Homework 2

This homework is due April 21st at the 4:00 P.M.
Read it over from start to end ASAP

I highly advise you read the section in Pederson on calculating drawdowns. Start from there and get the answers he gets. Then build your code so it is flexible in the type of returns you generate throughout. If you set your spreadsheet and code up correctly, this problem set becomes much easier.

There is also an Optional Section of the Homework. Not doing this section will have *not* be detrimental to your grade. I am providing these questions as you may find this material interesting and thought provoking. Thomas and I will read and provide you with comments on any portion of the Optional Section that you submit. Note again, this section is purely optional for those who wish to engage further with material. Regardless of whether or not you do the Optional Section, the required portion is due on Friday April 21st at 4:00 pm.

You have hired a just a hired a hedge fund manager to run a separately managed account for you. The manager is Ms. Polly Anna Afelis of Braggadocio Capital Management (BCM). She has showed you a series of backtests that you believe are very well-down and realistic, incorporating everything you have learned to date about good quantitative testing of models, including out-of-sample tests, numerous robustness checks, many techniques for minimizing the possibility of out-of-sample, and you believe it is innovative and using relatively unique data.

Your best estimate is that the true return generating capability of BCM will be 8% annualized returns with an annualized standard deviation of 4%. So the expected Sharpe Ratio of BCM is 2.0.

Note in all of these questions the *dollars gained or lost do not grow or shrink the size of the portfolio.* The AUM remains constant (gains are swept out and losses replenished. However, your returns take into account gains and loss, obviously. This is standard in the hedge fund industry. Also, ignore fees and trading costs).

A. Assume the returns for BCM are normally, independently and identically distributed with the expected returns and standard deviations described above.
   i. What is the probability that BCM will lose money in any given year?
   ii. What is the probability that BCM will lose money in any given quarter?
   iii. What is the probability that BCM will lose money in any given month?
   iv. What is the probability that BCM will lose money on any given day?
   v. If you invested $500 million with BCM, what is the expected value of the dollars of the PnL you expect to earn at the end of the first quarter? What is the +/- 2 standard deviation of dollars of PnL you would expect to earn?

B. Let's imagine you are comfortable with the probabilities and answers calculated above. So, you tell BCM that they can move from running with zero leverage to running with 100% leverage. In other words, they can borrow against the assets of the fund to run with twice the amount of current capital. From your and BCM's analysis, the efficacy of the strategy will in no way be degraded by this increase in capital.
  i.   What is the probability that BCM will lose money in any given year?
  ii.   What is the probability that BCM will lose money in any given quarter?
  iii.   What is the probability that BCM will lose money in any given month?
  iv.   What is the probability that BCM will lose money on any given day?
  v.   If you invested $500 million with BCM, what is the expected value of the dollars of the PnL you expect to earn at the end of the first quarter? What is the +/- 2 standard deviation of dollars of PnL you would expect to earn?

C. How do you understand the difference in your answers between (A) and (B)? How do you understand why any investor would want a fund manager to run with leverage? Or even higher leverage than in (B), say 300% or 500%? Why wouldn't you want to run with them with say 1,000% leverage? Please put them into the context of the calculations you did for the above ((a) through (f)).

D. For the general set-up of the investment strategies described in (A) and (B) please calculate the winning rate (i.e. percent of days your strategy earns money) and please calculate the up-down ratio (i.e. the average $ on PnL earned on up days divided by absolute value of $ PnL lost on down days).
  i.   Now if you vary the standard deviation by 0.25 (ranging from 2 to 12) but hold the expected return constant from above, how does the winning rate, up-down ratio and Sharpe-Ratio change? Please put this in the form of a table with the with winning rate going down the columns, the up-down ratio across the columns, and the Sharpe Ratio in the middle of the table.
  ii.   Now you vary the expected return by 0.25 (ranging from 2 to 12) but hold the standard deviation constant from above, how does the winning rate, up-down ratio and Sharpe-Ratio change? Please put this in the form of a table with the with winning rate going down the columns, the up-down ratio across the columns, and the Sharpe Ratio in the middle of the table.
  iii.   How do you understand the relationship winning-rate, up-down ratio and Sharpe Ratio? What assumptions in this problem do you think are responsible for these results?

E. Let's go back to the original set-up: Your best estimate is that the true return generating capability of BCM will be 8% annualized returns with an annualized standard deviation of 4%. So the expected Sharpe Ratio of BCM is 2.0. Assume the returns for BCM are normally, independently and identically distributed with the expected returns and standard deviations described above.
  i.   If you were to hire BCM and irrevocably commit to keeping your money invested with them for a 5 year period, what is the probability you will observe them having a 90 day drawdown?

a) Note: A drawdown is defined as the time between the fund peak $ PnL until it surpasses that previous peak $ PnL. For purposes of this calculation assume that the dollars invested in the fund remain constant (e.g. when you make money, the winnings are swept out of your account; when you lose money, the dollars are also replenished. So the dollars you are managing remain constant).

b) Hint: There is no closed-form mathematical solution to this problem. You must do this by running a series of simulations (e.g., Monte Carlo simulations). It is up to you to decide how many "simulations" you need to run before you answer converges.

ii. Instead of irrevocably committing your money to them for a 5 year period, imagine it is for a 10 year period and the other assumptions of (i) (a) remain the same. Over that period of time, what is the probability that you will observe them having a 90 day drawdown? What is the probability they would have 120 day drawdown? What is your *intuition* of observing a 90 day down, if you shorten and/or lengthen the period of time you can irrevocably commit to BCM? Please comment on each of these questions.

iii. Let's say BCM is the typical start-up hedge fund. It can only get investors to lock up their capital for 1 year but all of the other assumptions of (i) (a) remain the same. As the investor in the BCM, you are trying to learn what the true Sharpe Ratio of Ms. Afelis's team. You think the appropriate way to do this is through observing drawdowns – after all, if she is truly skilled she should lose you less money, that is have a lower probability of a drawdown. But how much lower?

So please calculate the following table. Across the columns, should be the set of Sharpe Ratios ranging from -1 to 3, incrementing by 0.5. Down the columns is the length of the drawdown. Let the set of lengths of the drawdown be the following: 1 day, 3 days, 5 days, 10 days, 20 days, 40 days, 60 days, 90 days, 120 days. In the middle of the table is the probability of observing a day drown of that length or longer, given the SR at the top of the column.

For the avoidance of any doubt, the following is the table you need to fill in:

	Sharpe Ratio								
	-1	-0.5	0	0.5	1	1.5	2	2.5	3

Drawdown Length (Days): 1, 3, 5, 10, 20, 40, 60, 90
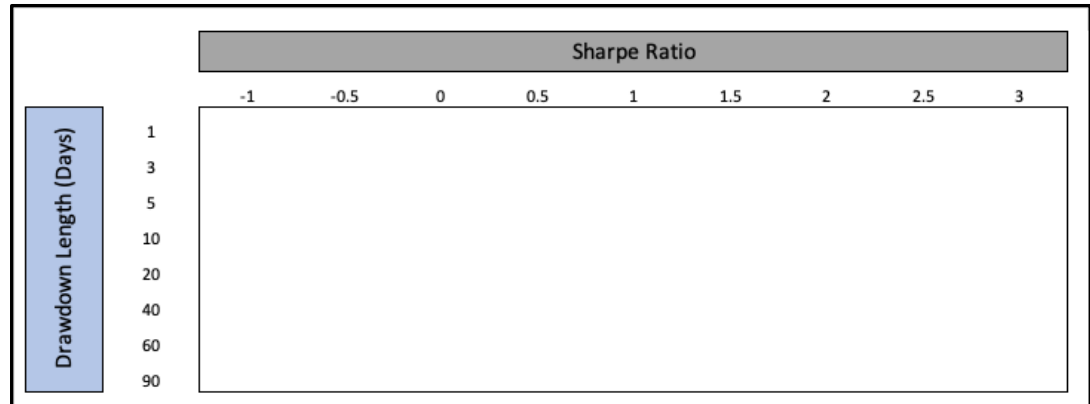
F. What do the above results about drawdown probabilities teach you about the usefulness of observing drawdowns as a way to measure a manager's skill? What lessons do you conclude?

If you were a manager of a hedge fund, how does this impact the way you want to set-up your fund? If you are an investor in a hedge fund, what do you think is appropriate amount of time you need before you can make any conclusions about a manager's skill? Does observing winning rates and up-down ratios help you learn about skill?

Please discuss in your answers here how this relates to the Limits of Arbitrage as discussed in class.
Please discuss in your answers here how this relates to the Fundamental Law of Active Management we have also discussed.

Please discuss in your answers here how the above also relate to how often a manager should be reporting results to investors. Discuss the pro's and con's of greater transparency (e.g. higher frequency of reporting performance statistics).

As we have discussed in class, some factors and strategies are non-symmetric. Some have fatter tails than others. Some have more skew. Certain types of assets maybe highly no linear. Factor exposures may not be continuous

a) Do the results of this exercise impact what type of risk model you think a manager should be using? Either in terms of the horizon of the risk model or the way you would believe they should be estimating risk?
b) How does this exercise influence the type of constraints that you believe a manager should be putting on their portfolio construction process?
c) What constraints do you specifically want to see them use? Or you can answer this in terms of the risk-guidelines you would give them.

d) If you do not implement any risk guidelines on your manager versus if you were to put on very strict guidelines, where would this difference manifest itself in this analysis you just completed?

As the world is ever changing, managers also need to be adapting their types of strategies and constantly innovating.

e) From the results of this homework, as an allocator to hedge funds, discuss the pro's and the con's on manager innovation?
f) How much transparency do you *now* believe you want into the underlying process and changes that manager is implementing?
g) As a manager, how much do you believe you know need to give to your investor?
h) Please discuss this with respect to the Muller's Law of Active Management reading.
i) Please again layer in the Limits of Arbitrage into your answer(s).
j) Has anything in this homework altered your belief regarding running linear models, "piece-wise" linear models or machine learning models? Why or why not?

G. Are there any other performance metrics, besides the ones you were asked to compute in this exercise that you would want your "dashboard" as you monitored your hedge fund manager's daily PnL and returns? Why or why not?

H. Finally, you are run a hedge fund that makes a profit every single day. That is, your fund makes money every single day. The dollars of PnL (after all costs) > 0. What is the minimum Sharpe Ratio of the fund?

APPENDIX Homework

This section of the homework is TRULY optional. Not doing will NOT be detrimental to your grade in this class in any fashion. It is here for those of you who want to continue learning and thinking about this material in a deeper manner. If you are intellectually curious and would like to do it and discuss the material with Thomas and me we would be happy to discuss with you. Moreover thinking through these problems will be helpful in your preparation for exam. So we will happily read any answers or thoughts you have on these questions and give you feedback. Again, this section is completely optional.

In reality, of course, returns are not Normally identically and independently distributed with no skew and no kurtosis (e.g. the third and fourth of returns are not zero). Let's try relaxing the simplest assumption: that there is skewness is returns.

There are many ways to model this but the easiest way to do it is to assume that return you are observing is actually a mixture of two independent Normal variables. Specifically, we can generate a skewed Normal distribution by combining normal distribution with the absolute value of a normal distribution using the following rule:

$$\frac{\alpha|X_1| + X_2}{\sqrt{1+\alpha^2}} \quad \text{where:} \quad X_1, X_2 \text{ are iid } \mathcal{N}(0,1)$$

I. To make sure you understand this, please generate and plot the probability distribution function (pdf) where the Sharpe Ratio is 2 and alpha is -10, 0, 10. Hint: when alpha=0, the pdf should look like your standard bell-shape symmetric Normal distribution function.

Now we want to understand how the skewness of returns impacts both the winning rate and the up-down ratio of the manager you are observing.

Again, the purpose of these exercises are that you are trying to match the observations you are seeing of BCM's return with the true skill of the manager actually may be.

II. In particular, we want to calculate the win rate percentages for various [Skew, Sharpe] combinations. Again, note there is no closed form solution here. This can only be calculated through Monte Carlo simulations.

For the avoidance of any doubt, please fill in the following table. It will show how Skewness and Sharpe Ratio's combine to impact the winning rate. In the center of the table, you should be calculating the winning rate

		Skew Parameter																			
		-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
Sharpe Ratio	0																					
	0.5																					
	1																					
	1.5																					
	2																					
	2.5																					
	3																					
	3.5																					
	4																					
	4.5																					
	5																					

III.   Now for the same table as above, please compute and put into the center of the table compute the theoretical observed up-down ratio.

IV.   Imagine you were to compute the drawdown probabilities as in table E (iii) now, where the Sharpe Ratio was 2 and across the columns you were to vary the Skew Parameter between -10 to 10. What you find is that changes in skew basically have extremely little impact on drawdown probabilities. Trust me, this is a true result. What is your intuition and reasoning for why you would get this result?

Now we want to calculate how varying the Skew Parameter and the Sharpe Ratio, can impact what the probability of observing a given winning percentage and up-down ratio. For example, we want to compute the probability of say observing a winning rate of 50% and up-down ratio of 1.25, if the manager's true Sharpe was 2 and the Skew Parameter was 2.

Generally speaking, you want to fill in the following table. Again, this needs to be done through Monte Carlo simulations.

		Up-Down Ratio													
		<0.85	0.85	0.90	0.95	1.00	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	>1.40
Winning Percentage	<40%														
	40%														
	42%														
	44%														
	46%														
	48%														
	50%														
	52%														
	54%														
	56%														
	58%														
	60%														
	>60%														

V.   Please calculate the following four tables for the following set of values:
    i.   Window Length for Calculations = 120 days; Sharpe Ratio = 2; Skew Parameter of 0
    ii.   Window Length for Calculations = 120 days; Sharpe Ratio = 1; Skew Parameter of 0
    iii.   Window Length for Calculations = 120 days; Sharpe Ratio = 2; Skew Parameter of 2
    iv.   Window Length for Calculations = 120 days; Sharpe Ratio = 1; Skew Parameter of 0

VI.   Lastly, let's revisit Question A but with a slightly different twist.

      i.   Please calculate the probability, that if BCM has a true Sharpe Ratio of 2 and a skew parameter of 0, for over some random given 60 day period, what is the probability that you observe them having a realized SR of less than zero? Of less than -1? Of less -3?

      ii.   Please repeat but now with the true skew parameter is +2

      iii.   Please repeat but now with the true skew parameter is -2

      iv.   Again, what is your intuition and reasoning for the difference in these sets of results?

            Note here you are just calculating 3 probability distribution functions with the different parameters. Make sure you put the parameters on the right time scale!

VII.   After now completing this exercise with skew introduced and taking into consideration your earlier analysis, please amend your answers to Question F as you see fit.

      i.   Place yourself in the shoes of an allocator to hedge funds.

          a) How do you want to structure your investment contracts with your hedge fund managers?

          b) What types of strategies would or would not have skew?

          c) How does skew impact the contract you would be willing to sign (i.e. lock-up periods during which you cannot withdraw your money, the type of return transparency you will want, the relative level of base management and incentive fees you will be willing to pay, etc)?

          d) Layer in Sharpe Ratio into answer c). Are there any type of investment strategies that might have high SR and lots of skew? Give examples. How does that impact the contract you want to sign?

          e) How does your answers above align (or not) with the type of contract the fund manager will want to sign?