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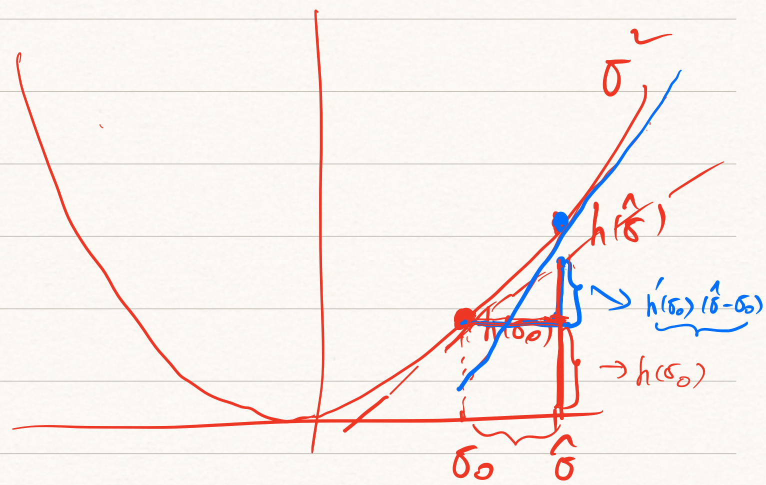
$$h(\theta) = \theta_1 - \theta_2 = A\theta \qquad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 1 & -1 \end{bmatrix}}_{A} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

$$h(\theta) = \frac{\theta_1}{\theta_2}$$

$\hat{\mu}$. $\boxed{\hat{\sigma}}$

$\boxed{\hat{\sigma}^2}$?



$$h(\hat{\sigma}) \approx h(\sigma_0) + h'(\sigma_0)(\hat{\sigma} - \sigma_0)$$

$h'(\hat{\sigma})$

$$\hat{\sigma} - \sigma_0 \sim N(0, -)$$

$$h(\hat{\sigma}) - h(\sigma_0) \approx \underbrace{\boxed{h'(\hat{\sigma})}}_{\text{Constant}} \underbrace{(\hat{\sigma} - \sigma_0)}_{N(0, \Omega)}$$

$$\hat{\sigma}^2 - \sigma_0^2 \approx 2\hat{\sigma}(\hat{\sigma} - \sigma_0)$$

$h(\sigma) = \sigma^2$

$h(\theta) - 0$

$$\hat{\sigma}^2 \sim N\left(\sigma_0^2,\ 4\sigma_0^2 \cdot Var(\hat{\sigma})\right)$$

$$h(\hat{\theta}) - h(\theta_0) \sim \underline{N(0,\ \hat{A}\hat{\Omega}A')}$$

$$\hat{A} = \left.\frac{\partial h(\theta)}{\partial \theta'}\right|_{\hat{\theta}}$$

$$\theta = \begin{bmatrix} \mu \\ \sigma \end{bmatrix} \qquad h(\theta) = \frac{\theta_1}{\theta_2} = \frac{\mu}{\sigma}$$

$$\hat{A} = \left.\frac{\partial h(\theta)}{\partial \theta'}\right|_{\hat{\theta}} = \left[\ \left.\frac{\partial h}{\partial \mu}\right|_{\hat{\theta}} \quad \left.\frac{\partial h}{\partial \sigma}\right|_{\hat{\theta}}\ \right]$$

$$= \underline{\left[\ \frac{1}{\hat{\sigma}} \quad -\frac{\hat{\mu}}{\hat{\sigma}^2}\ \right]}$$

$$\hat{\theta} = \boxed{\begin{bmatrix} \hat{\mu} \\ \hat{\sigma} \end{bmatrix}} \overset{a}{\sim} N\left(\begin{bmatrix} \mu_0 \\ \sigma_0 \end{bmatrix},\ \hat{\Omega}\right)$$

$$\frac{\hat{\mu}}{\hat{\sigma}} \overset{a}{\sim} N\left(\frac{\mu_0}{\sigma_0},\ \begin{bmatrix}\frac{1}{\hat{\sigma}} & -\frac{\hat{\mu}}{\hat{\sigma}^2}\end{bmatrix} \hat{\Omega} \begin{bmatrix}\frac{1}{\hat{\sigma}} \\ -\frac{\hat{\mu}}{\hat{\sigma}^2}\end{bmatrix}\right)$$

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

$$H_0:\ \beta_1 = 0\ ? \longrightarrow \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}\beta_0 \\ \beta_1\end{bmatrix} = 0$$

$$H_0:\ \begin{matrix}\beta_0 = 0 \\ \beta_1 = 0\end{matrix} \longrightarrow \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}\begin{bmatrix}\beta_0 \\ \beta_1\end{bmatrix} = 0$$

$$\frac{|\hat{\beta}_1|}{\sigma(\hat{\beta}_1)} > t_\alpha$$

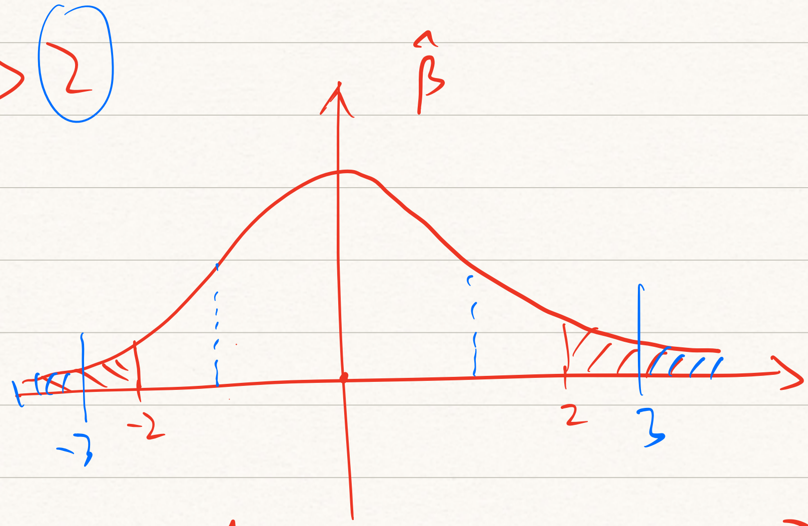
1.96

2

	$t<2$	$t>2$
	F.R.	R
Null is correct	✓	Type I
Null is incorrect	Type II	✓

$$\left|\frac{\hat{\beta}}{SE(\hat{\beta})}\right| > 2$$

1



$$\underline{\underline{|\hat{\beta}| > 2}}$$

$$Pr(|\hat{\beta}| > 2) = Pr(\text{Type I})$$

$$= \text{Test size}$$