Chapter 1: Mathematical Logics

Propositions: Arguments or statements that are either True or False clearly. (only 2 ans)

↓ denoted either P, Q, R, S or p, q, r, s

eg: 2>5 is false

Today is Sunday → T/F

2 is even → T

Connectives / Logical connectives: Used to connect the propositions.
(operators)
→ 'and', 'or', 'if and only if', 'if p then q'

Proposition calculus results in True or False.

Boolean Operators (Connectives):

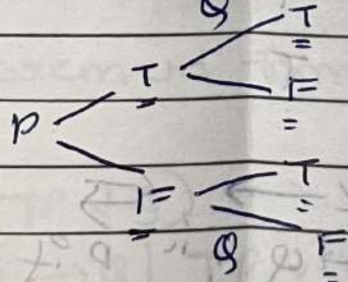
1) ¬, ~ → NOT (unary operator)

Binary operators: ii) ∧ → Conjunction

'P∧Q' called as 'P and Q'

Possibilities can be P: T / F   Q: T / F

Tree diagram



Truth values → values of Tree diagram

Truth Table:

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

iii) Disjunction : ∨

'p∨q' → 'p or q'

Truth Table.

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

iv) Conditional → 'if then'

p, q, 'p → q' 'if p then q'

p → hypothesis (premise), q → conclusion.

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Based on final conclusion i.e q truth value depends.

v) Biconditional ⟷, ⇆

p, q p ⟷ q, " p if and only if q"

" p iff q"

p	q	p ⟷ q
T	T	T
T	F	F
F	T	F
F	F	T

"Both p & q same truth values"

Statement Formula / composite statement:
Combination of many statements
eg: $(\neg P \to Q) \wedge R$.

Tautology: Final result or truth values will always be true.
" A statement formula which is true regardless of the truth values of the statements which replace the variables in it " is called ~~universally valid formula or~~ logical truth or tautology.

Contradiction: Final result or truth values will always be False.
" A statement formula which is false regardless of the truth values of the statements which replace the variables in it " is called ~~universally~~ contradiction.

i) $P \to (Q \to (P \wedge Q))$

P	Q	$P \wedge Q$	$Q \to (P \wedge Q)$	$P \to (Q \to P \wedge Q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	F	T	T

∴ This statement formula is tautology.

ii) $(P \to Q) \longleftrightarrow (\neg P \vee Q) \to$ tautology.

P	Q	$\neg P$	$P \to Q$	$\neg P \vee Q$	$(P \to Q) \longleftrightarrow (\neg P \vee Q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

iii) $(p \vee q) \vee (\sim q)$

p	q	$p \vee q$	$\sim q$	$(p \vee q) \vee (\sim q)$
T	T	T	F	T
T	F	T	T	T
F	T	T	F	T
F	F	F	T	T

∴ Follows tautology.

Satisfiable: mixture of true and false in the truth values then its satisfiable.

- If the truth values of composite statement are sometimes true and sometimes false irrespective of the truth values of the statements" then it is called as satisfiable.

iv) $(p \rightarrow q) \rightarrow (q \rightarrow p)$ ∴ The statement is satisfiable.

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

If $P \to Q$, P conditional Q

Converse: then $Q \to P$, Q conditional P.

Inverse: $\neg P \to \neg Q$ $(\sim P \to \sim Q)$

contrapositive: $\neg Q \to \neg P$

eg: P: Sun rises in east

Q: Ram did yoga

$P \to Q$: If sun rises in east then Ram did yoga

$Q \to P$: If ram did yoga then sun rises in east.

$\neg P \to \neg Q$: If sun does not rise in east then ram did not do yoga.

$\neg Q \to \neg P$:

vi Exclusive OR : 'XOR', '$\bar{\vee}$'

P	Q	$P \bar{\vee} Q$
T	T	F
T	F	T
F	T	T
F	F	F

"$P \bar{\vee} Q$ is true if either P or Q is true but not both otherwise false"

vii. NAND : 'NOT & AND' '$\uparrow$'

P	Q	$P \uparrow Q$
T	T	F
T	F	T
F	T	T
F	F	T

"$P \uparrow Q$ is true if either P or Q is false otherwise false"

$$P \uparrow Q \equiv \neg(P \wedge Q)$$

(equivalent)

viii NOR : NOT & OR ; $P \downarrow Q$

$P \downarrow Q \equiv \neg(P \vee Q)$

P	Q	$P \downarrow Q$
T	T	F
T	F	F
F	T	F
F	F	T

Evaluate the boolean expression where $a=2$, $b=3$, $c=5$ and $d=7$

i) $[\sim(a \geq b)] \vee [\sim(c < d)]$

ii) $\neg\{(a \leq b) \wedge [\neg(c > d)]\}$

1. $[\sim(a \geq b)] \vee [\sim(c < d)]$

$a \geq b = 2 \geq 3$ — F

$c < d = 5 < 7$ — T

$\sim a \geq b = T$

$\sim c < d = F$

$T \vee F = T$

ii. $\neg\{(a \leq b) \wedge [\neg(c \geq d)]\}$

$a \leq b = T$

$c \geq d = F$

$\neg(c \geq d) = T$

$\neg\{T \wedge T\} = F$

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Construct the truth table : $\veebar$ = XOR

Prove that, $P \wedge Q \equiv (P \veebar P) \veebar (Q \veebar Q)$

(i)

P	Q	$P \wedge Q$	$P \veebar P$	$Q \veebar Q$	$(P \veebar Q) \veebar (Q \veebar Q)$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	F	T	T	F

$\therefore$ Hence proved, $P \wedge Q = (P \veebar P) \veebar (Q \veebar Q)$

(ii) $P \bar{\vee} Q \equiv (P \bar{\vee} Q) \wedge \neg (P \wedge Q)$

P	Q	$P \bar{\vee} Q$	$P \vee Q$	$P \wedge Q$	$\neg (P \wedge Q)$	$(P \vee Q) \wedge \neg (P \wedge Q)$
T	T	F	T	T	F	F
T	F	T	T	F	T	T
F	T	T	T	F	T	T
F	F	F	F	F	T	F

$\therefore$ Hence proved

Check the following statement formula is tautology, contradiction or satisfiable.

i) $(P \rightarrow (Q \rightarrow (P \wedge Q))) \leftrightarrow P$.

P	Q	$P \wedge Q$	$Q \rightarrow (P \wedge Q)$	$P \rightarrow (Q \rightarrow (P \wedge Q))$	$P \rightarrow (Q \rightarrow (P \wedge Q)) \leftrightarrow P$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	F	F	T	F
F	F	F	T	T	F

$\therefore$ It is satisfiable.

ii) $\neg(p \vee q) \vee (\neg p \wedge q)$

$\neg p$	p	q	$p \vee q$	$\neg(p \vee q)$	$p \wedge q$	$\neg p \wedge q$	$\neg(p \vee q) \vee (\neg p \wedge q)$
F	T	T	T	F	T	F	F
F	T	F	T	F	F	F	F
T	F	T	T	F	F	T	T
T	F	F	F	T	F	F	T

$\therefore$ It is satisfiable.

iii) $((p \rightarrow q) \leftrightarrow q) \rightarrow p$

p	q	$p \rightarrow q$	$(p \rightarrow q) \leftrightarrow q$	
T	T	T	T	T
T	F	F	T	T
F	T	T	T	F
F	F	T	F	T

Laws:

Contrapositive law: $p \rightarrow q \equiv \sim q \rightarrow \sim p$

Duality: $p \wedge (p \vee q) \equiv p \vee (p \wedge q)$

$\rightarrow$ '$\vee$', '$\wedge$' the connectives AND & OR are dual to each other, this is duality law.

Algebra of propositions:

1. Commutative law: $p \wedge q \equiv q \wedge p$
   $p \vee q \equiv q \vee p$

2. associative law: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
   $(p \vee q) \vee r \equiv p \vee (q \vee r)$

3. Distributive laws: $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

4. Dual negation laws: $\sim(\sim p) \equiv p$

5. Identity laws: $p \wedge t \equiv p$ where, $t \to$ tautology - T
$p \vee f \equiv p$ $f \to$ contradiction - F

6. Inverse laws: $p \wedge \sim p \equiv f$
$p \vee \sim p \equiv t$

7. Domination laws: $p \vee t \equiv t$
$p \wedge f \equiv f$

8. $p \to q \equiv \sim p \vee q$ - Implication converse law

9. $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

10. Demorgan's laws: $\sim(p \wedge q) \equiv \sim p \vee \sim q$
$\sim(p \vee q) = \sim p \wedge \sim q$

11. Idempotent laws: $p \wedge p \equiv p$
$p \vee p \equiv p$

12. $p \to q \equiv (p \wedge \sim q) \to f$

13. Absorption laws: $p \wedge (p \vee q) \equiv p$
$p \vee (p \wedge q) \equiv p$

$\ast$ $p \uparrow q \equiv \neg(p \wedge q)$
$p \downarrow q \equiv \neg(p \vee q)$
$p \bar{\vee} q \equiv \neg(p \leftrightarrow q)$

Using the laws of logic simplify the boolean expression :

1. $p\wedge(p\vee\sim q)\wedge(\sim p\vee\sim q) \equiv$

$$p\wedge[(p\vee\sim q)\wedge(\sim p\vee\sim q)] \quad ①$$

$$\equiv p\wedge\left[[p\wedge(\sim p\vee\sim q)]\vee(\sim q\wedge(\sim p\vee\sim q))\right] \quad ②$$

$$\equiv p\wedge\{[\underbrace{(p\wedge\sim p)}_{f}\vee(p\wedge\sim q)]\vee[(\sim q\wedge\sim p)\vee\underbrace{(\sim q\wedge\sim q)}_{\sim q}]\}$$

$$\equiv p\wedge\{[f\vee(p\wedge\sim q)]\vee[\sim q\vee(\sim q\wedge\sim p)]\}$$ absorption law

$$\equiv p\wedge\{(f\vee p)\wedge f\vee\sim q]\vee\sim q$$

$$= p\wedge[(p\wedge\sim q)\vee\sim q]$$

$$\equiv p\wedge\sim q \quad \because \text{absorption law}.$$

2. $(p\wedge\sim q)\vee(\sim p\wedge q)\vee(\sim p\wedge\sim q)$

$(p\wedge\sim q)\vee[(\sim p\wedge q)\vee(\sim p\wedge\sim q)]$ ∵ associative law.

$(p\wedge\sim q)\vee[((\sim p\wedge q)\vee\sim p)\wedge(\sim p\wedge q)\vee\sim q)]$ ∵ distributi

$(p\wedge\sim q)\vee[\sim p\wedge((\sim p\wedge q)\vee\sim q)]$ ∵ absorption law

$(p\wedge\sim q)\vee[\sim p\wedge((\sim p\vee\sim q)\wedge(q\vee\sim q))]$

$(p\wedge\sim q)\vee[\sim p\wedge[(\sim p\vee\sim q)\wedge t]]$ ∵ Inverse law

$(p\wedge\sim q)\vee[\sim p\wedge(\sim p\vee\sim q)]$ ∵ Identity law

$(p\wedge\sim q)\vee\sim p$ ∵ absorption law

$(p\vee\sim p)\wedge(\sim q\vee\sim p)$ ∵ distributive

$t\wedge(\sim q\vee\sim p)$ ∵ Identity

$\sim q\vee\sim p$

iii) $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

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iii. $(p \wedge \sim q) \vee q \vee (\sim p \wedge q) \equiv$
$(p \wedge \sim q) \vee q$ ($\because$ absorption law)
$(p \vee q) \wedge (\sim q \vee q)$ ($\because$ distributive)
$(p \vee q) \wedge t$ ($\because$ inverse law)
$\cancel{q}$ ($\because$ ~~absorption~~)
$(p \wedge t) \vee (q \wedge t)$ ($\because$ distributive law)
$p \vee q$ ($\because$ Identity law)

Let x, y, z be any real numbers,
Represent each sentence symbolically where,
$p: x<y$
$q: y<z$
$r: x<z$

(i) $x \geq y$ or $y<z$ $\rightarrow \neg p \vee q$
(ii) $x<y$ or $[y \geq z$ and $z \geq x] \rightarrow p \vee [\neg q \wedge r]$
(iii) If $z \geq y$ and $x<y$, then $z \geq x \rightarrow (q \wedge p) \rightarrow r$
(iv) $x \geq y$ and $y \geq z$ iff $x \geq z \longrightarrow \neg p \wedge \neg q \longleftrightarrow \neg r$
(v) $x<z$ iff $x<y$ or $y<z \longrightarrow r \longleftrightarrow p \vee q$

If 2 statements are equivalent such that $A \equiv B$, then its biconditional, $A \longleftrightarrow B$ is always tautology.

Express $p \to q$ using $\uparrow$ & $\downarrow$ only.

Sdn: ① $p \to q \equiv \sim p \vee q$ ∵ Implication converse law.

$\equiv \sim p \vee \sim(\sim q)$ ∵ Double negation

$\equiv \sim(p \wedge \sim q)$ ∵ demorgans

$\equiv p \uparrow \sim q$ ∵ negation + and

$\equiv p \uparrow (\sim q \vee \sim q)$ ∵ idempotent

$\equiv p \uparrow [\neg(q \wedge q)]$ ∵ Demorgans

$\equiv p \uparrow (q \uparrow q)$

② $p \wedge q \equiv \neg(\neg(p \wedge q))$

$\equiv \neg(p \uparrow q)$

$\equiv \neg(p \uparrow q) \vee \neg(p \uparrow q)$ ∵ Idempotent

$\equiv \neg((p \uparrow q) \wedge (p \uparrow q))$ ∵ DM

$\equiv (p \uparrow q) \uparrow (p \uparrow q)$

$p \wedge q \equiv (p \downarrow p) \downarrow (q \downarrow q)$

H.W $p \leftrightarrow q$

③ $p \vee q \equiv \neg(\neg(p \vee q))$

$\equiv \neg(p \downarrow q)$

$\equiv \neg(p \downarrow q) \wedge \neg(p \downarrow q)$ ∵ Idempotent

$\equiv \neg(p \downarrow q \vee p \downarrow q)$ ∵ d.m.

$\equiv (p \downarrow q) \downarrow (p \downarrow q)$

$p \vee q \equiv (p \uparrow p) \uparrow (q \uparrow q)$

Show that $p \to (q \to R) \equiv (p \wedge q) \to R$ without
④ constructing truth table.
① with truth table.

①

P	Q	R	$Q \to R$	$P \to (Q \to R)$	$P \wedge Q$	$(P \wedge Q) \to R$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	F	T
F	F	F	T	T	F	T

② $P \to (Q \to R) \equiv \cancel{(P \to Q) \to R}$ $\quad \neg P \vee (\neg Q \vee R)$

$\equiv \cancel{(P \wedge \sim Q) \to R}$ $\quad (\neg P \vee \neg Q) \vee R$

$\equiv \cancel{P \wedge (\sim Q}$ $\quad \neg (P \vee Q) \vee R$

$\equiv (P \vee Q) \to R.$

$(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \equiv R$

$(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$

$\equiv (\neg P \wedge (\neg Q \wedge R)) \vee ((Q \wedge R) \vee (P \wedge R))$

$= \neg (P \vee Q) \wedge R \vee ((P \vee Q) \wedge R)$

$(A \wedge R) \vee (B \wedge R)$
$= (A \vee B) \wedge R$

$\because$ Distributive.

$\equiv [\neg (P \vee Q) \vee (P \vee Q)] \wedge R$

$\because \neg P \vee P \equiv t$

$\equiv t \wedge R$

$\equiv R$

* $P \uparrow P \equiv \neg (P \wedge P) \equiv \neg P$
* $P \downarrow P \equiv \neg (P \vee P) \equiv \neg P$
* $P \bar{\vee} Q \equiv Q \bar{\vee} P$
* $(P \bar{\vee} Q) \bar{\vee} R \equiv P \bar{\vee} (Q \bar{\vee} R)$

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Show that : $[(P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R))] \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology

I method:

$((P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R))] \vee [((\neg P \wedge \neg Q) \vee \neg P) \wedge (\neg P \wedge \neg Q) \vee \neg R$

$((P \vee Q) \wedge \sim(\sim P \wedge (\sim Q \vee \sim R))] \vee [\neg P \vee (\neg P \wedge \neg Q) \vee \neg R)$

$[(P \vee Q) \wedge \sim (\sim P \wedge (\sim Q \vee \sim R))] \vee (\neg P \vee \neg R)$

$[(P \vee Q) \wedge \sim[([\neg P \wedge \neg Q] \vee [\neg P \wedge \neg R]] \vee (\neg P \vee \neg R)$

$[(P \vee Q) \wedge \sim(\neg P \wedge \neg Q) \wedge \sim(\neg P \wedge \neg R) \vee (\neg P \vee \neg R)$

$(P \vee Q) \wedge \sim(\neg p \wedge \neg Q) \wedge [\sim(\neg P) \vee \sim(\neg R)] \vee [\neg p \vee \neg R]$

$\wedge [P \vee R] \vee [P \vee \neg R]$

$(P \vee Q) \wedge \sim(\sim(P \vee Q))$ | $\wedge [P \vee R] \vee [\neg p \vee \neg R]$

$(p \vee Q)$

$(p \vee Q) \wedge [p \vee R] \vee [\neg p \vee \neg R]$

$p \vee (Q \wedge R) \vee [\neg p \vee \neg R]$

$p \vee (Q \wedge R) \vee (\sim(p \wedge R))$

$p \vee (\sim(p \wedge R)) \vee (Q \wedge R)$

$p \vee [\sim(p \wedge R) \vee (Q \wedge R)]$

$[p \vee (\sim p) \vee \sim R)] \vee (Q \wedge R)$

$p \vee \sim p$
$(t \vee \sim R)$ $\quad t \vee (Q \wedge R)$

t

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$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$

$\equiv [[\sim p \vee q] \wedge \sim q] \vee [(\sim p \vee q) \wedge p)$

$\equiv [\sim p \vee (q \wedge \sim q)] \vee [(q \vee \sim p) \wedge p)$

$\equiv [\sim p \vee f] \vee [q \vee (\sim p \wedge p)]$

$\equiv [\sim p \vee f] \vee [q \vee f]$

$\equiv \sim p \vee q$

$\equiv p \to q$

$\equiv$ same as (1)

II method:

∵ Distribution

$\equiv [(P \vee Q) \wedge \sim[\sim P \wedge (\sim Q \vee \sim R)] \vee (\sim P \wedge (\sim Q \vee \sim R))$

∵ DM $\equiv [(P \vee Q) \wedge \sim[\sim P \wedge \sim(Q \wedge R)] \vee [\sim P \wedge (\sim Q \vee \sim R))$

$\equiv (P \vee Q) \wedge \sim[\sim [P \vee (Q \wedge R)]] \vee [\sim P \wedge (\sim Q \vee \sim R)]$

∵ Dual negation $\equiv (P \vee Q) \wedge [P \vee (Q \wedge R)] \vee [\sim P \wedge (\sim Q \vee \sim R)]$

∵ DM $\equiv [(P \vee Q) \wedge (P \vee (Q \wedge R))] \vee [\sim P \wedge (\sim(Q \wedge R))]$

∵ DM $\equiv [(P \vee Q) \wedge (P \vee Q) \wedge (P \vee R)] \vee [\sim P \vee (Q \wedge R))$

∵ associative $\equiv [(P \vee Q) \wedge (P \vee Q)] \wedge (P \vee R) \vee [\sim P \vee (Q \wedge R)$

$\equiv [P \vee Q] \wedge [P \vee R] \vee [\sim P \vee (Q \wedge R)]$ ∵ Idempotent

$\equiv [P \vee (Q \wedge R)] \vee \sim [P \vee (Q \wedge R)$ (∵ distribution)

$\equiv t$ (∵ Inverse law)

III method

$(\cancel{p \vee q}) \wedge \cancel{\dots}$

$\therefore$ DM $\equiv ((p \vee q) \wedge \sim(\sim p \wedge \sim(q \wedge r))) \vee \neg(p \vee q) \vee \neg(p \vee r)$

$\equiv [(p \vee q) \wedge (p \vee (q \wedge r))] \vee \neg[(p \vee q) \wedge (p \vee r)]$

$\equiv (p \vee (q \wedge r)] \vee \neg [p \vee (q \wedge r)]$

$\equiv t$

Express $p \uparrow (q \wedge \neg[R \downarrow p)]$ interms of $\neg, \wedge, \vee$ only.

soln:

$\equiv \neg(p \wedge (q \wedge \neg(R \downarrow p)]$

$\equiv \neg(p \wedge (q \wedge \neg(\neg(R \vee p))))$

$\equiv \neg(p \wedge (q \wedge (R \vee p)))$

$\equiv \neg p \vee \sim(q \wedge (R \vee p))$

Express $p \uparrow q$ interms of NOR only.

$p \uparrow q \equiv \neg(p \wedge q)$

$\equiv \neg[(p \downarrow p) \downarrow (q \downarrow q)]$

$(\because \neg p \equiv p \downarrow p) \equiv [(p \downarrow p) \downarrow (q \downarrow q)] \downarrow [(p \downarrow q) \downarrow (q \downarrow q)]$

Express $p \downarrow q$ interms of NAND

$p \downarrow q \equiv \neg(p \vee q)$

$\equiv \neg(p \uparrow p) \uparrow (q \uparrow q)$

$(\because \neg p \equiv p \uparrow p) \equiv [(p \uparrow p) \uparrow (q \uparrow q)] \uparrow [(p \uparrow p) \uparrow (q \uparrow q)]$

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Check true or false:

* If $q \equiv r$ then $p \wedge q \equiv p \wedge r$ — True
* If $q \equiv r$ then $p \to q \equiv p \to r$ — True
* If $p \to q \equiv p \to r$ then $q \equiv r$ — False
* If $q \equiv r$ then $p \vee q \equiv p \vee r$ — True

eg: $p \to F$ $q \to T$ $r \to F$

$p \to q \to T$

$p \to r \to T$

Find truth value of $p \to q$ iff $p \vee q$ is false.

Soln: $p \vee q$ — F

p — F

q — F

$p \to q$ — True.

(ii) $p \wedge (q \wedge r)$, iff $r \equiv s$ and s is not true

s — f

r — f

$q \wedge r$ — f

$p \wedge (q \wedge r)$ — false.

$\equiv p \wedge (q \wedge F)$

$\equiv p \wedge F$

$\equiv F$

(iii) $p \to \sim q$, if $q \to \sim p$ is false.

$T \to \sim(\text{True})$ $\sim p$ — False. q — True

$T \to$ false $p \to$ True

ans: false

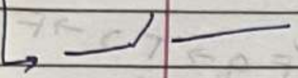
If $p \equiv q$ then $p \leftrightarrow q$ is Tautology T

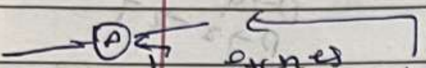
If $p \leftrightarrow q$ is tautology then $p \equiv q$ T

Applications: Switching Networks:

→ playing with wires and switches connected to two terminals.

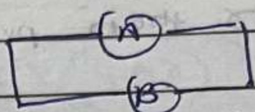
open, no current flow
close → current flows





closed/open switches represent as

① switches in series → $A \wedge B$ or $A \cdot B$

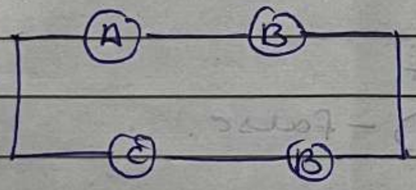


② switches connected in parallel → $A \vee B$ or $A + B$

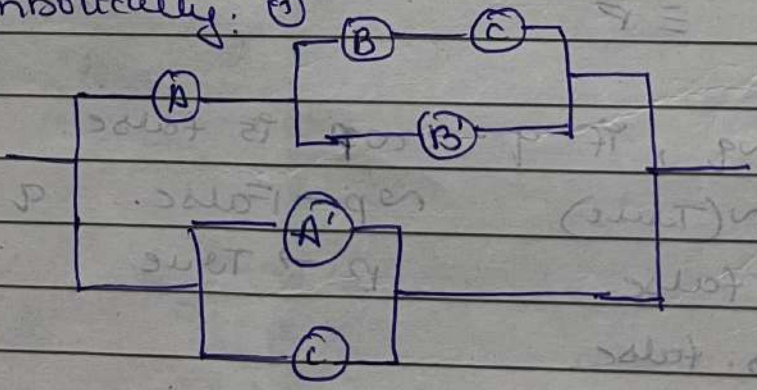
'¬', '∧', '∨' → logical
''', '.', '+' → in electricals and circuits
(not) (and) (or)

series - AND
parallel - OR

$(AB) + (CB)$ → symbolic representation of circuit

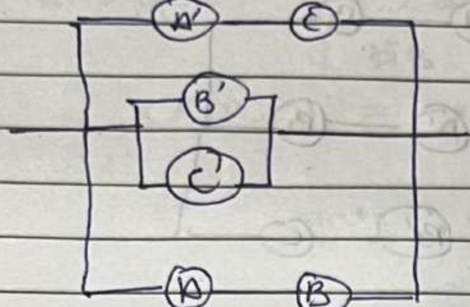
≡  → circuit.

write symbolically: ①



$$\Rightarrow \left[A \cdot (B \cdot C + B')\right] + (A' + C) \equiv \left\{A \wedge (B \wedge C \vee B')\right\} \vee \left[A' \vee C\right]$$

(2)



$\Rightarrow A'\cdot C + (B'+C') + A\cdot B$

$\Rightarrow A' \wedge C \vee (B' \vee C') \vee A \wedge B$

Simplify the above equation.

$(A' \wedge C) \vee (B' \vee C') \vee (A \wedge B)$

cannot be simplified.

(3) Draw a switching network by an equivalent simpler network.

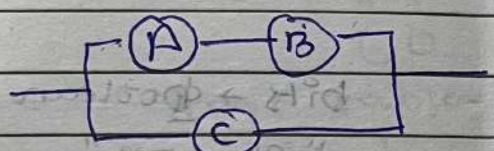
(i)

$(A \wedge B) \vee ((A \wedge B) \vee C)$

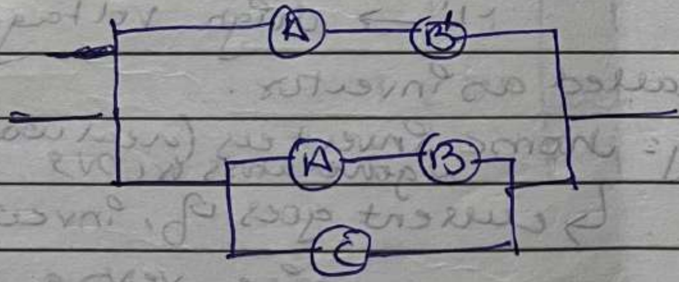
$\equiv [(A \wedge B) \vee (A \wedge B)] \vee C$

$\equiv (A \wedge B) \vee C$

$\equiv$



(ii)



$(A \wedge B') \vee ((A \wedge B) \vee C)$

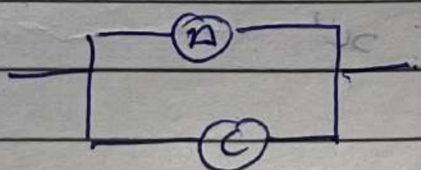
$\equiv (A \wedge B') \vee (A \wedge B) \vee C$

$\equiv A \wedge (B' \vee B) \vee C$ ∵ distributive law

$\equiv (A \wedge T) \vee C$ ∵ $p' \vee p = t$ Inverse law

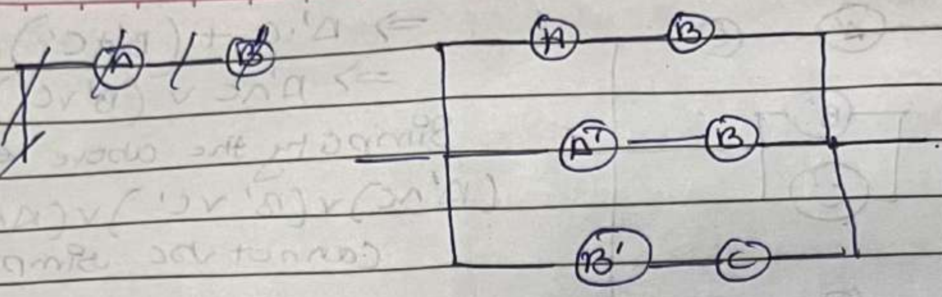
$\equiv A \vee C$

$\equiv$



"When same signs associative law holds good"

(vi)



$(A \wedge B) \vee (A' \wedge B) \vee (B' \wedge C)$

$B \wedge (A \vee A') \vee (B' \wedge C)$ ∵ Distributive law

$B \wedge T \vee (B' \wedge C)$ ∵ Inverse law

$= B \vee (B' \wedge C)$ ∵ distributive law

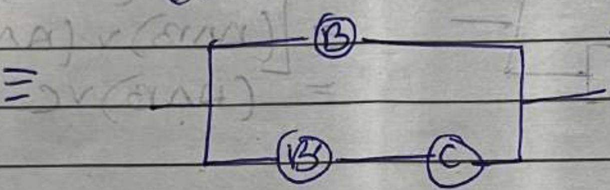
$\equiv (B \vee B') \wedge (B \vee C)$

$\equiv 1 \wedge (B \vee C)$

$\equiv B \vee C$

$= B + C$

$\equiv$



Logic gates:
one or many inputs
but only one output.

bits → boolean algebra
"Shanon"

'0' → low voltage → false
'1' → high voltage → true.

(1) NOT gate: Also called as inverter.

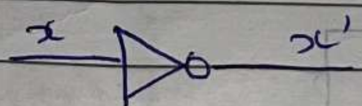
Logic table:

eg: home inverters (real world)
or generators & UPS
↳ current goes off, inverter on & vice versa
or streetlights.

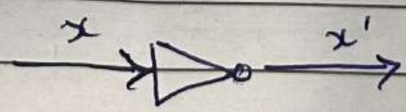
bits

x	x'
0	1
1	0.

NOT Gate:



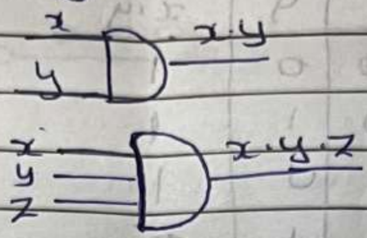
or



$$x' = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x = 1 \end{cases}$$

(2) AND gate: x and y are input signals in terms of bits.

$X \text{ AND } Y \equiv xy$

gate: 

Logic table:

x	y	$x \cdot y$
0	0	0
0	1	0
1	0	0
1	1	1

example: examination with internal and external markings.

$$xy = \begin{cases} 1 & \text{if } x = y = 1 \\ 0 & \text{otherwise.} \end{cases}$$

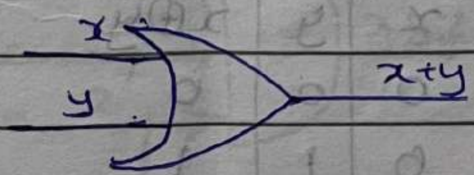
(3) OR gate: x and y are input signals in terms of bits

$x + y \equiv X \text{ OR } Y$

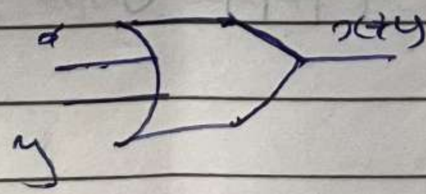
$$x+y = \begin{cases} 1 & \text{if } x = 1 \text{ or } y = 1 \\ 0 & \text{otherwise.} \end{cases}$$

Logic table:

x	y	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

gate: 

or.



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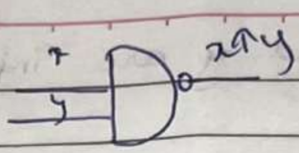
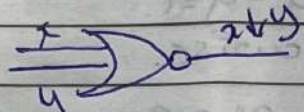
(4) NAND:



$x \uparrow y = \begin{cases} 0 & \text{if } x = y = 1 \\ 1 & \text{otherwise} \end{cases}$

x	y	$\overline{x \cdot y}$
0	0	1
0	1	1
1	0	1
1	1	0

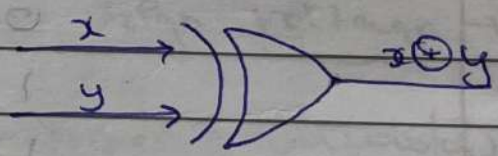
(5) NOR



$x \downarrow y \begin{cases} 1 & \text{if } x = y = 0 \\ 0 & \text{otherwise} \end{cases}$

x	y	$\overline{x+y}$
0	0	1
0	1	0
1	0	0
1	1	0

(6) XOR : $x \oplus y$
xoringsome why

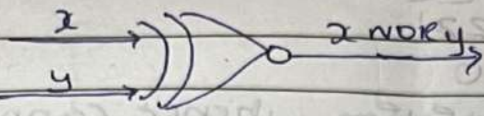


x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

$x \oplus y \begin{cases} 0 & x \text{ and } y \text{ same value} \\ 1 & \text{otherwise.} \end{cases}$

(7) XNOR : Exclusive NOR Gate. ≡ Biconditional

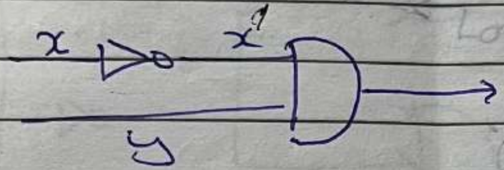
x XNOR y



x	y	x XNOR y
0	0	1
0	1	0
1	0	0
1	1	1

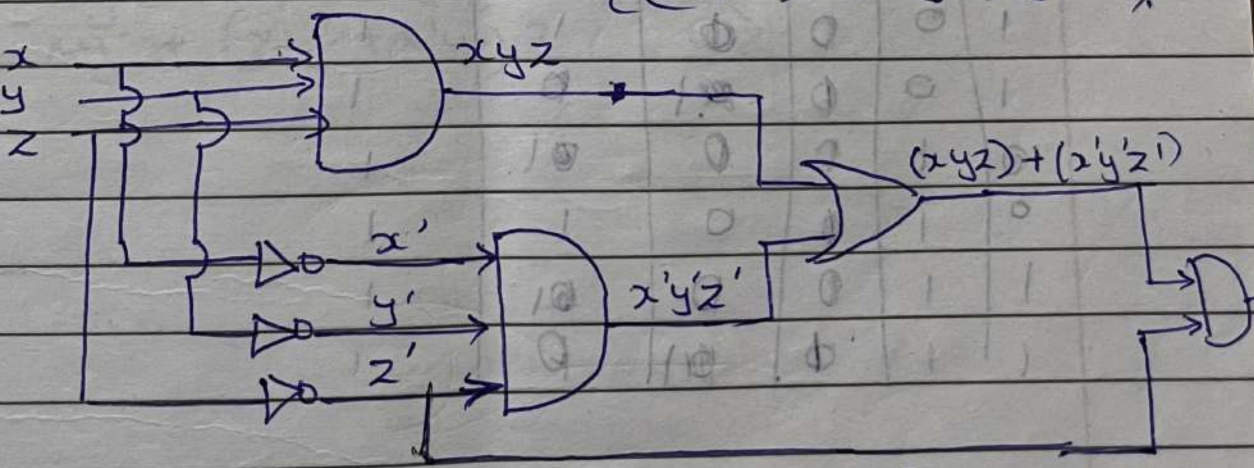
Circuits with logic gates → Combinatorial circuit
circuits with switches and wires → switching circuits

Combinatorial circuit:



$x'y$ — Boolean expression.

(8) $[(xyz) + (x'y'z')]\, \cancel{xy}\, z'$ (Actual question $[(xyz)+(x'y'z)]z'$.)



If possible write the simpler combinatorial circuit which is equivalent to given boolean expression.

Boolean function,

* $f(xyz) = [(xyz) + (x'y'z)]z'$

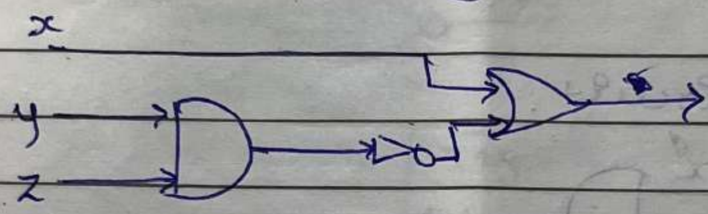
$= [z(xy + x'y')]z'$

$= zz'$

$= 0$

got contradiction hence cannot simplify and go ahead with the circuit.

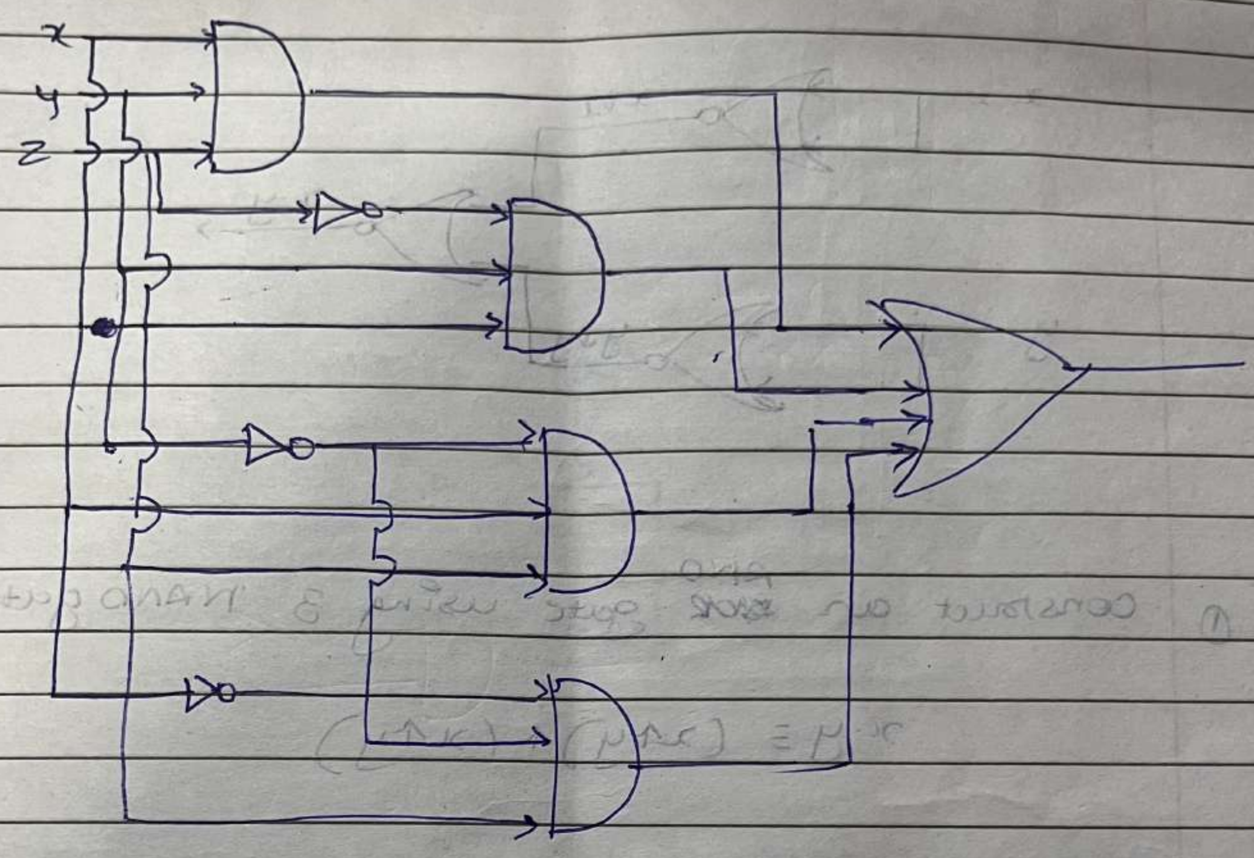
Find the output of the combinational circuit, also construct a logic table.



$x + (y \cdot z)'$

y	x	z	yz	(yz)'	x+(yz)'
0	0	0	0	1	1
0	0	ϕ	0	1	1
1	0	0	ϕ	1	1
1	0	ϕ	1	0	1
0	1	0	0	1	1
0	1	ϕ	0	1	1
1	1	0	0	1	1
1	1	ϕ	1	0	1

Find the output produced by the combinational circuit also find the simple cs equivalent to it.



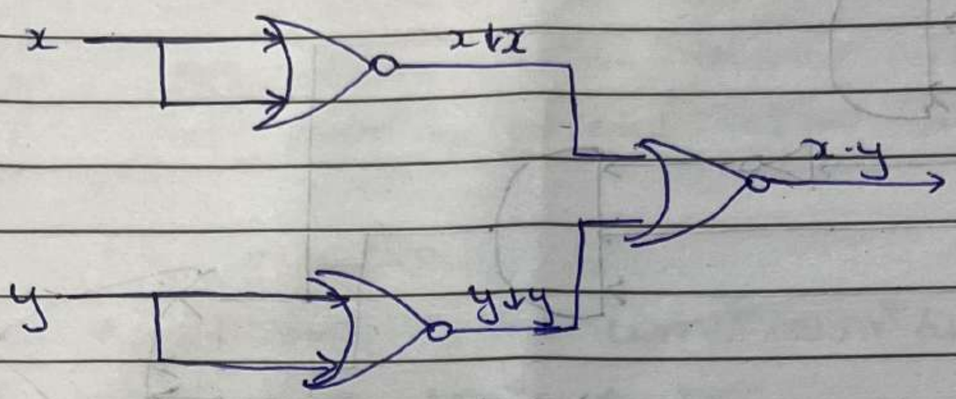
$f(xyz) \equiv (xyz) + (xyz') + (xy'z) + (x'y'z) \equiv xy + y'z$

$\equiv xy(z+z') + (xy' + x'y')z \; (y'z(x+x')$

$= xy + (xy' + x'y)z \; (\; y'z$

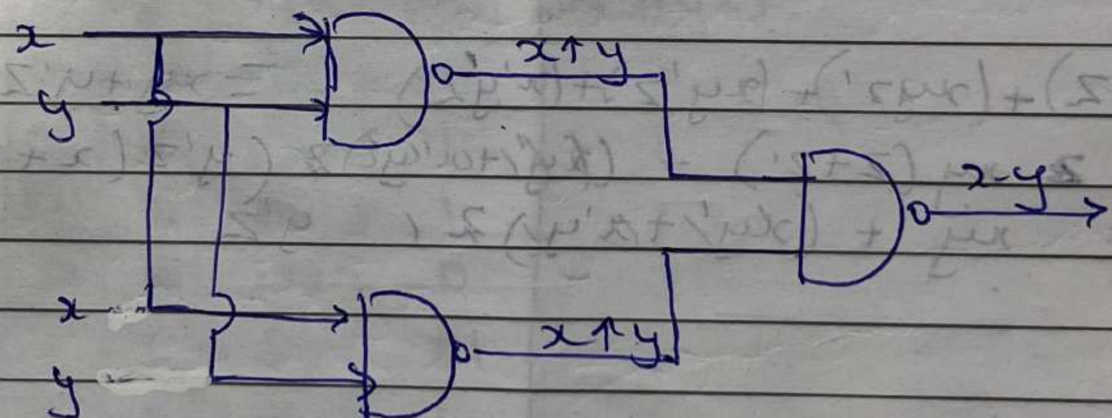
Construct an AND gate using 3 NOR gates.

$x\cdot y \equiv (x \downarrow x) \downarrow (y \downarrow y)$ $\qquad$ $xy = \overline{x+y}$

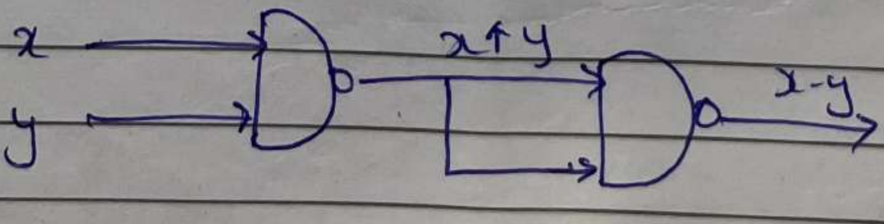


① construct an ~~NOR~~ AND gate using 3 NAND gate

$x\cdot y \equiv (x \uparrow y) \uparrow (x \uparrow y)$



④ AND by only 2 NAND gates



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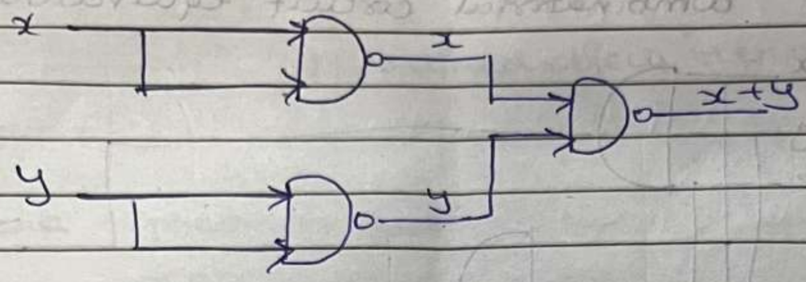
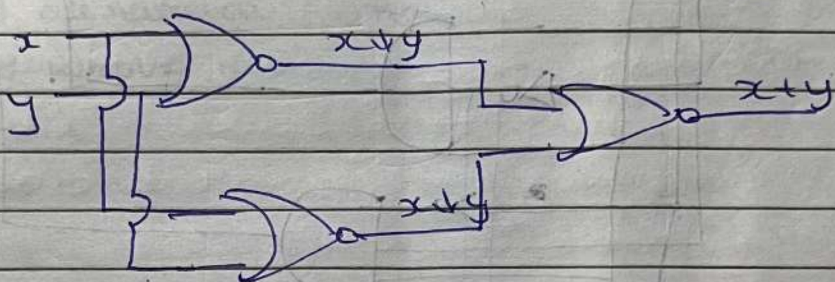
Construct OR gate using 3 NAND gate

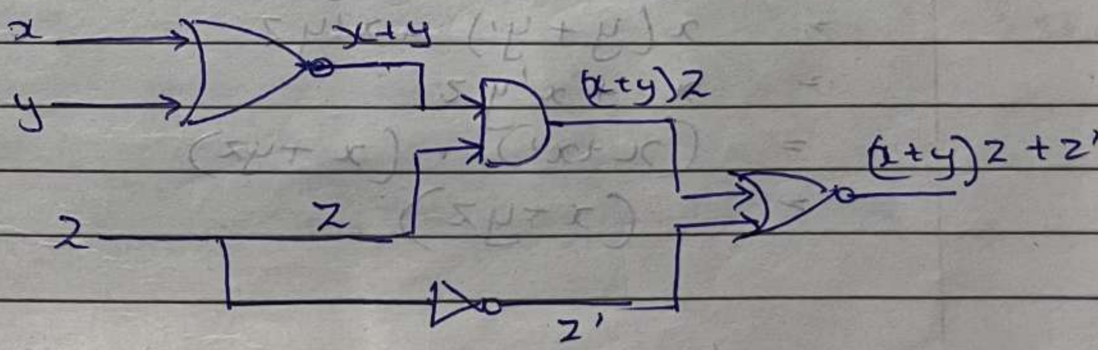
i) $x+y \equiv (x \uparrow x) \uparrow (y \uparrow y)$



ii) $x+y \equiv (x \downarrow y) \downarrow (x \downarrow y)$



make a combinational circuit that yields the boolean expression $(x+y)z + z'$ as its output.



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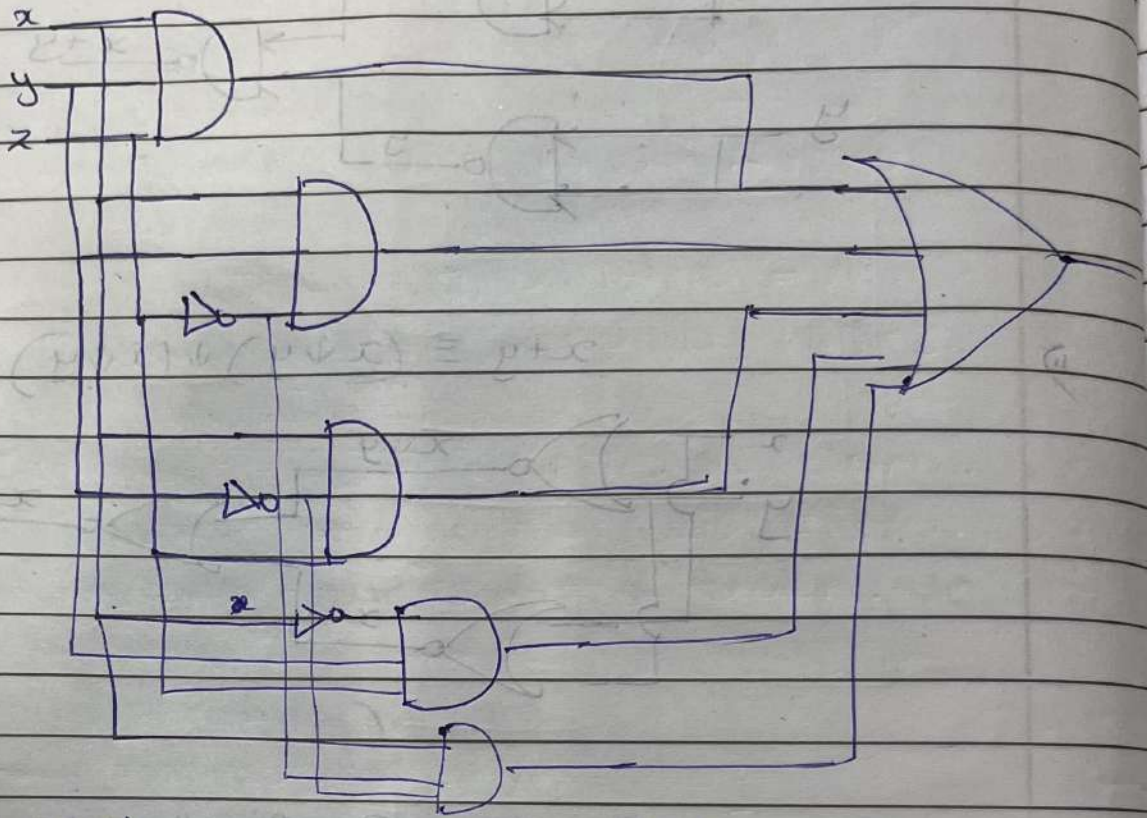
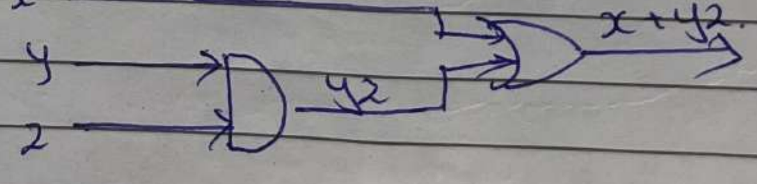
Construct a combinational circuit for the boolean expression $xyz + xyz' + xy'z + x'yz + xy'z'$. Find a simpler combinational circuit equivalent to it.

Soln:



$$f(x,y,z) = xyz + xyz' + xy'z + x'yz + xy'z'$$
$$= xy(z+z') + xy'(z+z') + x'yz$$
$$= xy + xy' + x'yz$$
$$= x(y+y') + x'yz$$
$$= x + x'yz$$
$$= (x+x')\cdot(x+yz)$$
$$= (x+yz)$$



$\therefore$ equivalent combinational circuit is