Resilience in Transportation Networks

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Problem Definition

Given a graph $G(V, E)$ and a set of paths $P = \{(s_1, t_1), (s_2, t_2), \ldots, (s_k, t_k)\}$, where each $p_i \equiv (s_i, t_i) \in P$ represents the source and destination node for a path p_i, find a subset $E' \subseteq E$ consisting of all edges which when removed from the graph, maximum number of paths are affected. Let a path be represented by an ordered list of edges $p_i = (e_{i1}, e_{i2}, \ldots, e_{ij})$. It is assumed that a path connecting the nodes (s_i, t_i) is affected if at least one of the edge $e \in p_i$ is attacked. Also p_i is the only path connecting the nodes (s_i, t_i), and there is no other path connecting (s_i, t_i). Further, $\sum_{e_i \in E'} b(e_i) \leq B$, where B is the budget of the adversary and $b(e_i)$ is the cost of removing the edge e_i.

Proposed Solution

Let us assume the graph has n nodes and m edges.
We construct a $m \times 1$ matrix $\mathbf{C}$ such that i^{th} element in $\mathbf{C}$ represents the number of paths in set P passing through the i^{th} edge. This can be found by iterating over all the paths in P.

Let $\mathbf{b}$ be a $m \times 1$ matrix where b_i represents the cost of removing the i^{th} edge.

Let $\mathbf{X}$ be a $m \times 1$ matrix such that $x_i = 1$ if i^{th} edge belongs to E', otherwise it is 0.

The solution to the above problem is equivalent to solving a linear program

$$\arg\max_X \mathbf{C^T} \cdot \mathbf{X} \tag{1}$$

$$\text{such that } \mathbf{b^T} \cdot \mathbf{X} \leq B \text{ and } x_i \in \{0, 1\}$$

which is equivalent to

$$\arg\min_X -\mathbf{C^T} \cdot \mathbf{X} \tag{2}$$

$$\text{such that } \mathbf{b^T} \cdot \mathbf{X} \leq B \text{ and } x_i \in \{0, 1\}$$