

INTRODUCTION TO DATA ANALYSIS_ UNDERSTANDING GROUPS

2nd Sem, MCA

CONTENT

❑ Understanding Groups in Data Analysis

- Clustering
- Association rules
 - Market Basket Analysis
 - Recommendation system
 - Apriori algorithm
 - FP Growth Algorithm

DATA ANALYSIS - GROUPING

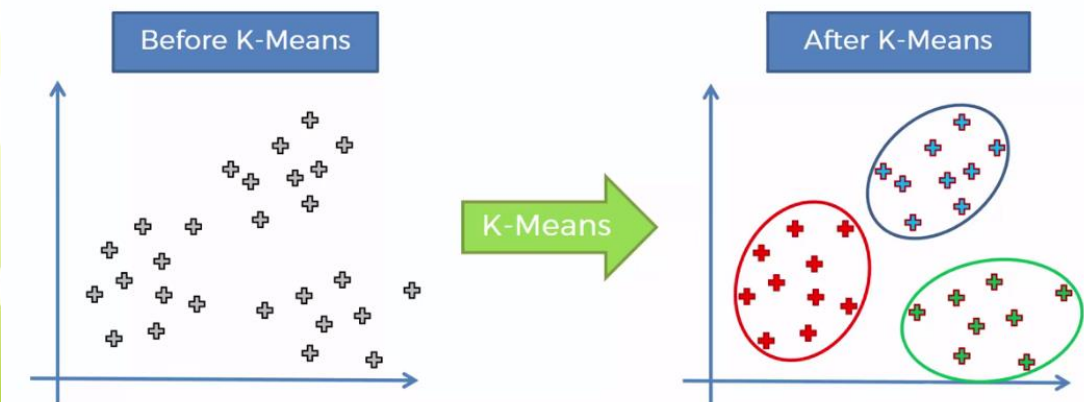
- Grouping and classification techniques are very important methods in data analysis.
- **Grouping Analysis** methods helps to determine natural groupings in data.
- Useful to decompose data set into simpler subsets → helps to make sense of entire collection of observations.
- For each group summary statistics, variety of graphs may help in better analysis
- Different ways to visualize and group observations,
 - *Clustering*: based on similarities of overall set of variables of interest.
 - *Association rule*: identify groups based on interesting combinations of predefined categories
 - *Decision tree*: groups observation based on combination of ranges of continuous variables or of specific categories.

DATA ANALYSIS – CLUSTERING

- **Cluster:** group of (similar) objects that belongs to same class.
- **Clustering:** process of making a group of abstract objects into classes of similar objects.
- Given a data set of items, with certain features, and values for these features; the task is to categorize those items into groups.
 - Used to find similarity as well as relationship patterns among data samples and then cluster those samples into groups having similarity based on features.
 - Clustering is important because it determines the intrinsic grouping among the present unlabeled data.

Clustering methods –

- Partitioning Method
- Hierarchical Method; Agglomerative Approach, Divisive Approach
- Constraint-based Method
- Density-based Method
- Grid-Based Method
- Model-Based Method



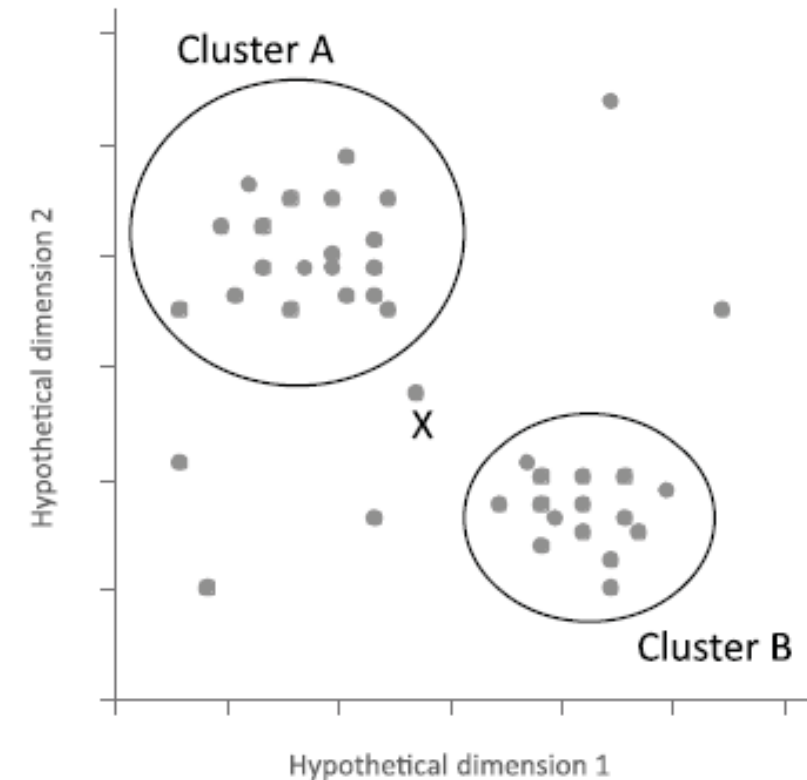
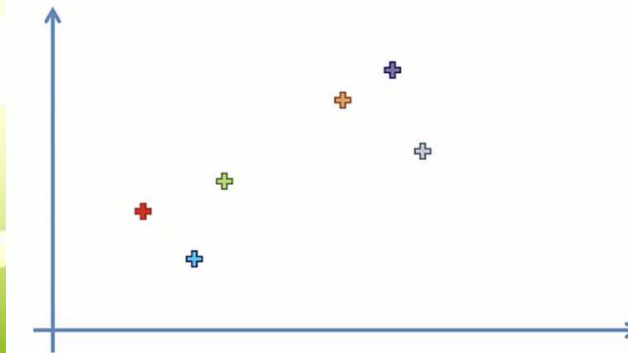
Clustering

Applications of Cluster Analysis

- *Collaborative systems and customer segmentation:* help marketers discover distinct groups in their customer base → characterize customer groups based on purchasing patterns.
- helps in classifying documents on the web for information discovery.
- used in outlier detection applications; example detection of credit card fraud.
- *Biological data analysis:* used to derive plant and animal taxonomies, categorize genes with similar functionalities and gain insight into structures inherent to populations.
- Data summarization and compression
- Trend detection in dynamic data
- Social network analysis

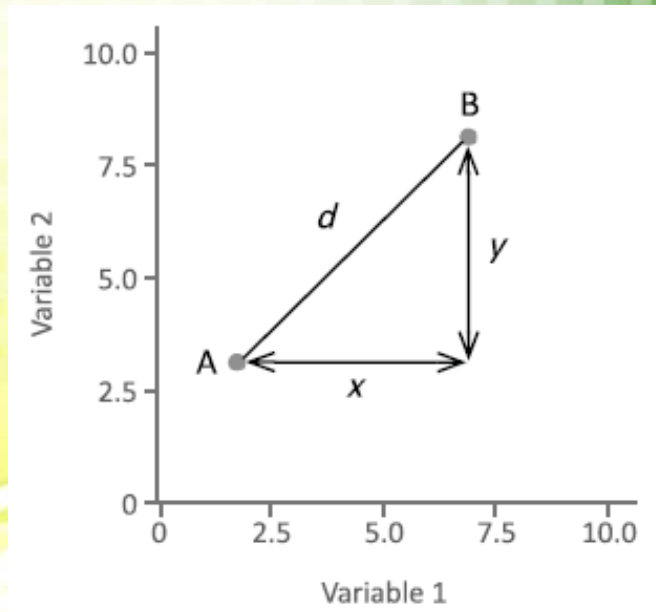
DATA ANALYSIS – CLUSTERING

- Clustering is an *unsupervised* method for grouping.
- **Unsupervised:** groups are not known in advance.
- Clustering method chosen to subdivide data into groups applies automated procedure to discover groups based on some criteria.
- Many clustering methods.
- Each method will group data differently based on criteria it uses.
- For clustering, there is no way to measure accuracy (usefulness matters).
- **Distance** between two observations defines how similar they are to be in same cluster or not.



DATA ANALYSIS – CLUSTERING

- Clustering needs a way to measure how similar the observations are to each other.
- To calculate similarity, distance between observations is computed.
- Simple distance between two observations can be calculated using simple trigonometry.



$$d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

d: Euclidean distance

$(x_1, y_1) \rightarrow$ coordinate of first point

$(x_2, y_2) \rightarrow$ coordinate of second point.

$$x = 7 - 2 = 5$$

$$y = 8 - 3 = 5$$

$$d = \sqrt{x^2 + y^2} = \sqrt{25 + 25} = 7.07$$

- Distance metrics: **Euclidean**, **Jaccard**, City Block, Minkowski, Cosine, Spearman, Hamming, Mahalanobis etc.

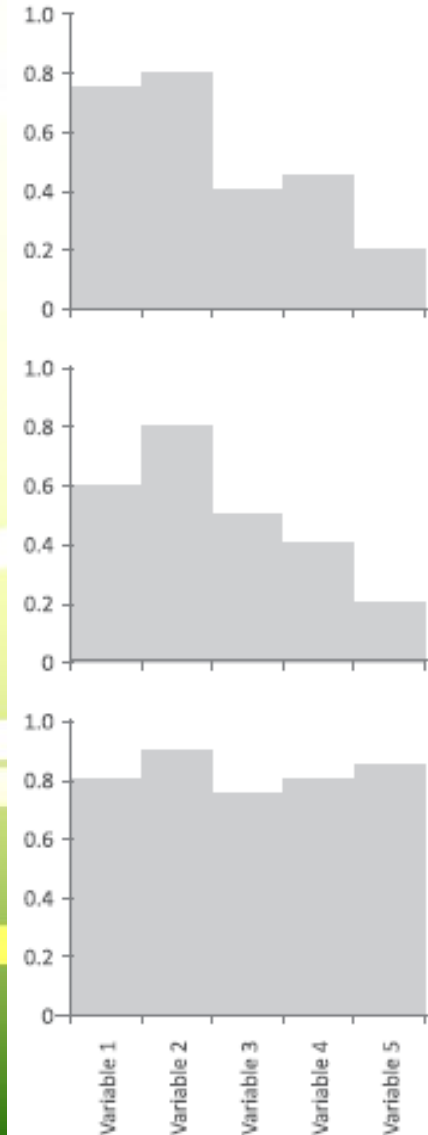
DATA ANALYSIS – CLUSTERING

- **Euclidian Distance:** calculate distances between observations with more than two variables.

$$d = \sqrt{\sum_{i=1}^n (p_i - q_i)^2}$$

$$d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$$

ID	Variable 1	Variable 2	Variable 3	Variable 4	Variable 5
A	0.7	0.8	0.4	0.5	0.2
B	0.6	0.8	0.5	0.4	0.2
C	0.8	0.9	0.7	0.8	0.9



DATA ANALYSIS – CLUSTERING

- **Euclidian Distance:** $d = \sqrt{\sum_{i=1}^n (p_i - q_i)^2}$

ID	Variable 1	Variable 2	Variable 3	Variable 4	Variable 5
A	0.7	0.8	0.4	0.5	0.2
B	0.6	0.8	0.5	0.4	0.2
C	0.8	0.9	0.7	0.8	0.9

- Used to calculate distance between two observations p and q where each observation has n variables.
- Euclidean distance between A and B; A and C; B and C.

$$d_{A-B} = \sqrt{(0.7 - 0.6)^2 + (0.8 - 0.8)^2 + (0.4 - 0.5)^2 + (0.5 - 0.4)^2 + (0.2 - 0.2)^2}$$

$$d_{A-B} = 0.17$$

- More similarity between observations A-B than A-C.
- C is not closely related to either A or B.

The Euclidean distances between A and C is

$$d_{A-C} = \sqrt{(0.7 - 0.8)^2 + (0.8 - 0.9)^2 + (0.4 - 0.7)^2 + (0.5 - 0.8)^2 + (0.2 - 0.9)^2}$$

$$d_{A-C} = 0.83$$

The Euclidean distance between B and C is

$$d_{B-C} = \sqrt{(0.6 - 0.8)^2 + (0.8 - 0.9)^2 + (0.5 - 0.7)^2 + (0.4 - 0.8)^2 + (0.2 - 0.9)^2}$$

$$d_{B-C} = 0.86$$

DATA ANALYSIS – CLUSTERING

- Euclidean distance metric can be used only for numerical variables.

- **Jaccard distance:** for binary variables.

	Variable 1	Variable 2	Variable 3	Variable 4	Variable 5
A	1	1	0	0	1
B	1	1	0	0	0
C	0	0	1	1	1

- This approach is based on number of common or different (0/1) values between corresponding variables across each pair of observations.
 - *Count₁₁: Count of all variables that are 1 in “Observation 1” and 1 in “Observation 2.”*
 - *Count₁₀: Count of all variables that are 1 in “Observation 1” and 0 in “Observation 2.”*
 - *Count₀₁: Count of all variables that are 0 in “Observation 1” and 1 in “Observation 2.”*
 - ~~*Count₀₀: Count of all variables that are 0 in “Observation 1” and 0 in “Observation 2.”*~~

- **Jaccard distance (d):**

$$d = \frac{\text{Count}_{10} + \text{Count}_{01}}{\text{Count}_{11} + \text{Count}_{10} + \text{Count}_{01}}$$

DATA ANALYSIS – CLUSTERING

• **Jaccard distance:**
$$d = \frac{\text{Count}_{10} + \text{Count}_{01}}{\text{Count}_{11} + \text{Count}_{10} + \text{Count}_{01}}$$

	Variable 1	Variable 2	Variable 3	Variable 4	Variable 5
A	1	1	0	0	1
B	1	1	0	0	0
C	0	0	1	1	1

- *Count11: Count of all variables that are 1 in “Observation 1” and 1 in “Observation 2.”*
- *Count10: Count of all variables that are 1 in “Observation 1” and 0 in “Observation 2.”*
- *Count01: Count of all variables that are 0 in “Observation 1” and 1 in “Observation 2.”*
- ~~*Count00: Count of all variables that are 0 in “Observation 1” and 0 in “Observation 2.”*~~

Jaccard distance (d):

- between A and B $d_{A-B} = (1 + 0)/(2 + 1 + 0) = 0.33$
- between A and C $d_{A-C} = (2 + 2)/(1 + 2 + 2) = 0.8$
- between B and C $d_{B-C} = (2 + 3)/(0 + 2 + 3) = 1.0$

DATA ANALYSIS – CLUSTERING

Example: Calculate the Jaccard distance (replacing **None** with **0**, **Mild** with **1**, and **Severe** with **2**) using the variables: Fever, Headaches, General aches, Weakness, Exhaustion, Stuffy nose, Sneezing, Sore throat, Chest discomfort, for the following pairs of patient observations:

- (a) 1326 and 398
- (b) 1326 and 1234
- (c) 6377 and 2662

Patient ID	Fever	Headaches	General Aches	Weakness	Exhaustion	Stuffy Nose	Sneezing	Sore Throat	Chest Discomfort
1326	None	Mild	None	None	None	Mild	Severe	Severe	Mild
398	Severe	Severe	Severe	Severe	Severe	None	None	Severe	Severe
6377	Severe	Severe	Mild	Severe	Severe	Severe	None	Severe	Severe
1234	None	None	None	Mild	None	Severe	None	Mild	Mild
2662	Severe	Severe	Mild	Severe	Severe	Severe	None	Severe	Severe
9477	None	None	None	Mild	None	Severe	Severe	Severe	None
7286	Severe	Severe	Severe	Severe	Severe	None	None	None	Severe
1732	None	None	None	None	None	Severe	Severe	None	Mild
1082	None	Mild	Mild	None	None	Severe	Severe	Severe	Severe
1429	Severe	Severe	Severe	Mild	Mild	None	Severe	None	Severe
14455	None	None	None	Mild	None	Severe	Mild	Severe	None
524	Severe	Mild	Severe	Mild	Severe	None	Severe	None	Mild
1542	None	None	Mild	Mild	None	Severe	Severe	Severe	None
8775	Severe	Severe	Severe	Severe	Mild	None	Severe	Severe	Severe
1615	Mild	None	None	Mild	None	Severe	None	Severe	Mild
1132	None	None	None	None	None	Severe	Severe	Severe	Severe
4522	Severe	Mild	Severe	Mild	Mild	None	None	None	Severe

$$d = \frac{\text{Count}_{10} + \text{Count}_{01}}{\text{Count}_{11} + \text{Count}_{10} + \text{Count}_{01}}$$

DATA ANALYSIS – CLUSTERING

Euclidean distance needs first the data to normalize before bring using. Also, as dimensionality increases, it becomes more complex and less useful.

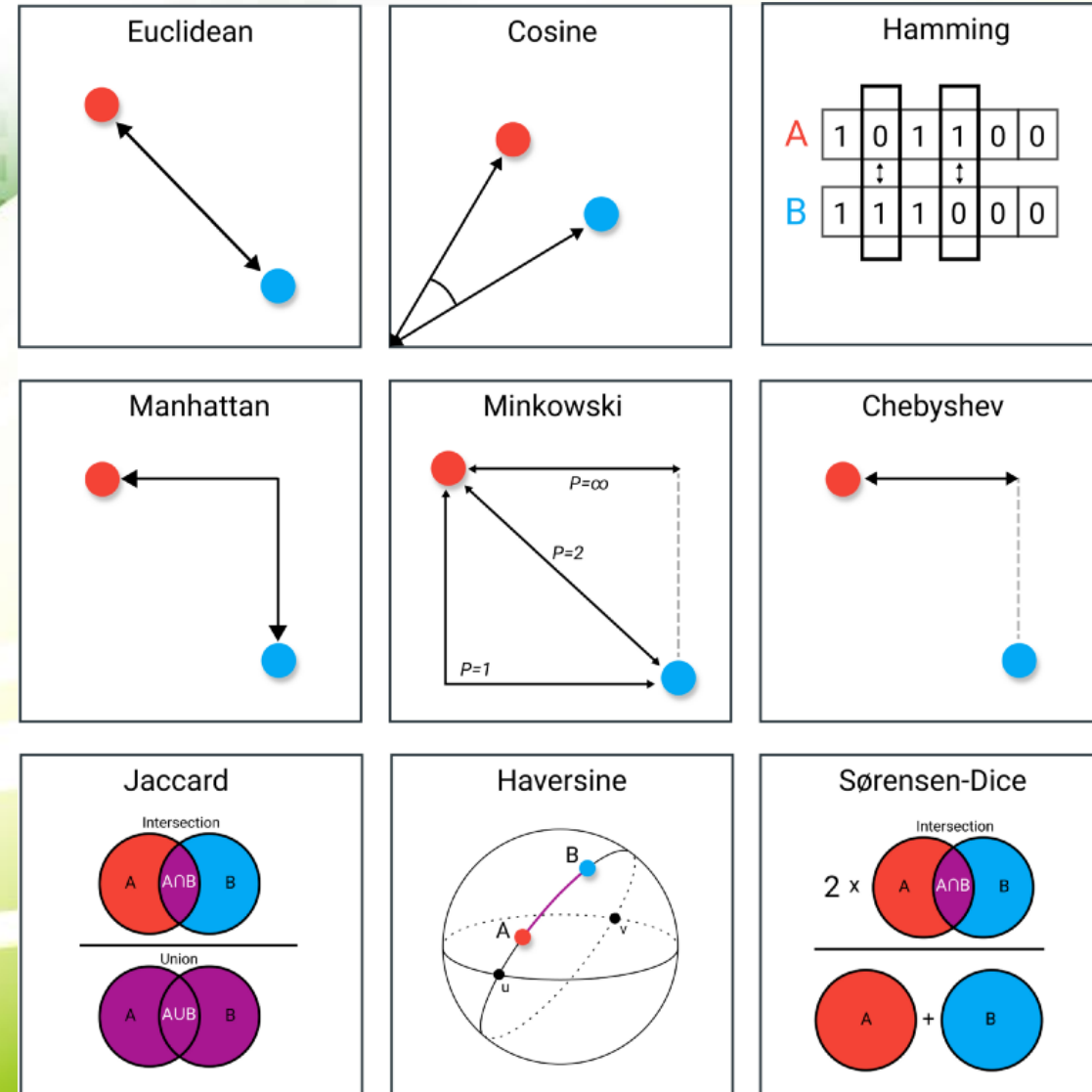
$$D(x, y) = \sqrt{\sum_{i=1}^n (x_i - y_i)^2}$$

Manhattan distance

- Taxicab distance or City Block distance

$$D(x, y) = \sum_{i=1}^k |x_i - y_i|$$

Hamming distance used to compare two binary strings of equal length (compares similarity by calculating number of characters that are different from each other).



DATA ANALYSIS – CLUSTERING

Chebyshev distance (Chessboard distance): maximum distance along one axis.

$$D(x, y) = \max_i (|x_i - y_i|)$$

Minkowski distance

$$D(x, y) = \left(\sum_{i=1}^n |x_i - y_i|^p \right)^{\frac{1}{p}}$$

$P = 1 \rightarrow$ Manhattan distance

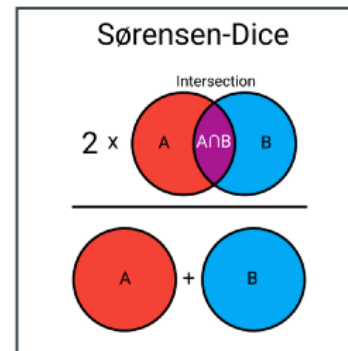
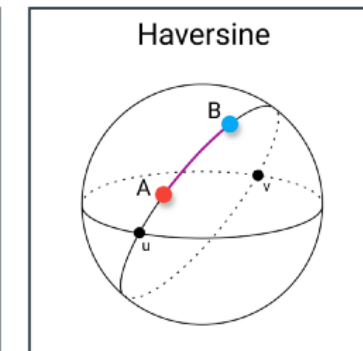
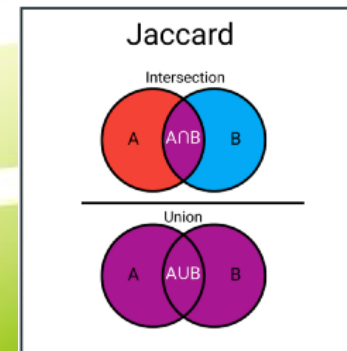
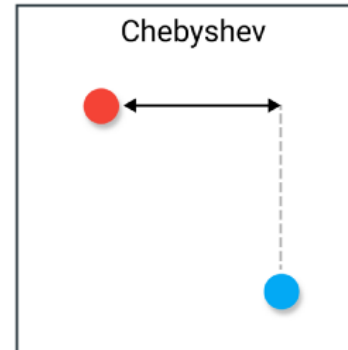
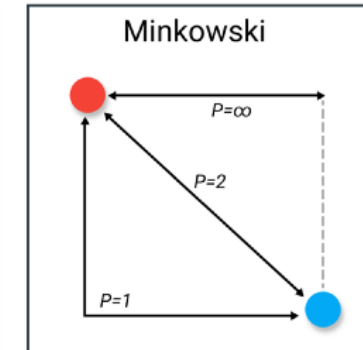
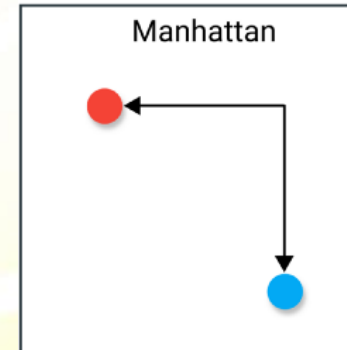
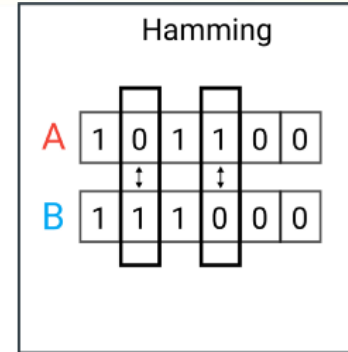
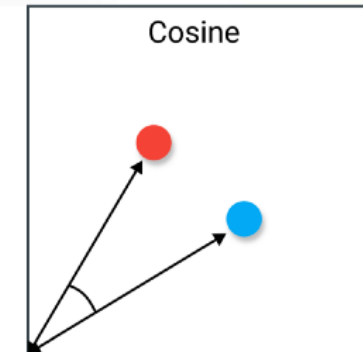
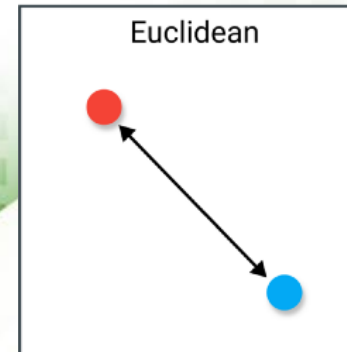
$P = 2 \rightarrow$ Euclidean distance

$P = \infty \rightarrow$ Chebyshev distance

Jaccard index calculates similarity and diversity as size of intersection divided by size of union.

$$D(x, y) = 1 - \frac{|x \cap y|}{|y \cup x|}$$

$$d = \frac{\text{Count}_{10} + \text{Count}_{01}}{\text{Count}_{11} + \text{Count}_{10} + \text{Count}_{01}}$$



DATA ANALYSIS - GROUPING

- Association rules method groups observations and attempts to discover links or associations between different attributes of the group.
- unsupervised grouping method
- **Association rule learning:** procedure to check for dependency of one data item on another data item and maps accordingly so that it can help in be more profitable analysis.
 - *If a customer buys bread, (s)he's 70% likely of buying milk.*
- **Association Rule:** simple If/Then statements that help discover relationships between seemingly independent relational databases or other data repositories.
 - Ways to find patterns in data; Helps in finding features (dimensions) which occur together (correlated).
 - Employed in *Market Basket analysis*, Web usage mining, Medical Diagnosis, etc.

*IF the customer is age 18 AND
the customer buys paper AND
the customer buys a hole punch
THEN the customer buys a binder*

<<A package of products can be created for college students.>>

DATA ANALYSIS - GROUPING

Advantages

- rules are easy to understand

Limitations

- Generating rules can be computationally expensive (especially when dataset has many variables or many possible values per variable)
- can generate large numbers of rules that must be prioritized and interpreted.
- There are ways to make analysis run faster but they often compromise final results.
- forces to either restrict analysis to variables that are categorical or convert continuous variables to categorical variables.

DATA ANALYSIS - GROUPING

Grouping by Combinations of Values

- Increasing number of variables or number of possible values for each variable or both → increases number of groups
- When number of groups becomes too large → impractical to generate all combinations

Customer ID	Gender	Purchase
932085	Male	Television
596720	Female	Camera
267375	Female	Television

Group Number	Count	Gender	Purchase
Group 1	16,099	Male	Camera or Television
Group 2	15,513	Female	Camera or Television
Group 3	16,106	Male or Female	Camera
Group 4	15,506	Male or Female	Television
Group 5	7,889	Male	Camera
Group 6	8,210	Male	Television
Group 7	8,217	Female	Camera
Group 8	7,296	Female	Television

Group Number	Count	Gender	Purchase	Income
Group 1	16,099	Male	Camera or Television	Below \$50K or Above \$50K
Group 2	15,513	Female	Camera or Television	Below \$50K or Above \$50K
Group 3	16,106	Male or Female	Camera	Below \$50K or Above \$50K
Group 4	15,506	Male or Female	Television	Below \$50K or Above \$50K
Group 5	15,854	Male or Female	Camera or Television	Below \$50K
Group 6	15,758	Male or Female	Camera or Television	Above \$50K
Group 7	7,889	Male	Camera	Below \$50K or Above \$50K
Group 8	8,210	Male	Television	Below \$50K or Above \$50K
Group 9	8,549	Male	Camera or Television	Below \$50K
Group 10	7,550	Male	Camera or Television	Above \$50K
Group 11	8,217	Female	Camera	Below \$50K or Above \$50K
Group 12	7,296	Female	Television	Below \$50K or Above \$50K
Group 13	7,305	Female	Camera or Television	Below \$50K
Group 14	8,208	Female	Camera or Television	Above \$50K
Group 15	8,534	Male or Female	Camera	Below \$50K
Group 16	7,572	Male or Female	Camera	Above \$50K
Group 17	7,320	Male or Female	Television	Below \$50K
Group 18	8,186	Male or Female	Television	Above \$50K
Group 19	4,371	Male	Camera	Below \$50K
Group 20	3,518	Male	Camera	Above \$50K
Group 21	4,178	Male	Television	Below \$50K
Group 22	4,032	Male	Television	Above \$50K
Group 23	4,163	Female	Camera	Below \$50K
Group 24	4,054	Female	Camera	Above \$50K
Group 25	3,142	Female	Television	Below \$50K
Group 26	4,154	Female	Television	Above \$50K

DATA ANALYSIS – ASSOCIATION RULE

- Association rule learning works on the concept of If (**antecedent**) and Else Statement (**Consequent**).

- *If a customer buys bread, (s)he's 70% likely of buying milk.*



- **Single cardinality:** Association or relation between two items
 - If number of items increases, then cardinality also increases accordingly.
- **Important association metrics:**
 - **Support** (frequency)
 - **Confidence** (paired/conditional occurrence)
 - **Lift** (strength of rule)

DATA ANALYSIS – ASSOCIATION RULE

- **Support:** frequency of an event in dataset.

- $Supp(e) = Freq(e) / Total\ transaction$



- **Confidence:** indicates how often the rule has been found to be true (*how often items X and Y occur together in dataset when occurrence of X is already given*).

- $Confidence(X,Y) = Freq(X,Y) / Freq(X)$

Confidence = Group support / IF-part support

- **Lift:** strength of any rule. *Ratio of observed support measure and expected support if X and Y are independent of each other.*

- $Lift(R) = Supp(X,Y) / (Supp(X)*Supp(Y))$

Lift = Confidence / THEN-part support

- **Lift= 1:** probability of occurrence of antecedent and consequent is independent of each other.

- **Lift>1:** determines the degree to which two itemsets are dependent to each other.

- **Lift<1:** tells that rule body and rule head appear less often together than expected (negative effect on occurrence).

Patient ID	Fever	Headaches
1326	None	Mild
398	Severe	Severe
6377	Severe	Severe
1234	None	None
2662	Severe	Severe
9477	None	None
7286	Severe	Severe
1732	None	None

DATA ANALYSIS – ASSOCIATION RULE

$Supp(e) = Freq(e) / \text{Total transaction}$

$Confidence(X,Y) = Freq(X,Y) / Freq(X)$

- **Confidence = Group support / IF-part support**

$Lift(R) = Supp(X,Y) / (Supp(X)*Supp(Y))$

- **Lift = Confidence / THEN-part support**



Rule: $X \Rightarrow Y$

$$Support = \frac{freq(X,Y)}{N}$$

$$Confidence = \frac{freq(X,Y)}{freq(X)}$$

$$Lift = \frac{Support}{Supp(X) \times Supp(Y)}$$

Rule 1

IF Class of work is Private AND
Education is Doctorate
THEN Income is <=50K

Total observations: 32,561

Class of work is Private: 22,696 observations

Education is Doctorate: 413 observations

Class of work is private and Education is Doctorate: 181 observations

Income is <=50K: 24,720 observations

Income is >50K: 7841 observations

Rule 2

IF Class of work is Private AND
Education is Doctorate
THEN Income is >50K

Association Rule Summary Table

	Rule 1	Rule 2
Count	49	132
Support	0.0015	0.0041
Confidence	0.27	0.73
Lift	0.36	3.03

DATA ANALYSIS - ASSOCIATION RULE

		LHS	RHS	rules	support	confidence	lift
		Ice Cream	Soda	{Ice Cream} => {Soda}	0.07	1.00	5.00
		Soda	Ice Cream	{Soda} => {Ice Cream}	0.07	0.33	5.00
		Ice Cream	Pie	{Ice Cream} => {Pie}	0.07	1.00	3.00
		Pie	Ice Cream	{Pie} => {Ice Cream}	0.07	0.20	3.00
		Ice Cream	Burger	{Ice Cream} => {Burger}	0.07	1.00	2.50
		Burger	Ice Cream	{Burger} => {Ice Cream}	0.07	0.17	2.50
		Ice Cream	Salad	{Ice Cream} => {Salad}	0.07	1.00	2.14
		Salad	Ice Cream	{Salad} => {Ice Cream}	0.07	0.14	2.14
		Ice Cream	Pizza	{Ice Cream} => {Pizza}	0.07	1.00	2.14
		Pizza	Ice Cream	{Pizza} => {Ice Cream}	0.07	0.14	2.14
		Soda	Chicken	{Soda} => {Chicken}	0.07	0.33	1.67
		Chicken	Soda	{Chicken} => {Soda}	0.07	0.33	1.67
		Soda	Chocolate Shake	{Soda} => {Chocolate Shake}	0.07	0.33	1.25
		Chocolate Shake	Soda	{Chocolate Shake} => {Soda}	0.07	0.25	1.25
		Soda	Pie	{Soda} => {Pie}	0.20	1.00	3.00
		Pie	Soda	{Pie} => {Soda}	0.20	0.60	3.00
		Soda	Burger	{Soda} => {Burger}	0.20	1.00	2.50
		Burger	Soda	{Burger} => {Soda}	0.20	0.50	2.50
		Soda	Bottled Water	{Soda} => {Bottled Water}	0.07	0.33	0.83
		Bottled Water	Soda	{Bottled Water} => {Soda}	0.07	0.17	0.83
		Soda	Salad	{Soda} => {Salad}	0.20	1.00	2.14
		Salad	Soda	{Salad} => {Soda}	0.20	0.43	2.14
		Soda	Pizza	{Soda} => {Pizza}	0.20	1.00	2.14

Transaction ID	Items List
1	Cookies, Egg, Milk, Sandwich
2	Bottled Water, Burger, Chicken, Egg, Pizza, Salad
3	Beacon, Bottled Water, Egg, Sandwich, Yogurt
4	Burger, Pie, Pizza, Salad, Soda
5	Burger, Ice Cream, Pie, Pizza, Salad, Soda
6	Chocolate Shake, Cookies, Egg, Milk, Sandwich
7	Beacon, Chocolate Shake, Cookies, Milk, Yogurt
8	Bottled Water, Burger, Chicken, Chocolate Shake, Egg, Pie, Pizza, S
9	Beacon, Bottled Water, Egg, Milk, Pizza, Salad, Yogurt
10	Chocolate Shake, Cookies, Egg, Milk, Sandwich
11	Beacon, Burger, Salad
12	Cookies, Egg, Milk, Sandwich, Yogurt
13	Beacon, Bottled Water, Egg, Pie, Pizza, Sandwich
14	Cookies, Egg, Milk, Sandwich
15	Bottled Water, Burger, Chicken, Egg, Pie, Pizza, Salad

DATA ANALYSIS - ASSOCIATION RULE

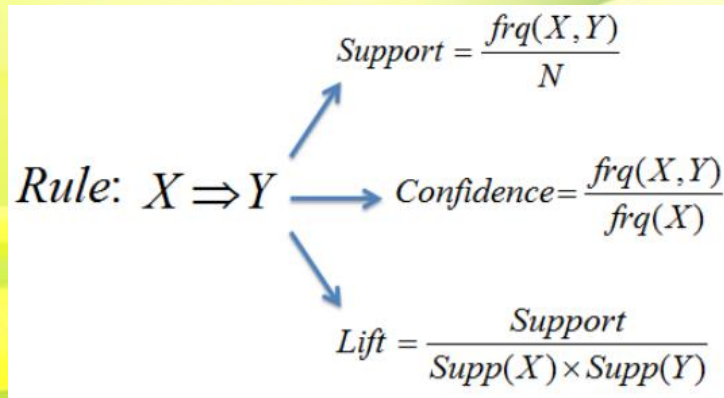
TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

	Beer	Bread	Milk	Diaper	Eggs	Coke
T_1	0	1	1	0	0	0
T_2	1	1	0	1	1	0
T_3	1	0	1	1	0	1
T_4	1	1	1	1	0	0
T_5	0	1	1	1	0	1

One-hot encoded Table

	Rule 1	Rule 2
Count	49	132
Support	0.0015	0.0041
Confidence	0.27	0.73
Lift	0.36	3.03

Association Rule Summary Table



- {Diaper, Beer} \Rightarrow Milk**
 - Support = 2/5, Confidence = 2/3
- {Milk} \Rightarrow {Diaper, Beer}**
 - Support = 2/5, Confidence = 2/4
- {Milk, Diaper} \Rightarrow Bread**
 - Support = 2/5, Confidence = 2/3

DATA ANALYSIS - ASSOCIATION RULE

Example: For the computer accessories purchase transaction, prepare the Association Rule Summary table by calculating the Support, Confidence & Lift for the following Association rules.

1. If customer purchases Laptop, then (s)he also buys Monitor.
2. If customer purchases Monitor & Tablet, then (s)he also buys headset.
3. If customer purchases Laptop & Monitor, then (s)he also buys headset.
4. If customer purchases Laptop & Monitor, then (s)he also buys Printer.

Rule: $X \Rightarrow Y$

$$\text{Support} = \frac{\text{freq}(X, Y)}{N}$$

$$\text{Confidence} = \frac{\text{freq}(X, Y)}{\text{freq}(X)}$$

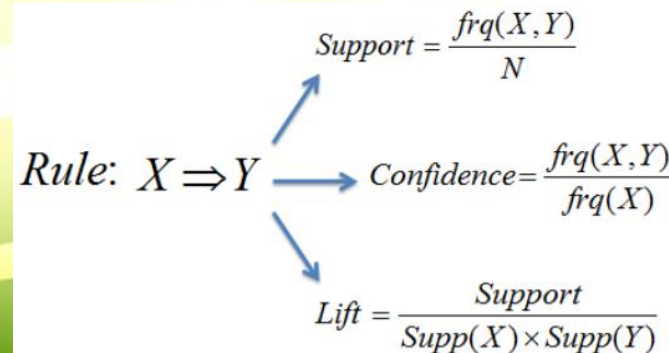
$$\text{Lift} = \frac{\text{Support}}{\text{Supp}(X) \times \text{Supp}(Y)}$$

Table 1. Market basket transactions

Transaction ID	Items Bought
1	{Laptop, Printer, Tablet, Headset}
2	{Printer, Monitor, Tablet}
3	{Laptop, Printer, Tablet, Headset}
4	{Laptop, Monitor, Tablet, Headset}
5	{Printer, Monitor, Tablet, Headset}
6	{Printer, Tablet, Headset}
7	{Monitor, Tablet}
8	{Laptop, Printer, Monitor}
9	{Laptop, Tablet, Headset}
10	{Printer, Tablet}

DATA ANALYSIS – ASSOCIATION RULE

- **Market Basket Analysis (MBA):** popular examples and applications of association rule mining.
- Technique used by big retailers to determine association between items, as a marketing strategy.
 - *If a customer buys bread, (s)he most likely can also buy butter, eggs, or milk → these products are stored within a shelf or mostly nearby.*
- **Steps in MBA:**
 - *Establish possible Rules.*
 - *Calculate support, confidence, lift for each rule.*
 - *Validate the Rule(s).*


$$\begin{aligned} \text{Rule: } X \Rightarrow Y & \begin{cases} \text{Support} = \frac{\text{frq}(X, Y)}{N} \\ \text{Confidence} = \frac{\text{frq}(X, Y)}{\text{frq}(X)} \\ \text{Lift} = \frac{\text{Support}}{\text{Supp}(X) \times \text{Supp}(Y)} \end{cases} \end{aligned}$$

DATA ANALYSIS – ASSOCIATION RULE

BENEFITS OF MARKET BASKET ANALYSIS

- Customer Behavior analysis
- Optimization of in-store operations & stock management.
- Campaigns and promotions
- Item Recommendations
- Increasing market share

DATA ANALYSIS – ASSOCIATION RULE

- **(Item) Recommendation system** makes prediction for user consumption.
- **Content-based** approach focused on information of items' own features, rather than using users' interactions and feedbacks.
 - *Example, movie attributes: genre, year, director, actor etc.*
- **Collaborative Filtering:** focused on users' historical preference.
 - Based on user's own past preference.
 - Basic assumption: *users who have agreed in past tend to also agree in future.*
 - *User regularly watches scientific videos → (s)he's recommended other such videos.*
 - Based on other users' (majority) past preference.
 - Collecting preferences or taste information from many users (collaborating).
 - Basic assumption: *if many have agreed in past, others will also agree in future.*
 - *Many users are watching budget analysis video → many other users also recommended same/such videos.*

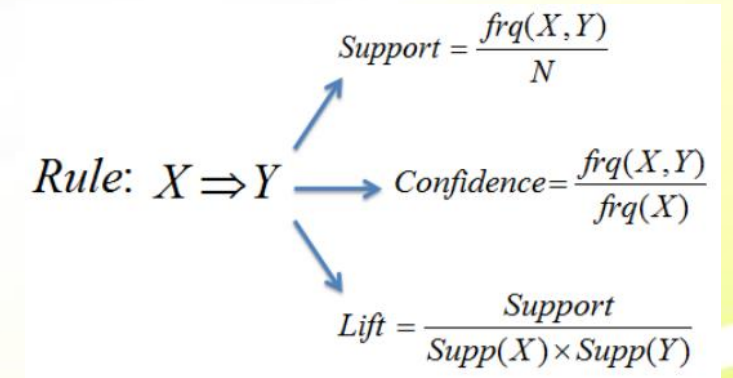
DATA ANALYSIS – ASSOCIATION RULE

- Collaborative Filtering focuses on users' historical preference, for Item Recommendation.
- User preference usually expressed by two categories.
- **Explicit Rating** given by user to an item on a sliding scale.
 - Example: 5 stars for a movie
 - Most direct feedback from users to show how much they like an item.
- **Implicit Rating** suggests users preference indirectly.
 - Example: page views, clicks, purchase records, whether or not listen to music track, etc.
 - Shows user's involvement/attention with the “content”, time spent, etc. → value of the “content”

DATA ANALYSIS – ASSOCIATION RULE

- **Steps in Grouping Analysis:**

- Association rule learning → *Establish possible Rules.*
- Calculate support, confidence, lift for each rule.
- Validate the Rule(s) with threshold/acceptance level.



Rule: $X \Rightarrow Y$

$$\text{Support} = \frac{\text{freq}(X, Y)}{N}$$
$$\text{Confidence} = \frac{\text{freq}(X, Y)}{\text{freq}(X)}$$
$$\text{Lift} = \frac{\text{Support}}{\text{Supp}(X) \times \text{Supp}(Y)}$$

- **Types of Association rule learning:**

- Apriori Algorithm
- F-P (Frequent Pattern) Growth Algorithm
- Eclat (Equivalence Class Transformation) algorithm

DATA ANALYSIS - APRIORI

- **Apriori algorithm** uses frequently purchased item-sets to **generate association rules**.
 - Used in market basket analysis.
 - Helps to find frequent item-sets in transactions and identifies association rules between these items.
- Named Apriori because it uses prior knowledge of frequent itemset properties.
- Limitation is *frequent itemset generation* → needs to scan database many times leading to increased time and reduce performance (computationally costly step).
- Basic Assumption:
 - *All subsets of a frequent itemset must be frequent.*
 - *If an itemset is infrequent, all its supersets will be infrequent.*

TID	ITEMSETS
T1	A, B
T2	B, D
T3	B, C
T4	A, B, D
T5	A, C
T6	B, C
T7	A, C
T8	A, B, C, E
T9	A, B, C

Apriori Algorithm

Step-1: Calculating C1 and F1:

- Create a table that contains each itemset's support count (frequency of each itemset individually in dataset). This table is called the **Candidate set or C1**.
- Take out all the itemsets that have the greater support count than the Minimum Support (2). It will give us the table for the **frequent itemset F1**.

TID	ITEMSETS
T1	A, B
T2	B, D
T3	B, C
T4	A, B, D
T5	A, C
T6	B, C
T7	A, C
T8	A, B, C, E
T9	A, B, C

Given: Minimum Support= 2, Minimum Confidence= 50%

Step-2: Candidate Generation C2, and F2:

- Create the pair of the itemsets of F1 in the form of subsets.
- Again find support count for these pairs from the main transaction table of datasets (**C2**).
- Compare the C2 Support count with minimum support count, and eliminate the itemsets with less support count (**in F2**).

Itemset	Support_Count
A	6
B	7
C	5
D	2
E	1

Itemset	Support_Count
{A, B}	4
{A, C}	4
{A, D}	1
{B, C}	4
{B, D}	2
{C, D}	0

Apriori Algorithm

Step-3: Candidate generation C3, and F3:

- Repeat the same two processes, but with subsets of three itemsets together.
- Create C3 and F3 accordingly.

Step-4: Finding the association rules for the subsets:

- To generate the association rules, first create a new table with all possible rules from the occurred combination {A, B, C}.
- For all the rules, calculate the Confidence using required formula.
- After calculating the confidence value for all rules, exclude the rules that have less confidence than the minimum threshold (50%).
- First three rules $A \wedge B \rightarrow C$, $B \wedge C \rightarrow A$, and $A \wedge C \rightarrow B$ can be considered as the strong association rules for the given problem.

TID	ITEMSETS
T1	A, B
T2	B, D
T3	B, C
T4	A, B, D
T5	A, C
T6	B, C
T7	A, C
T8	A, B, C, E
T9	A, B, C

Given: Minimum Support= 2, Minimum Confidence= 50%

Itemset	Support_Count
{A, B, C}	2
{B, C, D}	1
{A, C, D}	0
{A, B, D}	0

Rules	Support	Confidence
$A \wedge B \rightarrow C$	2	$\text{Sup}\{(A \wedge B) \wedge C\} / \text{sup}(A \wedge B) = 2/4 = 0.5 = 50\%$
$B \wedge C \rightarrow A$	2	$\text{Sup}\{(B \wedge C) \wedge A\} / \text{sup}(B \wedge C) = 2/4 = 0.5 = 50\%$
$A \wedge C \rightarrow B$	2	$\text{Sup}\{(A \wedge C) \wedge B\} / \text{sup}(A \wedge C) = 2/4 = 0.5 = 50\%$
$C \rightarrow A \wedge B$	2	$\text{Sup}\{(C \wedge (A \wedge B))\} / \text{sup}(C) = 2/5 = 0.4 = 40\%$
$A \rightarrow B \wedge C$	2	$\text{Sup}\{(A \wedge (B \wedge C))\} / \text{sup}(A) = 2/6 = 0.33 = 33.33\%$
$B \rightarrow B \wedge C$	2	$\text{Sup}\{(B \wedge (B \wedge C))\} / \text{sup}(B) = 2/7 = 0.28 = 28\%$

DATA ANALYSIS - APRIORI

Transaction ID	Items bought
1	(Apple x 3), (Cabbage x 1), (Donut x 2)
2	(Bread x 2), (Cabbage x 3), (Egg x 1)
3	(Apple x 1), (Bread x 1), (Cabbage x 1), (Egg x 2)
4	(Bread x 3), (Egg x 4)
5	(Apple x 2), (Cabbage x 2), (Egg x 1)

Transaction ID	Items bought
1	A A A C D D
2	B B C C C E
3	A B C E E
4	B B B E E E E
5	A A C C E

Transaction ID	Items bought
1	A C D
2	B C E
3	A B C E
4	B E
5	A C E
6	A B C D E

Iteration-1

C1	
Item-Set	Support
{A}	4
{B}	4
{C}	5
{D}	2
{E}	5

F1	
Item-Set	Support
{A}	4
{B}	4
{C}	5
{E}	5

- Fix a threshold support level.
- Generally, 50% of total number of transaction = 2.5 (3)
- Discard item/itemset with frequency < threshold (3)

DATA ANALYSIS - APRIORI

Transaction ID	Items bought
1	A C D
2	B C E
3	A B C E
4	B E
5	A C E
6	A B C D E

F1	
Item-Set	Support
{A}	4
{B}	4
{C}	5
{E}	5

Discard item/itemset with frequency < threshold (3)

Basic Assumption:

- All subsets of a frequent itemset must be frequent.
- If an itemset is infrequent, all its supersets will be infrequent.

C1	
Item-Set	Support
{A}	4
{B}	4
{C}	5
{D}	2
{E}	5

Iteration-2

		Only items in F1					
		C2				F2	
Transaction ID	Items bought	Item-Set	Support			Item-Set	Support
1	A C D	{A,B}	2			{A,C}	4
2	B C E	{A,C}	4			{A,E}	3
3	A B C E	{A,E}	3			{B,C}	3
4	B E	{B,C}	3			{B,E}	4
5	A C E	{B,E}	4			{C,E}	4
6	A B C D E	{C,E}	4				

DATA ANALYSIS - APRIORI

Transaction ID	Items bought
1	A C D
2	B C E
3	A B C E
4	B E
5	A C E
6	A B C D E

F1	
Item-Set	Support
{A}	4
{B}	4
{C}	5
{E}	5

F2	
Item-Set	Support
{A,C}	4
{A,E}	3
{B,C}	3
{B,E}	4
{C,E}	4

Discard item/itemset with frequency \geq threshold (3)

F3	
Item-Set	Support
{A,C,E}	3
{B,C,E}	3

Iteration-3

- Grouping is done in a way that each item-set contains three items in them.
- Further, these will be divided into their **sub-sets**.
- Also, those with support value less than threshold (3), will be omitted → This process is known as **Pruning**.

		C3	
Transaction ID	Items bought	Item-Set	In F2 ?
1	A C D	{A,B,C}, {A,B}, {A,C}, {B,C}	No
2	B C E	{A,B,E}, {A,B}, {A,E}, {B,E}	No
3	A B C E	{A,C,E}, {A,E}, {A,C}, {C,E}	Yes
4	B E	{B,C,E}, {B,C}, {B,E}, {C,E}	Yes
5	A C E		
6	A B C D E		



DATA ANALYSIS - APRIORI

Transaction ID	Items bought
1	A C D
2	B C E
3	A B C E
4	B E
5	A C E
6	A B C D E

F1	
Item-Set	Support
{A}	4
{B}	4
{C}	5
{E}	5

F2	
Item-Set	Support
{A,C}	4
{A,E}	3
{B,C}	3
{B,E}	4
{C,E}	4

Discard item/itemset with frequency < threshold (3)

F3	
Item-Set	Support
{A,C,E}	3
{B,C,E}	3

Iteration-4

Transaction ID	Items bought
1	A C D
2	B C E
3	A B C E
4	B E
5	A C E
6	A B C D E



C4	
Item-Set	Support
{A,B,C,E}	2
{A,B,C,D}	1
{B,C,D,E}	1
{A,C,D,E}	1



Omitting items that are already omitted	
C4	
Item-Set	Support
{A,B,C,E}	2

In iteration-4, support of the only item-set having 4 items is less than threshold value (3) → Stop iterations → Take final item set to be F3.

DATA ANALYSIS - APRIORI

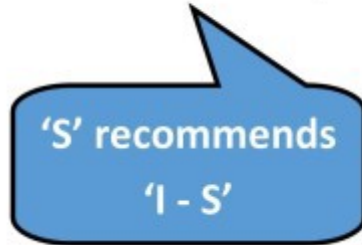
From F3,

- If $I = \{A, C, E\}$, then subsets are $\{A, C\}$, $\{A, E\}$, $\{C, E\}$, $\{A\}$, $\{C\}$ and $\{E\}$.
- If $I = \{B, C, E\}$, then subsets are $\{B, C\}$, $\{B, E\}$, $\{C, E\}$, $\{B\}$, $\{C\}$ and $\{E\}$.

Association Rules: In order to filter out relevant item-sets, create association rules and apply them to subsets. (assume minimum confidence value = 60%)

- For every subset S of I , association rule is:

$$S \longrightarrow (I - S) \text{ if } \frac{\text{Support}(I)}{\text{Support}(S)} \geq \text{Minimum confidence value i.e., 60\%}$$



Consider $\{A, C, E\}$ from F3.

Rule 1: $\{A, C\} \rightarrow (\{A, C, E\} - \{A, C\})$
which is $\{A, C\} \rightarrow \{E\}$

F3	
Item-Set	Support
$\{A, C, E\}$	3
$\{B, C, E\}$	3

F2	
Item-Set	Support
$\{A, C\}$	4
$\{A, E\}$	3
$\{B, C\}$	3
$\{B, E\}$	4
$\{C, E\}$	4

F1	
Item-Set	Support
$\{A\}$	4
$\{B\}$	4
$\{C\}$	5
$\{E\}$	5

DATA ANALYSIS - APRIORI



Consider {A,C,E} from F3.

Rule 1: {A,C} → ({A,C,E} — {A,C}) which is **{A,C} → {E}**

Confidence = Support{A,C,E} / Support{A,C} = 3/4 = 75% > 60%

So rule 1 i.e., {A,C} → {E} is valid.

Rule 2: {A,E} → ({A,C,E} — {A,E}) which is **{A,E} → {C}**

Confidence = Support{A,C,E} / Support{A,E} = 3/3 = 100% > 60%

So rule 2 i.e., {A,E} → {C} is valid.

Rule 3: {C,E} → ({A,C,E} — {C,E}) which is **{C,E} → {A}**

Confidence = Support{A,C,E} / Support{C,E} = 3/4 = 75% > 60%

So rule 3 i.e., {C,E} → {A} is valid.

Rule 4: {A} → ({A,C,E} — {A}) which is **{A} → {C,E}**

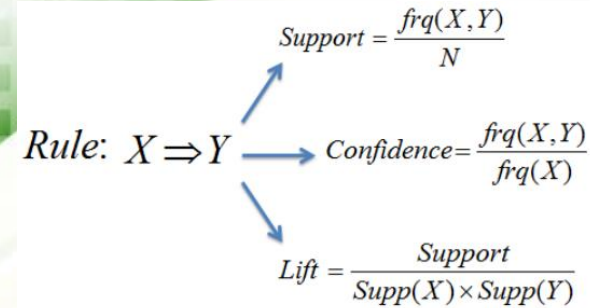
Confidence = Support{A,C,E} / Support{A} = 3/4 = 75% > 60%

So rule 4 i.e., {A} → {C,E} is valid.

Rule 5: {C} → ({A,C,E} — {C}) which is **{C} → {A,E}**

Confidence = Support{A,C,E} / Support{C} = 3/5 = 60% !> 60% (not greater than 60%)

So rule 5 i.e., {C} → {A,E} is **rejected**.



$S \rightarrow (I - S)$ if $\frac{\text{Support}(I)}{\text{Support}(S)} \geq \text{Minimum confidence value i.e., 60\%}$

'S' recommends
'I - S'

Rule 6: {E} → ({A,C,E} — {E}) which is **{E} → {A,C}**

Confidence = Support{A,C,E} / Support{E} = 3/5 = 60% !> 60%

(not greater than 60%)

So rule 6 i.e., {E} → {A,C} is **rejected**.

Transaction ID	Items bought
1	A C D
2	B C E
3	A B C E
4	B E
5	A C E
6	A B C D E

F2	
Item-Set	Support
{A,C}	4
{A,E}	3
{B,C}	3
{B,E}	4
{C,E}	4

F3	
Item-Set	Support
{A}	4
{B}	4
{C}	5
{E}	5

F1	
Item-Set	Support
{A}	4
{B}	4
{C}	5
{E}	5

F3	
Item-Set	Support
{A,C,E}	3
{B,C,E}	3

DATA ANALYSIS - APRIORI

FOR {A,C,E} in F3, association rules generated earlier.

Calculate strength of each one.

Rule 1: $\text{Lift} = \frac{\text{Support}\{A,C,E\}}{(\text{Support}\{A,C\} \times \text{Support}\{E\})} = \frac{3}{(4 \times 5)} = \frac{3}{20} = \mathbf{0.15}$

Rule 2: $\text{Lift} = \frac{\text{Support}\{A,C,E\}}{(\text{Support}\{A,E\} \times \text{Support}\{C\})} = \frac{3}{(3 \times 5)} = \frac{3}{15} = \mathbf{0.20}$

Rule 3: $\text{Lift} = \frac{\text{Support}\{A,C,E\}}{(\text{Support}\{C,E\} \times \text{Support}\{A\})} = \frac{3}{(4 \times 4)} = \frac{3}{16} = \mathbf{0.1875}$

Rule 4: $\text{Lift} = \frac{\text{Support}\{A,C,E\}}{(\text{Support}\{A\} \times \text{Support}\{C,E\})} = \frac{3}{(4 \times 4)} = \frac{3}{16} = \mathbf{0.1875}$

Rule 5: $\text{Lift} = \frac{\text{Support}\{A,C,E\}}{(\text{Support}\{C\} \times \text{Support}\{A,E\})} = \frac{3}{(5 \times 3)} = \frac{3}{15} = \mathbf{0.20}$

Rule 6: $\text{Lift} = \frac{\text{Support}\{A,C,E\}}{(\text{Support}\{E\} \times \text{Support}\{A,C\})} = \frac{3}{(5 \times 4)} = \frac{3}{20} = \mathbf{0.15}$

- Same steps can be applied to item-set {B,C,E}.
- Very small sample \rightarrow Lift values do not vary much here .
- **All Lift < 1 \rightarrow None of association rule is strong to be accepted.**
- For **larger data**; analysis is more evident.

F1		F2	
Item-Set	Support	Item-Set	Support
{A}	4	{A,C}	4
{B}	4	{A,E}	3
{C}	5	{B,C}	3
{E}	5	{B,E}	4
		{C,E}	4

F3	
Item-Set	Support
{A,C,E}	3
{B,C,E}	3

Transaction ID	Items bought
1	A C D
2	B C E
3	A B C E
4	B E
5	A C E
6	A B C D E

Rule: $X \Rightarrow Y$

$$\text{Support} = \frac{\text{freq}(X, Y)}{N}$$

$$\text{Confidence} = \frac{\text{freq}(X, Y)}{\text{freq}(X)}$$

$$\text{Lift} = \frac{\text{Support}}{\text{Supp}(X) \times \text{Supp}(Y)}$$

DATA ANALYSIS – APRIORI

Example: For the purchase transaction given below, establish the association rules with Support threshold=50%, Confidence= 60%.

Transaction	List of items
T1	I1,I2,I3
T2	I2,I3,I4
T3	I4,I5
T4	I1,I2
T5	I1,I2,I3,I5
T6	I1,I2,I3,I4

1-2	4
1-3	3
2-3	4
1-2-3	3

DATA ANALYSIS – F-P GROWTH

Shortcomings Of Apriori Algorithm

- Needs generation of large number of candidate itemsets (for huge database).
- Needs multiple scans of database to check support of each itemset generated and this leads to high costs.

Frequent Pattern (F-P) growth algorithm

- Improved version of Apriori Algorithm (overcome these shortcomings)
- No need for candidate generation to generate frequent pattern.
- Represents database in form of tree structure (F-P tree) to extract most frequent patterns.
- F-P tree structure maintains the association (frequency patterns) between itemsets.
- Database is fragmented using one frequent item. This fragmented part is called “pattern fragment”.
- Itemsets of these fragmented patterns are analyzed → Thus search for frequent itemsets is reduced comparatively.

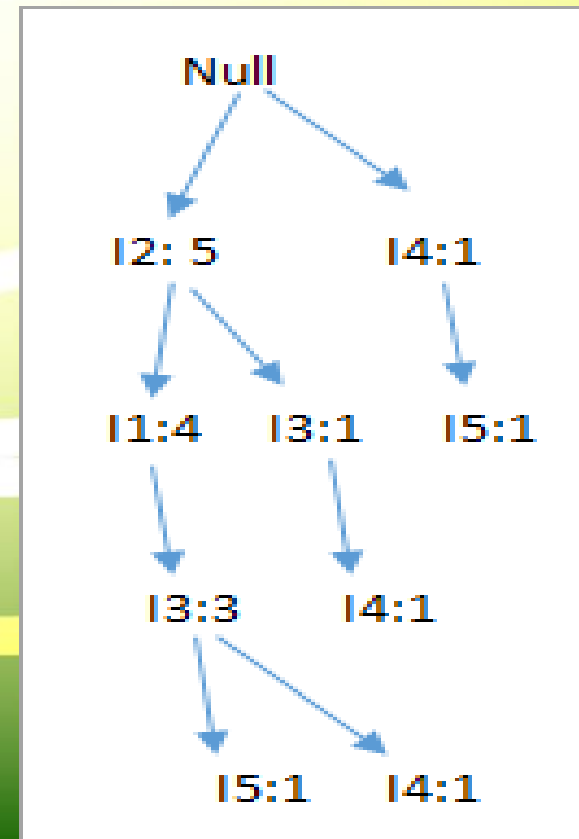
DATA ANALYSIS – F-P GROWTH

Frequent Pattern Tree

- Tree-like structure that is made with initial itemsets of database.
- Every node of FP tree represents an item of that itemset.
- Root node represents null value whereas lower nodes represent itemsets of the data.
- Association of these nodes with lower nodes that is between itemsets is maintained while creating the tree.

Transaction	List of items
T1	I1,I2,I3
T2	I2,I3,I4
T3	I4,I5
T4	I1,I2
T5	I1,I2,I3,I5
T6	I1,I2,I3,I4

Item	Count
I1	4
I2	5
I3	4
I4	3
I5	2



DATA ANALYSIS – F-P GROWTH

F-P Algorithm Steps

1. Scan database to find occurrences of itemsets.
2. Construct FP tree by creating root (null) of the tree.
3. Scan database again. Tree Branch is constructed with transaction itemsets in descending order of count.
 - Examine first transaction and find out itemset in it. Itemset with max count is taken at top, the next itemset with lower count and so on.
4. Next transaction in database is examined. Itemsets are ordered in descending order of count.
 - If any itemset of this transaction is already present in another branch, then this transaction branch would share a common prefix to root.
 - i.e. common itemset is linked to new node of another itemset in this transaction.
5. Count of itemset is incremented as it occurs in transactions. Both common node and node count is increased by 1 as they are created and linked according to transactions.
6. Once all the transactions are scanned iteratively → **FP tree is created.**
7. **Mine the created FP Tree.** i.e. lowest node is examined first along with the links of lowest nodes.
 - Lowest node represents frequency pattern length 1. From this, traverse the path in FP Tree. This path(s) are called conditional pattern base (a sub-database consisting of prefix paths in FP tree occurring with lowest node/suffix).
8. Construct a Conditional FP Tree, which is formed by a count of itemsets in the path (itemsets meeting threshold support are considered in Conditional FP Tree).
9. Frequent Patterns are generated from the Conditional FP Tree.

DATA ANALYSIS – F-P GROWTH

Support threshold=50% \rightarrow min_sup=3, Confidence= 60%

Transaction	List of items
T1	I1,I2,I3
T2	I2,I3,I4
T3	I4,I5
T4	I1,I2
T5	I1,I2,I3,I5
T6	I1,I2,I3,I4

Item	Count
I1	4
I2	5
I3	4
I4	3
I5	2

Freq count

Item	Count
I2	5
I1	4
I3	4
I4	3

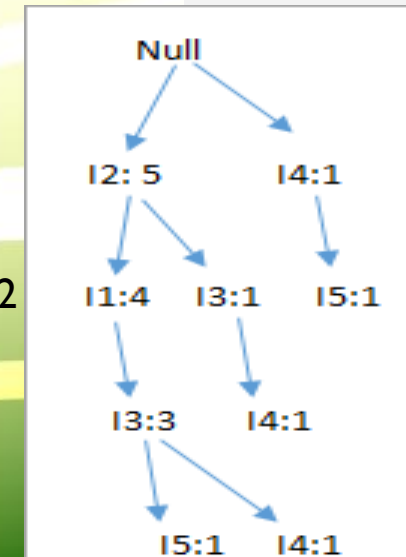
Threshold filter
& Sort

DATA ANALYSIS – F-P GROWTH

Build FP Tree

1. Considering the **root node** null.
2. First scan of Transaction **T1: I1, I2, I3** contains three items {I1:1}, {I2:1}, {I3:1}, where I2 is linked as a child to root, I1 is linked to I2 and I3 is linked to I1.
3. **T2: I2, I3, I4** contains I2, I3, and I4, where I2 is linked to root, I3 is linked to I2 and I4 is linked to I3. But this branch would share I2 node as common as it is already used in T1.
4. Increment the count of I2 by 1 and I3 is linked as a child to I2, I4 is linked as a child to I3. The count is {I2:2}, {I3:1}, {I4:1}.
5. **T3: I4, I5**. Similarly, a new branch with I5 is linked to I4 as a child is created.
6. **T4: I1, I2**. The sequence will be I2, and I1. I2 is already linked to the root node, hence it will be incremented by 1. Similarly I1 will be incremented by 1 as it is already linked with I2 in T1, thus {I2:3}, {I1:2}.
7. **T5: I1, I2, I3, I5**. The sequence will be I2, I1, I3, and I5. Thus {I2:4}, {I1:3}, {I3:2}, {I5:1}.
8. **T6: I1, I2, I3, I4**. The sequence will be I2, I1, I3, and I4. Thus {I2:5}, {I1:4}, {I3:3}, {I4:1}.

Transaction	List of items
T1	I1,I2,I3
T2	I2,I3,I4
T3	I4,I5
T4	I1,I2
T5	I1,I2,I3,I5
T6	I1,I2,I3,I4



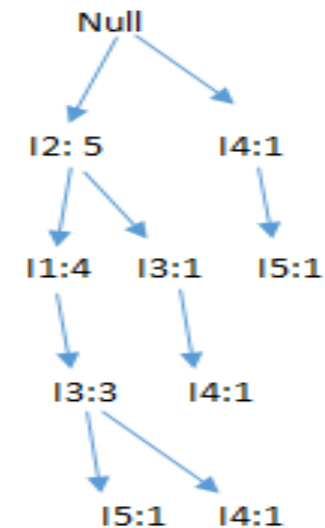
Item	Count
I2	5
I1	4
I3	4
I4	3
I5	2

DATA ANALYSIS – F-P GROWTH

Mining of FP-tree:

- Lowest node item I5 is deleted (threshold).
- Next lower node I4 occurs in 2 branches, {I2,I1,I3:1},{I2,I3:1}. This forms the conditional pattern base.
- Conditional pattern base is considered a transaction database & conditional FP-tree is constructed. This will contain {I2:2, I3:2, I1:1}. I1 is not considered as it does not meet the min support count.
- This path will generate all combinations of frequent patterns : {I2,I4:2},{I3,I4:2},{I2,I3,I4:2}
- For I3, prefix path is: {I2,I1:3},{I2:1}, this will generate a 2 node FP-tree : {I2:4, I1:3} and frequent patterns are generated: {I2,I3:4}, {I1:I3:3}, {I2,I1,I3:3}.
- For I1, prefix path is: {I2:4} this generates single node FP-tree: {I2:4} and frequent patterns are generated: {I2, I1:4}.

Transaction	List of items
T1	I1,I2,I3
T2	I2,I3,I4
T3	I4,I5
T4	I1,I2
T5	I1,I2,I3,I5
T6	I1,I2,I3,I4



Item	Count
I2	5
I1	4
I3	4
I4	4

Item	Conditional Pattern Base	Conditional FP-tree	Frequent Patterns Generated
I4	{I2,I1,I3:1},{I2,I3:1}	{I2:2, I3:2}	{I2,I4:2},{I3,I4:2},{I2,I3,I4:2}
I3	{I2,I1:3},{I2:1}	{I2:4, I1:3}	{I2,I3:4}, {I1:I3:3}, {I2,I1,I3:3}
I1	{I2:4}	{I2:4}	{I2,I1:4}

DATA ANALYSIS – F-P GROWTH

- Considering last column (frequent Pattern generation) & Support threshold=50% \rightarrow min_sup=3
 - 3-item frequent set generated $\{I2, I1, I3:3\}$
 - 2-Item frequent sets generated $\{I2, I3:4\}$, $\{I1, I3:3\}$, $\{I2, I1:4\}$.
- All these set (association rules) are distinct sets.
- Once association rules are generated; lift value can be calculated to further analysis (same like apriori).
- In comparison to Apriori Algorithm, only the frequent patterns are generated, NOT all combinations of different items & keep analyzing each (computationally costly).

				Transaction	List of items
Item	Conditional Pattern Base	Conditional FP-tree	Frequent Patterns Generated		
I4	$\{I2, I1, I3:1\}, \{I2, I3:1\}$	$\{I2:2, I3:2\}$	$\{I2, I4:2\}, \{I3, I4:2\}, \{I2, I3, I4:2\}$	T1	I1, I2, I3
I3	$\{I2, I1:3\}, \{I2:1\}$	$\{I2:4, I1:3\}$	$\{I2, I3:4\}, \{I1: I3:3\}, \{I2, I1, I3:3\}$	T2	I2, I3, I4
I1	$\{I2:4\}$	$\{I2:4\}$	$\{I2, I1:4\}$	T3	I4, I5
				T4	I1, I2
				T5	I1, I2, I3, I5
				T6	I1, I2, I3, I4

DATA ANALYSIS – F-P GROWTH

- Asparagus (A), Corn (C), Beans (B), Tomatoes (T) & Squash (S)
- minimum count=2

Item	Support Count
Asparagus (A)	7
Beans (B)	6
Squash (S)	6
Corn (C)	2
Tomatoes (T)	2

Transaction ID	List of items in transaction
T1	B , A , T
T2	A , C
T3	A , S
T4	B , A , C
T5	B , S
T6	A , S
T7	B , S
T8	B , A , S , T
T9	B , A , S

Item	Conditional Pattern base	Conditional FP tree	Frequent Pattern Generation
Tomatoes (T)	{{A,B:1},{A,B,S:1}}	<A:2,B:2>	{A,T:2},{B,T:2},{A,B,T:2}
Corn (C)	{{A,B:1},{A:1}}	<A:2>	{A,C:2}
Squash (S)	{{A,B:2},{A:2},{B:2}}	<A:4,B:2>,<B:2>	{A,S:4},{B,S:4},{A,B,S:2}
Bean (B)	{{A:4}}	<A:4>	{A,B:4}

