

$$E[\varepsilon_i] = 0, \text{Var}(\varepsilon_i) = \sigma^2$$

$$y_i = \beta x_i + \varepsilon_i \quad \hat{\beta}$$

$$(x_0, y_0)$$

$$E[(y_0 - \hat{y}_0)^2 | x_0] = E[(y_0 - \hat{\beta} x_0)^2 | x_0]$$

$$= E\left[\underbrace{(y_0 - \beta x_0)}_{\varepsilon_0} + \underbrace{\beta x_0 - \hat{\beta} x_0}_{\text{}}\right]^2 | x_0]$$

$$= E\left[\varepsilon_0^2 + (\beta x_0 - \hat{\beta} x_0)^2 + \underbrace{2\varepsilon_0(\beta x_0 - \hat{\beta} x_0)}_{\text{②}} | x_0\right]$$

$$= \sigma^2 + \underbrace{E[(\beta x_0 - \hat{\beta} x_0)^2 | x_0]}_{\text{}} + 0$$

$$= \sigma^2 + E\left[\underbrace{(\beta x_0 - E[\hat{\beta}] x_0)}_{\text{①}} + \underbrace{E[\hat{\beta}] x_0 - \hat{\beta} x_0}_{\text{②}}\right]^2 | x_0]$$

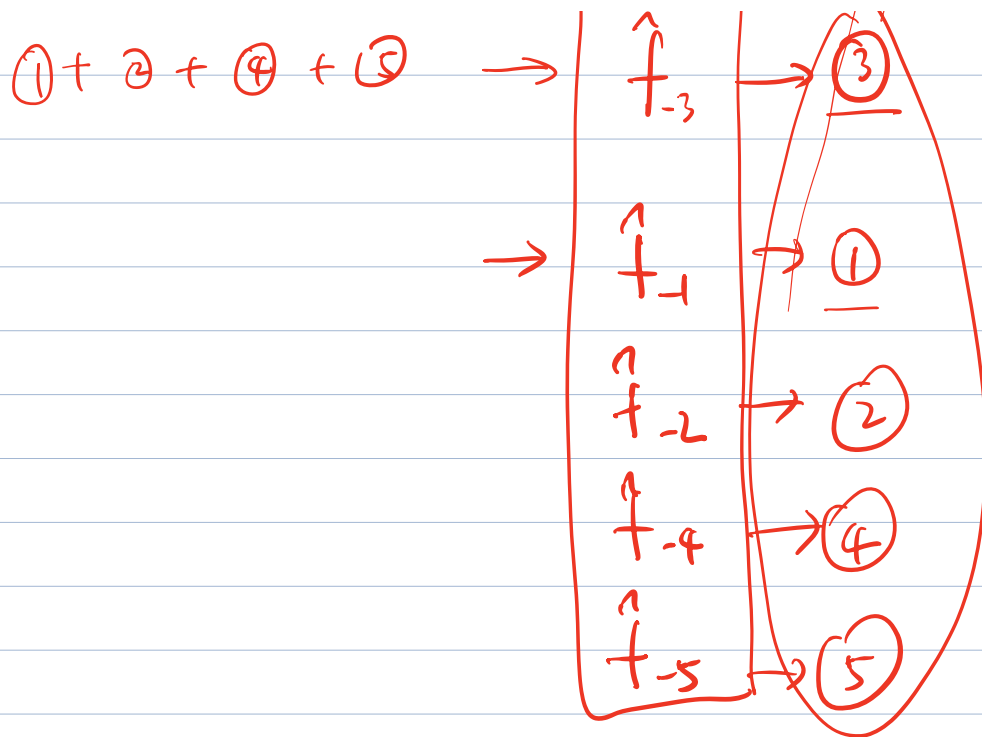
$$= \sigma^2 + \underbrace{(\beta x_0 - E[\hat{\beta}] x_0)^2}_{\text{}} + E[(\hat{\beta} x_0 - E[\hat{\beta}] x_0)^2 | x_0]$$

$$= \sigma^2 + \underbrace{[(\beta - E[\hat{\beta}]) x_0]^2}_{\text{}} + E[(\hat{\beta} - E[\hat{\beta}])^2 | x_0] x_0^2$$

$$= \sigma^2 + \underbrace{(\beta - E[\hat{\beta}])^2 x_0^2}_{\text{}} + \underbrace{\text{Var}(\hat{\beta}) x_0^2}_{\text{}}$$

bias

Variance



$P$        $x_1, x_2, \dots, x_P$

$$2 \times 2 \times \dots \times 2 = 2^P$$

$P=10$        $2^P \approx 1,000$

$M_0$	1		$\rightarrow$	$m_0^*$
<u><math>M_1</math></u>	$P$	$\begin{pmatrix} P \\ 1 \end{pmatrix}$	$\rightarrow$	$m_1^*$
<u><math>M_2</math></u>		$\begin{pmatrix} P \\ 2 \end{pmatrix}$	$\rightarrow$	$m_2^*$
$\vdots$				
$M_P$	1	$\begin{pmatrix} P \\ P \end{pmatrix}$	$\rightarrow$	$m_P^*$

1

$M_0 \rightarrow M_1 \rightarrow M_2 \quad M_3$

$\downarrow$

~~$(P_2)$~~

$x_1$

$x_1, x_2$

$x_1, x_3$

$\vdots$

$x_1, x_p$

$p-1$