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$$p(x_t|\mu) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_t-\mu)^2}{2\sigma^2}}$$

$x_1, x_2, \ldots, x_n$ iid

$$p(x_1|\mu) \times p(x_2|\mu) \times \cdots \times p(x_n|\mu)$$

$$p(X|\mu) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{\sum_{t=1}^{n}(x_t-\mu)^2}{2\sigma^2}}$$

$$\underline{\max \ln p(X|\mu)} = \underbrace{-n \ln \sqrt{2\pi\sigma^2}} - \frac{\sum_{t=1}^{n}(x_t-\mu)^2}{2\sigma^2}$$

$\hat{\mu}_{MLE}$

$$\frac{\partial \ln p(X|\mu)}{\partial\mu} = 0 + \boxed{\frac{\cancel{2}\sum_{t=1}^{n}(x_t-\mu)}{\cancel{2}\sigma^2} = 0}$$

$$= \frac{\left(\sum_{t=1}^{n} x_t - n\cdot\mu\right)}{\cancel{\sigma^2}} = 0$$

$$\hat{\mu} = \frac{\sum_{t=1}^{n} x_t}{n}$$

$$\frac{\partial^2 \ln p(X|\mu)}{\partial\mu^2} = \frac{\sum_{t=1}^{n}(-1)}{\sigma^2} = -\frac{n}{\sigma^2}$$

$$I = -E\left[\frac{\partial^2 \ln p(X|\mu)}{\partial \mu^2}\right] = \frac{n}{\sigma^2}$$

$$var(\hat{\mu}) = \frac{\sigma^2}{n}$$

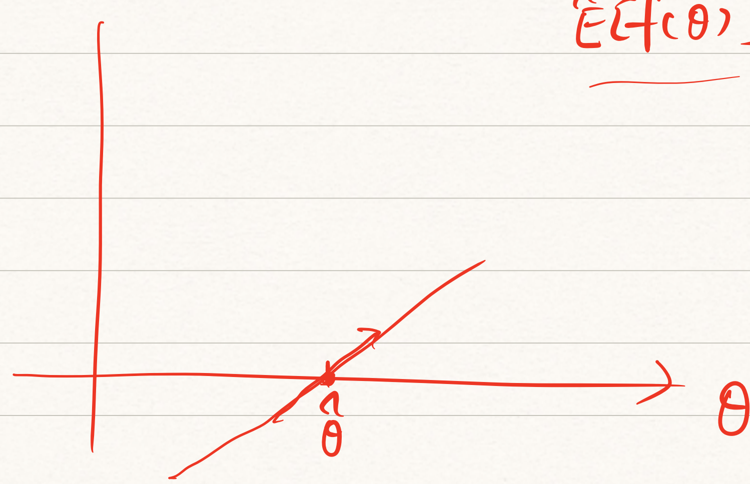
$x_1, x_2, \dots, x_T$ μ

σ^2

$$\begin{cases} E[X_t] = \mu \Leftarrow \\ E[(X_t-\mu)^2] = \sigma^2 \end{cases} \rightarrow f = \begin{bmatrix} X_t-\mu \\ (X_t-\mu)^2-\sigma^2 \end{bmatrix}$$ ✓

$$E[f(X_t|\mu,\sigma^2)] = 0$$ ✓

$\hat{E}[f(\theta)]$



$$E\left[\begin{bmatrix} f_1 \\ f_2 \end{bmatrix}\begin{bmatrix} f_1 & f_2 \end{bmatrix}\right]$$

$$= E\left[\begin{bmatrix} f_1^2 & f_1 f_2 \\ f_2 f_1 & f_2^2 \end{bmatrix}\right]$$

$$= \begin{bmatrix} \underset{Var(f_1)}{E[f_1^2]} & \underset{Cov(f_1, f_2)}{E[f_1 f_2]} \\ \underset{Cov(f_1, f_2)}{E[f_2 f_1]} & \underset{Var(f_2)}{E[f_2^2]} \end{bmatrix} = Var(f)$$

$$E[f_1] = 0$$

$$\frac{\partial \hat{E}[f(x_i, \theta)]}{\partial \theta'} \qquad \theta = \begin{bmatrix} \mu \\ \sigma \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\partial \hat{E}[f_1(x_t, \theta)]}{\partial \mu} \to -1 & \dfrac{\partial \hat{E}[f_1(x_t, \theta)]}{\partial \sigma} \to 0 \\ \dfrac{\partial \hat{E}[f_2(x_t, \theta)]}{\partial \mu} & \dfrac{\partial \hat{E}[f_2]}{\partial \sigma} \end{bmatrix}$$

$$\frac{\partial\left(\frac{1}{n}\sum_{t=1}^{n}\left((x_t-\mu)-\sigma^2\right)\right)}{\partial\sigma} = \frac{1}{n}\sum_{t=1}^{n}\left(-2(x_t-\mu)\right)$$

$$= -\frac{2}{n}\sum_{t=1}^{n}(x_t-\mu)$$

$$= -\frac{2}{n}\left(\sum_{t=1}^{n}x_t - n\cdot\mu\right)$$

$$= -2\left(\frac{\sum_{t=1}^{n}x_t}{n} - \mu\right)$$

$$\downarrow$$

$$\bar{x} - \hat{\mu}$$

$$\hat{\sigma} = 0.1936$$

$$\left(0.1936 - 1.96 \times 0.0142\right., \quad \left(\quad + 1.96 \times - \right)$$

$$\underset{n\times 1}{X} \sim N(\mu, \Omega) \qquad a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

$$\begin{bmatrix} a_1 x_1 + a_2 x_2 + \cdots + a_n \\ b_1 x_1 + \quad \cdots \quad b_n \\ c_1 x_1 \quad - \quad - \quad + c_n \end{bmatrix} \begin{matrix} x_n \\ x_n \\ x_n \end{matrix} = \quad a' X \sim N(\qquad)$$

$$A$$

$$\mathrm{Var}(Ax) = E[(Ax)\underline{(Ax)'}]$$

$$= E[Axx'A']$$

$$= A \; E[xx'] \; A'$$

$$= \overbrace{A \, \Omega \, A'}$$

$x \;,\; y \quad \sim N(0,1)$

$x+y \sim N \;?$

$x \sim N(0,1)$

$$y = \begin{cases} x & \text{if } z=0 \\ \underline{-x} & \text{if } \underline{z=1} \end{cases} \qquad z = \begin{cases} 0 & p \\ 1 & 1-p \end{cases}$$

$y \sim N(0,1)$

$Pr(x+y=0) = 1-p \Rightarrow x+y \not\sim N$

$x_1 \,,\, x_2 \,,\, \ldots \,,\, x_n$ indep. $N(0,1)$

$x_1^2 + x_2^2 + \cdots + x_n^2 \sim \chi(n)$

$\underset{n\times 1}{x} \sim \underline{N(0, \Omega)} \longrightarrow N(0, I)$

$$\boxed{\tilde{x} = \Omega^{-\frac{1}{2}} x} \qquad A = \Omega^{-\frac{1}{2}}$$

$$\tilde{x} \sim N(0, \Omega^{-\frac{1}{2}} \underset{\Omega^{\frac{1}{2}}\Omega^{\frac{1}{2}}}{\underset{\Downarrow}{\Omega}} \Omega^{-\frac{1}{2}}) \sim N(0, I)$$

$$\tilde{x}'\tilde{x} \sim \chi^2(n)$$

$$(\Omega^{-\frac{1}{2}}x)'(\Omega^{-\frac{1}{2}}x) = x'\underbrace{\Omega^{-\frac{1}{2}}\,\Omega^{-\frac{1}{2}}}x = \underline{x'\Omega^{-1}x}$$

$$x_1, x_2 \sim N\left(0, \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}\right)$$

$$\tilde{x}_1 = \frac{x_1}{\sigma_1} \qquad (\tilde{x}_1, \tilde{x}_2) \sim N\left(0, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

$$\tilde{x}_2 = \frac{x_2}{\sigma_2}$$

$$\tilde{x}_1^2 + \tilde{x}_1^2 \sim \chi^2(2)$$