Portfolio Construction & Risk Models

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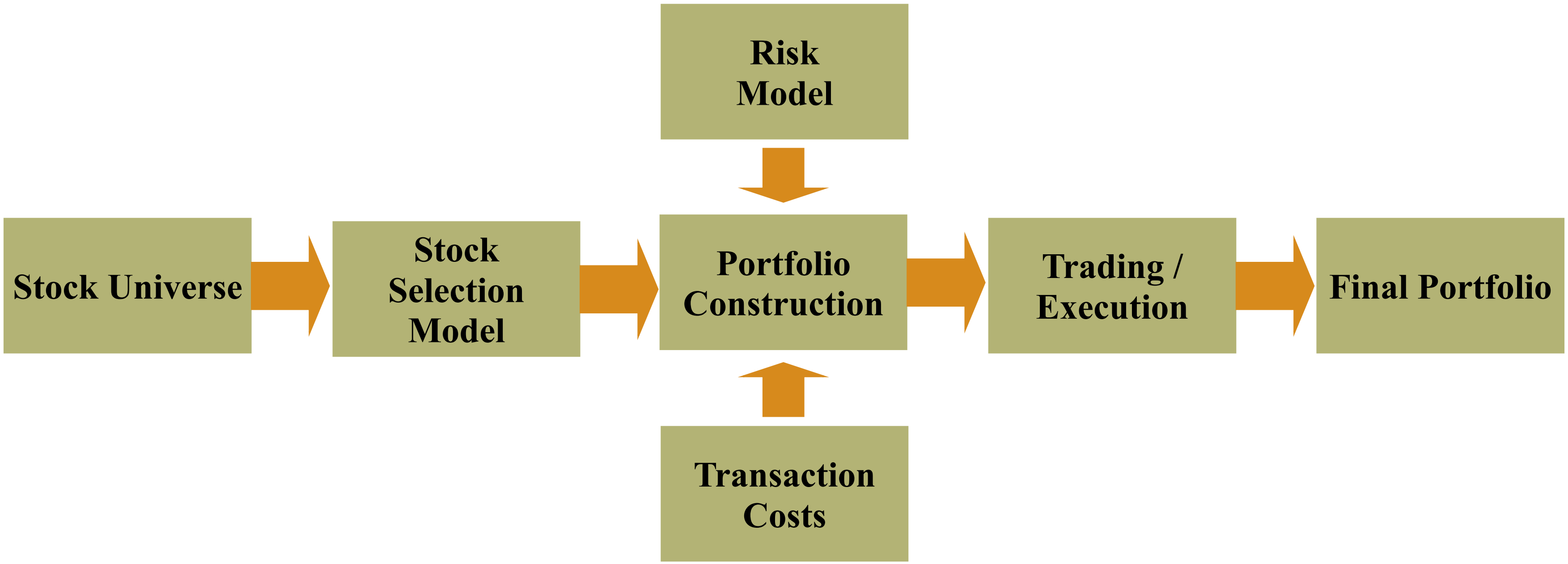
Outline

- Risk Models
- Portfolio Construction (Optimization)
- Summary

The Quantitative Investment Process



- Model portfolio subject to risk limits and transaction costs
- Risks include sector mismatch, style, individual position size
- Limit execution costs

Portfolio Construction

- Portfolio Construction is concerned primarily with two questions

  1. How do we choose which securities belong in our portfolio?
  2. How do we determine the right weight (position size) of the securities that belong in our portfolio?

- What distinguishes quantitative managers from many fundamental managers is that they try to bring scientific rigor to these questions, just as they bring scientific rigor to predicting the expected return of each stock.

  – Key observations:

    » A portfolio is not made up of your “best ideas” (e.g. stocks with the highest expected returns)!

    » Your largest positions may not be the ones with either the highest expected returns or the ones where you have the highest “conviction”.

Mean-Variance Utility

- Mean-Variance Utility:

$$U = E(r_p) - \frac{\gamma}{2}\,\mathrm{var}(r_p)$$

  where r_p = portfolio return, γ = risk aversion

- The more risk averse you are, the lower your E(r) target, or the lower your willingness to accept volatility/variance
- You can map risk aversion into E(r) or vol targets
- Most individuals would have risk aversions between 1 to 10 with it being very rare to have risk aversions greater than 10
- Represent mean-variance utility with indifference curves

Portfolio Construction

- So to create portfolios, one needs a method to trade off the expected return off each stock versus its expected risk. More specifically, in a portfolio context, its expected marginal expected return contribution versus its expected contribution to the risk.
- So we need to start by defining "risk" and then by forecasting risk.
- Main points:
  - Risk is the standard deviation of returns
  - Risks don't add
  - Most institutional investors care about active and residual risk and less about total risk
  - Risk models identify the important sources of risk and separate it into its components

Risk Models

Definition of Risk

Classical definition risk is the standard deviation of returns.

Not the only possible definition:

- Semi-variance (downside risk)
- Target Semi-variance (below target)
- Shortfall Probability (probability that return is below target)
- Value-at-Risk (amount at risk by a rare large loss)
- Tracking Error (standard deviation of active risk)

Power of Diversification

Standard Deviation does not have the portfolio property

$$\text{stdev(portfolio)} = \sqrt{w_1^2 \operatorname{var}(r_1) + w_1^2 \operatorname{var}(r_2) + 2 w_1 w_2 \rho_{r_1, r_2} stdev(r_1)\ stdev(r_2)}$$

and

$$\text{stdev(portfolio)} \leq w_1\ stdev(r_1) + \mathrm{w}_2\ stdev(r_2)$$

This is the key behind diversification!

Power of Diversification

Assume portfolio of N stocks, equally weighted, each with risk σ and uncorrelated returns, then

$$\sigma_p = \frac{\sigma}{\sqrt{N}}$$

So where average stock is σ, the portfolio risk is σ / $N^{1/2}$

More generally, assume correlation between all stocks is ρ.
Then the risk of an equally-weighted portfolio is:

$$\sigma_p = \sigma \sqrt{\frac{1 + \rho(N-1)}{N}}$$

So as N gets large, we get

$$\sigma_p \to \sigma \sqrt{\rho}$$

Risk Model: Estimation

To determine a portfolio's risk, must know (or estimate) each stock's standard deviation (i.e. it's σ) and each pair of stock's correlation with each other (i.e., $\rho_{i,j}$). That's what's you need to plug into the formula.

For two stock's not so bad. Four points to estimate.

BUT

For 100 stocks, that is 100 volatility estimates and 4,950 correlations estimates (or N(N-1)/2 correlation estimates).

For 500 stocks (i.e. S&P 500), that 500 volatility estimates and 125,250 correlation estimates!

As N gets larger, this gets huge!

Risk Model Estimation: Historical Approach

One approach is to estimate historical variance-covariance matrices.

- Need T periods to estimate an *NxN* variance-covariance matrix.
- Need for T>N it to make any sense (the number of observations must be greater than the number of quantities we need to estimate).
- We have N(N+1)/2 quantities to estimate. If we use T periods, then have NT observations. Since must have at least (!) two independent observations to estimate a standard deviation, this implies T>= N+1.
- So using monthly data and estimating a variance-covariance matrix of the S&P 500 requires more than 41 years of data….
- What's wrong with that, besides needing a ton of data?
  - Historical estimates cannot deal with changing nature of companies. Companies are not structurally stable; they change over time, including M&A and spinoffs.
  - Statistically meaningless estimates. Noise! S&P 500 requires 125,250 estimates. 5% (6262 observations) will be garbage.
  - Selection bias: no estimates of dead/failed companies. Massive understatement of risk.

Risk Model Estimation: Structural Risk Model

- So is there an easier / better way?
- Let's take all the things we know explain returns – the things that we know are responsible for the differences in returns (e.g. the factors) – and use the co-variation among those to predict the co-variation among returns.

Think in terms of a multi-factor risk model:

Let's say we think there are 20 factors that are important for the pricing of stocks. Then the structure is:

$$R_i(t) = \sum_k X_{i,k}(t)\,\gamma_k(t) \quad + \quad u_n(t)$$

where X is the exposure, or factor loading, or characteristic of the stock for the factor; γ is the factor return; and u is the stock's idiosyncratic risk.

Risk Model Estimation: Structural Risk Model

- *Why is this approach superior?*

    - U.S. market is 5,000 stocks. Instead of having to estimate (5000)(5001)/2, or 12,502,500, independent variance-covariances, I can just estimate it for the number of factors (and industries) and the sensitives I think explain returns.

- *What's the draw-back?*

    - Picking the factors! Anyone want to tell me what the important factors are? What book do I look that up in, again? What page of the *WSJ* is that on?

Risk Model Estimation: Structural Risk Model

Let's take an example.

Let's say I believe there are three factors: (1) a stock's covariance with the market; (2) a stock's book-to-market level; and (3) a stock's past 6 month return.

Then for each stock, *i*, it's expected return can be described by:

$$R_i(\mathrm{t}) = \beta_i \, \gamma_{market}(t) + \mathrm{B/P_i} \, \gamma_{book-to-price}(t) + R_{i,[0,-6]}\gamma_{6monthreturn}(t) + u_n(t)$$

Remember exposures are known at time t while returns, γ's, occur from t to t+1!

No cheating!

Risk Model Estimation: Structural Risk Model

- Now let's assume the factors are independent of each other – or we will make them so by construction – then we can estimate the risk model as follows:

$$V_{i,j} = \sum_{k_1,k_2} X_{i,k_1} \Sigma_{k_1 k_2} X_{j,k_2} \quad + \quad \Delta_{i,j}(t)$$

where X again is the exposure; Σ is the variance-covariance of the factors; and Δ is the specific variance-covariance of asset i with asset j.

- So we need to estimate the following quantities:

$$\mathbf{X}\,\boldsymbol{\Sigma}\,\mathbf{X}^{\mathbf{T}} + \Delta$$

where **X** the matrix of stock exposures' to the factors (*N x K* matrix);
Σ the variance-covariance matrix of factor returns (*K x K)*;
Δ is the vector of idiosyncratic risk (*N x 1* matrix).

- As long as the number of factors is much smaller then the number of stocks then this is much more efficient.

- For example 10 factor model can be used to run a risk model on S&P 500 or Russell 3000 pretty darn well!

Risk Model Estimation: Statistical Models of Risk

Take all the returns you can find and create statistical factors: use principal components analysis or maximum likelihood analysis. You can then estimate exposures (sensitivities) to these factors.

Okay now what are these factors? Don’t know.

How did my exposure to them change this period? Don’t know.

So how do I know if it captured something spurious? You can’t.

Can exposures change over time over the estimation period? Nope, must be constant.

Thought Questions: Did You Understand?

So we need to estimate the following quantities:

$$\mathbf{X}\,\boldsymbol{\Sigma}\,\mathbf{X}^{\mathbf{T}} + \Delta$$

where **X** the matrix of stock exposures' to the factors (*N x K* matrix);
Σ the variance-covariance matrix of factor returns (*K x K)*;
Δ is the vector of idiosyncratic risk (*N x 1* matrix).

What *exactly* are the **X'**s?
Are they characteristics? Are they sensitivities? Does it matter?

How exactly would you want to generate **Σ?**
What might be some bad ways? Better ways? Good ways?

- ARCH/GARCH
- Identifiable variation such as Business Cycles. What is the evidence for this?
- Identification of "similar securities"

Is Δ modellable? Or is random?
What kind of structure could you put on it?

Thought Questions: Did You Understand?

Should Sectors and Industries be a risk factor? Are they explainable source of risk? What might be the appropriate way to model this?

What about significant “transient” risks?

- Covid
- Debt Ceiling
- Brexit
- Regional Banking Crises

How could you make any of these into a risk factor?

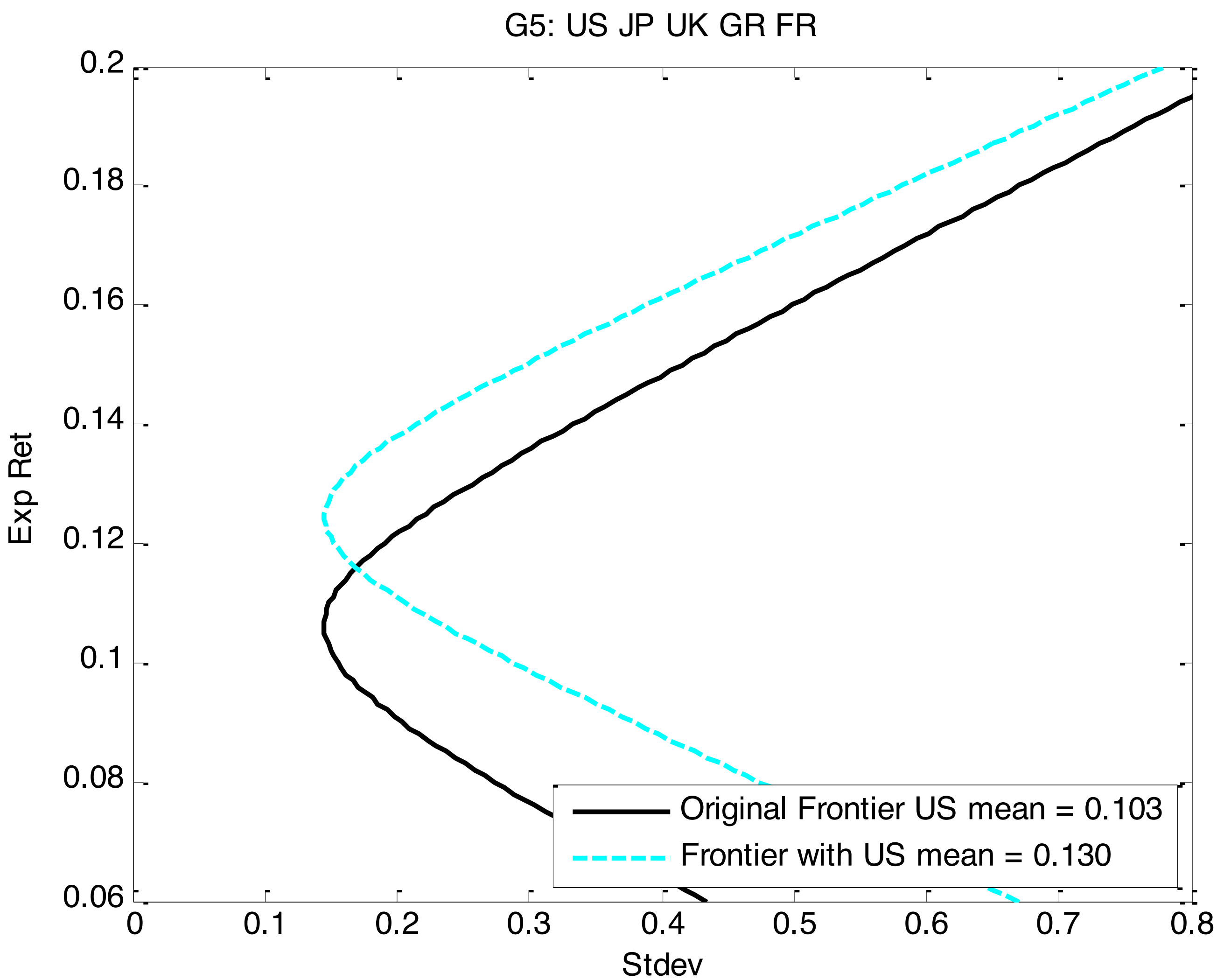
How could make a “BitCoin” risk factor?

Optimization

Sensitivity to Inputs

- Mean-variance frontiers are sensitive to inputs
  - Small changes in inputs result in extremely different portfolios with similar expected returns and volatilities
  - Errors are easily “magnified”. Garbage in, garbage out problem. Michaud (1989) calls mean-variance frontiers “error maximizing portfolios”.
  - For example, a small increase in one asset’s mean can drive half the asset positions to fall to zero with no short-sale constraints

Sensitivity to Inputs



Sensitivity to Inputs

- Suppose the target return = 14%

	Original US mean = 0.103	Altered US mean = 0.130
US	-1.2204	1.6521
JP	0.1612	-0.2663
UK	1.1270	-0.0598
GR	0.3304	-0.3443
FR	0.6018	0.0183

Solutions to Sensitivity to Inputs

Solutions to Sensitivity Problem

- Take account of sampling error in estimating the frontier
  - "Classical" statistics approaches (Ang and Bekaert (2002)) or by Monte Carlo (Michaud (1998))
- "Robust" estimates of means and covariances
  - Statistical methods which take care of outliers and extreme values. One method is Bayesian "shrinkage" methods. These shrink the means back to a model, e.g. CAPM, and the covariance back to a covariance matrix with the same volatilities and correlations (see e.g. Ledoit and Wolf (2003)). [Why is this reasonable?]
- Noisy estimates of means and covariances can lead to overtrading
  - Trade only when your "robust" stocks estimates are significantly different or when your projected portfolio return and variance are statistically different

Idiosyncratic Risk

Idiosyncratic is not explained by general factors but it can be modeled

- Idiosyncratic risk is higher during certain predictable times (e.g. earnings periods, analyst days, any scheduled announcements, non-deal roadshows)
- Certain types of stocks might have more idiosyncratic risk at times (e.g. energy stocks around OPEC announcements)
- There may be other cross-sectional differences across stocks that might make their “idiosyncratic” volatility correlated
- The time-series volatility of idiosyncratic volatility follows a pattern. Strong evidence that is follows an ARCH / GARCH process
- Idiosyncratic volatility shrinks in regards to other systemic shocks (e.g. GFC, Japanese tsunami, Covid, etc).
- Thematic factors may be being picked up in idio vol term, so a PCA decomposition of idiovol may be appropriate
- Past idiosyncratic volatility may not be representative of future idiosyncratic volatility. Should you have models for idiovol shocks?

Idiosyncratic Risk

Eastman Kodak Stock from 7/29/2018 to 7/29/2020

What is your estimate of its volatility and idiosyncratic volatility for tomorrow?

Price is from $1.50 to $4.50

Idiosyncratic Risk

Eastman Kodak Stock from 7/13/2018 to 7/31/2020

What is your estimate of its volatility and idiosyncratic volatility for tomorrow?

Price is from $1.50 to 22.50

Mean-Variance Utility

Shortcomings of Mean-Variance Utility

- Treats gains and losses symmetrically (around the mean)
- Only the first two moments matter
- Subjective vs objective probabilities
- Risk aversion is constant
- Most models are one period model
- Are they fast moving enough?
- Are not sophisticated around issues of outliers and robustness and idiovol handling

Summary

Summary

- How might one determine the size of a stock position in one's portfolio? What is important to consider? (see Pederson too)
- What does Pederson mean by "A trader must have no memory and forget nothing"? How does that apply to portfolio construction?
- Why might a manager use an optimizer in constructing a portfolio? What are the pro's and con's?
- What are alternatives to using an optimizer for a portfolio? What are the pro's and con's? Should Lee Ainslie use an optimizer? Or is his use of ad hoc rules which take account for correlation and many of the constraints used by an optimizer "good enough"?
- Why would one use a risk model? Are there uses aside from portfolio construction? What are the challenges in using it? How would you recommend constructing one? Be able to defend the pros and cons of your methodology.
- Why are the purposes of constraints when using an optimizer? How might they distort your solution? What challenges do they present when you use them? What can go wrong when using them?
- What is the impact of "false precision" in the optimization process? What might you do to combat it?
- If you see a stock with a low or negative alpha in your "trade list", why might it be there? And what would you do?
- Do you like tracking error as a definition of risk? What might your propose as an alternative?
- If your alphas estimates (expected return estimates) did not sum to zero over the benchmark you are using, what errors might that cause? How could you correct for this situation?
- What is the role of the portfolio manager in a quantitative investment process? What do you think should be the role?
- What are the pros and cons of different groups have control over the different functions of the investment process versus centralized control under the PMs?

Benchmarking and Tracking Error

I hate tracking error, in practical world, as a measure of risk.

Imagine a fund is simply a leveraged index fund – one of the ProFunds – that runs with a beta of two to the index.

$$
\begin{aligned}
\Psi_{Active} &= Stdev\left(\left(\alpha_{manager} + \beta_{mgr}(R_{Benchmark}) + \varepsilon_t\right) - R_{Benchmark}\right) \\
&= Stdev\left(\left(\beta_{mgr}(R_{Benchmark})\right) - R_{Benchmark}\right) \\
&= Stdev\left(2 \times R_{Benchmark} - R_{Benchmark}\right) \\
&= Stdev(R_{Benchmark})
\end{aligned}
$$

Lousy way to identify whether the fund is a closet indexer or if sticking to mandate.

Benchmarking and Tracking Error

Alternative: Use the Fund's R^2 with the index!

$$
\begin{aligned}
\Psi^2_{Active} &= Var\left(\left(\alpha_{manager} + \beta_{mgr}(R_{Benchmark}) + \varepsilon_t\right) - R_{Benchmark}\right) \\
&= Var\left(\alpha + \beta R_{Benchmark} + \varepsilon_t - R_{Benchmark}\right) \\
&= Var\left(\alpha + (1-\beta)R_{Benchmark} + \varepsilon_t\right) \\
&= (1-\beta)^2 \sigma^2_{Benchmark} + \left(1 - R^2\right)^2 \sigma^2_{portfolio}
\end{aligned}
$$

$$
\Rightarrow \quad \left(1 - R^2\right)^2 = \frac{\Psi^2_{Active} - (1-\beta)^2 \sigma^2_{Benchmark}}{\sigma^2_{portfolio}}
$$