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permutation & combina

Rule of sum ⇒ If the object A may be chosen in m ways & B in n ways then either A or B may be chosen in $(m+n)$ ways.

Rule of product ⇒ If the object A may be chosen in m ways & B in n ways then both A and B may be chosen in (mn) ways.

1. There are 42 ways to select a representative from class B

(i) In how many ways can you select the representatives of both class A & B?

Sol: $42 \times 50 = 2050$

(ii) In how many ways can you select the representatives for either A or B?

Sol: $42 + 50 = 92$

2. Suppose a licensed plate contains 2 letters followed by 4 digits with the first digit not zero. How many different license plate can be printed?

(a) with repeatation

⇒ $26^2 \times 9 \times 10^3$

(b) without repeatation

→ $26 \times 25 \times 9^2 \times 8 \times 7$

PERMUTATION & COMBINATION

1. Suppose you have 4 distinct objects a, b, c & d. Selec^n of 2 objects at a time (i) without repeatation.

Sol ⇒ $^4C_2 = \frac{4 \times 3}{2 \times 1} = 6$

$^4P_2 = 4 \times 3 = 12$

(ii) with repeatition

Sol → 16 [you add aa, bb, cc & dd]

- Permutation ⇒ Permutation is an ordered selec^n of r objects taken at a time out of n objects.

It is given by $\boxed{^nP_r = \frac{n!}{(n-r)!}}$

- Permutation with unlimited repeatition is given by

$\boxed{n \times n \times n \times \ldots = n^r}$

2. Consider objects of which m_1 are of first kind, m_2 are of second kind & m_k are of k^{th} kind such that then the no of permutation of all objects in this case is given by

$$\frac{n!}{m_1! m_2! \ldots m_k!}$$

2. There are 7 rooms & we want to assign 4 of them to 4 programmers as offices & use remaining 3 rooms for computer terminals. In how many ways can we arrange this?

Sol: $^7P_4 \times {}^3C_3 = \frac{7!}{4!} \times \frac{3!}{3!}$

$= \frac{7 \times 6 \times 5 \times \cancel{4!} \cdot \cancel{3!}}{\cancel{4!}\ \cancel{3!}}$

$\boxed{= 210}$

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3. How many 6 digit telephone no have 1 or more repeated digits?

Sol: A telephone no including 0 in the beginning ~~is~~ $= 10^6$

The 6 digit telephone no without any repeatation

$= {}^{10}P_6$

Required $= 10^6 - {}^{10}P_6$

$= 10^6 - \frac{10!}{6!}$

$= 10^6 - \frac{10 \times 9 \times 8 \times 7 \times \cancel{6!}}{\cancel{6!}}$

$= 10^6 - 720$

$\boxed{= 999280}$

4. Suppose 00 doesn't take first 2 places.

(i) With repeatition

$10^4 \times 9^2 = 810000$

(ii) Without repeatition

$9 \times 8^2 \times 7 \times 6 \times 5 = 120960$

5. A bit has either zero or 1, a byte is a sequence of 8 bits. Find no of bytes
(a) That can be formed
(b) Find the no of bytes that begin with 11 & end with 11.
(c) Begins with 11 & do not end with 11.
(d) Begins with 11 or ends with 11.

Sol: (a) $2^8 = 256$

(b) $2^4 = 16$

(c) $2^4 \times 3 = 48$

(d) $2^6 + 2^6 - \cancel{2^4}\,2^4 = 64 + 64 - \cancel{24}\,2^4 = \cancel{104}\ 112$

6. Find the sum of all 4 digit numbers that can be obtained using 1, 2, 3, 4 once in each.

Sol: Sum of digits in units place + tens place + hundreds place + thousands place

$6\times4 + 6\times3 + 6\times2 + 6\times1 + 6\times10 + 6\times20 + 6\times30 + 6\times40$
$+ \cancel{6\times00}\ 6\times100 + 6\times200 + 6\times300 + 6\times400 + 6\times1000$
$+ 6\times2000 + 6\times3000 + 6\times4000$

$\Rightarrow 6(4+3+2+1+40+30+20+10+400+300+200+100+4000+3000+2000+1000)$

$\Rightarrow 6(10+100+1000+10000)$

$\Rightarrow 6(11110)$

$\boxed{\Rightarrow 66660}$

7. In how many ways can the letters of English alphabets be arranged so that there are exactly 5 letters b/w a & b (without repeatition).

Sol ⇒ ∵ we can arrange the letters in 5 spaces, without repeatition and the no. of letters is 24 ($26-2=24$).

∴ The no. of ways 5 letters can be arranged b/w a & b is

$^{24}P_5$

But we have to count case 2 where b comes before a, so no. of ways $= 2 \times {}^{24}P_5$

Now, we take a _ _ _ _ _ b as a unit.
Total letters remaining $= 26 - 7 = 19$
adding the a _ _ _ _ _ b as a unit, total letters to be arranged 20.

$\therefore Req^d = \boxed{2 \times {}^{24}P_5 \times 20!}$

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8. How many +ve integers less than 1 million can be formed using

(a) 7, 8, 9 only
(b) 0, 8, 9 only

Sol: (a)

Case 1: integers with 1 digit

$$^3C_1 = \frac{3!}{1!\,2!} = 3$$

Case 2: integers with 2 digits

$$^3C_1 \times {}^3C_1 = \frac{3!}{2!\,1!} \times \frac{3!}{2!\,1!} = 3^2$$

case 3: integers with 3 digits

$$^3C_1 \times {}^3C_1 \times {}^3C_1 = 3^3$$

case 4: integers with 4 digits

$$^3C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 = 3^4$$

case 5: integers with 5 digits

$$^3C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 = 3^5$$

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case 6: integers with 6 digits

$$^3C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 = 3^6$$

$\therefore$ Req^d $= 3 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6$

$= 1092$

(b) case 1: +ve integers with 1 digit

$$^2C_1 = 2$$

case 2: +ve integers with 2 digits

$$^2C_1 \times {}^3C_1 = 2 \times 3$$

case 3: +ve integers with 3 digits

$$^2C_1 \times {}^3C_1 \times {}^3C_1 = 2 \times 3^2$$

case 4: +ve integers with 4 digits

$$^2C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 = 2 \times 3^3$$

case 5: +ve integers with 5 digits

$$^2C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 = 2 \times 3^4$$

case 6: +ve integers with 6 digits

$$^2C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 \times {}^3C_1 = 2 \times 3^5$$

$Req^d = 2 + 2\times3 + 2\times3^2 + 2\times3^3 + 2\times3^4 + 2\times3^5$

$\boxed{= 728}$

9. How many times is the digit 5 written when listing all the numbers from 1 to 1 lakh?

Sol: ~~Case 1: 5 appears only once~~

$\cancel{{}^5C_1 \times 9^4 \times 1}$

case 1: If 5 ~~occurs~~ once then we have ${}^5C_1$ & the remaining 4 places can be occupied by remaining 9 digits

${}^5C_1 \times 9^4 \times 1$ (1 cuz 5 appears once)

Case 2: If 5 occurs twice then we have ${}^5C_2$ & the remaining 3 places can be occupied by remaining 9 digits.

${}^5C_2 \times 9^3 \times 2$

Case 3: If 5 occurs thrice then we have ${}^5C_3$ & remaining 2 place are filled by remaining 9 digits

${}^5C_3 \times 9^2 \times 3$

Case 4: If 5 occurs 4 times then ${}^5C_4$ & remaining 1 place filled by 9 digit

${}^5C_4 \times 9 \times 4$

Case 5: If 5 occurs 5 times the ${}^5C_5$

${}^5C_5 \times 5$

Req^d: ${}^5C_1 \times 9^4 \times 1 + {}^5C_2 \times 9^3 \times 2 + {}^5C_3 \times 9^2 \times 3 + {}^5C_4 \times 9 \times 4 + {}^5C_5 \times 5$

$\boxed{= 49190}$

10. How many times is the digit 2 written when listing all numbers from 1 - 1000.

Sol: Case 1: If 2 appears once then we have ${}^3C_1$ & the remaining 2 places are filled by remaining 9 digits.

${}^3C_1 \times 9^2 \times 1$

Case 2: If 2 appears twice then we have ${}^3C_2$ & the remaining 1 place can be filled by 9 digits

${}^3C_2 \times 9 \times 2$

Case 3: If 2 appears thrice then we have ${}^3C_3$

${}^3C_3 \times 3$

Req^d: ${}^3C_1 \times 9^2 \times 1 + {}^3C_2 \times 9 \times 2 + {}^3C_3 \times 3$

$\boxed{= 300}$

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- Selection + Repeatition = ${}^{n+r-1}C_r$

- The r combination of n objects with unlimited repeatition is given by

$$\boxed{{}^{n+r-1}C_r}$$

11. Consider the no of permutation of letters of the word INSTITUTION

(a) How many of these begin with I

(b) How many of these begin with I & end with N

(c) How many permutations are there in which 3 T's aren't together.

Sol: (a) There are 11 letters in the word, so if we fix 1 I then we'll have 10 words remaining.

Also, there are 3 T's, 2 I's (other than the one fixed I) & 2 N's.

$$\therefore Req^d = \frac{10!}{3!\,2!\,2!} = \frac{10\times9\times8\times7\times6\times5\times\cancel{4}\times\cancel{3!}}{\cancel{3!}\,\cancel{2!}\,\cancel{2!}}$$

$$\boxed{= 151200}$$

(b) Now, if we fix I & N, then we'll have 9 words

And, 3T's and 2I's.

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$$\therefore Req^d = \frac{9!}{3!\,2!} = \frac{9\times8\times7\times6\times5\times\cancel{4}^{2}\times\cancel{3!}}{\cancel{3!}\,\cancel{2!}}$$

$$\boxed{= 30240}$$

(c) Now, The total no. of permutation in the word INSTITUTION is

$$\frac{11!}{3!\,3!\,2!} = \frac{11\times10\times9\times8\times7\times\cancel{6}\times5\times\cancel{4}^{2}\times\cancel{3!}}{\cancel{3!}\,\cancel{3}\times\cancel{2}\times\cancel{2!}}$$

$$\boxed{= 554400}$$

Now, we will bunch all the 3 T's together & treat them as a single unit.

So now we have 9 total letters.

The permutation =

$$\frac{9!}{3!\,2!} = \frac{9\times8\times7\times6\times5\times\cancel{4}^{2}\times\cancel{3!}}{\cancel{3!}\,\cancel{2!}}$$

$$\boxed{= 302400}$$

∴ The permuta^n in which 3T's aren't together

$= 554400 - 302400$

$$\boxed{= 524160}$$

12. A shop sells 6 different flavours of ice cream. In how many ways can a customer choose 4 ice cream cones if

(a) They're all of diff. flavours
(b) Not necessarilly diff. flavours
(c) They contain only 3 or 2 diff flavours

Sol: (a) Flavours = 6
Cones = 4

$^6C_4 = \frac{6!}{4!\,2!}$

$= \frac{\cancel{6}^3 \times 5 \times \cancel{4!}}{\cancel{4!}\ \cancel{2!}}$

$\boxed{= 15 \text{ ways}}$

(b) ∴ Combination with repeatition

$^{6+4-1}C_4 = {}^9C_4$

$= \frac{9 \times \cancel{8}^2 \times 7 \times \cancel{6} \times \cancel{5!}}{\cancel{5!} \times \cancel{4} \times \cancel{3} \times \cancel{2}}$

$\boxed{= 126}$

(c) Req^d = With only 2 flavours + With only 3 flavours

$\Rightarrow {}^6C_2 \times {}^3C_2 \left({}^{2+2-1}C_2\right) + {}^6C_3 \times {}^3C_1$

$\Rightarrow \frac{\cancel{6}^3 \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{2}} \times \frac{3 \times \cancel{2!}}{\cancel{2!}} \times \frac{3 \times \cancel{2!}}{\cancel{2!}} + \frac{\cancel{6} \times 5 \times 4 \times \cancel{3!}}{\cancel{3!} \times \cancel{3} \times \cancel{2}} \times \frac{3 \times \cancel{2!}}{\cancel{2!}}$

Note: If <gle b/w any 2 sides is not more than 180, its called a simple decagon or convex decagon

$\Rightarrow 15 \times 9 + 20 \times 3$

$\Rightarrow 135 + 60$

$\Rightarrow \boxed{195}$

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13. In how many ways can a lady wear 5 rings in 4 fingers (not thumb) on her right hand.

14. In how many ways can examiner can assign 30 marks to 8 questions so that no questions reeieve less than 2 marks.

Sol ⇒ 30 marks in total
We have to assign at least 2 marks to 8 diff. question.

$8 \times 2 = 16$

$30 - 16 = 14$ marks remaining to assign

$^{8+14-1}C_{14} = {}^{21}C_{14} = \frac{\cancel{21}^3 \times 20 \times 19 \times \cancel{18} \times 17 \times \cancel{16}^4 \times \cancel{15}^3 \times \cancel{14!}}{\cancel{14!} \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times 2}$

=

14. If there are no diagonals of a convex decagon at which meet at the same pt. inside the decagon, into how many ways line segments are the diagonals divided by their intersection.

Sol: 7 diagonals
$7 \times 10 = 70$ diagonals total
$\frac{70}{2} = 35$ ———〃
common diagonal

Formula of no. of diagonals = $^xC_2 - x$

OR $^{10}C_2 - 10 = \frac{\cancel{10}^{5} \times 9 \times \cancel{8!}}{\cancel{8!}\, 2!} - 10$

$= 45 - 10$

$\boxed{= 35}$

4 points are needed for intersection

$= {}^{10}C_4$

Since every 4 vertices give 1 intersecⁿ b/w the diagonals, Thus there are total of $^{10}C_4$ intersecⁿ.

∵ Diagonal is divided into k+1 line segment when there are k intersecⁿ lying in it.

And since each intersection pt. lies on 2 diagonals.

Total no. of straight line segments in which diagonals are divided is given by

$\boxed{2 \times 210 + 35}$

15. How many ways are there to distribute 4 identical oranges & 6 different apples into 5 distint boxes such that each box gets exactly 2 fruits.

Sol: Case I = We first choose 1 box ~~of~~ out of 5 = $^5C_1$, then 2 oranges, 2 apples and 1 distinct pair

$\underbrace{^5C_1}_{\text{box}} \times \underbrace{^6C_2}_{\text{apples}} \times \underbrace{^4P_4}_{\text{arrangement of apples}} \times \underbrace{^4C_4}_{\text{oranges}} = 75$

Case II = $\overset{\text{Box}}{^5C_2} \times \overbrace{^6C_2 \times {}^4C_2}^{\text{apples}} \times {}^3P_2 \times 1$

$= \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{3!} \times \cancel{2!}} \times \frac{\cancel{6}^{3} \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{2}} \times \frac{\cancel{4}^{2} \times 3 \times \cancel{2!}}{\cancel{2!}\,\cancel{2!}} \times \frac{3 \times \cancel{2!}}{\cancel{2!}}$

$= 10 \times 15 \times 6 \times 3$

$= 2700$

Case III = $^5C_3 \times {}^6C_2 \times {}^4C_2 \times {}^2C_2 \times {}^2P_2$

$= \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{3!}\, \cancel{2!}} \times \frac{\cancel{6}^{3} \times 5 \times \cancel{4!}}{\cancel{4!}\,\cancel{2!}} \times \frac{\cancel{4}^{2} \times 3 \times \cancel{2!}}{\cancel{2!}\,\cancel{2!}} \times \frac{\cancel{2!}}{\cancel{2!}} \times \frac{\cancel{2!}}{\cancel{2!}}$

$= 10 \times 15 \times 6$

$= 900$

Reqᵈ = ~~75~~ Case 1 + Case 2 + Case 3

$= 75 + 2700 + 900$

$\boxed{= 3675}$

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probability

Formula

$$P = \frac{\text{No of favourable cases}}{\text{Total no of cases}}$$

1. Find the probability of getting heads in tossing of a coin

Sol⇒ $P = \frac{1}{2}$

2. Find probability of getting (a) King (b) King or queens when a card is drawn at random from a pack of 52 cards.

Sol: (a) No of kings : 4
Total cards : 52

$$P = \frac{4}{52} = \frac{1}{13}$$

(b) No of kings or queens : 8

$$P = \frac{8}{52} = \frac{2}{13}$$

3. Find probability of getting (a) total more than 10 (b) getting a prime no (c) Total < 5, when 2 dice are thrown simultaneously.

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Sol: (a) $A = 3$
$SS = 36$

$$P(A) = \frac{3}{36} = \frac{1}{12}$$

(b) $A = \cancel{8}\ 9$

$$P(A) = \frac{9}{36} = \frac{1}{4}$$

(c) $A = 6$

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

4. Find probability that leap year will have 53 sundays.

Sol: Leap year = 366 days
A week = 7 days

Week in a leap year $= \frac{366}{7} = 52\frac{2}{7}$

Meaning 2 extra days
So, getting 53 Sundays $= \frac{2}{7}$

5. Out of digits 0,1,2,3,4 (w/o repeatiⁿ) a 5 digit no is formed. Find probability that the number formed is divisible by 4.

Sol: $SS = 4 \times 4 \times 3 \times 2 \times 1$
$= 96$

A no. is divisible by 4 if the last 2 no are divisible by 4

So $A = 04, 12, 20, 24, 32, 40$

Case 1: Ends with 04

$$\underbrace{\boxed{\ }\boxed{\ }\boxed{\ }}_{3!}\boxed{0}\boxed{4}$$

$= 3!$

Case 2: Ends with 12

$$\underset{2}{\boxed{\ }}\underset{2}{\boxed{\ }}\underset{1}{\boxed{\ }}\boxed{1}\boxed{2}$$

$= 2 \times 2 \times 1$

Case 3: Ends with 20

$$\underbrace{\boxed{\ }\boxed{\ }\boxed{\ }}_{3!}\boxed{2}\boxed{0}$$

$= 3!$

Case 4: Ends with 40

$$\underbrace{\boxed{\ }\boxed{\ }\boxed{\ }}_{3!}\boxed{4}\boxed{0}$$

$= 3!$

Case 5: Ends with 24

$$\underset{2}{\boxed{\ }}\underset{2}{\boxed{\ }}\underset{1}{\boxed{\ }}\boxed{2}\boxed{4}$$

$= 2 \times 2$

Case 6: Ends with 32

$$\underset{2}{\boxed{\ }}\underset{2}{\boxed{\ }}\underset{1}{\boxed{\ }}\boxed{3}\boxed{2}$$

$= 2 \times 2$

A: Total ~~possibilities~~ favourable outcomes $= 3! + 3! + 3! + 2^2 + 2^2 + 2^2$
$= 3 \times 3! + 3 \times 2^2$
$= 3(6+4)$
$= 30$

$$\boxed{P(A) = \frac{30}{96} = \frac{5}{16}}$$

6. A and B stand in a line at random with 12 other people
(a) What is P that there will be 4 people b/w A & B?
(b) What is P that they are standing together?

Sol: $SS = 14!$ $\quad [\because A + B + 12 = 14]$

Favourable outcomes

7. A committee of 4 people is to be formed from 3 representative of parent teacher, 4 representatives of teacher faculty, 2 representatives of non-teaching faculty & 1 students rep. Find P of forming committee if there must be at least 1 rep of teaching faculty.

Sol: $P(T \geq 1)$: A least 1 rep of teacher faculty

$= 1 - P(T=0)$

No of ways to choose no teaching faculty $= {}^{6}C_4$

Total ways $= {}^{10}C_4$

$$P(T=0) = \frac{{}^{6}C_4}{{}^{10}C_4}$$

$$\therefore P(T \geq 1) = 1 - P(T=0)$$

$$= 1 - \frac{{}^{6}C_4}{{}^{10}C_4}$$

$$= 1 - \frac{\frac{6 \times 5 \times 4!}{4! \times 2}}{\frac{10 \times 9 \times 8 \times 7 \times 6!}{6! \times 4 \times 3 \times 2}}$$

$$= 1 - \frac{3 \times 5}{\frac{10}{2} \times 3 \times 7}$$

$$= 1 - \frac{1}{14}$$

$$\boxed{P(T \geq 1) = \frac{13}{14}}$$

- The probability of happening of events A or B is denoted by $P(A \cup B)$ or $P(A+B)$

- Probability of happening of both the events A and B

$$P(A \cap B) \text{ and } P(AB)$$

- If A and B are not necessarily mutually exclusive then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

8. A student takes his examination in 4 subjects P, Q, R, S. He estimates his chances of passing in P is 4/5, in Q is 3/4, R = 5/6 and S = 2/3. To qualify he must pass in P and at least 2 other subjects. What is the probability that he qualifies?

Sol: $P(P) = \frac{4}{5}$ $\quad P(\bar{P}) = \frac{1}{5}$

$P(Q) = \frac{3}{4}$ $\quad P(\bar{Q}) = \frac{1}{4}$

$P(R) = \frac{5}{6}$ $\quad P(\bar{R}) = \frac{1}{6}$

$P(S) = \frac{2}{3}$ $\quad P(\bar{S}) = \frac{1}{3}$

$$P(\text{Qualifying}) = P(PQRS) + P(PQR\bar{S}) + P(PQ\bar{R}S) + P(P\bar{Q}RS)$$

$$= \left(\frac{4}{5} \times \frac{3}{4} \times \frac{5}{6} \times \frac{2}{3}\right) + \left(\frac{4}{5} \times \frac{3}{4} \times \frac{5}{6} \times \frac{1}{3}\right) + \left(\frac{4}{5} \times \frac{3}{4} \times \frac{1}{6} \times \frac{2}{3}\right)$$

$$+ \left(\frac{4}{5} \times \frac{1}{4} \times \frac{5}{6} \times \frac{2}{3}\right)$$

$$= \frac{1}{3} + \frac{1}{6} + \frac{1}{15} + \frac{1}{9}$$

$$= \frac{30+15+6+10}{90}$$

$$\boxed{P(\text{Qual}) = \frac{61}{90}}$$

9. A man tosses a coin and throws die, beginning with coin. What is the probability that he will get a heads before he will get a 5 or 6.

Sol: $P(H) = \frac{1}{2}$

$P(\bar{H}) = \frac{1}{2}$

$P(5 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$

$P(\overline{5 \text{ or } 6}) = \frac{4}{6} = \frac{2}{3}$

$Req^d = P(H) + P(\bar{H})P(\overline{5 or 6})P(H) + P(\bar{H})P(\overline{5 or 6})P(\bar{H})P(\overline{5 or 6})P(H) + \dots$

$$= \frac{1}{2} + \frac{1}{2}\times\frac{2}{3}\times\frac{1}{2} + \frac{1}{2}\times\frac{2}{3}\times\frac{1}{2}\times\frac{2}{3}\times\frac{1}{2} + \dots$$

$$= \frac{1}{2}\left(1 + \frac{2}{3}\times\frac{1}{2} + \frac{2}{3}\times\frac{1}{2}\times\frac{2}{3}\times\frac{1}{2} + \dots\right)$$

$\therefore Req^d = \frac{1}{2}\left(\frac{1}{1-1/3}\right)$ $\quad \left[S_n = \frac{a}{1-r}\right]$

$$= \frac{1}{2}\left(\frac{1}{2/3}\right)$$

$$= \frac{1}{2}\left(1 \times \frac{3}{2}\right) = \boxed{\frac{3}{4}}$$

10. Two person A and B toss an unbiased coin alternatively with the understanding that first who gets the heads wins (a) If A starts the game find the respective chances of winning.
(b) What's the probability of respective chances of winning if B starts the game.

Sol: $P(H) = \frac{1}{2}$ $\quad P(\bar{H}) = \frac{1}{2}$

(a) A: A wins

$$P(A) = \frac{1}{2} + \frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} + \dots$$

$$= \frac{1}{2}\left[1 + \frac{1}{2} + \frac{1}{2^2} + \dots\right]$$

$$= \frac{1}{2}\left(\frac{1}{1-1/4}\right)$$

$$= \frac{1}{2}\left(\frac{1}{3/4}\right)$$

$$= \frac{1}{\cancel{2}}\left(1 \times \frac{\cancel{4}\,2}{3}\right)$$

$$\boxed{= \frac{2}{3}}$$

(b) B: B wins

$P(H) = \frac{1}{2}$ $\quad P(\bar{H}) = \frac{1}{2}$

Same a part

11. In general when A & B play 12 games of chess 6 are won by A, 4 won by B and 2 end in a draw. They agree to play a tournament consist of 3 games. Find the probability that
(a) A wins all 3 games
(b) ~~Wins~~ 2 games end ~~is~~ in a draw
(c) A & B win ~~coin~~ alternately
(d) B wins at least 1 game

Sol: A: A wins

$P(A) = \frac{6}{12} = \frac{1}{2}$

B: B wins

$P(B) = \frac{4}{12} = \frac{1}{3}$

D: draw

$P(D) = \frac{2}{12} = \frac{1}{6}$ $\quad P(\bar{D}) = \frac{5}{6}$

(a) E1: A wins all 3 ~~clas~~ games

$P(E1) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$\boxed{P(E1) = \frac{1}{8}}$

(b) E2: 2 games end in a draw

$P(E2) = \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}$

$\boxed{P(E2) = \frac{5}{216}}$

(c) E3: A & B win alternatively

~~E~~ $P(E3) =$

(d) E4: B wins at least 1 game

$P(E4) =$

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12. Three winning tickets are to be selected from a box of 100 tickets. What is the P of winning for a person who buys
(a) 4 tickets
(b) 1 ticket

Sol: (a) W: Winning
W4: Winning by 4 tickets

$P(W) = \frac{3}{100}$

$P(\overline{W}) = \frac{{}^{97}C_4}{{}^{100}C_4}$

$= \dfrac{\frac{\cancel{97} \times 96 \times 95 \times 94}{\cancel{4!}}}{\frac{100 \times 99 \times 98 \times \cancel{97}}{\cancel{4!}}}$

$= \frac{96 \times 95 \times 94}{100 \times 99 \times 98}$

$= \frac{8 \times 19 \times 47}{5 \times 33 \times 49}$

$P(4W) = 1 - \frac{8 \times 19 \times 47}{5 \times 33 \times 49}$

$= \frac{5 \times 33 \times 49 - 8 \times 19 \times 47}{5 \times 33 \times 49}$

$\boxed{P(4W) = \frac{941}{5 \times 33 \times 49}}$

(b) $\boxed{P(W) = \frac{3}{100}}$

13. Find P of drawing ace or spade or both from deck of cards.

Sol: A: Card is ace
B: Card is spade

$P(A) = \frac{4}{52}$

$= \frac{1}{13}$

$P(B) = \frac{13}{52}$

$= \frac{1}{4}$

$\boxed{P(A \cap B) = \frac{1}{52}}$

$P(A \cup B) = \frac{16}{52}$

$= \frac{8}{26}$

$\boxed{P(A \cup B) = \frac{4}{13}}$

14. A box contains tags marked 1 - n. 2 tags are chosen at random. Find P that the no on tags will be consecutive integers if
(a) Tags are chosen with replacement
(b) Without replacement

Sol: $E = (1,2)\ (2,3)\ (3,4)\ (4,5) \ldots (9,10)$
$= 18$

(a) $\frac{18}{{}^{10}C_1 \times {}^{10}C_1} = \frac{2(n-1)}{{}^{n}C_1 \times {}^{n}C_1} = \frac{18}{10 \times 10} = \frac{18}{100}$

$\boxed{= \frac{9}{50}}$

(b) $\frac{2\times 9}{{}^{10}C_1 \times {}^{9}C_1} = \frac{\cancel{18}^{2}}{10 \times \cancel{9}} = \boxed{\frac{1}{5}}$

15. A bag contains 40 tickets numbered 1-40. Of which 4 are drawn at random & arranged in an ↑sing order. Find P of t_3 being 25.

Sol: 4 tickets are drawn from a box of 40
$\therefore$ denominator $= {}^{40}C_4$

We know that tickets are arranged in ↑sing order
$\therefore \quad t_1 < t_2 < t_3 < t_4$

So, for $t_3 = 25$, t_1 & $t_2 < 25$ & $t_4 > 25$.

~~P(A) =~~
No. less than 25 are 1-25
& no greater than 25 are 26-40 ⇒ 15

$\therefore \quad P(A) = \frac{{}^{24}C_2 \times 1 \times {}^{15}C_1}{{}^{40}C_4}$ [1 for 25]

$= \frac{12 \times 23 \times 1 \times 15}{\frac{40 \times 38 \times 39 \times 37}{4 \times 3 \times 2}}$

$= \cancel{12}^{6} \times 23 \times 1 \times \cancel{15}^{3} \times \frac{\cancel{4} \times \cancel{3} \times \cancel{2}}{\cancel{40}_{\cancel{10}\,2} \times \cancel{38}_{19} \times \cancel{39}_{13} \times 37}$

$= \frac{6 \times 23 \times 3}{19 \times 13 \times 37} =$

CONDITIONAL PROBABILITY

- Conditional probability of B given A is the P of happening of B when it is known that A has already occured. It is denoted by

$$\boxed{P(B|A) = \frac{P(A \cap B)}{P(A)}}$$

- Unconditional probability of B is the P of happening of B when nothing is known about happening of A & is denoted by

$$\boxed{P(B)}$$

- If 2 events A & B are independent the

$$\boxed{P(B|A) = P(B)}$$

16. A dice is tossed. If the no is odd on face what is probability that it is prime.

Sol: A : The no is odd

$P(A) = \frac{3}{6} = \frac{1}{2}$

$P(A \cap B) = \frac{2}{6}$

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$$P(B|A) = \frac{\cancel{1/2}\ 2/6}{1/2}$$

$$\boxed{P(B|A) = \frac{2}{3}}$$

17. A card is drawn at random from a pack of cards
(a) What is the P that it is a heart
(b) If it is known that the card drawn is red. What is P that it is heart.

Sol: (a) A : Card is a heart
A = 13 hearts
SS = 52 cards

$$P(A) = \frac{13}{52}$$

$$\boxed{P(A) = \frac{1}{4}}$$

(b) A : Card drawn is heart
B : Card draw is red

$$P(B) = \frac{26}{52} = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{4} \qquad [\text{13 hearts}]$$

$$P(A|B) = \frac{1/4}{1/2}$$

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$$\boxed{P(A|B) = \frac{1}{2}}$$

18. Two fair dice are thrown. If sum of no. obtained is 4, find P that no. obtained on both dice are even.

Sol: A : Sum of no obtained = 4

$$P(A) = \frac{3}{36} = \frac{1}{12}$$

$$P(A \cap B) = \frac{1}{36} \qquad [\because (2,2) \text{ are only even}]$$

$$P(B|A) = \frac{1/36}{1/12}$$

$$\cancel{P(\ } = \frac{1}{3}$$

$$\boxed{P(B|A) = \frac{1}{3}}$$

19. A box contains 4 bad & 6 good tubes. 2 tubes are drawn together. One of them is tested & found to be good. What is P that other one is also good?

Sol: A : 1st tube is good
B : 2nd — " —

$$P(A) = \frac{6}{10} = \frac{3}{5}$$

$$P(A\cap B) = \frac{{}^6C_2}{{}^{10}C_2}$$

$$= \frac{\cancel{6}\times 5}{3\cancel{9}\times\cancel{5}}$$

$$= \frac{1}{3}$$

$$P(B|A) = \frac{1/3}{3/5}$$

$$= \frac{1}{3}\times\frac{5}{3}$$

$$\boxed{P(B|A) = \frac{5}{9}}$$

TOTAL PROBABILITY

Partitions: The events $B_1, B_2, \ldots, B_k$ represent a partition of a sample space S if

(i) $B_i \cap B_j$ is a null set

(ii) $\bigcup_{i=1}^{k} B_i = S$

(iii) $P(B_i) > 0, \forall i$

For any event A, $P(A)$ is given by

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \ldots + P(A|B_k)P(B_k)$$

where $B_1, B_2, \ldots, B_k$ = partitions

20. The P that student passes an exam is 0.8. Given that he has studied. The P that he passes without studying is 0.2. What is P that he passes in exam. Assume that the P of student studying for exam is 0.6.

Sol: A : Student will pass
B_1: Student will study
B_2: Student won't study

$$P(B_1) = 0.6 = \frac{6}{10}$$

$$P(B_2) = 0.4 = \frac{4}{10}$$

$$P(A|B_1) = 0.8$$

$$P(A|B_2) = 0.2$$

$$P(A) = 0.8\times 0.6 + 0.2\times 0.4$$
$$= 0.48 + 0.08$$

$$\boxed{P(A) = 0.56}$$

21. If a old factory machine A, B & C manufacture 25%, 35% & 40% of total o/p resp. 5%, 4% & 2% resp are defective bolts from each machine. A bolt chosen at random & is found to be defective. What is P of getting a defective bolt.

Sol: A: Getting defective bolt

B_1: Bolt is from A

B_2: Bolt is from B

B_3: Bolt is from C

$$P(B_1) = \cancel{\tfrac{1}{3}}\ \frac{25}{100}$$

$$P(B_2) = \frac{35}{100}$$

$$P(B_3) = \frac{40}{100}$$

$$P(A|B_1) = \frac{5}{100}$$

$$P(A|B_2) = \frac{4}{100}$$

$$P(A|B_3) = \frac{2}{100}$$

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3)$$

$$= \frac{5}{100} \times \frac{25}{100} + \frac{4}{100} \times \frac{35}{100} + \frac{2}{100} \times \frac{40}{100}$$

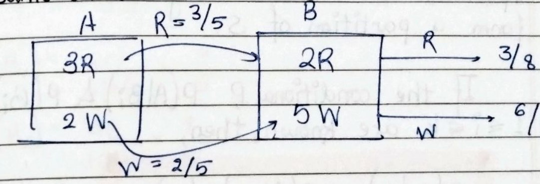
$$= \cancel{5\times}\ \frac{125 + 140 + 80}{100 \times 100}$$

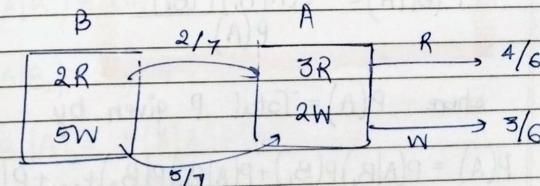
$$= \frac{345}{100 \times 100}$$

$$\boxed{P(A) = 0.0345}$$

22. Box A contains 3 red & 2 white balls. Box B contains 2 R & 5W balls. A box is selected at random then a ball is drawn from that box & put into the other then ball is drawn from that box. Find P that both balls are same color.

Sol:





box choosen

$$Req^d = \frac{1}{2}\left[\frac{3}{5} \times \frac{3}{8} + \frac{2}{5} \times \frac{6}{8} + \frac{2}{7} \times \frac{4}{6} + \frac{5}{7} \times \frac{3}{6}\right]$$

$$= \frac{1}{2}\left[\frac{9+12}{40} + \frac{8+15}{42}\right]$$

$$= \frac{1}{2}\left[\frac{21}{40} + \frac{23}{42}\right]$$

$$= \frac{1}{2}\left(\frac{901}{840}\right)$$

$$\boxed{Req^d = \frac{901}{1680}}$$

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~~Sol:~~ BAYE'S THEOREM

THEOREM OF INVERSE PROBABILITY

Let S be a sample space associated with an experiment E. Let A be any event. Let $B_1, B_2, \ldots B_k$ form a partition of S.

If the conditional P $P(A|B_i)$ & $P(B_i)$ for $1 \leq i \leq k$ are known, then,

$$\boxed{P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(A)}}$$

where $P(A)$ = Total P given by

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + \ldots + P(A|B_k)P(B_k)$$

Proof: By def^n that $P(B_i|A) = \frac{P(B_i \cap A)}{P(A)}$

$$\because P(A|B_i) = \frac{P(A \cap B_i)}{P(B_i)}$$

$$P(A \cap B_i) = P(A|B_i)P(B_i)$$

23. When a certain has 4% of boys, 1% of girls are taller than 1.8m. For the vote 60% students are girls. If a student is chosen at random & found to be taller than 1.8 m, what is the probability of it being girls?

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Sol: A: {Taller than 1.8 m}

B_1: {girls}

B_2: {Boys}

$$P(B_1) = \frac{60}{100} = \frac{3}{5}$$

$$P(B_2) = \frac{40}{100} = \frac{2}{5}$$

$$P(A|B_1) = 0.01$$

$$P(A|B_2) = 0.04$$

$$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{P(A)}$$

$$P(A) = P(A|B_1)P(B_1) + P(A|B_2)P(B_2)$$

$$= 0.01 \times \frac{3}{5} + 0.04 \times \frac{2}{5}$$

$$= \frac{0.03 + 0.08}{5}$$

$$= \frac{0.11}{5}$$

$$= 0.022$$

$$P(B_1|A) = \frac{0.01 \times 3/\cancel{5}}{0.11/\cancel{5}} = \frac{3}{11}$$

24. A card from pack of 52 playing cards is lost. 2 cards are drawn from remaining cards & are found to be hearts. Find P of missing card is also a heart.

Sol: A : Two cards drawn are heart

B_1 : Missing card is heart

B_2 : Missing card is not heart

$P(B_1) = \frac{1}{4}$ $\quad$ $P(A|B_1) = \frac{^{12}C_2}{^{51}C_2}$

$P(B_2) = \frac{3}{4}$ $\quad$ $P(A|B_2) = \frac{^{13}C_2}{^{51}C_2}$

~~$P(A) = \frac{^{12}C_2}{^{51}C_2} \times \frac{1}{4}$~~

$$P(A) = \frac{1}{4} \times \frac{^{12}C_2}{^{51}C_2} + \frac{3}{4} \times \frac{^{13}C_2}{^{51}C_2}$$

$$= \frac{1}{4} \times \frac{6\times 11}{51\times 25} + \frac{3}{4} \times \frac{6\times 13}{51\times 25}$$

$$= \frac{6\times 11 + 3\times 6\times 13}{4\times 51\times 25}$$

$$= \frac{66 + 234}{4\times 51\times 25}$$

$$= \frac{\cancel{300}\ \cancel{12}\ 3}{\cancel{4}\times 51\times \cancel{25}}$$

$$= \frac{3}{51}$$

$$P(B_1|A) = \frac{^{12}C_2 / {^{51}C_2} \times \cancel{1}/4}{\cancel{1}/4 \left(\frac{^{12}C_2}{^{51}C_2} + \frac{3\,^{13}C_2}{^{51}C_2} \right)}$$

$$= \frac{\frac{^{12}C_2}{\cancel{^{51}C_2}}}{\frac{^{12}C_2 + 3\,^{13}C_2}{\cancel{^{51}C_2}}}$$

$$= \frac{^{12}C_2}{^{12}C_2 + 3\,^{13}C_2}$$

$$= \frac{6\times 11}{6\times 11 + 3\times 6\times 13}$$

$$= \frac{\cancel{66}\ \cancel{22}\ 11}{\cancel{300}\ \cancel{100}\ 50}$$

$$= \frac{11}{50}$$

$$\boxed{P(B_1|A) = 0.22}$$

25. Economist believes that during times of high economic growth $ appreciates with P 0.7, during moderate ~~low~~ economic growth $ appreciates with P 0.4 & during low economic growth $ appreciates with P 0.2. During any time period the P of high economy growth is 0.3, moderate = 0.5 & low = 0.2. Suppose $ has been appreciating during the present period. What is P we are experiencing a period of HEG.

Sol: A : Dollar is appreciating

B_1 : HEG
B_2 : MEG
B_3 : LEG

$P(B_1) = 0.3$ $\quad$ $P(A|B_1) = 0.7$
$P(B_2) = 0.5$ $\quad$ $P(A|B_2) = 0.4$
$P(B_3) = 0.2$ $\quad$ $P(A|B_3) = 0.2$

$$P(A) = 0.3 \times 0.7 + 0.5 \times 0.4 + 0.2 \times 0.2$$
$$= 0.21 + 0.20 + 0.04$$
$$= 0.45$$

$$P(B_1|A) = \frac{P(A|B_1)\,P(B_1)}{P(A)}$$
$$= \frac{0.7 \times 0.3}{0.45}$$
$$= \frac{0.\cancel{21}\,7}{0.\cancel{45}\,15}$$

$$\boxed{P(B_1|A) = \frac{7}{15}}$$

RANDOM VARIABLES

Let E be an event & S be sample space associated with E. A function X assigning to every element.

$$X : S \longrightarrow R_X$$

a real number $R_X \in S$ is called random variable

i.e. $$S \xrightarrow{X} R$$

26 Consider the tossing of 2 coins

Sol: $S = \{HH, HT, TH, TT\}$

$X = \{\text{no of heads appearing}\}$

$X\{HH\} = 2$ $\quad$ $X\{TH\} = 1$

$X\{HT\} = 1$ $\quad$ $X\{TT\} = 0$

$X(S) = R_X$

$$\boxed{R_X = \{0, 1, 2\}}$$

TYPES OF RANDOM VARIABLLES

There, are 2 types of random variables

(i) Discrete random variable
(ii) Continuous random variable

1. Discrete Random Variable: • Let X be a RV. If the no of possible values of X is finite or countably infinite then X is called discrete.

Example: (i) The no of members in a family
(ii) Dice rolled twice.

2. Continuous Random Variables: A RV is said to be continuous if it can take all possible values in a continuous interval of a real line. Here the range space can be infinite.

Example: Life span of a person

PROBABILITY MASS FUNCTION (PMF)

Suppose X takes values $x_1, x_2, \dots, x_n$ such that $P(X = X_i) = P_i$, the P_i is called the probability mass function if P_i satisfies the following conditions:

(i) $P_i \geq 0 \; ; \forall i$
(ii) $\sum P_i = 1$

PROBABILITY DENSITY FUNCTION (PDF)

Let X be a RV defined in interval B, then the probability density function of X is denoted by $F(X)$ & has to satisfy the following conditions:

(i) $F(x) >= 0$
(ii) $\int_a^b F(x)\,dx = 1$

Note: If X is continuous

$$P\{a \leq x \leq b\} = \int_a^b f(x)\,dx = P\{a < x < b\}$$

27. A bag of 10 items containing 3 defectives. A sample of 4 is drawn at random without replacement. Let RV x denote the no of defective items, in that
(i) Find PMF of X
(ii) Find $P\{X \leq 1\}$, $P\{X<1\}$, $P\{0 < x < 2\}$

Sol: $X = \{0, 1, 2, 3\}$

$$P(X=0) = \frac{{}^7C_4}{{}^{10}C_4} = \frac{\frac{7\times6\times5}{3\times2}}{\frac{10\times9\times8\times7}{4\times3\times2}} = \frac{\cancel{7}\times\cancel{6}\times\cancel{5}}{\cancel{10}\times\cancel{9}\times\cancel{8}\times\cancel{7}}$$

$$= \frac{1}{6}$$

$$P(X=1) = \frac{{}^3C_1 \times {}^7C_3}{{}^{10}C_4}$$

$$= \frac{3 \times \frac{7\times6\times5}{3\times2}}{\frac{10\times9\times8\times7}{4\times3\times2}} = \frac{\cancel{3}\times\cancel{7}\times\cancel{6}\times\cancel{5}}{\cancel{10}\times\cancel{9}\times2\times\cancel{4}}$$

$$= \frac{1}{2}$$

$P(X=2) = \frac{{}^3C_2 \times {}^7C_2}{{}^{10}C_4}$

$= \dfrac{3 \times \frac{7\times 6}{\cancel{2}}}{\frac{10\times\cancel{9}\times\cancel{8}\times 7}{\cancel{4}\times\cancel{3}\times\cancel{2}}} = \frac{\cancel{3}\times\cancel{7}\times\cancel{6}}{10\times\cancel{3}\times\cancel{2}\times\cancel{7}}$

$= \frac{3}{10}$

$P(X=3) = \frac{{}^3C_3 \times {}^7C_1}{{}^{10}C_4}$

$= \dfrac{7}{\frac{10\times\cancel{9}\times\cancel{8}\times 7}{\cancel{4}\times\cancel{3}\times\cancel{2}}} = \frac{\cancel{7}}{10\times 3\times\cancel{7}}$

$= \frac{1}{30}$

X	0	1	2	3
P	0.166	0.5	0.3	0.033

28. The PMF of RV is given by

X	0	1	2	3	4	5	6
P(X)	k	3k	5k	7k	9k	11k	13k

(i) Find the $P(x) < 4$

(ii) $P(x) >= 5$

(iii) $P(3 < x \leq 6)$

Sol: $49k = 1$

$\boxed{k = \frac{1}{49}}$

(a) $P(X<4) = P(0) + P(1) + P(2) + P(3)$

$P(X=0) = k = \frac{1}{49}$

$P(X=1) = 3k = \frac{3}{49}$

$P(X=2) = 5k = \frac{5}{49}$

$P(X=3) = 7k = \frac{7}{49}$

$P(X<4) = \frac{1}{49} + \frac{3}{49} + \frac{5}{49} + \frac{7}{49}$

$\boxed{P(X<4) = \frac{16}{49}}$

(ii) $P(X>=5) = P(5) + P(6)$

$P(X=5) = 11k = \frac{11}{49}$

$P(X=6) = 13k = \frac{13}{49}$

$P(X>=5) = \frac{13}{49} + \frac{11}{49}$

$\boxed{P(X>=5) = \frac{24}{49}}$

(iii) $P(3<X\leq 6) = P(X=4) + P(X=5) + P(X=6)$

$$P(X=4) = 9k = \frac{9}{49}$$

$$P(X=5) = 11k = \frac{11}{49}$$

$$P(X=6) = 13k = \frac{13}{49}$$

$$P(3<X\leq 6) = \frac{9+11+13}{49}$$

$$\boxed{P(3<X\leq 6) = \frac{33}{49}}$$

29 Diameter of an electric ~~₹~~ cable is assummed to be a continuous RV with pdf

$$F(x) = \begin{cases} 6x(1-x), & 0<x<1 \\ 0, & \text{elsewhere} \end{cases}$$

Check whether $F(x)$ is valid pdf

Sol: $\int_a^b f(x)\,dx = 1$

$$\int_a^b 6x(1-x)\,dx$$

$$\int_0^1 (6x-6x^2)\,dx$$

$$\left[\frac{6x^2}{2} - \frac{6x^3}{3}\right]_0^1$$

$$3-2$$
$$= 1$$

$\therefore$ $F(x)$ is a valid pdf

30. Compute $P\left(X \leq \frac{1}{2} \middle| \frac{1}{3} \leq x \leq \frac{2}{3}\right)$

Sol: $$\frac{P\left(\left(x\leq\frac{1}{2}\right)\cap\left(\frac{1}{3}\leq x\leq\frac{2}{3}\right)\right)}{P\left(\frac{1}{3}\leq x\leq\frac{2}{3}\right)}$$

$$= \frac{P\left(\frac{1}{3}\leq x\leq\frac{1}{2}\right)}{P\left(\frac{1}{3}\leq x\leq\frac{2}{3}\right)}$$

30/8/23

30. 3 balls are randomly selected from a box of 3 white, 3 red and 5 black balls. The person who selects a ball, wins 1 dollar for each white ball & loses 1 dollar for each red ball selected. If X is the total winning from the experiment, find P func^n of X

Sol:

Cases	Result
3W	3
3R	-3
3B	0
2W 1R	1
2W 1B	2

2R 1W	-1
2R 1B	-2
2B 1W	1
2B 1R	-1
1B 1R 1W	0

$X = \{-3, -2, -1, 0, 1, 2, 3\}$

$$P(-3) = \frac{{}^3C_3}{{}^{11}C_3} = \frac{1}{165}$$

$$\cancel{P(-2) = \frac{{}^3C_2 \times {}^3C_1}{{}^{11}C_3} + \frac{{}^5C_2 \times}{{}^{11}C_3} + \frac{{}^3C_1}{{}^{11}C_3}}$$

$$\cancel{= \frac{3 \times 3 + 10 + 3}{165}}$$

$$\cancel{= \frac{22}{165}} \quad \cancel{\frac{2}{15}}$$

$$\cancel{= \frac{2}{15}}$$

$$P(-2) = \frac{{}^3C_2 \times {}^5C_1}{{}^{11}C_3}$$

$$= \frac{\cancel{3} \times \cancel{5}}{\cancel{165}\ \cancel{55}\ 11}$$

$$= \frac{1}{11}$$

$$P(-1) = \frac{{}^3C_2 \times {}^3C_1}{{}^{11}C_3} + \frac{{}^5C_2 \times {}^3C_1}{{}^{11}C_3}$$

$$= \frac{3 \times 3}{165} + \frac{10 \times 3}{165}$$

$$= \frac{9 + 30}{165}$$

$$= \frac{\cancel{39}\ 13}{\cancel{165}\ 55}$$

$$= \frac{13}{55}$$

$$P(0) = \frac{{}^5C_3}{{}^{11}C_3} + \frac{{}^3C_1 \times {}^3C_1 \times {}^5C_1}{{}^{11}C_3}$$

$$= \frac{10 + 3 \times 3 \times 5}{165}$$

$$= \frac{10 + 45}{165}$$

$$= \frac{\cancel{55}\ \cancel{5}\ 1}{\cancel{165}\ \cancel{15}\ 3}$$

$$= \frac{1}{3}$$

$$P(1) = \frac{{}^3C_2 \times {}^3C_1}{{}^{11}C_3} + \frac{{}^5C_2 \times {}^3C_1}{{}^{11}C_3}$$

$$= \frac{3 \times 3 + 10 \times 3}{165}$$

$$= \frac{9 + 30}{165}$$

$$= \frac{39}{165}$$

$$= \frac{13}{55}$$

$$P(2) = \frac{{}^3C_2 \times {}^5C_1}{{}^{11}C_3} = \frac{\cancel{3} \times \cancel{5}}{\cancel{165}\ \cancel{55}\ 11}$$

$$= \frac{1}{11}$$

$$P(3) = \frac{^3C_3}{^{11}C_3}$$

$$= \frac{1}{165}$$

31. $f(x) = \begin{cases} k(x^3), & 0<x<1 \\ 0, & \text{elsewhere} \end{cases}$

Find (i) $P\{1/4 < x < 3/4\}$
(ii) $P\{X < \frac{1}{2}\}$
(iii) $P\{x > 0.8\}$

Sol: Since $f(x)$ is pdf

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^1 kx^3\,dx = 1$$

$$\left[\frac{kx^4}{4}\right]_0^1 = 1$$

$$\frac{k}{4} = 1$$

$$\boxed{k=4}$$

(i) $P\{\frac{1}{4} < x < \frac{3}{4}\} = \int_{1/4}^{3/4} 4x^3\,dx$

$$= \cancel{4}\left[\frac{x^4}{\cancel{4}}\right]_{1/4}^{3/4}$$

$$= \left[\frac{3}{4}\right]^4 - \left(\frac{1}{4}\right)^4$$

$$= \frac{81}{216} - \frac{1}{216}$$

$$= \frac{\cancel{80}\ \cancel{40}\ \cancel{20}\ 10}{\cancel{216}\ \cancel{108}\ \cancel{54}\ 27}$$

$$= \frac{10}{27}$$

(ii) $P\left(x<\frac{1}{2}\right) = \int_{-\infty}^{1/2} f(x)\,dx$

$$= \int_{\infty}^{0} 4x^3\,dx + \int_0^{1/2} 4x^3\,dx$$

$$= 0 + \left[\frac{4x^4}{\cancel{4}}\right]_0^{1/2}$$

$$= \left[\frac{1}{2}\right]^4 - 0$$

$$= \frac{1}{16}$$

(iii) $P\{x>0.8\} = \int_{0.8}^{\infty} 4x^3\,dx$

$$= \int_{0.8}^{1} 4x^3\,dx + \int_1^{\infty} 4x^3\,dx$$

$$= \left[\frac{\cancel{4}x^4}{\cancel{4}}\right]_{0.8}^{1} + 0$$

$$= 1 - 0.4096$$

$$= 0.5904$$

32. The time that a person has to wait for a train at the station is observed to be a random phenomenon with the following pdf:

$$f(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{9}(x+1), & 0 \le x < 1 \\ \frac{4}{9}\left(x - \frac{1}{2}\right), & 1 \le x < \frac{3}{2} \\ \frac{4}{9}\left(\frac{5}{2} - x\right), & \frac{3}{2} \le x < 2 \\ \frac{1}{9}(4-x), & 2 < x < 3 \\ \frac{1}{9}, & 3 \le x < 6 \\ 0, & x \ge 6 \end{cases}$$

If A is the event that one waits b/w $0 \le x \le 2$ & B is event that one waits b/w $1 \le x \le 3$. Find $P(B|A)$ & $P(\bar{A} \cap \bar{B})$.

Sol: $P(A) = \int_0^2 f(x)\,dx$

$$= \int_0^1 \frac{1}{9}(x+1)\,dx + \int_1^{3/2} \frac{4}{9}\left(x-\frac{1}{2}\right)dx + \int_{3/2}^2 \frac{4}{9}\left(\frac{5}{2}-x\right)dx$$

$$= \frac{1}{9}\int_0^1 (x+1)\,dx + \frac{4}{9}\int_1^{3/2}\left(x-\frac{1}{2}\right)dx + \frac{4}{9}\int_{3/2}^2\left(\frac{5}{2}-x\right)dx$$

$$= \frac{1}{9}\left[\int_0^1 (x+1)\,dx + 4\int_1^{3/2}\left(x-\frac{1}{2}\right)dx + 4\int_{3/2}^2\left(\frac{5}{2}-x\right)dx\right]$$

$$= \frac{1}{9}\left[\left[\frac{x^2}{2}+x\right]_0^1 + 4\left[\frac{x^2}{2}-\frac{x}{2}\right]_1^{3/2} + 4\left[\frac{5x}{2}-\frac{x^2}{2}\right]_{3/2}^2\right]$$

$$= \frac{1}{9}\left(\frac{1}{2}+1+4\left(\left(\frac{9/4}{2}-\frac{3/2}{2}\right)-\left(\frac{\cancel{1}}{2}-\frac{\cancel{1}}{2}\right)\right) + 4\left[\left(\frac{5\times\cancel{2}}{\cancel{2}}-\frac{\cancel{2}\times 2}{\cancel{2}}\right)-\left(\frac{3/2\times 5}{2}-\frac{9/4}{2}\right)\right]\right)$$

$$= \frac{1}{9}\left(\frac{3}{2}+4\left(\left(\frac{9}{4}\times\frac{1}{2}-\frac{3}{2}\times\frac{1}{2}\right)\right)+4\left[(5-2)-\left(\frac{3}{2}\times\frac{5\times 1}{2} - \frac{9}{4}\times\frac{1}{2}\right)\right]\right)$$

$$= \frac{1}{9}\left(\frac{3}{2}+4\left(\frac{9}{8}-\frac{3}{4}\right)+4\left(3-\left(\frac{15}{4}-\frac{9}{8}\right)\right)\right)$$

$$= \frac{1}{9}\left(\frac{3}{2}+\cancel{4}\left(\frac{9-6}{\cancel{8}\,2}\right)+4\left(3-\left(\frac{30-9}{8}\right)\right)\right)$$

$$= \frac{1}{9}\left(\frac{3}{2}+\frac{3}{2}+4\left(3-\frac{21}{8}\right)\right)$$

$$= \frac{1}{9}\left(\frac{3}{2}+\frac{3}{2}+\cancel{4}\left(\frac{24-21}{\cancel{8}\,2}\right)\right)$$

$$= \frac{1}{9}\left(\frac{6}{2}+\frac{3}{2}\right)$$

$$= \frac{1}{\cancel{9}}\left(\frac{\cancel{9}}{2}\right)$$

$$= \frac{1}{2}$$

$$= \cancel{\frac{4}{9}}_{3}\left(\frac{\cancel{54}^{\cancel{18}^{\cancel{6}\,3}}}{\cancel{8}_{2}}\right)$$

$$=$$

$$P(A\cap B) = \int_1^2 f(x)\,dx$$

$$= \int_1^{3/2} \frac{4}{9}\left(x-\frac{1}{2}\right)dx + \int_{3/2}^{2} \frac{4}{9}\left(\frac{5}{2}-x\right)dx$$

$$= \frac{4}{9}\left[\frac{9}{4}\times\frac{1}{2}-\frac{3}{2}\times\frac{1}{2}+3-\left(\frac{15}{4}-\frac{9}{8}\right)\right]$$

$$= \frac{4}{9}\left[\frac{9}{8}-\frac{3}{4}+3-\left(\frac{30-9}{8}\right)\right]$$

$$= \frac{4}{9}\left(\frac{9-6}{8}+3-\frac{21}{8}\right)$$

$$= \frac{4}{9}\left(\frac{3}{8}+\frac{3}{8}\right)$$

$$= \cancel{\frac{4}{9}}_{3}\left(\frac{\cancel{6}^{3}}{\cancel{8}_{2}}\right)$$

$$= \frac{1}{3}$$

(ii) $P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B})$

$$= 1 - P(A\cup B)$$

$$= 1 - \int_0^3 f(x)\,dx$$

$$=1-\left[\int_0^1 \frac{1}{9}(x+1)\,dx + \int_1^{3/2}\frac{4}{9}\left(x-\frac12\right)dx + \int_{3/2}^{2}\frac49\left(\frac52 - x\right)dx + \int_2^3 \frac19(4-x)\,dx\right]$$

$$=1-\left[\frac19\left[\frac{x^2}{2}+x\right]_0^1 + \frac49\left[\frac{x^2}{2}-\frac{x}{2}\right]_1^{3/2} + \frac49\left[\frac{5x}{2}-\frac{x^2}{2}\right]_{3/2}^{2} + \frac19\left[4x-\frac{x^2}{2}\right]_2^3\right)$$

$$\cancel{=1-\left[\frac19\right.}$$

$$=1-\frac19\left[\left(\frac12+1\right)+4\left[\left(\frac{(3/2)^2}{2}-\frac{3/2}{2}\right)-\left(\frac12-\frac12\right)\right] + 4\left[\left(\frac{5\times2}{2}-\frac{2\times2}{2}\right)-\left(\frac{5(3/2)}{2}-\frac{(3/2)^2}{2}\right)\right] + \left(\left(4(3)-\frac92\right)-\left(4(2)-\frac{2\times2}{2}\right)\right)\right]$$

$$=1-\frac19\left(\frac32+4\left(\frac{3\times3}{2\times2}\times\frac12-\frac32\times\frac12\right)+4\left(5-2-\left(\frac{5\times3}{2\times2}-\frac{3\times3}{2\times2\times2}\right)\right)+\left(\left(12-\frac92\right)-(8-2)\right)\right]$$

$$=1-\frac19\left(\frac32+4\left(\frac98-\frac34\right)+4\left(3-\left(\frac{15}{4}-\frac98\right)\right)+\left(\frac{15}{2}-6\right)\right]$$

$$=1-\frac19\left(\frac32+\cancel{4}\left(\frac{9-6}{\cancel{8}\,2}\right)+4\left(3-\left(\frac{30-9}{8}\right)\right]+\frac32\right]$$

$$=1-\frac19\left(\frac32+\frac32+\cancel{4}\frac32+\frac32\right)$$

$$=1-\frac{1}{\cancel{9}\,3}\left(\frac{\cancel{12}^{\cancel{4}\,2}}{\cancel{2}}\right)$$

$$=1-\frac23$$

$$=\frac13$$

CUMMULATIVE DISTRIBUTION FUNCTION

$$F(x)=\Sigma P(X\le x)$$

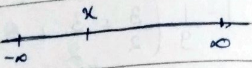
$$\boxed{F(x_j)-F(x_{j-1})=P(x_j)}$$

33 Consider the PMS given by

x	$P(x)$	$F(x)$
0	$1/49$	$1/49$
1	$3/49$	$4/49$
2	$5/49$	$9/49$
3	$7/49$	$16/49$
4	$9/49$	$25/49$
5	$11/49$	$36/49$
6	$13/49$	$49/49$

For Continuous

$F(x) = \int_{-\infty}^{x} f(x)\,dx$



$F(x) = P(X \le x)$

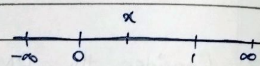
This is why we take the limits from $-\infty$ to x.

34. Consider the pdf of continuous RV

$$f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0, & \text{~~ot~~ elsewhere} \end{cases}$$

Find cdf of $f(x)$.

Sol: $F(x) = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{x} 6x(1-x)\,dx$



$= 0 + 6\int_{0}^{x} (x - x^2)\,dx$

$= 6\int_{0}^{x} (x - x^2)\,dx$

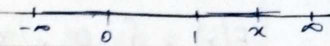
$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^x$

$= \frac{\cancel{6}^{3} x^2}{\cancel{2}} - \frac{\cancel{6}^{2} x^3}{\cancel{3}}$

$= 3x^2 - 2x^3$

$F(x) = 1 \qquad (\because \text{the cdf at last element} = 1)$

or $= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} 6x(1-x)\,dx + \int_{1}^{x} 0$



$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= 6\left(\frac{1}{2} - \frac{1}{3}\right)$

$= \cancel{6}\left(\frac{3-2}{\cancel{6}}\right)$

$= 3 - 2$

$= 1$

$$\therefore F(x) = \begin{cases} 0, & x \le 0 \\ 3x^2 - 2x^3, & 0 \le x \le 1 \\ 1, & x \le \infty \text{ or } x \ge 1 \end{cases}$$

35. Consider RV x satisfying the pdf

$$f(x) = \begin{cases} kx^3, & 0 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

Find $F(x)$

Sol: $\because f(x) = \text{pdf}$

$\int_{0}^{2} kx^3 = 1$

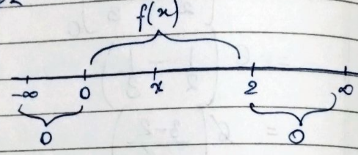
$k\left[\frac{x^4}{4}\right]_0^2 = 1$

$k\left[\frac{\cancel{16}^{4}}{\cancel{4}}\right] = 1$

$4k = 1$

$$\boxed{k = \frac{1}{4}}$$

$$F(x) = \begin{cases} 0, & x \le 0 \\ x^4/16, & 0 \le x \le 2 \\ 1, & x \ge 2 \end{cases}$$



$$F(x) = \int_{-\infty}^{0} f(x) + \int_{0}^{x} f(x)$$

$$= 0 + \int_0^x kx^3\,dx$$

$$= \frac{1}{4}\left[\frac{x^4}{4}\right]_0^x$$

$$= \frac{x^4}{16}$$

2/9/23

36. Let x be a continuous RV with pdf

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ -ax+3a, & 2 \le x < 3 \\ 0, & \text{elsewhere} \end{cases}$$

Determine the constant a & hence find the cdf

Sol: $a\int_0^1 x\,dx + a\int_1^2 1\,dx + a\int_2^3 (-x+3)\,dx = 1$

$$\left[\frac{ax^2}{2}\right]_0^1 + \left[ax\right]_1^2 - \left[\frac{ax^2}{2}\right]_2^3 + \left(3ax\right)_2^3 = 1$$

$$\frac{a}{2} + 2a - 1a - \left(\frac{9a}{2} - \frac{4a}{2}\right) + 9a - 6a = 1$$

$$\frac{a}{2} + a - \frac{5a}{2} + 3a = 1$$

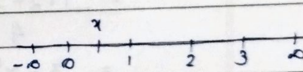
$$4a - \frac{\cancel{4}a}{\cancel{2}} = 1$$

$$2a = 1$$

$$\boxed{a = \frac{1}{2}}$$

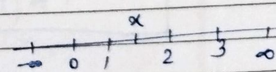
$$f(x) = \begin{cases} x/2, & 0 \le x \le 1 \\ 1/2, & 1 \le x \le 2 \\ -x/2 + 3/2, & 2 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

$$F(x) = \int_{-\infty}^{0} f(x)\,dx + \int_0^x \frac{x}{2}\,dx$$



$$= 0 + \left[\frac{x^2}{4}\right]_0^x$$

$$= \frac{x^2}{4}$$

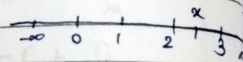


$$F(x<2) = \int_{-\infty}^{0} f(x) + \int_0^1 \frac{x}{2}\,dx + \int_1^x \frac{1}{2}\,dx$$

$$= 0 + \left[\frac{x^2}{4}\right]_0^1 + \left[\frac{x}{2}\right]_1^x$$

$$= \frac{1}{4} + \frac{x}{2} - \frac{1}{2}$$

$$= \frac{x}{2} - \frac{1}{4}$$



$$F(x<3) = \int_{-\infty}^{0} f(x) + \int_{0}^{1} \frac{x}{2}\,dx + \int_{1}^{2} \frac{1}{2}\,dx + \int_{2}^{x} \left(\frac{-x}{2} + \frac{3}{2}\right) dx$$

$$= 0 + \left[\frac{x^2}{4}\right]_0^1 + \left[\frac{x}{2}\right]_1^2 + \left[\frac{-x^2}{4}\right]_2^x + \left[\frac{3x}{2}\right]_2^x$$

$$= \cancel{0} + \frac{1}{4} + 1 - \frac{1}{2} + \left(\frac{-x^2}{4} + \frac{4}{4}\right) + \left(\frac{3x}{2} - 3\right)$$

$$= \frac{1}{4} + \frac{1}{2} - \frac{x^2}{4} + 1 + \frac{3x}{2} - 3$$

$$= \frac{1+2-x^2+4+6x-12}{4}$$

$$= \frac{3-x^2-8+6x}{4}$$

$$= \frac{6x}{4} - \frac{x^2}{4} - \frac{5}{4}$$

$$\boxed{= \frac{3x}{2} - \frac{x^2}{4} - \frac{5}{4}}$$

$$F(x) = \begin{cases} 0, & x \le 0 \\ x/4, & 0 \le x \le 1 \\ x/2 - 1/4, & 1 \le x \le 2 \\ 3x/2 - x^2/4 - 5/4, & x \ge 3 \end{cases}$$

37. A RV x has the follow probability func^n

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

If $P(x \le a) > \frac{1}{2}$. Find min value of k. Also find cdf.

Sol: $k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$$10k^2 + 9k = 1$$

$$10k^2 + 9k - 1 = 0$$
$$10k^2 + 10k - k - 1 = 0$$
$$10k(k+1) - 1(k+1) = 0$$
$$(10k-1)(k+1) = 0$$

$$\boxed{k = \frac{1}{10}, -1}$$

$\because$ Probability cannot be -ve

$$\boxed{\therefore k = \frac{1}{10}}$$

$F(0) = 0$
$F(1) = 1/10$
$F(2) = 3/10$
$F(3) = 5/10$
$F(4) = 8/10$
$F(5) = 81/100$
$F(6) = \frac{83}{100}$

$F(7) = 1$

$P(x \le a) \ge \frac{1}{2}$

$\therefore$ The min value of $a = 4$

MEAN AND VARIANCE

Mean & variance of a Random variable X denoted by

$$\boxed{E(X) = \mu = \text{Mean}}$$

$$\boxed{V(X) = \sigma^2 = \text{Variance}}$$

If x is discrete then

$$\boxed{E(X) = \Sigma x p(x)}$$

$$\boxed{V(X) = E(x^2) - (E(X))^2}$$

If x is continuous then

$$\boxed{E(x) = \int_{-\infty}^{\infty} x f(x) dx}$$

$$\boxed{V(x) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)}$$

$$\text{Def}^n = \sum_{i=1}^{n} (x_i - \mu)^2 * p(x)$$

38. Find the mean & SD for the following P distribution table

x	-3	-2	-1	0	1	2	3
$p(x)$	k	$2k$	$3k$	$4k$	$3k$	$2k$	k

Sol→ $k + 2k + 3k + 4k + 3k + 2k + k = 1$

$16k = 1$

$$\boxed{k = \frac{1}{16}}$$

E(

x	x^2	$P(x)$	$x(P(x))$	$x^2(P(x))$
-3	9	$1/16$	$-3/16$	$9/16$
-2	4	$2/16$	$-4/16$	$8/16$
-1	1	$3/16$	$-3/16$	$3/16$
0	0	$4/16$	0	0
1	1	$3/16$	$3/16$	$3/16$
2	4	$2/16$	$4/16$	$8/16$
3	9	$1/16$	$3/16$	$9/16$
			$\Sigma x p(x) = 0$	$\Sigma x^2 p(x) = 40/16$

$E(x) = 0$

$E(x^2) = \frac{40}{16}$

$(E(x))^2 = 0$

$$\sigma^2 = \frac{40}{16} - 0$$

$$\sigma = \sqrt{\frac{40}{16}}$$

$$= \frac{2\sqrt{10}}{4}$$

$$\boxed{\sigma = \frac{\sqrt{10}}{2}}$$

39. Find μ & SD of sum obtained in tossing a pair of fair dice.

Sol:

x	$p(x)$	$x(p(x))$	x^2	$x^2(p(x))$
2	1/36	2/36	4	4/36
3	2/36	6/36	9	18/36
4	3/36	12/36	16	48/36
5	4/36	20/36	25	100/36
6	5/36	30/36	36	180/36
7	6/36	42/36	49	294/36
8	5/36	40/36	64	320/36
9	4/36	36/36	81	324/36
10	3/36	30/36	100	300/36
11	2/36	22/36	121	242/36
12	1/36	12/36	144	144/36
		$\Sigma = 7$		54.833

$$E(x) = \frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36}$$

$$= \frac{252}{36}$$

$$= 7$$

$$\sigma^2 = E(x^2) - (E(x))^2$$

$$= 54.833 - 49$$

$$= 5.833$$

40. A coin is tossed until a head appears what is the expectation of no of tosses required.

i.e. mean

Sol:

x	1	2	3	4	5	6	...	n
$p(x)$	1/2	1/4	1/8	1/16	1/32	1/64	...	

$$E(x) = 1\times\frac{1}{2} + 2\times\frac{1}{4} + 3\times\frac{1}{8} + 4\times\frac{1}{16} + 5\times\frac{1}{32} + \dots$$

$$= \frac{1}{2}\left[1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2^3}\right) + \dots\right)$$

$$\boxed{S_n = \frac{a}{1-r} + \frac{rd}{(1-r)^2}}$$

$$= \frac{1}{2}\left(\frac{1}{1-1/2} + \frac{1/2 \times 1}{(1-1/2)^2}\right)$$

$$= \frac{1}{2}\left(\frac{1}{1/2} + \frac{1/2}{1/4}\right)$$

$$E(X) = \frac{1}{2}(2+2)$$
$$= 2$$

41. Suppose that a game is to be played with a single dice assumed fair. A player wins Rs 20 if 2 turns up, Rs 40 if 4 turns up, loses Rs 30 if 6 turns up. While the player neither wins nor loses if any other face turns up. Find the expected sum of money to be won.

Sol:

X	$P(X)$	$x\,p(x)$
-30	1/6	-30/6
0	3/6	0
20	1/6	20/6
40	1/6	40/6
		30/6
		=5

$$E(x) = 5$$

PROPERTIES OF MEAN AND VARIANCE

1. Expectation of a constant = $E(a) = a$
2. Expectation of expectation of x = $E(E(x)) = E(x)$
3. $E(aX) = aE(X)$
4. $E(aX+b) = aE(X) + b$
5. $E(x_1+x_2) = E(x_1) + E(x_2)$
6. $V(a) = 0$
7. $V(x_1+x_2) = V(x_1) + V(x_2)$
8. $V(aX) = E((aX)^2) - (E(aX))^2$

$$= E(a^2X^2) - (aE(X))^2$$
$$= a^2E(x^2) - a^2E(X)^2$$
$$\boxed{V(aX) = a^2V(X)}$$

42. If the RV x has a pdf

$$f(x) = \begin{cases} \frac{3+2x}{18}, & 2 < x \le 4 \\ 0, & \text{otherwise} \end{cases}$$

Find mean & SD.

Sol: $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$$= \int_2^4 x\left(\frac{3+2x}{18}\right)dx$$
$$= \frac{1}{18}\int_2^4 (3x+2x^2)\,dx$$
$$= \frac{1}{18}\left(\left[\frac{3x^2}{2}\right]_2^4 + \left[\frac{2x^3}{3}\right]_2^4\right)$$

$$= \frac{1}{18}\left[\frac{3(16)}{2} - \frac{3(4)}{2} + \left(\frac{128}{3} - \frac{16}{3}\right)\right]$$

$$= \frac{1}{18}\left[\frac{48}{2} - \frac{12}{2} + \left\{\frac{112}{3}\right)\right.$$

$$= \frac{1}{18}\left(\frac{36}{2} + \frac{112}{3}\right)$$

$$= \frac{1}{18}\left(\frac{108+224}{6}\right)$$

$$= \frac{1}{\cancel{18}_{9}}\left(\frac{\cancel{332}^{\cancel{166}\,83}}{\cancel{6}\,3}\right)$$

$$= \frac{83}{27}$$

$$\boxed{E(x) = 3.074}$$

$$\sigma^2 = \int_2^4 (x-3.074)^2\left(\frac{3+2x}{18}\right)$$

$$= \cancel{\int_2^4 \left[\frac{3x+2x^3}{18}\right.}$$

$$= \int_2^4 \left(x^2 - 6.148x + 9.44\right)\left(\frac{3+2x}{18}\right)$$

$$= \frac{1}{18}\int_2^4 \left(3x^2 - 18.44x + 28.32 + 2x^3 - 12.29 + 18.88\right)dx$$

$$= \frac{1}{18}\int_2^4 \left(2x^3 + 3x^2 - 18.44x + 34.91\right)$$

$$= \frac{1}{18}\left\{\left[\frac{\cancel{2}x^4}{\cancel{4}_2} + \frac{\cancel{3}x^3}{\cancel{3}} - \frac{\cancel{18.44}^{9.22}x^2}{\cancel{2}} + 34.91x\right]_2^4\right.$$

$$= \frac{1}{18}\left[\frac{x^4}{2} + x^3 - 9.22x^2 + 34.91x\right]_2^4$$

$$\cancel{= \frac{1}{18}\left[\left(\frac{4\times4\times4\times4}{2}\right.\right.}$$

$$= \frac{1}{18}\left[\left(\frac{\cancel{4}^{2}\times4\times4\times4}{\cancel{2}} - \frac{\cancel{2}\times2\times2\times2}{\cancel{2}}\right) + \left[(4)^3 - (2)^3\right] - \left[9.22(4)^2 - 9.22(2)^2\right] + \left(34.91(4) - 34.91(2)\right)\right]$$

$$= \frac{1}{18}\left[(128-8) + (64-8) - (147.52 - 36.88) + (139.64 - 69.82)\right]$$

$$= \frac{1}{18}\left(120 + 56 - 110.64 + 69.82\right)$$

$$= \frac{1}{18}\left(135.18\right)$$

$$\sigma^2 = 7.51$$

$$\boxed{\sigma = 2.7404}$$

$$E(x^2) = \int_2^4 x^2 f(x)$$

$$= \int_2^4 x^2\left(\frac{3+2x}{18}\right)$$

$$= \frac{1}{18}\int_2^4 3x^2 + 2x^3$$

$$\lim_{x\to\infty} e^{-ax} = 0$$

$$= \frac{1}{18}\left[\frac{\cancel{3}x^3}{\cancel{3}} + \frac{\cancel{2}x^4}{\cancel{4}\,2}\right]_2^4$$

$$= \frac{1}{18}\left[x^3 + \frac{x^4}{2}\right]_2^4$$

$$= \frac{1}{18}\left[(4^3 - 2^3) + \left(\frac{\cancel{4}\times4\times4\times4}{\cancel{2}} - \frac{\cancel{2}\times2\times2\times2}{\cancel{2}}\right)\right]$$

$$= \frac{1}{18}\left((64-8) + (128-8)\right)$$

$$= \frac{1}{18}(56+120)$$

$$= \frac{1}{18}(176)$$

$$\boxed{E(x^2) = 9.778}$$

$$\sigma^2 = 9.778 - (3.074)^2$$

$$= 9.778 - 9.449$$

$$\boxed{\sigma^2 = 0.329}$$

$$\boxed{\sigma = 0.5735}$$

43. $f(x) = \begin{cases} 2e^{-2x} & ; x>0 \\ 0 & ; x\le 0 \end{cases}$

Sol: $E(X) = \int_0^\infty 2xe^{-2x}\,dx = 2\int_0^\infty xe^{-2x}\,dx$

$$\int f(x)g(x) = f(x)\int g(x) - \int\left(\frac{d}{dx}(f(x)) \cdot \int g(x)\right)$$

$$\cancel{= 2\left[\frac{xe^{-2x}}{2}\right]}$$

$$= 2\left[x\int e^{-2x}\,dx - \int\left(\frac{dx}{dx}\cdot\int e^{-2x}\,dx\right)dx\right]_0^\infty$$

$$= 2\left[x\,\frac{e^{-2x}}{-2} - \int\frac{e^{-2x}}{-2}\,dx\right]_0^\infty$$

$$= 2\left[\frac{xe^{-2x}}{-2} - \frac{1}{2}\times\frac{e^{-2x}}{2}\right]_0^\infty$$

$$= 2\left[\frac{xe^{-2x}}{-2} - \frac{e^{-2x}}{4}\right]_0^\infty$$

$$= 2\left(\frac{-xe^{-2x}}{2} - \frac{e^{-2x}}{4}\right)_0^\infty$$

$$= \left(-xe^{-2x} - \frac{e^{-2x}}{2}\right)_0^\infty$$

$$= [0-0] - \left[0 - \frac{1}{-2}\right]$$

$$\boxed{E(x) = \frac{1}{2}}$$

$$E(x^2) = \int_0^\infty x^2 f(x) = 2\int_0^\infty x^2 e^{-2x}\,dx$$

$$= 2\left[\frac{x^2 e^{-2x}}{-2} - \frac{1}{-2}\times\frac{2xe^{-2x}}{-2} + \frac{1}{4}\times\frac{2e^{-2x}}{-2}\right]_0^\infty$$

$$= 2\left[\frac{x^2 e^{-2x}}{-2} - \frac{2xe^{-2x}}{4} - \frac{2e^{-2x}}{8}\right]_0^\infty$$

$$= \cancel{2}\left(\frac{-4x^2e^{-2x} - 4xe^{-2x} - 2e^{-2x}}{\cancel{8}\ 4}\right]_0^\infty$$

$$= \frac{1}{4}\left(-4x^2e^{-2x} - 4xe^{-2x} - 2e^{-2x}\right]_0^\infty$$

$$= \frac{1}{2}\left(-2x^2e^{-2x} - 2xe^{-2x} - e^{-2x}\right]_0^\infty$$

$$= \frac{1}{2}\left((0-0)-(0-0)-(0-1)\right]$$

$$= \frac{1}{2}(1)$$

$$\boxed{E(x^2) = \frac{1}{2}}$$