# Axioma World-Wide Equity Factor Risk Model, Version 4 Model Update

January 2018

## Contents

# 1 Overview

The AX-WW4 suite of factor risk models forecasts risk for equities listed on global exchanges. Available in two horizons (Medium and Short) and two factor model variants (Fundamental and Statistical), it caters to different investment objectives and quantitative needs.

Highlights of AX-WW version 4 (WW4) fundamental model include:

- New Methodology
  - Implemented using our new model framework and latest research methodology
  - Improved our factor return estimation methodology, yielding superior attribution results
- Insightful and Transparent Factor Exposures
  - Quality and stability of exposures
  - Substantially more balance sheet information in fundamental style exposures than WW21 model
  - Used new weighting scheme for estimating market-based exposures
- Intuitive Factor selection
  - New and improved style and industry factor structure
  - Three new style factors: Profitability, Market Sensitivity and Dividend Yield
  - Incorporation of the GICS 2016 Industry Classification
- Actionable Results
  - New factors provide more informative and intuitive results for performance and risk attribution
  - Actionable information about portfolios/strategies: superior risk attribution results

# 2 Methodology Changes

This section describes changes made to our risk model estimation methodology. For a full description of our methodology, see the accompanying Axioma Robust™ Risk Model Version 4 Handbook.

## 2.1 New Methodology

### 2.1.1 Estimation Universe Selection Algorithm

The estimation universe selection methodology has been updated to include:

*Exclusion of illiquid returns*: A stock will be excluded from the estimation universe if more than 25% of its returns are missing over either the previous 250 days, or the life of the asset, whichever is longer. Conversely, a newly-listed asset is required to have at least 21 days (approximately one month) of suitably liquid returns before it is considered for inclusion. Exclusion of illiquid returns prevents assets with non-daily returns entering the regression and distorting the model's factor returns. The criteria are relaxed if necessary for Frontier markets to avoid empty factors. While WW21 excluded foreign listings and ADRs, WW4 allows some, for instance where the underlying is illiquid.

Figure 1 shows how the WW4 estimation universe evolves in comparison to the market cap of the total coverage universe over time. We see that throughout the entire history, WW4 estimation universe consistently covers a similar proportion of market cap as WW21 does and is close to the eligible universe of securities.

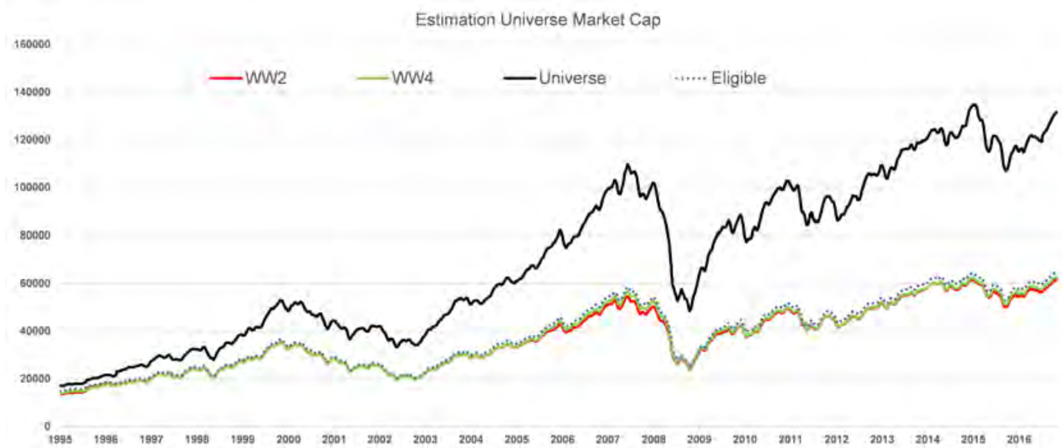


Figure 1: Estimation Universe Market Cap

### 2.1.2 Estimating Factor Returns

In WW4, we use a single-stage regression with constraints on the industry and country factor returns. The Global Market factor return, country returns, industry returns and style factor returns are estimated from a single-stage root-cap weighted regression (with constraint on industries and countries) of the daily excess local returns. We use a secondary regression on the residual returns to estimate the Domestic China factor.

### 2.1.3 Outlier Detection

In WW4, we introduce a new multi-dimensional outlier detection and trimming algorithm that can detect outliers along either the asset dimension, the time-dimension or both together. The reason for this is that a cross-sectional extreme return may not actually be an outlier when viewed along the time dimension for all returns for that particular asset. Thus, depending on the situation, we may or may not wish to treat this particular data point as an outlier. The new WW4 model takes account of this by identifying outliers along one or both dimensions depending on the context.

#### 2.1.4 Shrinkage for Illiquid Assets

Assets with illiquid returns histories, and hence more uncertain specific risk have their specific risk shrunk towards a cross-sectionally computed average depending on their degree of illiquidity.

We first fit a cross-sectional model to the specific risks of "good" assets, i.e. those with liquid returns series via robust regression, as shown below:

$$\sigma_L = W_L y + \mu.$$

Here, $\sigma_L$ are the predicted specific risks for the good assets and $W_L$ are exposures to known characteristics such as sector, size etc. We solve for $y$, and then compute a set of estimates for the illiquid assets, $\sigma_I$ as:

$$\bar{\sigma}_I = W_I y.$$

Finally, for each illiquid asset, $i$, if $\kappa_i$ represents the proportion of missing returns, $\sigma_i$ its original specific risk, and $\bar{\sigma}_i$ the estimate via the cross-sectional model, we compute an adjusted specific risk as:

$$\hat{\sigma}_i = \kappa_i \bar{\sigma}_i + (1 - \kappa_i)\sigma_i.$$

#### 2.1.5 Proxy Total Returns for Illiquid Assets

For computing the components that make up the style exposures, the WW4 fundamental model uses a missing returns filler that generates a series of synthetic returns for each and every asset that are used wherever the real return is missing, whether due to the asset being a recent IPO, or simply illiquid. Illiquid assets suffer from their returns not being truly daily, so a proxy return is generated for missing days, and non-daily returns are scaled accordingly. This yields more stable and smooth asset correlations and reduces the risk of pollution of factor returns by non-daily returns.

Proxy returns are also used in the Statistical model to fill in the missing asset returns prior to APCA (Asymptotic Principal Component Analysis) modeling. For more details on our APCA methodology, please refer to the Axioma Robust Risk Model Version 4 Handbook.

### 2.2 Quality and Stability of Exposures

#### 2.2.1 Trapezoidal Weighting Scheme

Many of our market-based descriptors, such as momentum, market sensitivity and volatility, rely on a rolling history of asset returns for their computation. If we take the case of medium-term momentum, a 230-day history of asset returns is extracted for each asset, and its cumulative return computed. This gives the raw exposure for the current day. The next trading day, this history is advanced by one day, and the oldest datum dropped from the series. And then the computation repeats.

Typically, the returns histories used for these descriptors have been equal-weighted — that is, every return in the series contributes equally to the descriptor value. This, however, introduces potential instability, as exposures may exhibit spurious jumps when a large return enters or exits the series.

To counter this, we introduce a new weighting scheme for such descriptors. We down-weight returns in the beginning and end of the series, while leaving the central portion of the series with equal weights of one, forming a trapezoid. Down-weighting is done in a linear fashion, with the first and last return in the series having the smallest weight, the second and penultimate return having the next smallest weight, and so on, until the central portion of the series is reached. Figure 2 demonstrates how this works in practice for a 250-day history of data.

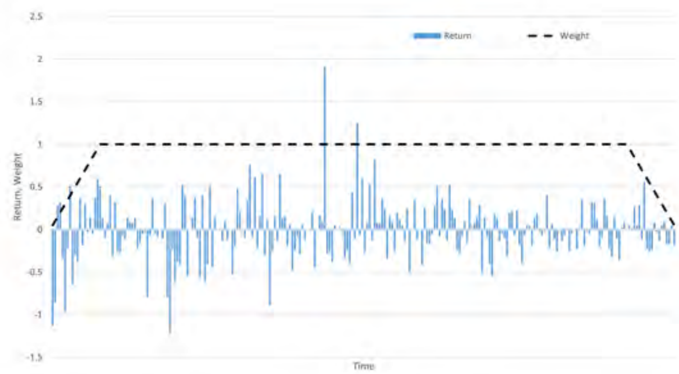


Figure 2: Trapezoidal Weighting Scheme

At a model level, we measure the stability of exposures by computing the correlation coefficient between exposures on dates $t$ and $t-21$. These correlation coefficients are sometimes referred to as exposure "churn" or exposure turnover statistics. A correlation coefficient that is close to 1 suggests that exposures are very similar on dates $t$ and $t-21$ (and churn is, therefore, relatively low), and a correlation coefficient that is close to zero suggests that exposures are dissimilar on dates $t$ and $t-21$ (and churn is, therefore, relatively high). Figure 3 show how exposure churn is improved with the use of trapezoid weights for the Medium-Term Momentum (MTM) factor.

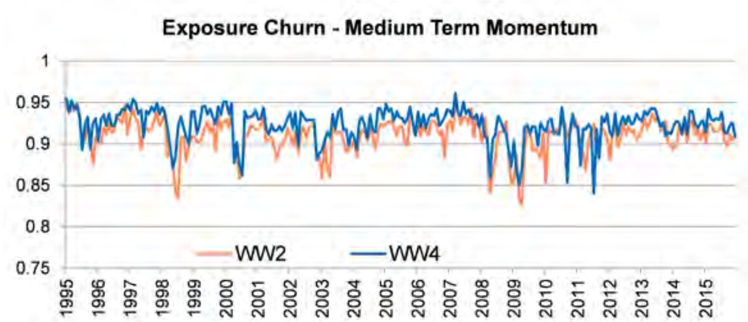


Figure 3: Exposure Churn of Momentum Factor

### 2.2.2 Exposures for Assets with Insufficient Data (IPOs)

Many factor exposures, such as momentum, market sensitivity and volatility, rely on a fixed history of asset returns for their computation. In the case of an IPO, no previous history will exist. While we can and do generate a proxy history of returns for such assets as a starting point, returns-based exposures for IPOs are still likely to suffer from instability and unintuitive values. We therefore shrink exposures of IPOs toward the cap-weighted average sector or region exposure. The shrinkage parameter is a linear function of the number of days on which actual returns are available.

To illustrate, let $X$ denote the raw style exposure for a recently listed asset, and let $M$ denote the cap-weighted average exposure, computed for the sector to which the asset belongs. Let $T$ be the number of returns required for the style factor computation, and let $t$ denote the actual number of genuine returns available. Then the shrunk exposure, $\tilde{X}$, is given by:

$$\tilde{X} = \frac{((T - t) * M + t * X)}{T}$$

Thus, at $t = 0$ (the IPO date), the asset's exposure is the cap-weighted average sector or region exposure, while at $t = T$, the exposure is determined entirely by the asset's returns.

Figure 4 and 5 compare volatility and momentum exposures for SCB (Standard Chartered Bank PLC) for WW4 and WW21. In both cases, the WW4 exposures are smoother in comparison to WW21 and they converge once we have accumulated enough data points to reliably estimate the exposures.

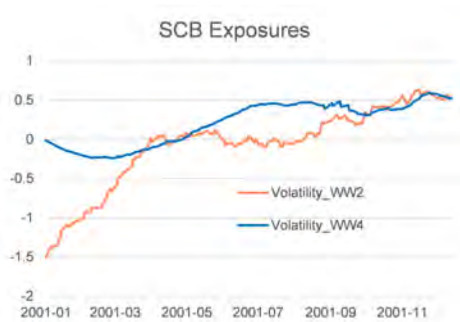


Figure 4: Volatility Exposure


Figure 5: Medium Term Momentum Exposure

## 2.3 Estimating Fundamental Style Exposures

The WW4 fundamental model uses substantially more fundamental data than its predecessor when constructing style exposures. For example, in the WW4 model, Profitability factor exposures are constructed as a linear combination of six financial ratios (cash flow to assets, cash flow to income, gross margin, return on assets, return on equity, and sales to assets). Leverage factor exposures are constructed as a linear combination of two ratios (debt to assets and debt to equity).

When constructing financial ratios from financial statement data, we use the following methodology:

- For ratios with flow data (sales, earnings, etc) in the numerator and point-in-time data (common equity, total assets, etc) in the denominator, we compute the numerator as the most recently reported annual value, and we compute the denominator as the average of the two most recently reported values from annual reports.
- For ratios with flow data in the numerator and denominator (for example, gross margin), we compute the numerator and the denominator as the most recently reported annual values.

Computing flow data as the most recently reported annual value ensures that we use the most current information in our models. Computing point-in-time data as averages mitigates issues associated with marginal point-in-time fundamental data, resulting in smoother ratios over time.

## 3 Style Factor Changes

This section lists new factors in the WW4 model, followed by improvements to existing factors. Table 7 provides a brief overview of the the factors in WW4 and WW21 and how they compare between the medium- and short-horizon models. For factor definitions, see the accompanying WW4 Model Supplement Handbook.

| Style factor | WW21 – Short and Medium Horizons | WW4 – Medium Horizon | WW4 – Short Horizon |
|---|---|---|---|
| Value | Book-to-price (B/P) and Earnings to Price (E/P) | Book-to-Price | Book-to-Price |
| Leverage | Total debt / assets | (Debt / Assets) + (Debt/Equity) | (Debt / Assets) + (Debt/Equity) |
| Growth | Sales Growth + Earnings Growth + Plowback times return-on-equity | Earnings growth + Sales growth | Earnings growth + Sales growth |
| Size | Natural logarithm of market capitalization | Same | Same |
| Liquidity | 1-month average daily volume over market capitalization | log of proportion of trading activity of past 3 months<br>Inverse of 6-month Amihud ratio<br>Proportion of returns traded over past year | log of proportion of trading activity of past 1 month<br>Inverse of 3-month Amihud ratio<br>Proportion of returns traded over past year |
| Market Sensitivity | Absent | 2 year weekly beta | 1 year weekly beta |
| Medium Term momentum | Cumulative returns over past 12 months excluding the most recent month | Same (trapezoidal weighting scheme) | Same (trapezoidal weighting scheme) |
| Short term momentum | Cumulative return over past month | Absent | Cumulative return over past month |
| Volatility | 3-month average of absolute return over cross-sectional standard deviation | Uses 6 month data, and orthogonal to market sensitivity | Uses 3 month data, and orthogonal to market sensitivity |
| Exchange rate sensitivity | 6-month beta to SDR returns | 2 year weekly beta to SDR returns | 1 year weekly beta to SDR returns |
| Earnings Yield | Absent | (Earnings / Price) * .75 + (12 m forward looking estimated Earnings/Price) * 0.25 | (Earnings / Price) * .75 + (12 m forward looking estimated Earnings/Price) * 0.25 |
| Dividend Yield | Absent | Dividend Yield | Dividend Yield |
| Profitability | Absent | Combines 6 descriptors: Return-on-Equity, Return-on-Assets, CashFlow-to-Assets, CashFlow-to-Income, Sales-to-Assets, Gross Margin | Combines 6 descriptors: Return-on-Equity, Return-on-Assets, CashFlow-to-Assets, CashFlow-to-Income, Sales-to-Assets, Gross Margin |

Figure 6: Style Factor Comparison

### 3.1 Factor Significance

As seen in Figure 7, most of the style factors are significant (proportion of days where T-Stat is greater than 2). On the lower end of the spectrum, there is Growth with significance around 30%, while on the higher end of the spectrum, there is Market Sensitivity with significance around 72%.

### 3.2 Factor Performance

It is helpful to look at cumulative style factor returns and relate them to different investing periods. Figure 8 shows that the cumulative factor returns for fundamental-based style factors and Figure 9 is for market-based style factors. As can be seen, Value and Profitability are the top performing fundamental factors while Medium-Term Momentum outperformed all other market-based style factors.

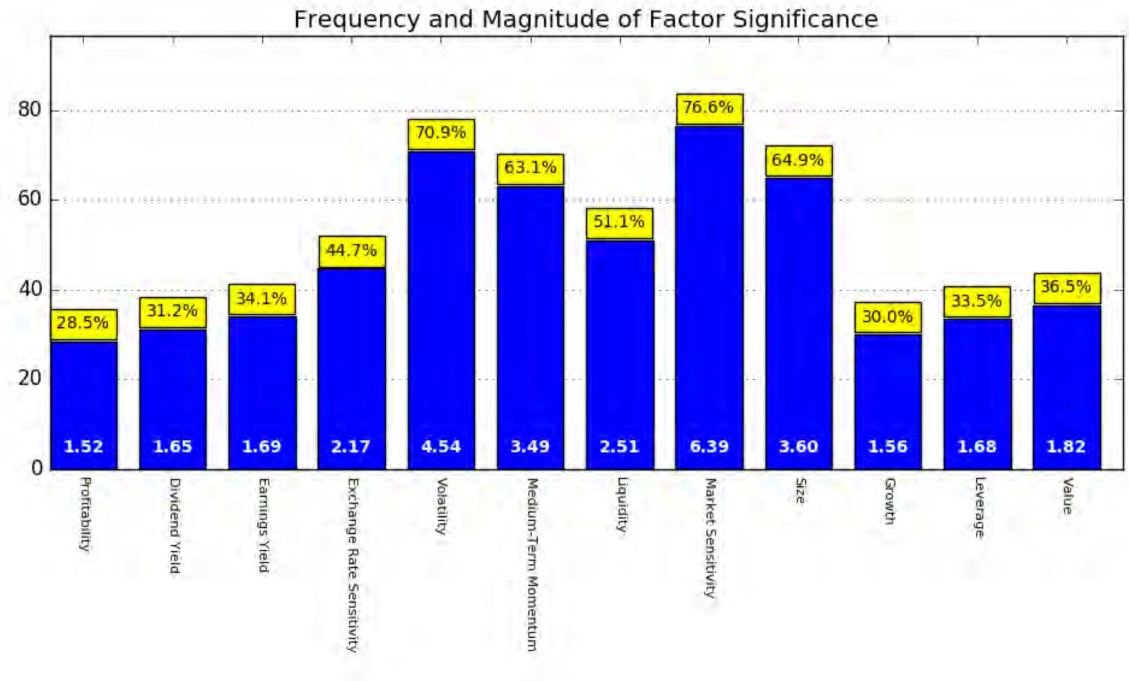


Figure 7: Style Factors - Frequency of Factor Significance

## 3.3 New Factors

### 3.3.1 Profitability

WW4 fundamental model takes return on equity (ROE) from the Growth factor and incorporates it into a new factor, Profitability, constructed as a linear combination of the return on equity, return on assets, cash flow to assets, cash flow to income, gross margin, and sales to asset descriptors (Figure 10). It is motivated by research on Quality and Profitability investing [[1]].

The introduction of Profitability factor leads to a superior returns model, as well as better performance and risk attribution, particularly for portfolios that have Profitability or quality tilts. We discuss some of these improvements in section 6 (Testing Portfolios with Alpha Signals).

[1] AsnessFrazziniPedersen2014, NovyMarx2013, NovyMarx2014

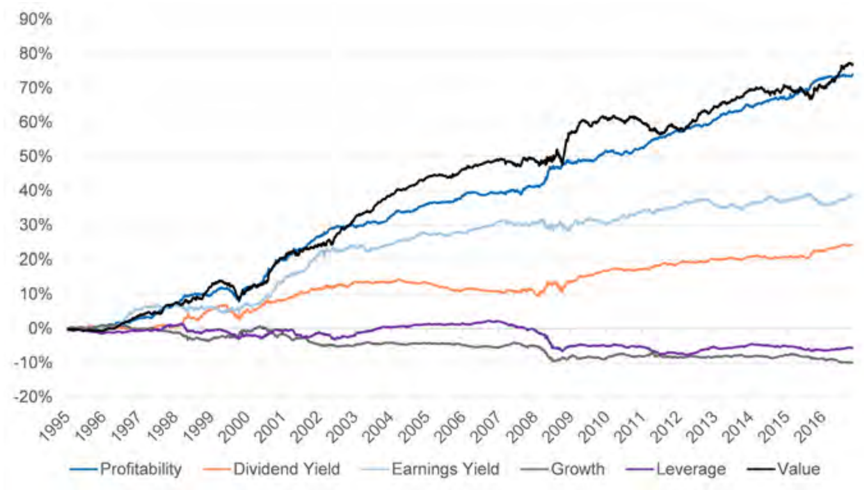


Figure 8: Factor Cumulative Returns - Fundamental

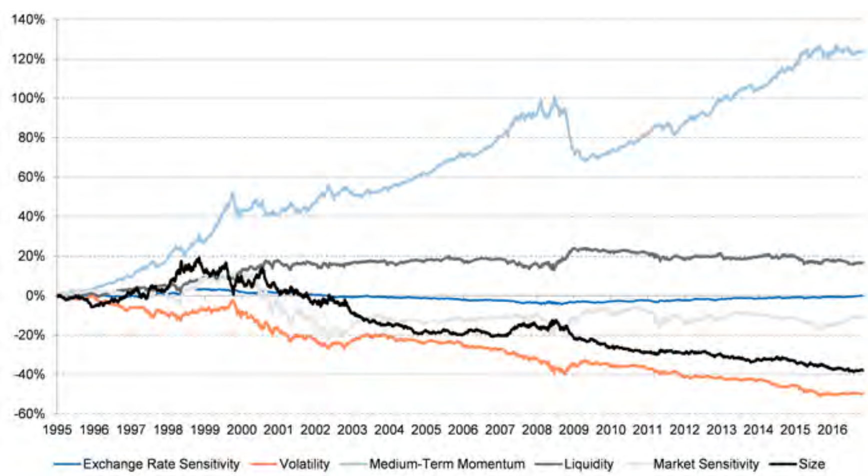


Figure 9: Factor Cumulative Returns - Market-based


Figure 10: Descriptors used for Profitability Factor

Figures 11 and 12 show the Profitability cumulative factor returns and information ratio respectively.

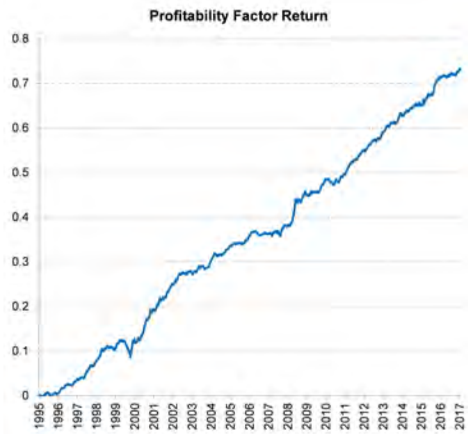


Figure 11: Profitability: Cumulative Factor Return

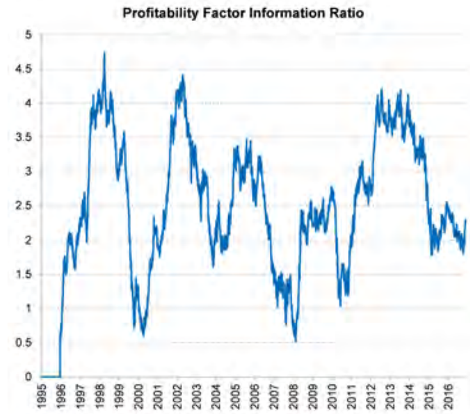


Figure 12: Profitability: Factor Information Ratio

### 3.3.2 Market Sensitivity

WW4 Market Sensitivity exposures are estimated using the trapezoid weighting scheme discussed in section 2.2.1 of this document, which improves the stability of exposures over time. Figure 13 depicts the cumulative factor returns which drifts downwards signifying low beta stocks outperform high beta stocks over time.

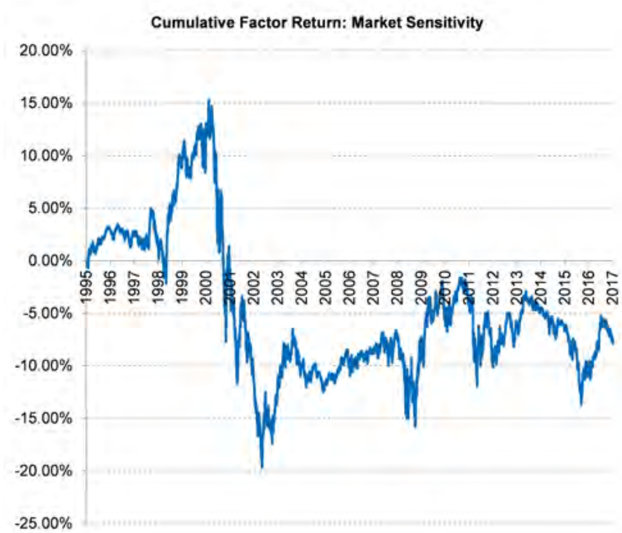


Figure 13: Market Sensitivity: Cumulative Factor Return

### 3.3.3 Dividend Yield

Dividend yield is a measure of how much a company pays out in dividends each year relative to its share price. Yields for a current year are estimated using the latest year's dividend yield.

Figures 14 and 15 respectively show the dividend yield cumulative factor returns and dividend yield factor information ratio respectively. Figure 16 shows that the factor significance (average of absolute T-stats).

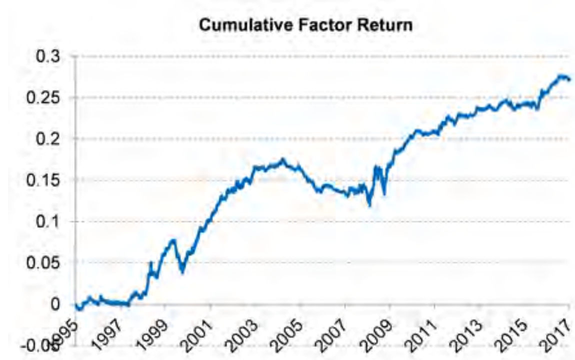


Figure 14: Dividend Yield:Cumulative Factor Return

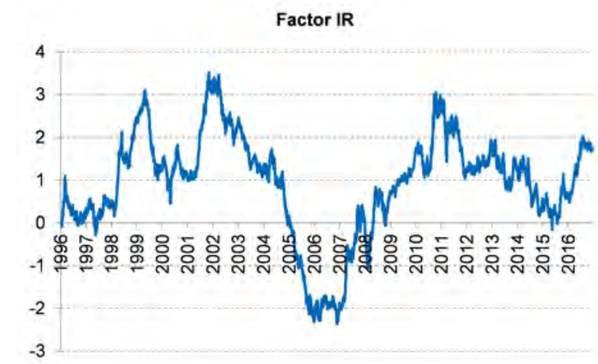


Figure 15: Dividend Yield: Factor Information Ratio

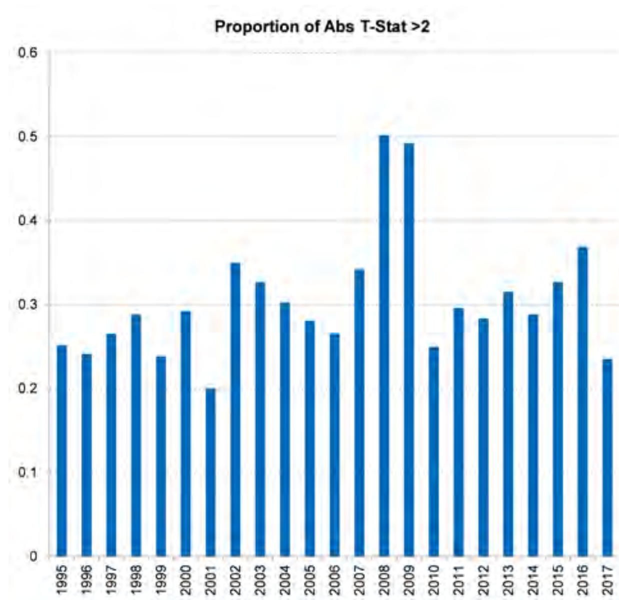


Figure 16: Dividend Yield: Absolute $T$-Stats

## 3.4 Improved Factors

### 3.4.1 Earnings Yield and Value

WW4 model introduces two value factors - earnings yield and book-to-price, as shown in Figure 17. In comparison, the WW21 fundamental model has a single value factor (book-to-price and earnings-to-price combined). To estimate Earnings Yield factor exposure in WW4, we use a combination of trailing and forecast earnings in the ratio of 1:3.

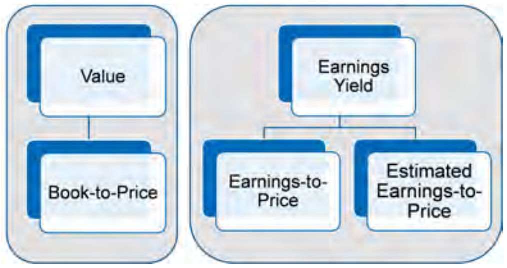


Figure 17: Value and Earnings Yield: Descriptors

In case of WW4, we had to decide whether it makes sense to combine the two value factors or keep them as separate standalone factors. The decision to separate earnings yield and book-to-price into two factors is justified for the following reasons:

Earnings yield and book-to-price exposures are both sufficiently correlated with asset returns, and sufficiently uncorrelated with each other, to suggest that they represent relevant and distinct dimensions of the returns model. Figure 18 presents the proportion of factor returns that are significantly different from zero when earnings yield and book-to-price are incorporated as separate factors. Figure 19 depicts the cross-sectional correlation between earnings yield and book-to-price exposures; here we observe that the exposures are relatively uncorrelated with each other.

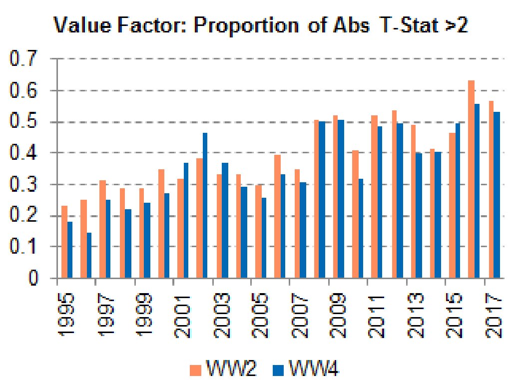


Figure 18: Absolute $T$-Stats

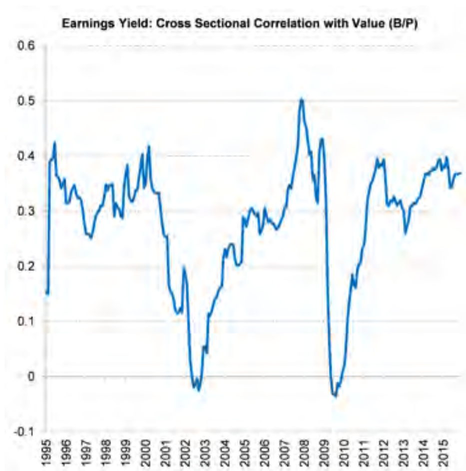


Figure 19: Cross-sectional Correlations

Estimated factor returns for earnings yield and book-to-price behave in different ways, indicating that the factors represent different dimensions of value. Figure 20 depicts cumulative returns and Figure 21 depicts predicted volatility of estimated factor returns. Here we observe periods where earnings yield and book-to-price returns move in opposite directions, and periods where book-to-price returns appear more volatile than earnings yield returns. A model in which earnings yield and book to price are combined would not capture this divergent information.

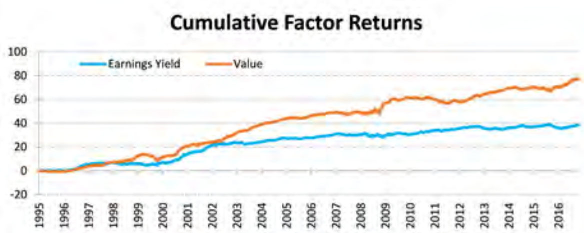


Figure 20: Earnings Yield and Value Factor Cumulative Returns

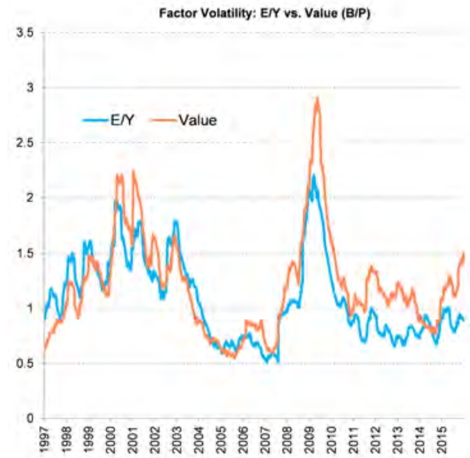


Figure 21: Earnings Yield and Value Factor Predicted Volatility

### 3.4.2 Leverage

The new WW4 Leverage factor is a combination of (Debt / Assets) and (Debt / Equity) compared with the old definition of (Debt / Assets). The inclusion of debt to equity ratio boosts the exposure of Financial assets. Using an Asian Financial benchmark as an example, we see exposure to leverage for WW4 is much higher than WW2, as shown in Figure 22. Figure 23 shows the exposure of TSE Electronics portfolio to illustrate that the exposures didn't change for non-financial industries.

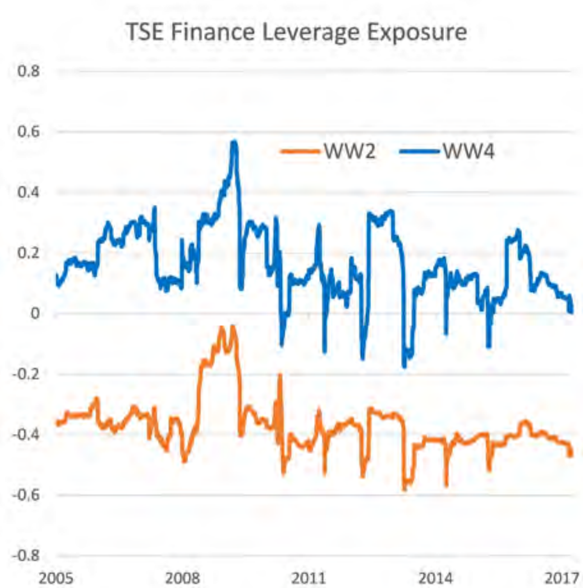


Figure 22: Leverage Exposures - TSE Finance Industry

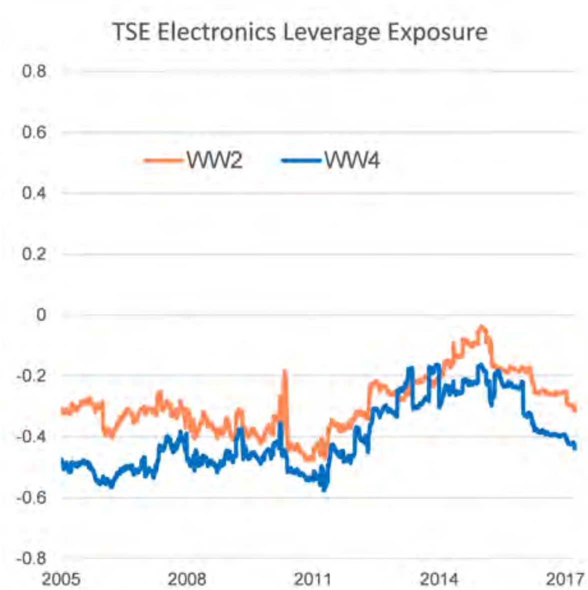


Figure 23: Leverage Exposures - TSE Electronics Industry

### 3.4.3 Growth

Growth is not a new factor, but the underlying descriptors have changed. While the previous definition uses 5 year sales and earnings growth, ROE and plowback, the new definition is based on traditional measures of sales and earnings growth. It is estimated as the equal-weighted average of the earnings growth rate and the sales growth rate.

The earnings growth descriptor is calculated by regressing five years of realized earnings and one year of forecast earnings against time and an intercept term,

$$earnings_{i,t} = \alpha_i + \beta_i t + \epsilon_{i,t}, \text{ for } t = 1, ..., T,$$

obtaining the estimated coefficient $\hat{\beta}_i$, which measures the average earnings per year in currency units. The estimated coefficient is then standardized by the average absolute earnings value used in the regression,

$$\frac{\hat{\beta}_i}{(1/T)\sum_{t=1}^{T} |earnings_{i,t}|}.$$

The sales growth descriptor is calculated in the same manner as earnings growth, but using realized and forecasted sales data.

Figure 24 depicts factor significance and Figure 25 depicts predicted volatility of estimated factor returns. Here we observe that the new growth factor is less significant and has a lower predicted volatility than before. The reason being much of the significance of WW21 Growth factor was driven by the ROE component.

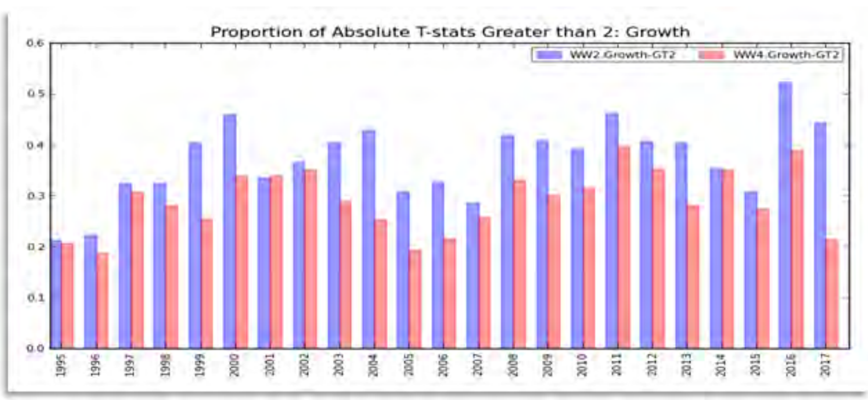


Figure 24: Growth T-Stat

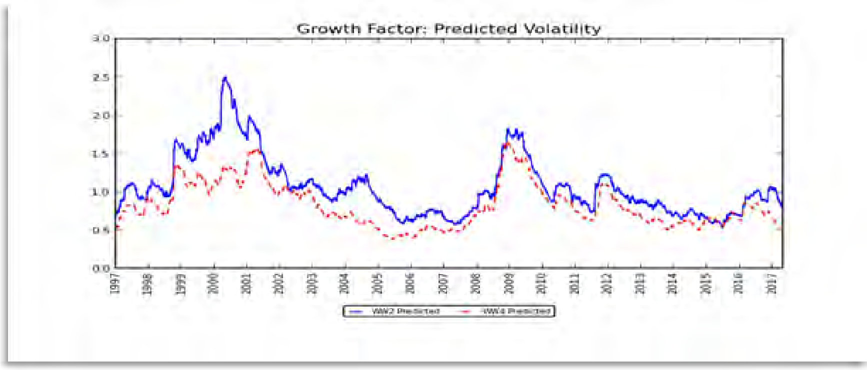


Figure 25: Growth Predicted Volatility

### 3.4.4 Volatility

WW4 Volatility exposures are estimated using the trapezoid weighting scheme discussed in section 2.2.1 of this document, which improves the stability of exposures over time. Since Market Sensitivity and Volatility are highly correlated as shown in Figure 26, we have made Volatility orthogonal to Market Sensitivity, by regressing its cross-sectional standardized exposures against those of Market Sensitivity, as shown below:

$$x_{vol} = \beta x_{ms} + \epsilon,$$

where $x_{vol}$ is the original (standardized) volatility exposure, $x_{ms}$ is the standardized market sensitivity factor and $\epsilon$ is the new volatility exposure. In addition, the regression is weighted using square-root of market cap.

Figure 27 depicts the cumulative factor returns for WW4 and WW21 volatility factors.


Figure 26: Market Sensitivity and Volatility:Cross-sectional Correlation

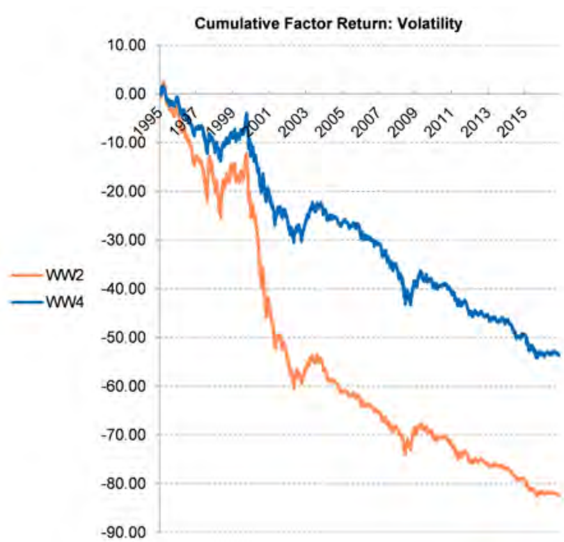


Figure 27: Volatility: Cumulative Factor Returns

### 3.4.5 Liquidity

The WW4 liquidity factor is calculated as the equal weighted average of Volume/Market Cap, inverse of Amihud illiquidity ratio and percent of returns traded. All the three descriptors are standardized prior to summation.

Figure 28 depicts how liquidity exposures vary with size. We observe that the new definition is more intuitive and consistent across the market cap spectrum in the sense that large caps are more liquid than mid caps which in turn are more liquid than small caps, which was not the case with WW21 liquidity exposures.

Next we look at the liquidity exposures for some benchmarks to get an understanding of how the new liquidity factor behaves. The liquidity exposure for a highly liquid benchmark such as Russell 1000 Index is similar for both WW4 and WW21, as shown in Figure 29. But when we look at an illiquid benchmark such as FTSE Xinhua B-Share, WW4 liquidity exposures are lower (more accurate reflection of the liquidity of the index) than WW21 liquidity exposures, as shown in Figure 30.

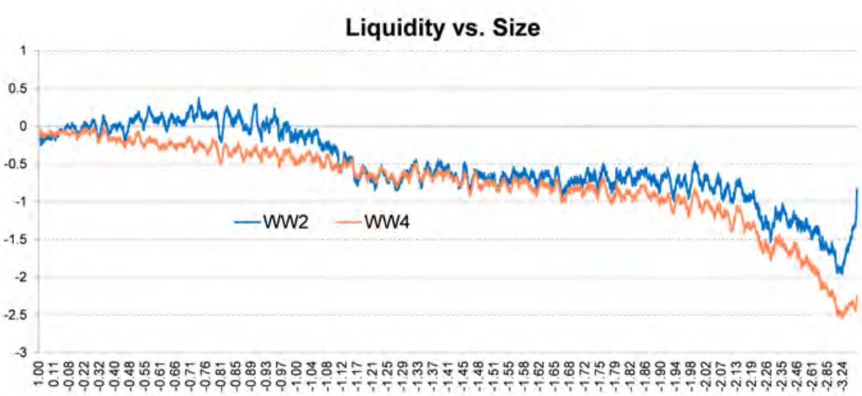


Figure 28: Liquidity vs. Size

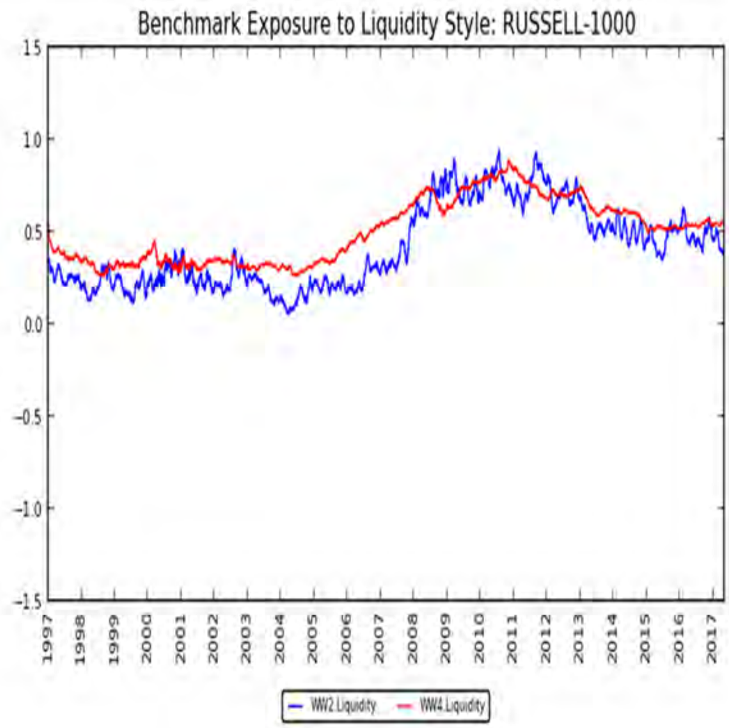


Figure 29: Russell 1000 Index Liquidity Exposure

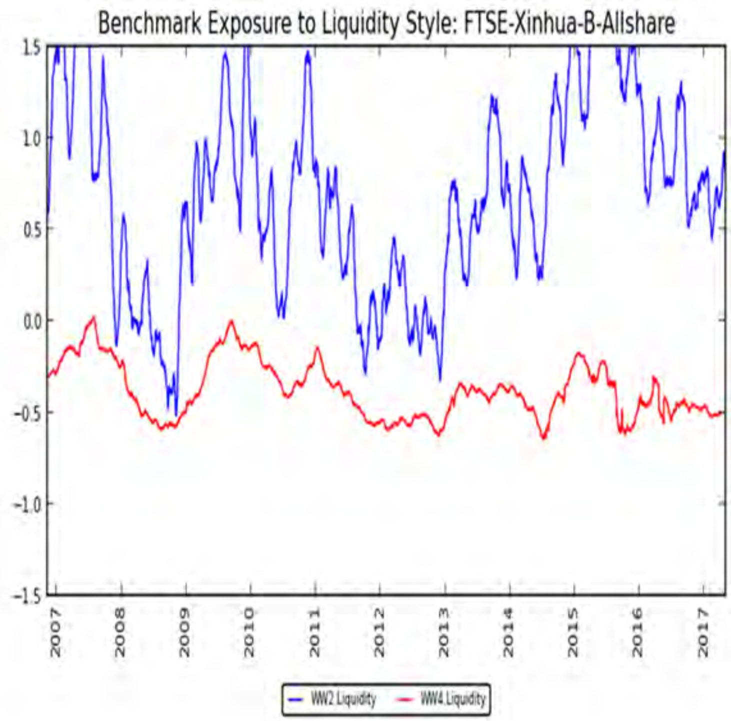


Figure 30: FTSE Xinhua B-Share Index Liquidity Exposure

## 3.5 Other Factors

### 3.5.1 Exchange Rate Sensitivity

Exchange Rate Sensitivity (XRS) exposures in the WW4 fundamental model have two improvements relative to its predecessor model. First, we estimate exposures from two years of weekly returns, rather than six months. Second, we apply the trapezoidal weighting scheme discussed in section 2.2.1 of this document to downweight returns at the beginning and end of the input series. The result of these changes is that exposures are more stable. Figure 31 shows the exchange rate sensitivity of TSE Semiconductor Industry of WW4 and WW21, respectively.

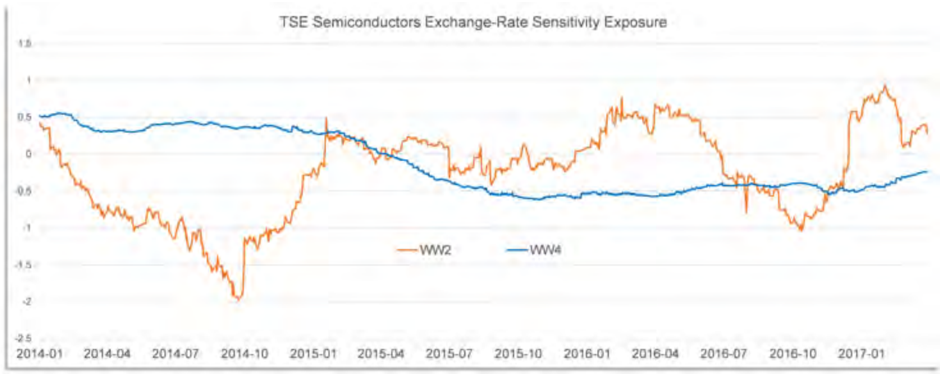


Figure 31: Exchange Rate Sensitivity Exposure

### 3.5.2 Medium-Term Momentum

WW4 Medium-Term Momentum exposures are estimated using the trapezoid weighting scheme discussed in section 2.2.1 of this document, which improves the stability of exposures over time.

### 3.5.3 Removal of Short-Term Momentum from MH model

We have removed short-term momentum (STM) from the MH model. As shown in Figure 32, short-term momentum factor is significant in explaining returns in the cross-section (proportion of days where T-Stat is greater than 2 is around 67%), when compared to other factors.

It gives a measure of a stock's recent performance, and it is defined as an asset's cumulative return over the last 20 trading days (approximately one month). Since the factor exposure changes from day to day, it could lead to high turnover, as shown in Figure 33. We deem that this makes it unsuitable for models with horizons beyond a month. Also, regressing against longer-horizon returns has shown that STM contains no long-term information.

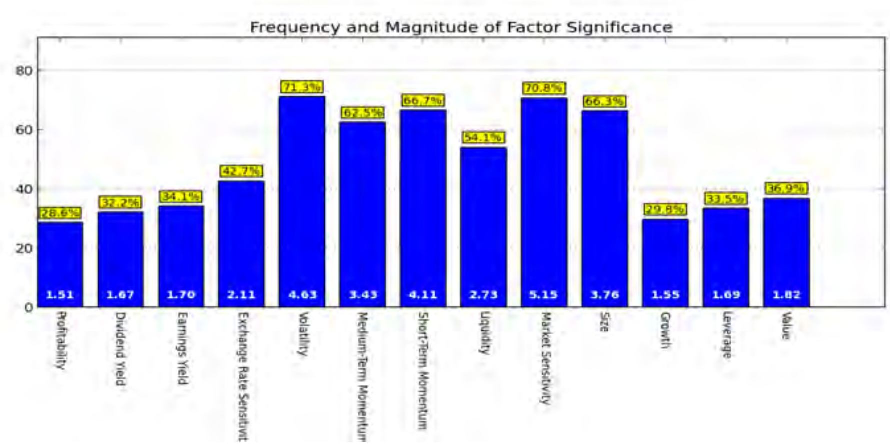


Figure 32: Factor Significance

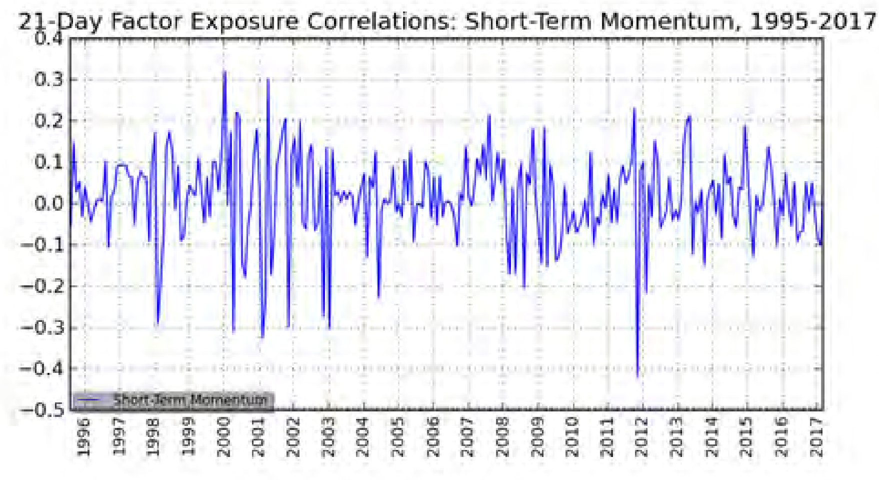


Figure 33: StyleFac Short-Term Momentum Churn Rate

### 3.5.4 Size

Our WW4 Size factor definition is same as in WW21.

## 4 Industry Changes

WW4 uses a customized industry classification based on the GICS 2016 classification. WW4 introduces a new Real Estate Sector with two industries: Equity Real Estate Investment Trusts (REITs) and Real Estate Management & Development.

Figure 34 shows the cumulative factor returns for REITs, Equity REITs and Mortgage REITs. Here we observe that the Mortgage REITs are much more volatile than Equity REITs and the REIT sector. This is consistent with the factor volatility chart as show in Figure 35.

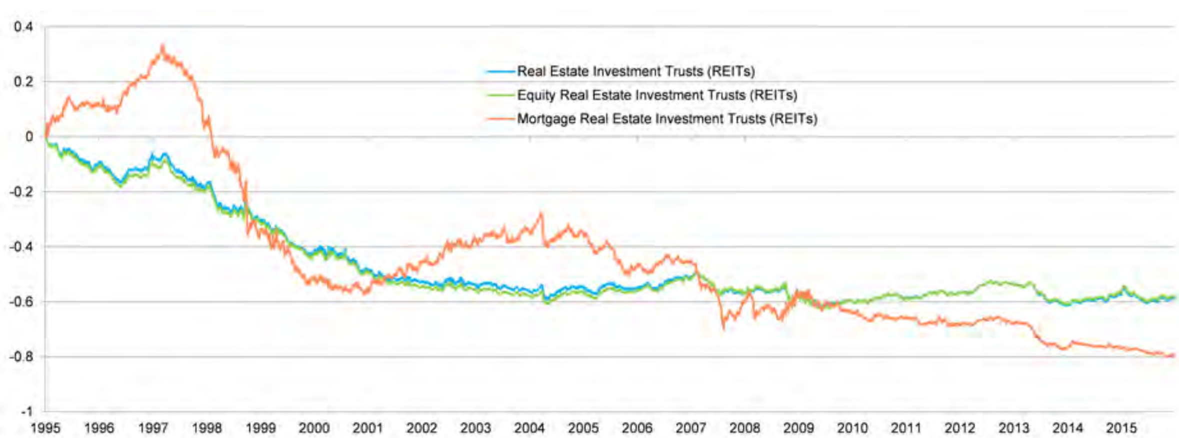


Figure 34: Cumulative Factor Returns

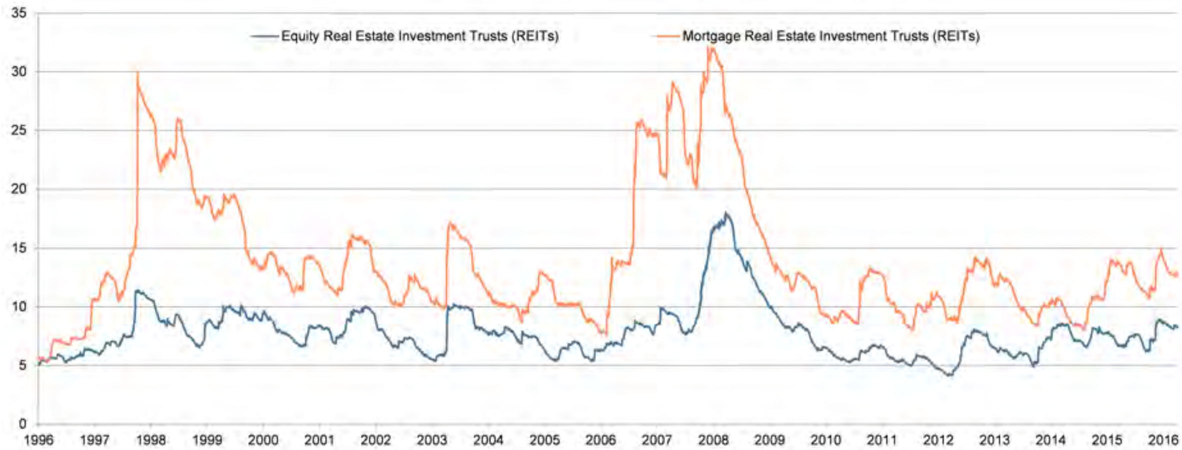


Figure 35: Factor Voaltility

# 5 Risk Analysis of Alpha Strategies, Global ETFs, and Benchmarks

To compare performance across models, we construct monthly active portfolios benchmarked against Russell Global Index. We create factor tilted portfolios using a third party alpha signal, based on Momentum, Value, Growth and Quality. To test models, we implement different investment strategies, such as Top/Bottom Long Only Quintile Portfolio (Quintile), Long Short Mean Variance Portfolio (MV), Long Only Mean Variance Portfolio (LOMV), Long Only Mean Variance Portfolio with Turn Over constraint=15% (LOMV_TO), and Minimum Volatility Portfolio (minVol). The time period for analysis was from 2005 to 2016. We consider both WW4-MH and WW21-MH models for constructing optimized portfolios and for portfolio return/risk attribution.

## 5.1 Risk Analysis, Portfolio Turnover and Transfer Coefficient

We analyze the portfolio turnover and transfer coefficient for portfolios constructed by the two models. First we compare the turnover of alpha tilted (minimum and maximum exposure) optimized strategies without controlling turnover. Figure 36 shows that the turnover is consistently lower for portfolios optimized by WW4 model than WW21. Next we construct the same alpha tilted portfolios with a turnover constraint of 15% and check which model leads to a higher transfer coefficient. We infer from Figure 37, across all portfolios, that optimizing with WW4 leads to a higher transfer coefficient than WW21. These results show that the new WW4 model leads to lower turnover for optimized portfolios and is able to capture more information from the alpha factors (higher transfer coefficient and better portfolio construction efficiency) than WW21.

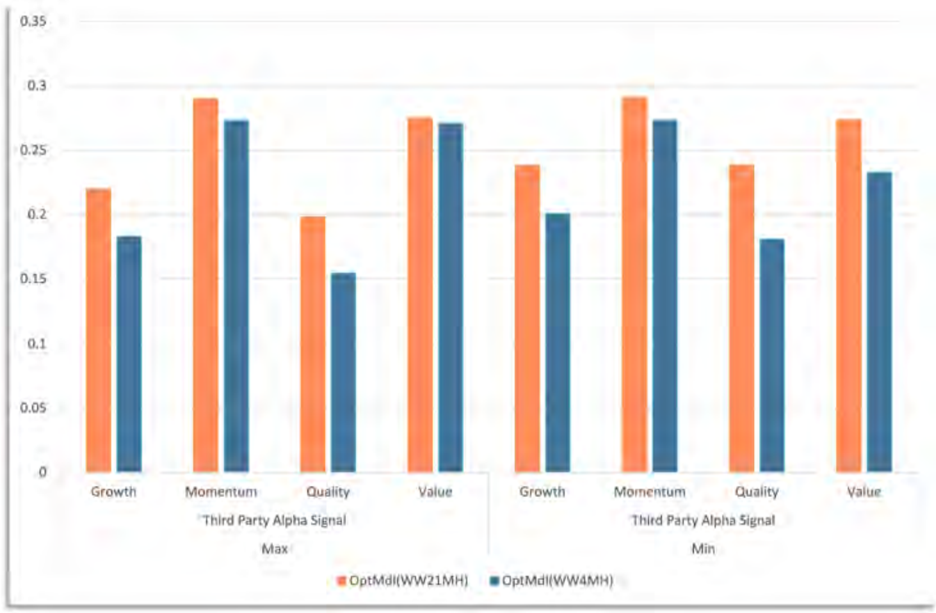


Figure 36: Portfolio Turnover

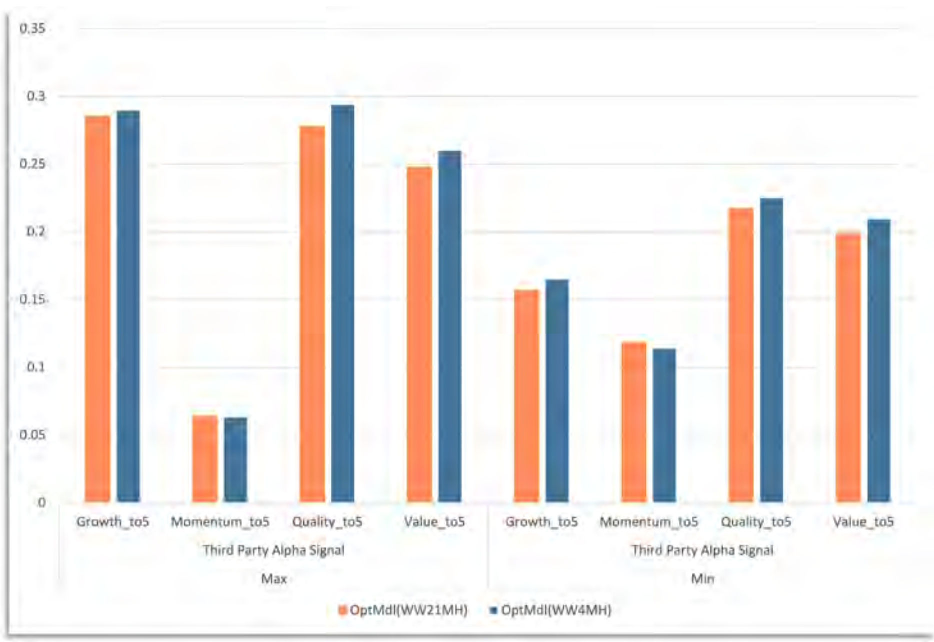


Figure 37: Portfolio Transfer Coefficient

Figures 38 and 39 show a similar story when we analyze portfolio turnover of Minimum Volatility and Long Only Minimum Volatility strategies constructed using WW4 and WW21. In both cases turnover is consistently lower for portfolios optimized by WW4 than WW21.

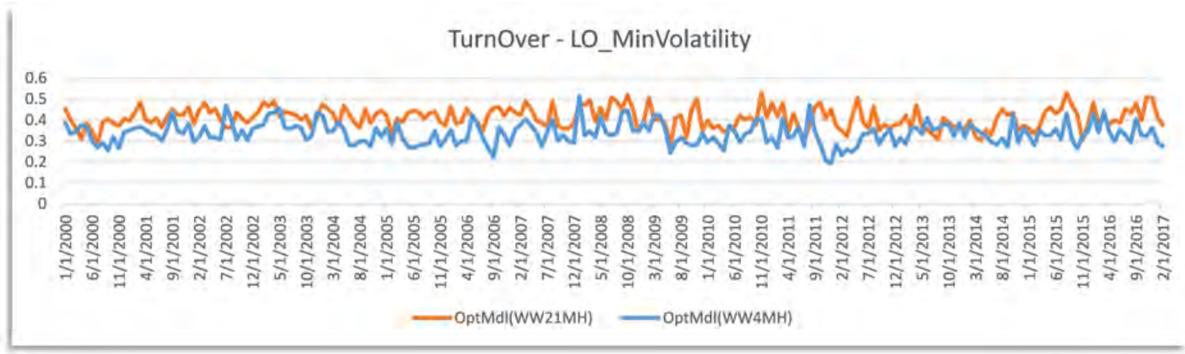


Figure 38: Long Only Min Vol Portfolio Turnover

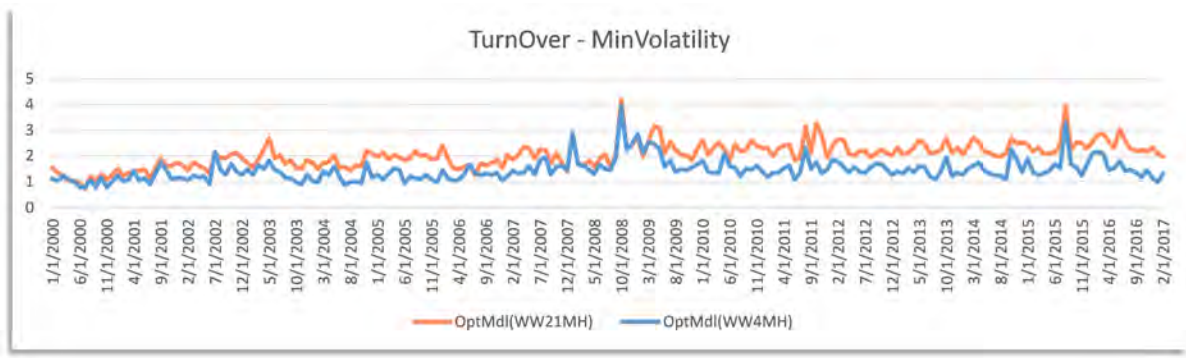


Figure 39: Min Vol Portfolio Turnover

Next we construct minimum variance portfolios from Russell Emerging, Russell Developed and Russell Global indices using WW4 and WW21. Figure 40 compares the total risk of these portfolios and we observe that portfolios optimized with WW4 have lower total risk than WW21. Figure 41 shows the Sharpe ratios of these portfolios, and we observe that Sharpe ratios of portfolios constructed with WW4 are higher than WW21.

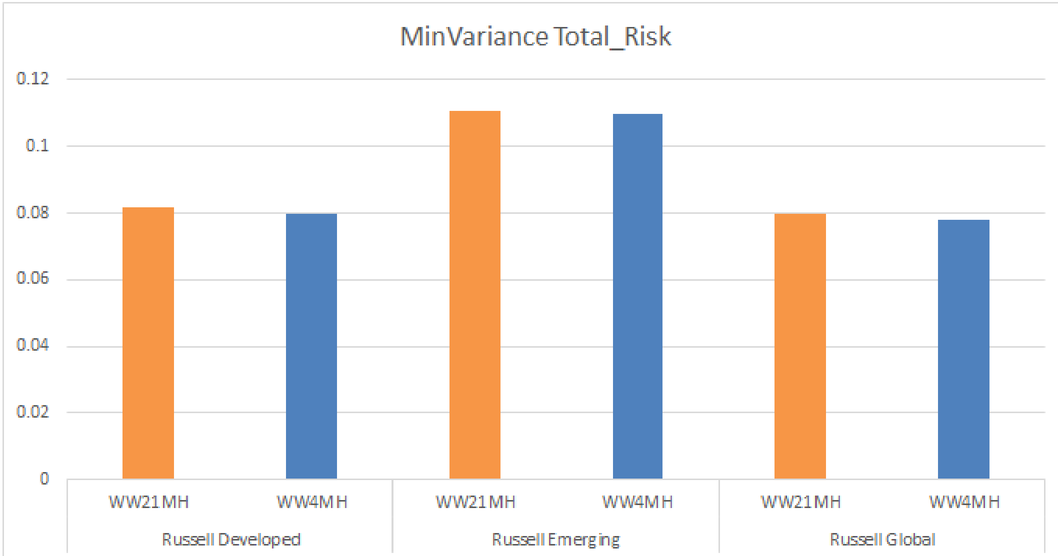


Figure 40: Total Risk

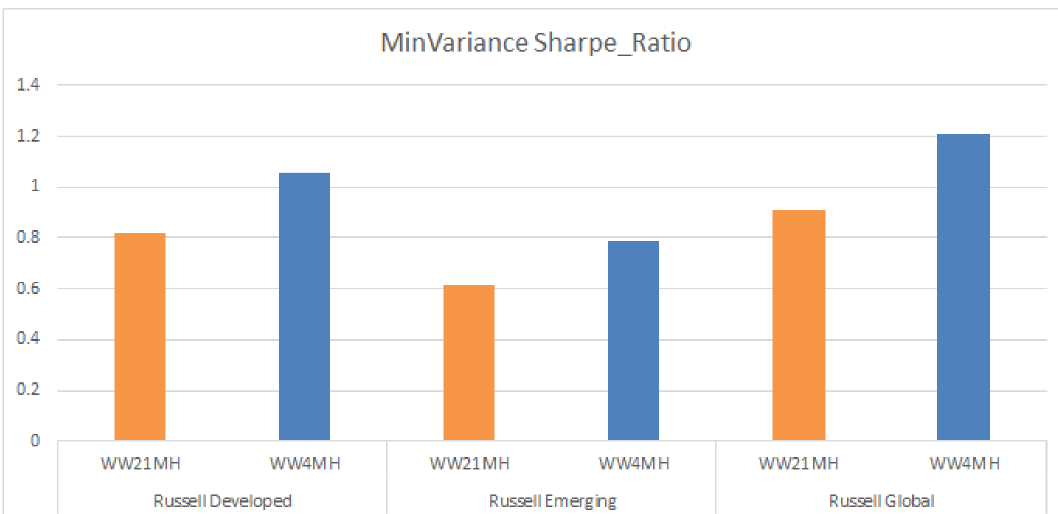


Figure 41: Sharpe Ratio

## 5.2 Return Attribution

Next we consider factor-based attribution results to compare WW4-MH and WW21-MH models. Figure 42 shows the factor return attribution of the top quintile quality portfolio. We see that for the WW4 model, the cumulative factor return contribution (blue line) closely tracks the active return of the portfolio (grey line), thus making the specific return contribution (dark green line) hover close to zero, which is desirable. In this case, the WW4 model is able to attribute most of the returns coming from the active portfolio to its factors, thus reducing the volatility of the specific return contribution. In contrast, for WW21 model, the factor contribution (orange line) is overshooting the active return of the portfolio (grey line), thus making the specific return contribution (light green line) steadily drift downwards. When compared to WW21 model, WW4 is able to better capture the active returns of the quintile portfolio.

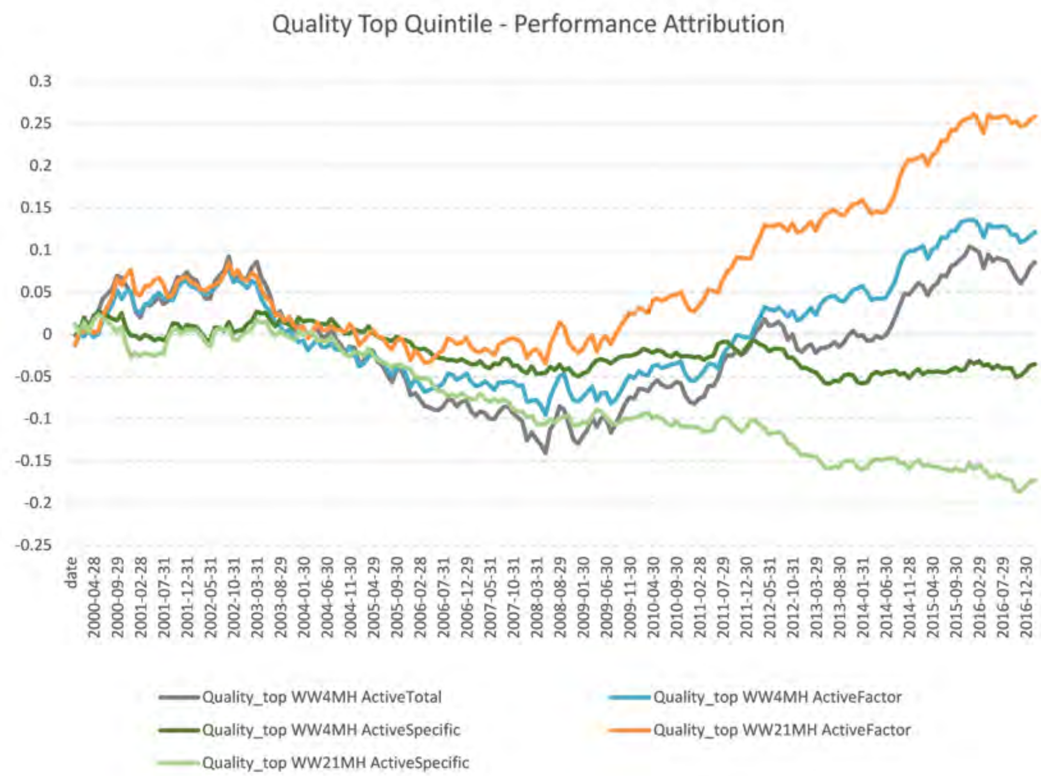


Figure 42: Return Attribution for Quintile Strategy

## 5.3 Risk Decomposition

In this section, we construct factor-tilted portfolios using both WW4-MH and WW21-MH models to compare risk attribution results.

First, we consider the LOMV Strategy Quality portfolio. Figure 43 shows active exposures and risk contribution of style factors. For WW4-MH, we notice that the Profitability factor (being a component of quality) has a high exposure to this portfolio, which is intuitive, followed by Growth and Earnings Yield. It is interesting to note that the Growth factor (traditional sales and earnings growth) has a significant exposure to this portfolio. Leverage and Value factors have the most negative exposures. From a risk contribution viewpoint, Value, Profitability and Earnings Yield contribute most of the risk to this portfolio. For WW21-MH Model, the Growth factor (that has ROE as one of the descriptors), has a high exposure followed by Size and Medium-Term Momentum. Leverage and Value factors have the top negative exposures. From a risk contribution viewpoint, Growth, Value and Size contribute most of the risk to this portfolio.

The new model (WW4) is better able to capture the different dimensions of the quality-tilted portfolio (Profitability and traditional growth) than WW21.

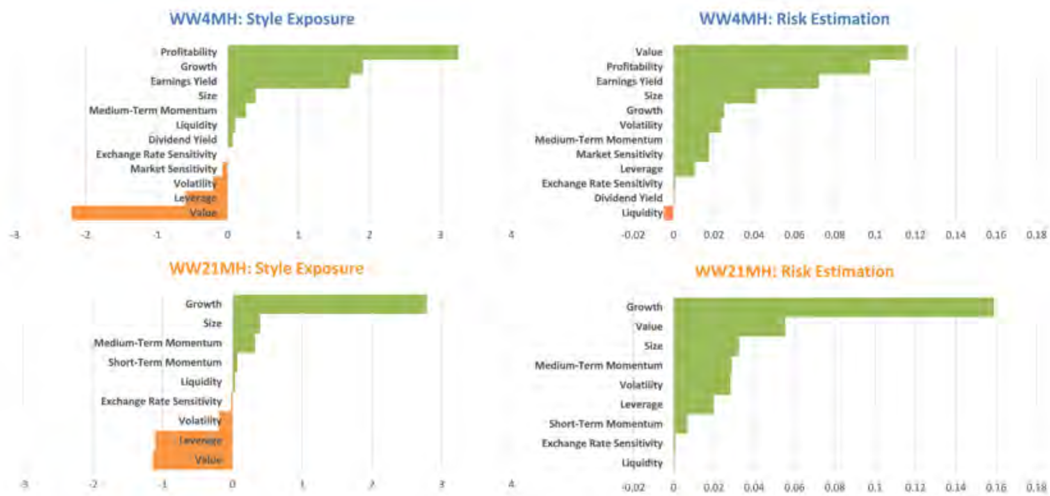


Figure 43: Risk Decomposition for LOMV Quality portfolio

Next we look at an All Country Minimum Volatility ETF. Figure 44 shows active exposures and the risk contribution of style factors. For WW4-MH, we notice that the Market Sensitivity factor has a high negative exposure to this portfolio, which is intuitive, followed by Volatility and Value. It is interesting to note that the Dividend Yield factor has a significant positive exposure to this portfolio. From a risk contribution viewpoint, Market Sensitivity and Volatility contribute most of the risk to this portfolio. For WW21-MH Model, the Volatility factor has a high negative exposure followed by Value and Liquidity. From a risk contribution viewpoint, Volatility is the main driver of risk to this portfolio.

The new model (WW4) is better able to capture the the different dimensions of the Min Vol ETF than WW21 by providing extra granularity in the factor structure. In addition to Volatility, the new Market Sensitivity factor is able to capture another risk dimension of Min Vol portfolios.


Figure 44: Risk Decomposition for All Country Min Vol ETF

Next we look at a Dividend ETF. Figure 45 shows active exposures and the risk contribution of style factors. For WW4-MH, we notice that the Dividend Yield factor has a high exposure to this portfolio, which is intuitive, followed by Leverage and Value. From a risk contribution viewpoint, Dividend Yield and Size contribute most of the risk to this portfolio. For WW21-MH Model, the Leverage factor has a high exposure followed by Value. From a risk contribution viewpoint, Size is the main driver of risk to this portfolio.

The new model (WW4) is better able to capture the different dimensions of the Dividend ETF than WW21.

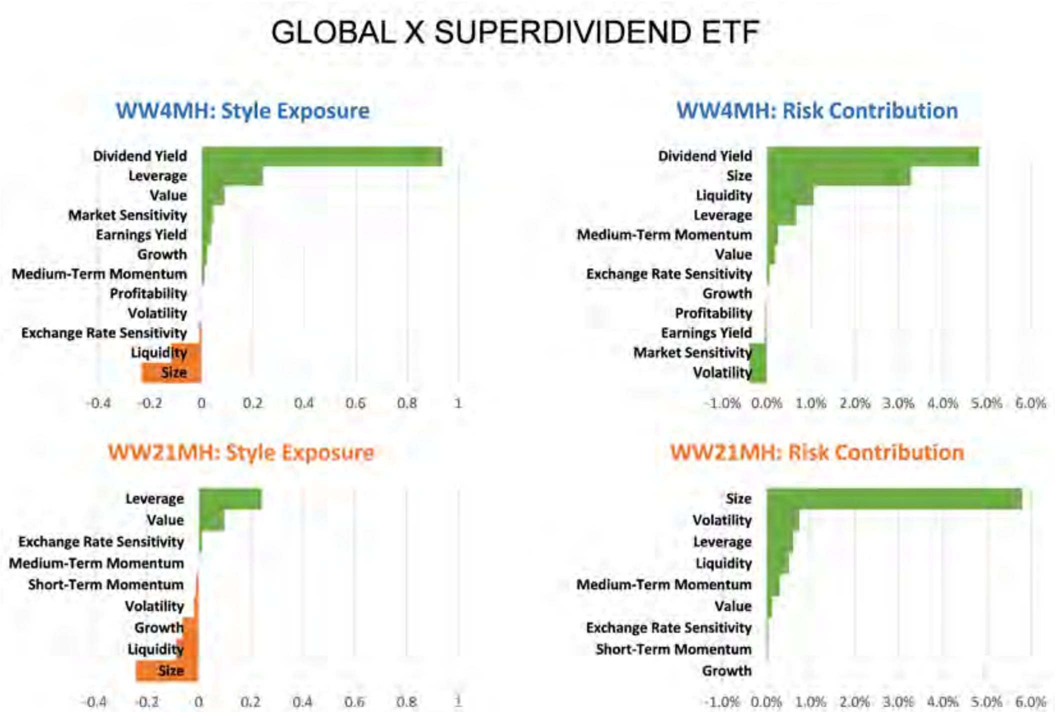


Figure 45: Risk Decomposition for Dividend ETF

## 6 Short-Horizon Model

The WW4 Short-Horizon model has the same factor structure as the WW4 Medium Horizon model, with one additional style factor (Short-Term Momentum). The market-based style factors in the Short-Horizon model are designed to be more reactive to changes in overall risk levels than the Medium-Horizon model. The model uses less history and shorter half-lives to make it more responsive to the overall changes in market volatility. Please refer to the WW4 Model Supplement Handbook and WW4 Factsheet for additional details.

# 7 Statistical Model

The WW4 statistical model provides similar risk forecasts as WW21, but with less noise, as shown in the risk forecast comparison plots below.

Figure 46 shows the total risk bias stat for WW4 and WW21. We observe that there is not much difference in risk forecast accuracy between the two models.

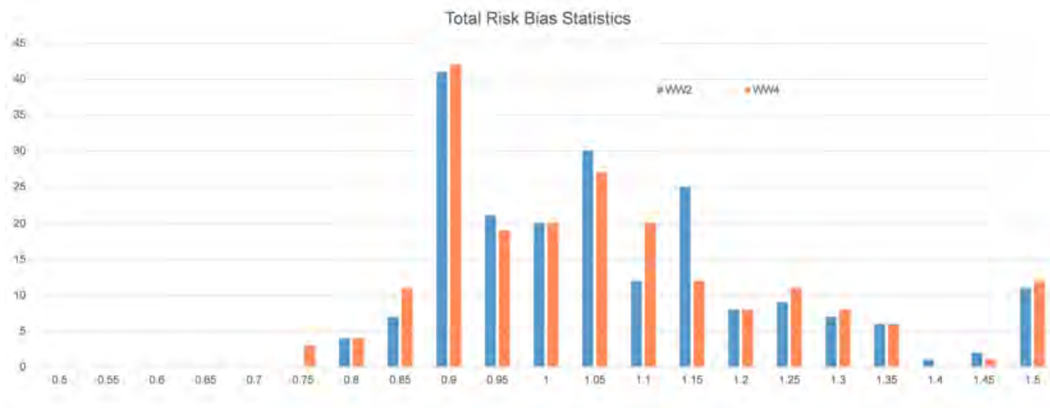


Figure 46: Statistical Model: Total Risk Bias Stat

We also show comparison of Sharpe ratios for minimum variance and tracking portfolios constructed using WW4 and WW21 (see Figures 47 and 48). In both cases, the results are comparable across WW4 and WW21.

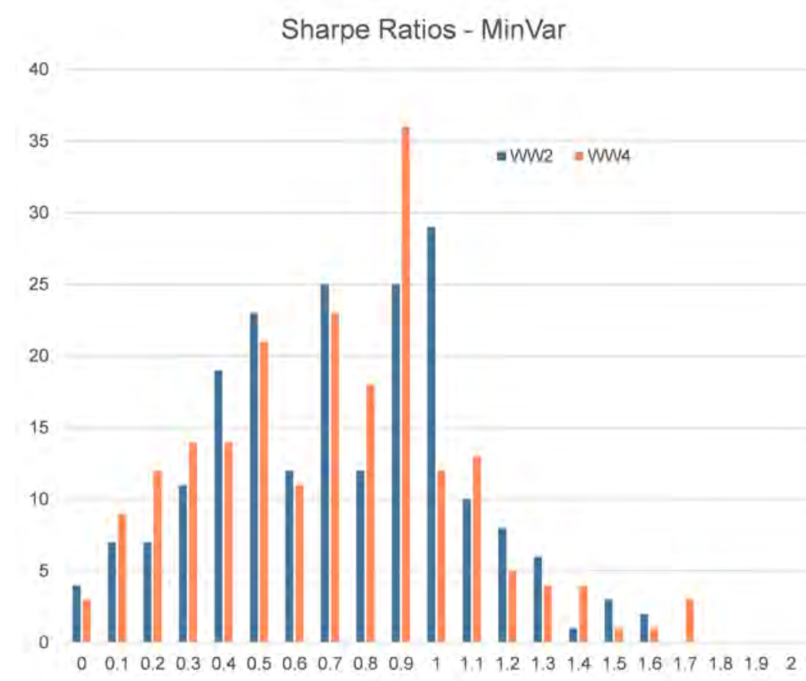


Figure 47: Statistical Model: Sharpe Ratio of Minimum Variance Portfolio

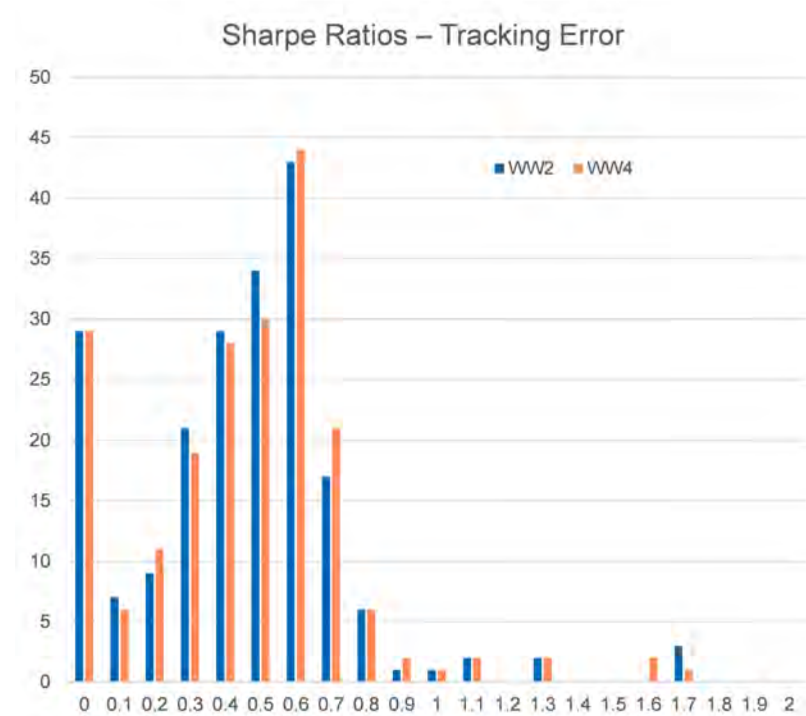


Figure 48: Statistical Model: Sharpe Ratio of Minimum Tracking Error Portfolio

The Statistical model history is updated to keep asset coverage consistent with WW4 Fundamental model.

# 8 Appendix A: Fundamental Descriptors

This section defines the descriptors used to generate fundamental style factor exposures.

| Factor | Descriptor | Definition |
|---|---|---|
| Dividend Yield | Dividend Yield | Dividend yield is calculated as the sum of the dividends paid (excluding non-recurring, special dividends) over the most recent year, divided by the average total issuer market capitalization computed over the last 30 calendar days. |
| Growth | Earnings Growth | Earnings growth is calculated by regressing five years of realized earnings and one year of forecast earnings against time and an intercept term, $earnings_{i,t} = \alpha_i + \beta_i t + \epsilon_{i,t}$, for $t = 1, ..., T$, obtaining the estimated coefficient $\hat{\beta}_i$, which measures the average earnings per year in currency units. The estimated coefficient is then standardized by the average absolute earnings value used in the regression, $\frac{\hat{\beta}_i}{(1/T)\sum_{t=1}^{T} \lvert earnings_{i,t} \rvert}$. Note that the growth rate is computed from annual earnings data. |
| | Sales Growth | Sales growth is calculated in the same manner as earnings growth, but using realized and forecast sales data. |
| Leverage | Debt to Assets | Debt to assets is calculated as the ratio of long-term and short-term debt to total assets, where total assets is the most recently reported annual value. |
| | Debt to Equity | Debt to equity is calculated as the ratio of long-term and short-term debt to common equity, where common equity is computed as the average of the two most recently reported annual values. |
| Profitability | Cash Flow to Assets | Cash flow to assets is calculated as the most recently reported annual operating cash flow divided by the average of the two most recently reported annual total assets values. |
| | Cash Flow to Income | Cash flow to income is calculated as the average of the two most recently reported annual operating cash flows, divided by the average of the two most recently reported annual income values. |

| | | |
|---|---|---|
| | Gross Margin | Gross margin is calculated as net sales (sales minus the cost of goods sold), divided by sales. Both the numerator and the denominator values are computed as the most recently reported annual values. |
| | Return on Assets | Return on assets is calculated as the most recently reported annual earnings value, divided by the average of the two most recently reported annual total assets values. |
| | Return on Equity | Return on equity is calculated as the most recently reported annual earnings value, divided by the average of the two most recently reported annual common equity values. |
| | Sales to Assets | Sales to assets is calculated as the most recently reported annual sales value, divided by the average of the average of two most recently reported annual total assets values. |
| Value | Book-to-Price | Book to price is calculated as the most recently reported common equity divided by the average total issuer market capitalization computed over the last 30 calendar days. |
| | Earnings-to-Price | Realized earnings to price is calculated as the most recently reported annual net income value, divided by the average total issuer market capitalization computed over the last 30 calendar days. |
| | Forecast Earnings-to-Price | Forecast earnings to price is calculated as the forecasted earnings estimate, divided by the average total issuer market capitalization computed over the last 30 calendar days. |

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