Optimization Programs

Matthew Rothman

MIT Sloan School of Management

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Benchmarking and Tracking Error

In practice, one compare's a manager to a benchmark, an index, comparable to her mandate

So perhaps if you want to know how your large-cap stock manager is doing, compare her to Russell 1000. Assuming you are in a CAPM world then:

$$R_{manager,t} = \alpha_{manager} + \beta_{mgr}(R_{R1000}) + \varepsilon_t$$

Then look at the difference in returns, or *active return*:

$$\begin{aligned} R_{Active} &= \mathrm{R}_{\mathrm{mgr}} - \mathrm{R}_{\mathrm{Benchmark}} \\ &= \left(\alpha_{manager} + \beta_{mgr}(R_{Benchmark}) + \varepsilon_t\right) - \mathrm{R}_{\mathrm{Benchmark}} \end{aligned}$$

And *active risk* is, or *tracking error:*

$$\begin{aligned} \Psi_{Active} &= \mathrm{Stdev}\left(\mathrm{R}_{\mathrm{mgr}} - \mathrm{R}_{\mathrm{Benchmark}}\right) \\ &= Stdev\left(\left(\alpha_{manager} + \beta_{mgr}(R_{Benchmark}) + \varepsilon_t\right) - \mathrm{R}_{\mathrm{Benchmark}}\right) \end{aligned}$$

And residual risk is:

$$\omega_{Active} = \sqrt{\sigma^2_{manager} - \beta^2_{manager}\sigma^2_{bemchmark}}$$

Benchmarking and Tracking Error

I hate tracking error, in practical world, as a measure of risk.

Imagine a fund is simply a leveraged index fund – one of the ProFunds – that runs with a beta of two to the index.

$$
\begin{aligned}
\Psi_{Active} &= Stdev\left(\left(\alpha_{manager} + \beta_{mgr}(R_{Benchmark}) + \varepsilon_t\right) - R_{Benchmark}\right) \\
&= Stdev\left(\left(\beta_{mgr}(R_{Benchmark})\right) - R_{Benchmark}\right) \\
&= Stdev\left(2 \times R_{Benchmark} - R_{Benchmark}\right) \\
&= Stdev(R_{Benchmark})
\end{aligned}
$$

Lousy way to identify whether the fund is a closet indexer or if sticking to mandate.

Benchmarking and Tracking Error

Alternative: Use the Fund's R^2 with the index!

$$\begin{aligned}
\Psi^2_{Active} &= Var\left(\left(\alpha_{manager} + \beta_{mgr}(R_{Benchmark}) + \varepsilon_t\right) - R_{Benchmark}\right) \\
&= Var\left(\alpha + \beta R_{Benchmark} + \varepsilon_t - R_{Benchmark}\right) \\
&= Var\left(\alpha + (1-\beta) R_{Benchmark} + \varepsilon_t\right) \\
&= (1-\beta)^2 \sigma^2_{Benchmark} + \left(1 - R^2\right)^2 \sigma^2_{portfolio}
\end{aligned}$$

$$\Rightarrow \quad \left(1 - R^2\right)^2 = \frac{\Psi^2_{Active} - (1-\beta)^2 \sigma^2_{Benchmark}}{\sigma^2_{portfolio}}$$

Optimization

Portfolio Construction: Risk Models

- In practice, to create a diversified portfolio, asset managers will use a tool called an optimizer that **maximizes return subject to a level of risk and a number of other constraints**.

- Optimizers work great as a mathematical tools. But they are only as good as there inputs. If the inputs are mis-specified in any way then you can get unintuitive results. Garbage in leads to garbage out.
    - It takes very seriously the difference between two stocks with expected returns of 1.0000000000000001 and 1.0.
    - Very large alpha estimates can radically skew the optimization results. Should you "winsorize" your alpha estimates?
    - It is very sensitive to estimates of correlation. Should you use some "shrinkage" estimates for variances and correlations?
    - Constraints can help keep the optimizer from implementing "corner solutions" but at the same time can lead to results that are unintuitive. They must be very well thought-out.
    - Tracking error is a non-linear constraint is often very hard for an optimizer to solve.

Solutions to Sensitivity to Inputs

Solutions to Sensitivity Problem

- Take account of sampling error in estimating the frontier
  - "Classical" statistics approaches (Ang and Bekaert (2002)) or by Monte Carlo (Michaud (1998))
- "Robust" estimates of means and covariances
  - Statistical methods which take care of outliers and extreme values. One method is Bayesian "shrinkage" methods. These shrink the means back to a model, e.g. CAPM, and the covariance back to a covariance matrix with the same volatilities and correlations (see e.g. Ledoit and Wolf (2003)). [Why is this reasonable?]
- Noisy estimates of means and covariances can lead to overtrading
  - Trade only when your "robust" stocks estimates are significantly different or when your projected portfolio return and variance are statistically different

Portfolio Construction: Long Only Funds

- Examples of classical constraints that are used in practice:
  - Tracking Error <= “Upper Limit” (set by mandate)
  - Maximum active stock position size <= ±2%
  - Maximum active industry weight <= ± 2.5%
  - Maximum active sector weight <= ±5%
  - Maximum active country weight <= ± 5%
  - Maximum “individual factor exposure” <= ± “limit”
  - Market Beta of the portfolio must be ~1 (0.97 to 1.03)
  - Maximum contribution to active risk from a “stock”, “industry”, “sector”, “country” must be <= “limit”
  - Minimum number of stocks held in the portfolio >= 300 (depending on universe)
  - Minimum active position size >= ± .05%

Portfolio Construction: Levered Funds

- More Complex Constraints for Stat Arb Managers & Equity Mkt Neutral
  - **Volatility <=** "Upper Limit" **(set by risk manager)**
  - Maximum active stock position size <= **±1%**
  - Maximum active industry weight <= **± 1%**
  - Maximum active sector weight <= **±2.5%**
  - Maximum active country weight <= **± 3%**
  - Market Beta of the portfolio must be **~0**
  - **Net market exposure ($ exposure) should be ~0**
  - **Control (hedge) factor exposures**
  - **More names on short side vs long side due to path dependencies and larger number of names generally**
  - **Exposure to "macro risk factor" (non-alpha factors such as interest rates, GDP, etc) must be small**
  - **Statistical model to control idiosyncratic risk**

Portfolio Construction: Levered Funds

- More Complex Constraints for Stat Arb Managers & Equity Mkt Neutral
  - **Control (hedge) factor exposures**

How do you this? What *exactly* does this mean? Please write down in math the actual constraint!

If you can't do this then you haven't actually understood this lecture today.

Write the whole optimization problem is math (in code better yet!) when you can do that then you know you understand this!

Mean-Variance Utility

Shortcomings of Mean-Variance Utility

- Treats gains and losses symmetrically (around the mean)
- Only the first two moments matter
- Subjective vs objective probabilities
- Risk aversion is constant
- Most models are one period model
- Are they fast moving enough?
- Are not sophisticated around issues of outliers and robustness and idiovol handling

Summary

Summary

- How might one determine the size of a stock position in one's portfolio? What is important to consider? (see Pederson too)
- What does Pederson mean by "A trader must have no memory and forget nothing"? How does that apply to portfolio construction?
- Why might a manager use an optimizer in constructing a portfolio? What are the pro's and con's?
- What are alternatives to using an optimizer for a portfolio? What are the pro's and con's? Should Lee Ainslie use an optimizer? Or is his use of ad hoc rules which take account for correlation and many of the constraints used by an optimizer "good enough"?
- Why would one use a risk model? Are there uses aside from portfolio construction? What are the challenges in using it? How would you recommend constructing one? Be able to defend the pros and cons of your methodology.
- Why are the purposes of constraints when using an optimizer? How might they distort your solution? What challenges do they present when you use them? What can go wrong when using them?
- What is the impact of "false precision" in the optimization process? What might you do to combat it?
- If you see a stock with a low or negative alpha in your "trade list", why might it be there? And what would you do?
- Do you like tracking error as a definition of risk? What might your propose as an alternative?
- If your alphas estimates (expected return estimates) did not sum to zero over the benchmark you are using, what errors might that cause? How could you correct for this situation?
- What is the role of the portfolio manager in a quantitative investment process? What do you think should be the role?
- What are the pros and cons of different groups have control over the different functions of the investment process versus centralized control under the PMs?