

Assignment 3: MCA I Sem MAT 4111.

Submit on or before 20th November, 2023.

- 1) At an art exhibition there are 12 paintings of which 10 are original. A visitor selects a painting at random and before he decides to buy, he asks the opinion of an expert about the authenticity of the painting. The expert is right in 9 out of 10 cases on an average. i) Given that the expert decides that the painting is authentic, what is the probability that this is really the case. ii) If the expert decides that the painting is a copy, then the visitor returns it and chooses another one, what is the probability that his second choice is actually the original.
- 2) A, B, C play a game and the chances of their winning it in an attempt are $\frac{2}{3}$, $\frac{1}{2}$ and $\frac{1}{4}$ respectively. A has the first chance, followed by B and then by C. This cycle is repeated till one of them wins the game. Find their respective chances of winning the game.
- 3) Box A contains 5 red and 4 white balls. Box B contains 3 red and 7 white balls. A box is selected at random. A ball is drawn and put into the other. Then a ball is drawn from that box. Find the prob that both balls are of same colour.
- 4) Each of the two persons toss 3 coins. What is the probability that they get the same number of heads.
- 5) In a coin tossing experiment, if the coin shows head, one die is thrown and the result is recorded. But if the coin shows tail, two dice are thrown and the sum of the numbers is recorded. What is the probability that the recorded number is 2?

6) The chances of X, Y and Z becoming managers of a certain company are 4:2:3.

The probability that bonus scheme will be introduced if X, Y and Z become managers are 0.3, 0.5 and 0.8. The bonus scheme has been introduced. What is the probability that X is appointed the manager?

7) Suppose the pdf of a random variable X is given by $f(x) = \begin{cases} 4x - 4x^3, & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$.

Find the mean and Variance of X.

8) a) If the probability that A solves a problem is $\frac{1}{2}$ and that for B is $\frac{3}{4}$ and if they aim at solving a problem independently. What is the probability that the problem is solved?

b) Let G be a simple graph with 6 vertices. The degree of 6 vertices are (2,3,3,3,4,5). Then number of edges of graph G_____.

c) The graph in which, there is a walk which includes every vertex of the graph exactly once is known as _____.

d) Generators of the group (G, .), where $G = \{1, -1, i, -i\}$ is _____

e) Write all the subgroups of (G, .), where $G = \{1, -1, i, -i\}$.

9) For $a, b \in \mathbb{Z}$ define the binary operation $*$ as $a | b$ to mean that a divides b.

Is $(\mathbb{Z}, *)$ a group. Also verify if $(\mathbb{R}, *)$ is a group under same binary operation.

10) Check which of the following Binary Operations is a group under \mathbb{Z} , \mathbb{Q} ,

\mathbb{N} and \mathbb{R} . i) $a*b = (a-b)$ ii) $a*b = (a^2+b)$ iii) $a*b = (ab)/2$