

# MIT SLOAN SCHOOL OF MANAGEMENT

Analytics of Finance  
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15.450  
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## Problem Set 2

(Due: 1:00 PM, Tuesday, March 7)

### Instructions:

- Please submit your homework on Canvas. List the names of all of your team members and their IDs in your writeup. Each team only needs to submit once.
- Submit the following files: (1) a PDF file for the writeup of your answers to all the questions; (2) for each question that involves coding, a separate zip file with the code and data for that question.

### 1. Interview questions.

- (a) You are offered to play the following game. You keep rolling a die until a number other than one appears (on the top face of the die). The first time that occurs, you would be paid the same number of dollars as the number of dots on the upturned face of the die. How much would you be willing to pay to play this game?
  - (b) Would you use daily, weekly, or monthly returns to estimate the market beta of a stock? Please explain.
2. Consider a simple linear regression  $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ . Show that the  $R^2$  statistic of this regression equals to the squared value of the correlation between  $x$  and  $y$ ,

$$R^2 = \widehat{\text{Corr}}^2(x, y)$$

where

$$\widehat{\text{Corr}}(x, y) = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}.$$

3. **Constructing an investable cyclical and defensive sector index.** In class, we have seen how the market model can be used to measure how strongly different industry portfolio returns comove with the market. For a real life example of cyclical and defensive sector index, see the MSCI document (<https://tinyurl.com/rqxncj3>).

In this exercise, let's construct our own "cyclical" and "defensive" sector indices.

- (a) From Ken French’s website (<https://goo.gl/TrqEYX>), download (i) monthly market excess returns and one-month Treasury bill rates, and (ii) monthly returns of the 30 industry portfolios. We will define the “full sample” as the period from 01/1971 to 12/2022. Make a plot of the mean (on the y-axis) against the variance (on the x-axis) of the returns of 30 portfolios.
- (b) Use the full sample, estimate the market model to each of the 30 industry portfolios. Plot the data and fitted regression line (as in the lecture notes) for the top 5 and bottom 5 industries ranked by their market betas. In addition, plot the estimated  $\hat{\alpha}_i$  against  $\hat{\beta}_i$  for the 30 industries. Comment on your findings.
- (c) We would like to form a “**cyclical sector index**” as an equal-weight portfolio of the top 5 industries by market beta, and a “**defensive sector index**” as an equal-weight portfolio of the bottom 5 industries. Unlike in Part (b), we want to sort the industries dynamically so as to capture the potential changes in cyclicity for a given sector over time. So here is what we need to do:
  - i. Starting in 01/1981, at the beginning of each month, use data from the previous 5 years (this is called “rolling-window”) to estimate the industry betas and then update the two indices according to the new market beta ranking. (If the components of the top and bottom industries change, you would be rebalancing the portfolio by selling the existing positions on some industries and buying stocks of new industries.)
  - ii. Report the average excess return, market beta, alpha, Sharpe ratio, information ratio (using market portfolio as benchmark), and maximum drawdown for the two portfolios. Comment on your findings.