).(A+\bar{B}+C).(A+\bar{C})$

- <u>Self Dual</u> - If dual of function is same function then It is reffered as self dual.

<u>e.g. 3</u> $F = AB + BC + AC$

Find given function F is self dual or not.

$F_d = (A+\underline{B}).(\underline{B}+C).(A+C)$

$= (B + A.C).(A+C)$ [As per distribution rule $(A+B)(B+C) = (B+AC)$]

$= BA + BC + \underline{A}.C.\underline{A} + A.\underline{C}.\underline{C}$

$= BA + BC + \underline{AC} + \underline{AC}$

$= BA + BC + AC$

$\boxed{F_d = F}$ So function F is self dual.

- For n number of variable, total self dual $= 2^{2^{n-1}}$

<u>e.g. 4</u> For $n = 2$ variable findout total self dual.

$= 2^{2^{n-1}} = 2^{2^{2-1}} = 2^{2} = 4$

→ we have variable A & B.

$A \longleftrightarrow A$

$B \longleftrightarrow B$

$\bar{A} \longleftrightarrow \bar{A}$

$\bar{B} \longleftrightarrow \bar{B}$

<u>e.g. 5</u> If $n = 5$ variables, then find total self dual.

$= 2^{2^{n-1}} = 2^{2^{5-1}} = 2^{2^{4}} = 2^{16} = 65536$

SOP, POS & Canonical Form of Boolean function Representation

SOP - Sum of Product [DNF - Disjunctive Normal Form]

- It is a summation of Product terms.

Summation

$$\text{Eg}\quad Y = A.B + A.\bar{B}.C + A\bar{B}\bar{C}$$

Product terms.

POS - Product of sum [CNF - Conjunctive Normal Form].

- It is a product of sum terms.

Product.

$$\text{Eg.}\quad Y = (A+B).(A+C).(\bar{A}+\bar{B})$$

Summation terms.

Canonical Form

Standard SOP (SSOP)	Standard POS (SPOS)
- Each product term contains all the variables of the function.	- Each sum terms contains all the variables of the function.
Eg. $F(A,B,C) = A.B.C + \bar{A}.\bar{B}.C + A.B.\bar{C}$	Eg. $F(A,B,C) = (A+B+\bar{C}).(\bar{A}+B+C)$
SSOP ✓	SPOS ✓
SOP ✗	POS ✗

Engineering Funda YouTube Channel

Minterms and Maxterms in Boolean function Representation

Minterms

- Each Individual term in SSOP is called as Minterms

Maxterms

- Each Individual term in SPOS is called as Maxterm

For 3 variables

F	ABC	Minterms.	Maxterms
0	000	$\bar{A}\bar{B}\bar{C} \rightarrow m_0$	$(A+B+C) \rightarrow m0$
1	001	$\bar{A}\bar{B}C \rightarrow m_1$	$(A+B+\bar{C}) \rightarrow m1$
0	010	$\bar{A}B\bar{C} \rightarrow m_2$	$(A+\bar{B}+C) \rightarrow m_2$
1	011	$\bar{A}BC \rightarrow m_3$	$(A+\bar{B}+\bar{C}) \rightarrow m3$
1	100	$A\bar{B}\bar{C} \rightarrow m_4$	$(\bar{A}+B+C) \rightarrow m4$
0	101	$A\bar{B}C \rightarrow m_5$	$(\bar{A}+B+\bar{C}) \rightarrow m5$
1	110	$AB\bar{C} \rightarrow m_6$	$(\bar{A}+\bar{B}+C) \rightarrow m6$
1	111	$ABC \rightarrow m_7$	$(\bar{A}+\bar{B}+\bar{C}) \rightarrow m7$

$F = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + ABC$

$= \Sigma m(1,3,4,6,7)$

$F = (A+B+C).(A+\bar{B}+C)(\bar{A}+B+\bar{C})$

$= \Pi M(0,2,5)$

- Max-terms and Min-terms are compliment to each other.

$F_{ABC} = \Sigma m(0,2,3,5) \longrightarrow F_{ABC} = \Pi M(1,4,6,7)$

↑

3 variables $\longrightarrow$ total min & max terms $= 2^3 = 8$ $(0,1,2,3,4,5,6,7)$

SOP to SSOP Conversion 49

Step-1 - Indentify the missing variables in product terms.

$$F(A,B,C) = \underbrace{A.B}_{\substack{\uparrow \\ C \text{ is missing}}} + \underline{A.\bar{B}.C} + \underbrace{A.C}_{\substack{\uparrow \\ B \text{ is missing}}}$$

Step 2 - Multiply [missing Variable + it's complement]

$$F(A,B,C) = A.B.[C+\bar{C}] + A\bar{B}.C + A.C.[B+\bar{B}]$$

$$= A.B.C + A.B.\bar{C} + A.\bar{B}.C + \cancel{A.B.C} + \cancel{A.\bar{B}.C}$$

Step 3 - Neglect the repeated terms.

$$F(A,B,C) = \underset{m_7}{A.B.C} + \underset{m_6}{A.B.\bar{C}} + \underset{m_5}{A.\bar{B}.C}$$

$$= \Sigma m(5,6,7)$$

$$F(A,B,C,D) = \underbrace{A.B}_{\substack{C \,\&\, D \\ \text{is missing}}} + \underbrace{A.C}_{\substack{B \,\&\, D \\ \text{is missing.}}} + \underline{ABC\bar{D}}$$

$$= A.B.(C+\bar{C})(D+\bar{D}) + A.C.(B+\bar{B})(D+\bar{D}) + ABC\bar{D}$$

$$= ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + \cancel{ABCD} + \cancel{ABC\bar{D}} + A\bar{B}CD + AC\bar{B}\bar{D} + \cancel{ABC\bar{D}}$$

$$= \underset{m_{15}}{ABCD} + \underset{m_{14}}{ABC\bar{D}} + \underset{m_{13}}{AB\bar{C}D} + \underset{m_{12}}{AB\bar{C}\bar{D}} + \underset{m_{11}}{A\bar{B}CD} + \underset{m_{10}}{A\bar{B}C\bar{D}}$$

$$= \Sigma m(10, 11, 12, 13, 14, 15)$$

$$\boxed{\underline{HW} \qquad F(A,B,C,D) = A + CD}$$

↑ Solve this in SSOP

POS to SPOS Conversion SO

Step 1 - Identify the missing Varible.

$$F(A,B,C) = \underline{(\bar{A}+\bar{B})}\,.\,\underline{A}\,.\,\underline{(A+B+\bar{C})}$$

C is missing — B & C is missing

Step 2 - Add with that Variable & it's complement Seperatly.

$$F(A,B,C) = (\bar{A}+\bar{B}+C).(\bar{A}+\bar{B}+\bar{C}).$$
$$(A+B+C).(A+B+\bar{C}).(A+\bar{B}+C).(A+\bar{B}+\bar{C})$$
$$\cancel{(A+B+\bar{C})}.$$

Step 3 - Neglect repeated terms.

$$F(A,B,C) = \underbrace{(\bar{A}+\bar{B}+C)}_{m6}\underbrace{(\bar{A}+\bar{B}+\bar{C})}_{m7}\underbrace{(A+B+C)}_{m0}.\underbrace{(A+B+\bar{C})}_{m1}$$
$$\underbrace{(A+\bar{B}+C)}_{m2}\underbrace{(A+\bar{B}+\bar{C})}_{m3}$$

$$= \Pi M(0,1,2,3,6,7)$$

e.g. $F(A,B,C) = (A+\bar{B})(\bar{A}+C)(\bar{A}+\bar{B}+C)$

C is missing — B is missing.

$$= (A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+C)$$
$$(\bar{A}+\bar{B}+C)$$

$$= \underset{m2}{(A+\bar{B}+C)}\ \underset{m3}{(A+\bar{B}+\bar{C})}\ \underset{m4}{(\bar{A}+B+C)}\ \underset{m6}{(\bar{A}+\bar{B}+C)}$$

$$= \Pi M(2,3,4,6)$$

$$\boxed{\text{HW}\quad F(A,B,C) = B(A+C)}$$

↑ Convert into SPOS.

SSOP to SPOS Conversion & SPOS to SSOP Conversion 5]

1) $F(A,B,C) = \sum m(0,1,4,7)$. Convert SSOP to SPOS..

$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + ABC$

$n = 3$, $[0, 1, \underline{2}, \underline{3}, 4, \underline{5}, \underline{6}, 7]$

$F(A,B,C) = \prod m[\underline{2}, \underline{3}, \underline{5}, \underline{6}]$

$= (A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$

2) $F(A,B,C) = \prod m(1,2,6)$. Convert SPOS to SSOP.

$= (A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+\bar{B}+C)$

$n = 3$, $[0, \underline{1}, \underline{2}, 3, 4, 5, \underline{6}, 7]$

$F(A,B,C) = \sum m(0,3,4,5,7)$

$= \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC$

3) $F(A,B,C) = \underset{m_7}{ABC} + \underset{m_6}{AB\bar{C}} + \underset{m_3}{\bar{A}BC}$, Convert SSOP to SPOS.

$= \sum m(3,6,7)$

$n = 3$, $[0, 1, 2, \underline{3}, 4, 5, \underline{6}, \underline{7}]$

$F(A,B,C) = \prod m(\underline{0}, \underline{1}, \underline{2}, \underline{4}, \underline{5})$

$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+B+C)(\bar{A}+B+\bar{C})$

4) $F(A,B,C) = \underset{m_2}{(A+\bar{B}+C)}\underset{m_5}{(\bar{A}+B+\bar{C})}\underset{m_1}{(A+B+\bar{C})}$, Convert SPOS to SSOP.

$F(A,B,C) = \prod m(1,2,5)$

$n = 3$, $[0, \underline{1}, \underline{2}, 3, 4, \underline{5}, 6, 7]$.

$F(A,B,C) = \sum m(0,3,4,6,7)$

$= \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C} + ABC$

Examples on SOP & POS S2

1) If $n=3$ variables then total minterms is $\underline{8}$

total maxterms is $\underline{8}$

total terms is $\underline{256}$

- total minterms $= \boxed{2^n} = 2^3 = 8$
- total maxterms $= \boxed{2^n} = 2^3 = 8$
- total terms $= \boxed{2^{2^n}} = 2^{2^3} = 2^8 = 256$

- total self dual $= \boxed{2^{2^{n-1}}}$

2) $Y(A,B,C) = A + \overline{B}C$. Find SSOP & SPOS.

A: B & C is missing; $\overline{B}C$: A is missing.

$= A.(B+\overline{B})C(C+\overline{C}) + \overline{B}.C(A+\overline{A})$

$= A.B.C + A.B.\overline{C} + A.\overline{B}.C + A.\overline{B}.\overline{C} + \cancel{A.\overline{B}.C} + \overline{A}.\overline{B}.C$

$= \underset{m7}{ABC} + \underset{m6}{AB\overline{C}} + \underset{m5}{A\overline{B}C} + \underset{m4}{A\overline{B}\overline{C}} + \underset{m1}{\overline{A}\overline{B}C}$

$= \Sigma m(1,4,5,6,7)$

$n=3$, $[0, \underline{1}, 2, 3, \underline{4}, \underline{5}, \underline{6}, \underline{7}]$

$Y(A,B,C) = \Pi m[0,2,3]$

$= (A+B+C).(A+\overline{B}+C).(A+\overline{B}+\overline{C})$

3) $Y = (A+B)(A+C)$. find total minterms (5) & maxterms (3).

$(A+B)$: C is missing; $(A+C)$: B is missing.

$= (A+B+C).(A+B+\overline{C}).\cancel{(A+B+C)}(A+\overline{B}+C)$

$= \underset{m0}{(A+B+C)}.\underset{m1}{(A+B+\overline{C})}.\underset{m2}{(A+\overline{B}+C)}$.

$= \Pi m(0,1,2)$

$n=3$, $[\underline{0}, \underline{1}, \underline{2}, 3, 4, 5, 6, 7]$.

$Y = \Sigma m(3,4,5,6,7)$

$= \overline{A}BC + A\overline{B}\overline{C} + A\overline{B}C + AB\overline{C} + ABC$