MIT Sloan School of Management

Analytics of Finance — 15.450

Hui Chen — Spring 2023

Problem Set 3

Due: 1:00 PM, Thursday, April 6

Instructions:

- Please submit your homework on Canvas. List the names of all of your team members and their IDs in your writeup. Each team only needs to submit once.
- Submit the following files: (1) a PDF file for the writeup of your answers to all the questions; (2) for each question that involves coding, a separate zip file with the code and data for that question.

1. **Interview questions.**

   (a) Return of stock X has mean 20% and volatility 50%. Market return has mean 8% and volatility 20%. The correlation between stock X and market is 50%. If we run regression of stock X return on market return (with intercept), what is the regression coefficient on market return (i.e. beta)?

   (b) You want to predict Walmart's sales using retail traffic activities. You just learned that one can count the number of cars in Walmart's parking lots using satellite data. Clearly, the car counts are a very noisy measure of the actual level of retail traffic. If you run a linear regression of sales on car counts, how would the noise affect the results?

2. **Comparing two strategies.** You work for the asset management department of a large bank. You have the historical returns of 2 trading strategies over the same time period, $x_{i,t}$, with $i = 1, 2$ and $t = 1, \cdots, T$. You want to see whether the two strategies produce the same average returns.

   (a) State the null hypothesis of your test.

   (b) Assume that returns are independent and identically distributed over time but potentially correlated contemporaneously. Derive the estimator of the average returns for the 2 strategies, $\widehat{\mu}$ (this is a 2×1 vector, with $\widehat{\mu}_i$ being the average return estimator for manager i). Derive also the asymptotic covariance matrix $\widehat{\Omega}$ for the estimated mean vector. Argue that the estimator $\widehat{\mu}$ has multivariate normal distribution asymptotically.

(c) Define $\widehat{\delta} = \widehat{\mu}_2 - \widehat{\mu}_1$. Argue that the asymptotic distribution of $\widehat{\delta}$ is normal. What is the mean and variance of this distribution under the null hypothesis? Derive the variance of $\widehat{\delta}$ from $\widehat{\Omega}$, and explain how to estimate it directly from return data $x_{i,t}$.

(d) Denote the variance of the distribution of $\widehat{\delta}$ in (2c) by $\widehat{V}$. The following test statistic

$$W = \frac{\widehat{\delta}^2}{\widehat{V}}$$

is distributed as $\chi^2(1)$, and can be used to test the null hypothesis in (2a). Construct a test of the null hypothesis with 5% size.

3. **Performance evaluation.** You run a large university endowment that invests with outside portfolio managers. Two final candidates have been presented to you. The PMs' track records from 2005 to 2022 are included in the spreadsheet "pm.xlsx." You might also need the data for monthly market excess returns, returns for the SMB and HML factors, and one-month Treasury bill rates from Ken French's website (https://goo.gl/xtDApU). Please answer the following questions.

   (a) Summarize the performances of the two managers:

   - Make scatter plots of the managers' monthly excess returns against the market excess returns.
   - Make a boxplot of the two managers' monthly returns and comment on the differences. If you notice any outliers, discuss how you would treat them.
   - Report the mean and standard deviation of monthly returns; compute the Sharpe ratio, information ratio, and maximum drawdown. For the information ratio, use market portfolio as the benchmark.

   (b) For each manager, is there any evidence for the ability to outperform the market? Is there any evidence of ability to generate alpha (relative to the Fama-French 3-factor model)?

$$R^e_{i,t} = \alpha_i + \beta_i R^e_{m,t} + \gamma_i R_{hml,t} + \delta_i R_{smb,t} + \epsilon_{i,t}$$

   (c) Is there any evidence that one PM outperforms the other based on average return? How about 3-factor model alpha? Answer these questions through formal statistical tests.